

STRONG COUPLING ALGORITHM TO SOLVE FLUID-STRUCTURE-INTERACTION PROBLEMS WITH A STAGGERED APPROACH

Vaassen Jean-Marc⁽¹⁾, DeVincenzo Pascal⁽²⁾, Hirsch Charles⁽³⁾, Leonard Benoit⁽⁴⁾

⁽¹⁾*Open Engineering S.A.
Rue des Chasseurs Ardennais 4, 4031 Angleur (Belgium)
Email: jm.vaassen@open-engineering.com*

⁽²⁾*Open Engineering S.A.
Rue des Chasseurs Ardennais 4, 4031 Angleur (Belgium)
Email: p.devincenzo@open-engineering.com*

⁽³⁾*Numeca International
Chaussée de la Hulpe 187-189, B-1170 Watermael Boitsfort, Brussels (Belgium)
Email: charles.hirsch@numeca.be*

⁽⁴⁾*Numeca International
Chaussée de la Hulpe 187-189, B-1170 Watermael Boitsfort, Brussels (Belgium)
Email: benoit.leonard@numeca.be*

INTRODUCTION

During the last decades, numerical analysis of multi-physics problems has been increasingly used, especially for space applications. A particularly challenging problem is the FSI (Fluid-Structure Interaction), where a fluid flow (liquid or gas) induces forces and thermal fluxes on a solid structure, which modifies in return the fluid domain, the velocities and the temperature fields at the fluid-structure interfaces.

To solve an FSI problem, two different approaches can be used. The first one is the monolithic approach, where a single solver is in charge of the resolution of the complete system of equations. The second one is the staggered approach [1], where two solvers deal respectively with the fluid and the structure equations, and exchange information at the interfaces to ensure continuity of the variables (velocity and temperature) and compatibility of the charges (forces and heat fluxes). The main advantage of this second approach is that optimized existing solvers can be reused and coupled.

A very important concept for multi-physics unsteady problems is the strong coupling. Such a coupling requires that the solutions for all the physics must be synchronized at every time step. For a staggered approach, using high order implicit schemes is not sufficient because, as the fluid and the structure domains are advanced in time successively, there will always be lags between both solutions. Achieving strong coupling by advancing respectively the fluid and the structure solutions once per time step (Loosely-Coupled Staggered Approach) is thus impossible. Strong coupling can however be achieved if several fluid and structure computations are performed at every time step, until synchronization is obtained between the solutions (Strongly-Coupled Staggered Approach). This allows to achieve strong coupling even if explicit time-marching methods are used to integrate the equations for each domain. The disadvantage of such a method is that, as several computations are done at each time step, the computational cost is much larger. However, strong coupling allows to ensure second order time accuracy, increases the stability of the coupled algorithm, and is the only way (except if monolithic approach is considered) to ensure energy conservation at the fluid-structure interfaces [2]. It is then highly recommended for problems with large deformations where both domains influence each other, especially if the chosen time step is quite large.

The ESA Flow-SCHyp research project, started in 2006, is dedicated to the resolution of FSI couplings. A Strongly-Coupled Staggered Approach has been developed in this project. The CSM (Computational Structure Mechanics [4]) solver Oofelie®, developed at Open Engineering, has been coupled with the CFD (Computational Fluid Dynamics [5]) solver FINE™/Hexa, developed at Numeca International. The exchange of information at the interfaces is done using the MPI (Message Passing Interface) protocol.

COUPLED SYSTEMS

To study staggered schemes, a generic coupled system is going to be envisaged [9].

$$\begin{cases} \dot{\mathbf{x}}_1 = \mathbf{f}_1(\mathbf{x}_1, \mathbf{x}_2) \\ \dot{\mathbf{x}}_2 = \mathbf{f}_2(\mathbf{x}_1, \mathbf{x}_2) \end{cases} \quad (1)$$

This is named a partitioning [6], i.e. a way to write an equation $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$ where the variables have been split into two groups $\mathbf{x} = (\mathbf{x}_1, \mathbf{x}_2)^T$. Such a decomposition generally appears if there are several physics, several spatial domains, or for numerical purposes (parallel computation).

