INDIRECT PLANETARY CAPTURE VIA PERIODIC ORBITS ABOUT LIBRATION POINTS

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ABSTRACT

The libration points and their periodic orbits possess unique dynamics properties in multi-body system, which have been exploited to design low-energy transfer in space missions. In this paper, we investigate the periodic orbits for planetary capture and propose an indirect planetary capture method. The periodic orbit is used as a park orbit during the capture, which connects with the interplanetary trajectory and the mission orbit by stable and unstable manifolds, respectively. The parking orbit selection is investigated under Sun-Mars system. The candidate planar and spatial orbits are obtained. The simulation shows that compared with direct capture the indirect capture via periodic orbits could save velocity increment. Better efficiency can be found for high altitude and high v_∞ orbit capture. This capture strategy may be of interest for future exploration missions because of low capture velocity, flexibility of transfer and extra scientific returns.

Index Terms— Planetary capture, periodic orbit, invariant manifolds

1. INTRODUCTION

The planetary capture is a key process for exploration mission, which transfers the spacecraft from interplanetary trajectories to mission orbits around planets. The capture trajectory strategy plays an important role in trajectory design. It not only affects the spacecraft design but also relate to the mission planning. So far, several capture strategies have been proposed and applied to planetary capture, including direct capture, aerocapture and ballistic capture.

The direct capture is the traditional capture method, which directly inserts the spacecraft into target orbit by performing single impulse maneuver. Direct capture is easy to design but the cost is generally large. The aero-brake technique is another kind of capture method, which takes advantage of the aerodynamic force to reduce the velocity of spacecraft or change the orbital parameters. It has been successfully applied to Mars exploration [1-2]. Though aero-braking shows high efficiency, it produces high temperature and overload during the capture process, which leads to high requirements for spacecraft design.

With the understanding of multi-body dynamics and the definition of weak stability boundaries and stable sets, another capture method, so-called ballistic capture has been proposed [3-5]. The ballistic capture exploits the gravitational force of planets to capture a spacecraft. Several lunar missions have used ballistic capture, showing good performance in fuel consumption [6-7]. However, it is found that ballistic capture for planetary capture is hard to achieve, if the excess velocity is high [8-11].

The libration points, periodic orbits and their associated manifolds are also unique dynamics properties in multi-body system. They have been exploited to design low-energy lunar transfer [12], interplanetary transfer [13] and transfer between different orbits [14]. The capture trajectories to periodic orbits are also studied. Nakamiya and Scheeres et al. investigated the capture to periodic orbit using impulsive maneuvers at periapsis of manifold [15] and apply to Earth-Mars transfer [16]. Wang provided more capture opportunity by performing extra more impulse maneuver [17]. However, capture the spacecraft into planetary orbits via periodic orbits has not been further studies.

In this paper, based on the work of Nakamiya, Scheeres and Wang, we extend the periodic capture to planetary orbit capture. The indirect planetary capture via libration points and periodic orbits is proposed and the design process is given. The spacecraft is firstly captured into periodic orbit using stable manifolds and then is transferred into the mission orbit by unstable manifolds.

The indirect capture method is researched under the background of Mars. Firstly, the dynamic model of CRTBP in Sun-Mars system is established. Secondly, the process for indirect capture is described. The candidate parking orbits are obtained in planar and spatial conditions. Finally, the efficiency of indirect capture method is evaluated and compared with other capture methods. The results show that indirect capture method requires less velocity increment than direct capture. Better efficiency can be found for high altitude and high v_∞ orbit capture. It also shows advantages in flexibility of transfer, extra scientific returns. The indirect planetary capture can also be applied to other planets and provide reference for future exploration mission.

2. EQUATIONS OF DYNAMICS

In this paper, the circular restrict three body model are used, which describes the motion of a massless particle under the gravity attraction of two primaries moving on a circular orbit around their common center of mass. Here, the two primaries are the Sun (m_1) and Mars (m_2), and the particle is the spacecraft. The motion is studied under the rotating reference frame, in which the origin is located at the barycenter of system The X-axis points from m_1 to m_2, the Z axis aligns with the direction of angular momentum of the system and Y axis complete the coordinate frame.