For fluid-structure interaction problems, there is a spatial and a physical decomposition :

- the first equation represents the evolution of the fluid variables \mathbf{x}_1 (velocities, pressure, density, temperature).
- the second equation represents the evolution of the structure variables \mathbf{x}_2 (displacements, velocities, temperature).
- the variables \mathbf{x}_2 in the first equation represents the displacements, velocities and temperature at the interfaces, which are transferred on the fluid interfaces.
- the variable \mathbf{x}_1 in the second equation represents the forces and the heat flux acting to the structure (functions of pressure, gradients of velocity and temperature) at the interfaces.

The definitions that are used in this report are the ones from Michler [7]. Some authors do not distinguish monolithic and strong coupling. It is assumed here that a strong coupling can be achieved with a staggered approach. A strong coupling is thus defined as a coupling by which all the variables are computed to verify simultaneously the equations at every time step. In our problem, that means that from a solution at iteration $N-1$, we can compute a solution at iteration N for which

- the fluid values are computed on the adapted mesh, and from the displacement and the velocity of the structure computed at the same iteration,
- the displacement of the structure is evaluated from the values of the fluid forces at the same iteration,
- the fluid domain (at thus the mesh) deformation is computed from the displacement of the structure at the same iteration.

WEAKLY-COUPLED STAGGERED SCHEME

The weakly-coupled staggered scheme can be summarized by Fig. 1.

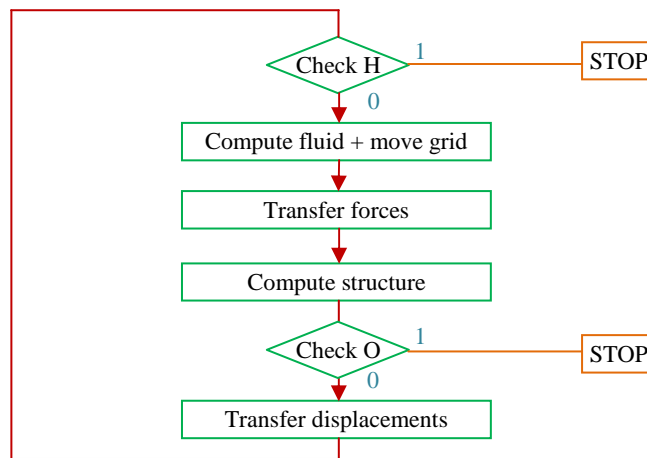


Fig. 1. Weak coupling representation

At time step N ,

- the fluid is computed (with the grid moved), depending on the displacements and velocities computed at time step $N-1$,
- the forces computed by the fluid solver on the interfaces are transferred to the structure solver,
- the structure is computed, depending on the forces computed at time step N ,
- the displacements (and velocities) computed by the structure solver on the interfaces are transferred to the fluid solver.

Considering the generic system presented above, the weak coupling could be written

- if explicit schemes are used in the solvers

$$\begin{cases} \mathbf{x}_1^N = \boldsymbol{\varphi}_1(\mathbf{x}_1^{N-1}, \mathbf{x}_2^{N-1}) \\ \mathbf{x}_2^N = \boldsymbol{\varphi}_2(\mathbf{x}_2^{N-1}, \mathbf{x}_1^{N-1}) \end{cases} \quad (2)$$

Such schemes introduce limitations in the possible time steps

- if implicit schemes are used in the solvers

$$\begin{cases} \mathbf{x}_1^N = \boldsymbol{\varphi}_1(\mathbf{x}_1^N, \mathbf{x}_1^{N-1}, \mathbf{x}_2^{N-1}) \\ \mathbf{x}_2^N = \boldsymbol{\varphi}_2(\mathbf{x}_2^N, \mathbf{x}_2^{N-1}, \mathbf{x}_1^{N-1}) \end{cases} \quad (3)$$

The resolution of algebraic linear systems is then required.

Such systems can be solved in parallel. However, if it is allowed to sacrifice this parallel mechanism, the second system can be solved using the variables from the first one at the same iteration. It is the case in the F.S.I. scheme, because the forces that are acting on the structure are computed at time step N . Such a system would be written

$$\begin{cases} \mathbf{x}_1^N = \boldsymbol{\varphi}_1(\mathbf{x}_1^N, \mathbf{x}_1^{N-1}, \mathbf{x}_2^{N-1}) \\ \mathbf{x}_2^N = \boldsymbol{\varphi}_2(\mathbf{x}_2^N, \mathbf{x}_2^{N-1}, \mathbf{x}_1^N) \end{cases} \quad (4)$$

This system is the best way to represent the F.S.I. system solved by a weakly-coupled staggered scheme. It can be seen that, even if both solvers are implicit, the coupled system still contains a \mathbf{x}_2^{N-1} term in the first equation, which means that the variables \mathbf{x}_1^N are computed from the variables \mathbf{x}_2 only at the previous time step. This implies that the full system is not implicitly solved. As it is still explicit, it is expected that there are limitations in the time steps and possible stability issues [6]. Moreover, as it looks like a backward Euler explicit scheme, only first order accuracy is possible for the coupled system, whatever the accuracy of the solvers [3].

STRONGLY-COUPLED STAGGERED SCHEME

General description

With the previous discretized system, there can be no synchronization of the variables. No strong coupling (in the sense it is defined previously) is then possible. To achieve strong coupling, it is necessary to have a staggered scheme that would be such that

$$\begin{cases} \mathbf{x}_1^N = \boldsymbol{\varphi}_1(\mathbf{x}_1^N, \mathbf{x}_1^{N-1}, \mathbf{x}_2^N) \\ \mathbf{x}_2^N = \boldsymbol{\varphi}_2(\mathbf{x}_2^N, \mathbf{x}_2^{N-1}, \mathbf{x}_1^N) \end{cases} \quad (5)$$

It can be noticed that such a system would be obtained with a monolithic approach and an implicit solver [8].

The strongly-coupled staggered schemes have not been described yet. All is known are the following points.

- They must ensure synchronization of the variables (by definition)
- As the system that will be solved is the same one as the one from implicit monolithic approach, the solution will be the same as the one that would be obtained if a monolithic solver had solved the equations in a single shot.
- It has been demonstrated [10] that strong coupling
 - ensures a better energy and momentum conservation at the fluid-structure interfaces. Weakly-coupled schemes are often energy-increasing,
 - improves stability,
 - allows to use large time steps,
 - provides a better accuracy (never higher than the one from the less accurate solver).

Construction of the method

Starting from the initial solution $(\mathbf{x}_1^{N,0}, \mathbf{x}_2^{N,0})^T = (\mathbf{x}_1^{N-1}, \mathbf{x}_2^{N-1})^T$, subiterations are performed within time step N to provide intermediate solutions k . Once these solutions do not change anymore, the solution at time step N must be obtained $(\mathbf{x}_1^N, \mathbf{x}_2^N)^T = (\mathbf{x}_1^{N,k_{conv}}, \mathbf{x}_2^{N,k_{conv}})^T$.

As the weakly-coupled scheme can be written with (4), it is interesting to use this scheme for the subiterations because it is already implemented. By considering the subiterations of (6), it is easy to see that, when the subiterations have converged, as (7) is verified, (5) will be solved.

$$\begin{cases} \mathbf{x}_1^{N,k} = \Phi_1(\mathbf{x}_1^{N,k}, \mathbf{x}_1^{N-1}, \mathbf{x}_2^{N,k-1}) \\ \mathbf{x}_2^{N,k} = \Phi_2(\mathbf{x}_2^{N,k}, \mathbf{x}_2^{N-1}, \mathbf{x}_1^{N,k}) \end{cases} \quad (6)$$

$$(\mathbf{x}_1^{N,k}, \mathbf{x}_2^{N,k})^T = (\mathbf{x}_1^{N,k-1}, \mathbf{x}_2^{N,k-1})^T = (\mathbf{x}_1^{N,k_{conv}}, \mathbf{x}_2^{N,k_{conv}})^T = (\mathbf{x}_1^N, \mathbf{x}_2^N)^T \quad (7)$$