In the CRTBP, the non-dimensional equations of motion for the spacecraft can be written as,

$$\begin{cases} \ddot{x}-2\dot{y}=x-\dfrac{(1-\mu)(x+\mu)}{r_1^3}-\dfrac{\mu(x-1+\mu)}{r_2^3} \\ \ddot{y}+2\dot{x}=y-\dfrac{(1-\mu)y}{r_1^3}-\dfrac{\mu y}{r_2^3} \\ \ddot{z}=-\dfrac{(1-\mu)z}{r_1^3}-\dfrac{\mu z}{r_2^3} \end{cases} \tag{1}$$

where $\mu=m_1/(m_1+m_2)$ is the mass parameter of the system, r_1 and r_2 are the distances from the spacecraft to Sun and the Mars, respectively.

Five libration points exist in the CRTBP, three of them are collinear libration points (namely L1, L2, L3) and the other two are triangular libration points (namely L4, L5). Spacecraft on those points keeps balance under the mutual influence of primaries. In this paper, we pay attention to collinear equilibria L1 and L2.

There are several kinds of periodic orbits around L1 and L2, including planar Lyapunov orbits, vertical Lyapunov orbits and halo orbits. The stability of periodic orbit could be obtained by the Monodromy of orbits, which defined as the state transition matrix of orbits in one orbital period. If any eigenvalue of Monodromy larger than 1, the orbit is unstable. The direction of corresponding eigenvector is noted as unstable direction. According to former research, all periodic orbits around L1/L2 are unstable. Small perturbations along unstable direction cause the spacecraft asymptotically depart from periodic orbits. The set of those trajectories are known as unstable invariant manifolds. On the other hand, the period orbit also possesses stable direction, and stable invariant manifolds. Spacecraft on those trajectories will asymptotically arrive onto the periodic orbit. In this paper, both stable and unstable invariant manifolds are employed during the capture process.

3. CONCEPT OF INDIRECT PLANETARY CAPTURE

The concept of indirect planetary capture is as follow,

(1) The first impulse is performed at the periapsis of interplanetary trajectory to inject the spacecraft into stable manifold of periodic orbits. Then the spacecraft could asymptotically arrive onto periodic orbits.

(2) After one or several periods on the parking orbit, the spacecraft will depart from the periodic orbit and approach the Mars along unstable manifolds.

(3) When the unstable manifold arrives its periapsis, with the third impulsive maneuver, spacecraft is captured into the mission orbit.

The indirect planetary capture can be separated to three patches, the periodic parking orbit and its corresponding stable and unstable manifolds. The stable manifolds should connect with hyperbolic trajectory and the unstable manifolds should intersect with mission orbits. The indirect capture usually involves three impulsive maneuvers. The first maneuver and the third maneuver happen at the corresponding periapsides. The magnitude of two maneuvers depends on the excess velocity v_∞ and the parameters of mission orbits. The second maneuver is usually small but necessary to generate the unstable manifold with specific parameters. One extra maneuver is optional if the perturbed stable manifolds are chosen. The process of indirect planetary capture is illustrated in Fig. 1.

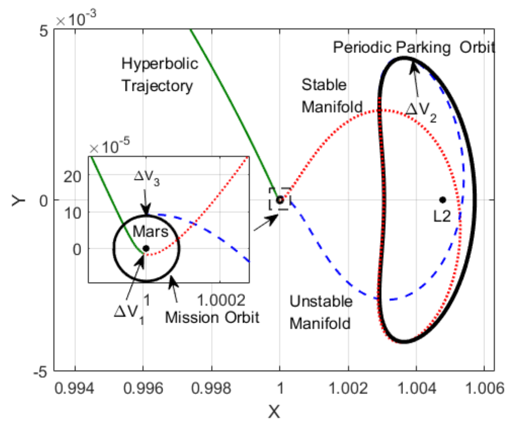


Fig. 1 Indirect planetary capture via periodic orbit

In order to utilized the indirect planetary capture, both the back and forward time integration are used. Once the parameter of mission orbit is determined, the design process could describe as follow,

Step 1. Construct the periodic parking orbit.