The strongly-coupled staggered scheme consists in starting from the solution at the previous time step and in solving several times the coupled equations of the weakly-coupled staggered scheme, modified in the sense that the solution at time step $N-1$ is never changed. The solutions at the previous subiterations only appear in the evaluation of $\mathbf{x}_2^{N,k-1}$. Fig. 2 represents the strongly-coupled staggered scheme implemented [10].

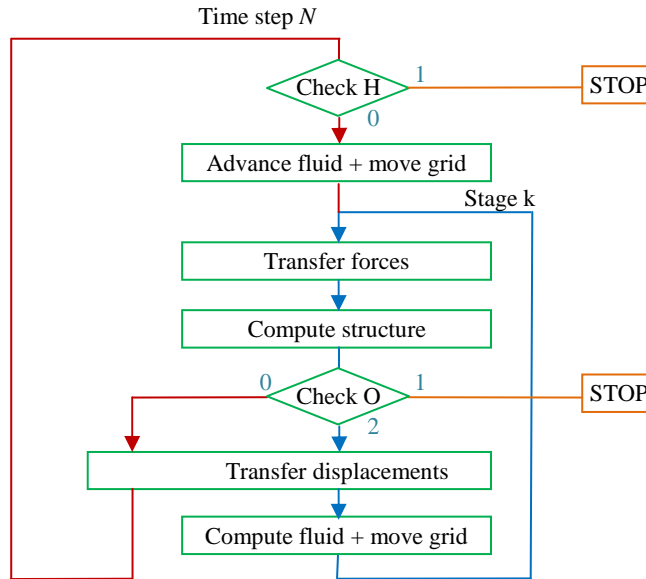


Fig. 2. Strong coupling representation

APPLICATION

Presentation of the problem

The 2D problem that is presented in this paper is a classical benchmark widely used in the literature [12]. Several versions exist, for different properties of the fluid and the structure. The computational domain also can vary: the obstacle can be a disk [11] or a square.

The computational domain is illustrated on Fig. 3.

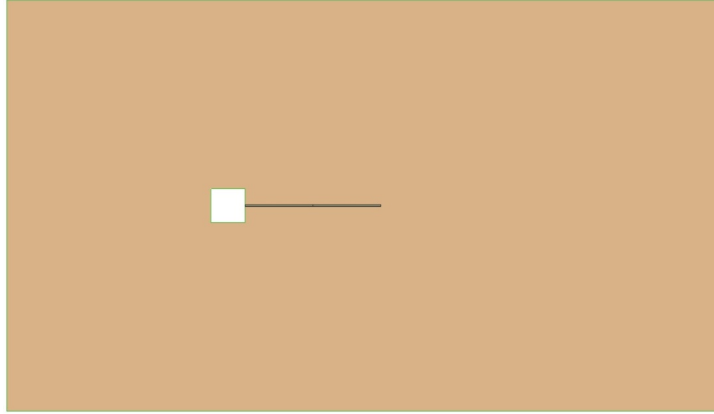


Fig. 3. Computational domain

The structure consists in a beam represented by square elements. The Young modulus is small (200 000) which does not correspond to a physical material. The Poisson coefficient is equal to 0.35. There are 3 fluid-structure interfaces.

An incompressible fluid flow comes from the left boundary. The density of the fluid is equal to 1.18 kg/m^3 and the inlet velocity is equal to 0.315 m/s . The fluid mesh is represented on Fig. 4. There are 9835 quadrangular cells.

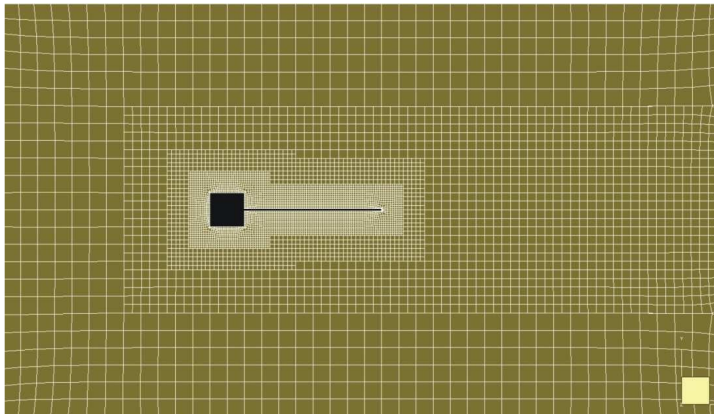


Fig. 4. Fluid domain mesh

The flow is supposed viscous laminar. The Reynolds number is such that no steady solution exists. Vortices are created on both parts of the beam. It is then not possible to compute an initial fluid solution. This computation is stopped after a certain number of iterations. The initial solution for the F.S.I. problem is thus dependent on the moment the first computation is stopped.

For the F.S.I. computation, the structure is going to vibrate because of the periodic variation of the pressure on both parts of the beam. After some time, a new periodic solution is going to be developed. This new solution is such that there is no amplification of the movement of the beam.

Both solvers use second order accurate schemes. For the fluid domain, a multigrid strategy with 4 meshes is used to accelerate the convergence to the next time step.

Solution of the problem

Fig. 5 represents the solution obtained at several times t . The beam is coloured by the displacement magnitude. The fluid domain is coloured by the velocity magnitude. Moreover, arrows represent the direction of the velocity vectors. The oscillation of the beam is clearly visible. Vortices, which already existed for the initial solution, are bigger and velocities are higher than in the initial solution.

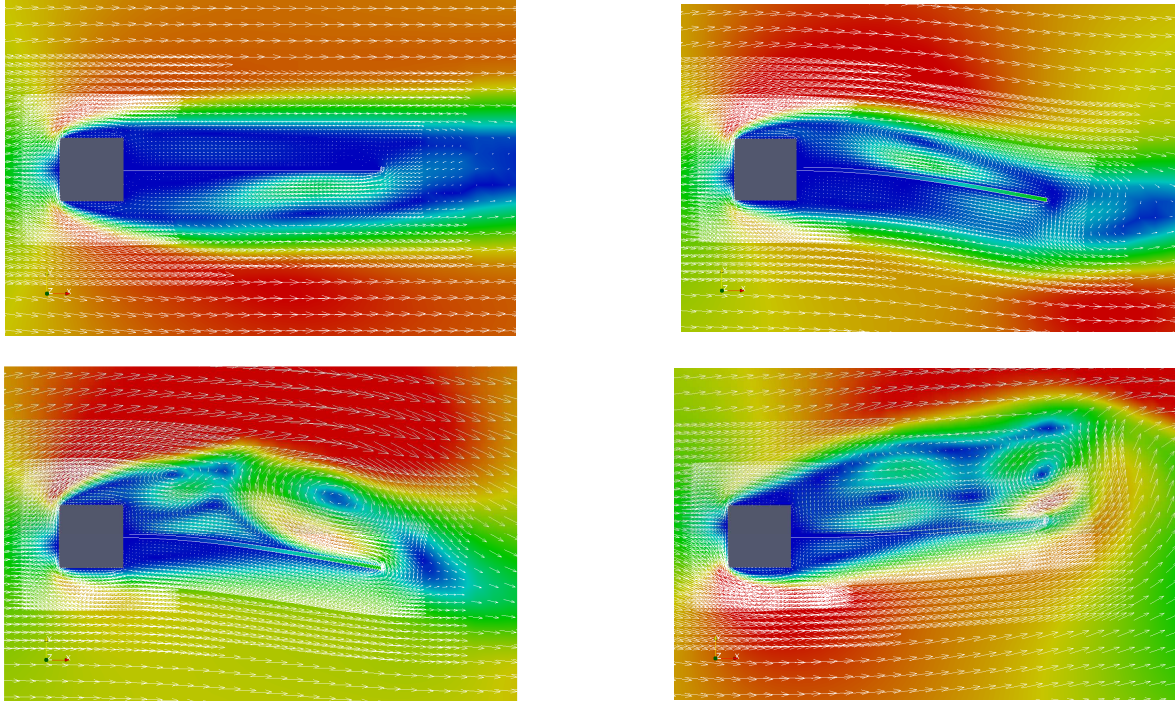


Fig. 5. Solution at $t = 0 - 1.3 - 2.5 - 3.8$

Fig. 6 illustrates the Y displacement at the extremity of the beam if a time step equal to 0.1 is used (quite a huge value). Weakly-coupled and strongly-coupled schemes are used successively.

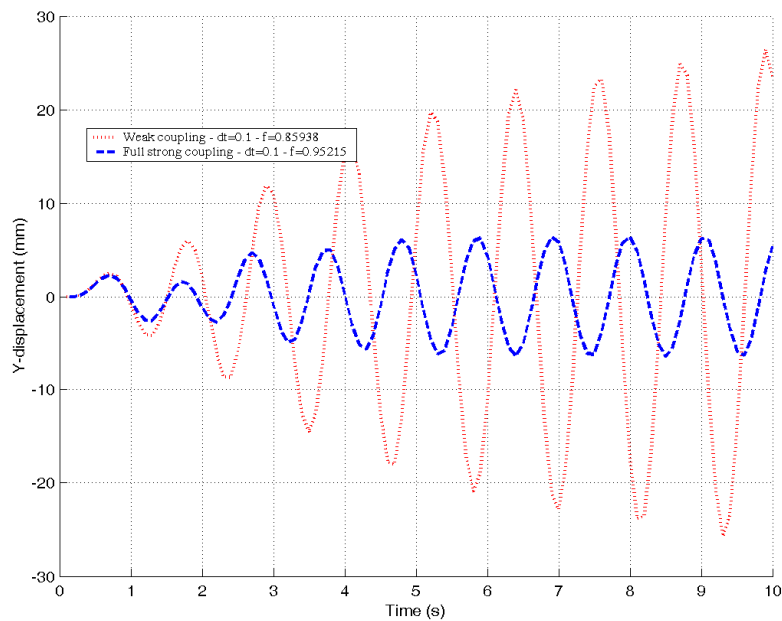


Fig. 6. Y-displacement at the extremity for weak and strong coupling

The weakly-coupled scheme clearly increases the amplitude in time, while the movement should be periodic. Strong coupling provides a much better time accuracy for this value of the time step.

In the previous figure, the strong coupling scheme let the subiterations converge until the difference in the displacement between 2 subiterations is 1000 times lower than the difference between the first two ones. Typically, 6 subiterations are performed. Table 1 gives the force applied on the structure for the first seven time steps and for the each subiteration.

Table 1 : Forces during subiterations for the first time steps

	Time step 1	Time step 2	Time step 3	Time step 4	Time step 5	Time step 6	Time step 7
Subiterations	-0.00915745	-0.00777485	-0.00684028	0.00610549	-0.00934753	-0.00575341	-0.00569087
	-0.00983359	-0.00756653	-0.00677907	-0.00121833	-0.00820871	-0.00238763	-0.00321541
	-0.00997949	-0.0075222	-0.00676655	-0.00271725	-0.00897300	-0.00272705	-0.00350493
	-0.01001090	-0.00751249	-0.00676388	-0.00229006	-0.00874235	-0.00231356	-0.00300954
	-0.01001760	-0.00751031	-0.00676323	-0.00224681	-0.00871223	-0.00225245	-0.00292958
	-0.01001900	-0.00750979	-0.00676306	-0.00222716	-0.00870005	-0.00223328	-0.00290585
	-0.01001930	-0.00750967	-0.00676301	-0.00222395	-0.00869778	-0.0022298	-0.00290150
					-0.00869726		-0.00290052

After 3 subiterations (the fourth value), there is almost no change in the values. Of course, such a behaviour is probably problem dependent. The ideal number of subiterations is impossible to be determined a priori.

A solution was then computed for different time steps. Fig. 7 illustrates the solutions that were obtained for the Y displacement at the extremity of the beam.

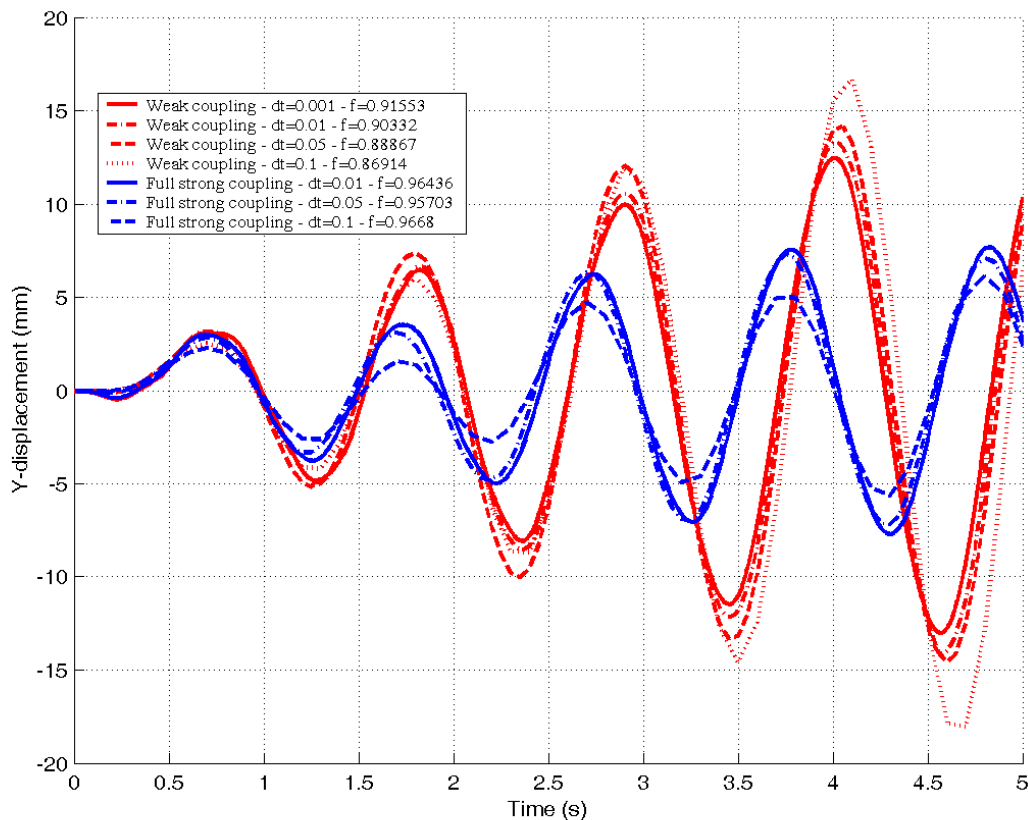


Fig. 7. Solution for different time steps and couplings

It can be seen that when the time step decreases, both solutions are closer. The differences are essentially in the amplitude, but also in the frequency of the vibration.

CONCLUSION

A strong coupling scheme has been implemented in a fluid-structure interaction solution using two separate solvers working in a staggered fashion. Strong coupling is necessary to stabilize the iterative process and ensure second order accuracy for the coupled problem (both fluid and structure solvers must of course be second order accurate). The conclusions are the following ones.

- It is the only way to ensure synchronization between the fluid and the structure solutions.
- The weak coupling behavior comes from the fact that the forces induced by the fluid on the structure are computed from the displacements and the velocities computed at the previous time step.
- The subiterations mechanism must then ensure that, at the convergence, the equation that are solved are the ones that are solved for a monolithic problem using an implicit integration scheme.
- Results have been obtained and show that strong coupling allows computing a more accurate solution even if the computation cost per iteration is higher.

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