Step 2. Compute the unstable manifolds of the periodic orbit. The particular stable manifold, which possess a similar periapsis condition as the mission orbit, such as periapsis distance, are chosen as the initial value. A differential correction process is used to obtained the accurate solution. The second and third impulsive maneuver $\Delta v_2, \Delta v_3$ could be calculated.

Step 3. Compute the stable manifolds of the periodic orbit to its periapsis and record the periapsis state. The first impulsive maneuver Δv_1 can be obtained.

4. SELECTION FOR PERIODIC ORBITS

As mentioned above, the selection of periodic orbits is a key step in capture trajectory design, which determines the periapsis state of invariant manifolds, including the periapsis distance and periapsis velocity. There are two criteria to select the periodic orbits.

(1) Energy constrain: The insertion velocity to the stable manifold should be as low as possible.

If we assume the periapsis distance of stable manifold is r_{ps}, the velocity at periapsis v_{ps} could approximately represent as the escape velocity at such distance $v_{es} = \sqrt{2\mu/r_{ps}}$. The velocity of the hyperbolic trajectory with excess velocity v_∞ at periapsis is, $v_{ex} = \sqrt{v_\infty^2 + 2\mu/r_{ps}}$ .The cost for the first maneuver is approximately,

$$\Delta v_1 = v_{ex} - v_{ps} \approx v_{ex} - v_{es} = \sqrt{v_\infty^2 + \frac{2\mu}{r_{ps}}} - \sqrt{\frac{2\mu}{r_{ps}}} \qquad (2)$$

The derivate of Eq. (2) to r_{ps} is expressed as,

$$\frac{\partial \Delta v_1}{\partial r_{ps}} = -\frac{1}{\sqrt{v_\infty^2 + \frac{2\mu}{r_p}}}\frac{\mu}{r_p^2} + \sqrt{\frac{\mu}{2r_p^3}} = \frac{\sqrt{v_\infty^2 + \frac{2\mu}{r_p}} - \sqrt{\frac{2\mu}{r_p}}}{\sqrt{v_\infty^2 + \frac{2\mu}{r_p}}} \bullet \sqrt{\frac{\mu}{2r_p^3}} \qquad (3)$$

It is clear to show that $\frac{\partial \Delta v_1}{\partial r_{ps}} > 0, \forall v_\infty > 0$. That means the first impulsive maneuver Δv_1 increases as the periapsis distance increases. The result shows that stable manifold at low periapsis requires less cost. Therefore, the periodic orbits which have low manifold periapsis close to the surface of Mars are priority selections.

(2) State constrain: The periapsis distance of natural unstable manifolds should close to that of mission orbit. Otherwise, the spacecraft need large perturbation maneuver Δv_2 to correct the periapsis distance.

Based on the two criteria, suitable periodic orbits for indirect planetary capture are investigated in the following parts.

1. Selection in planar orbits

The planar Lyapunov orbit is firstly investigated. Here we use the amplitude A_y to express the orbits. Based on numerical computation. The L1 Lyapunov orbits from amplitude $A_y = 7.3 \times 10^4 km$ to $A_y = 7.5 \times 10^5 km$ and L2 Lyapunov orbits from amplitude $A_y = 1.0 \times 10^5 km$ to $A_y = 1.5 \times 10^6 km$ are produced. 400 points are chosen equally along each periodic orbits. The stable manifolds are generated from each points and integrated to periapsis backwardly. The periapsis distance for each manifold is recorded. According to the analysis above, low periapsis distances are preferred. Therefore, the periapsides close to the surface of the Mars are shown. Fig. 2 and 3 illustrated the correspondence between periapsis distances and orbit amplitude for L1 and L2 points, respectively.

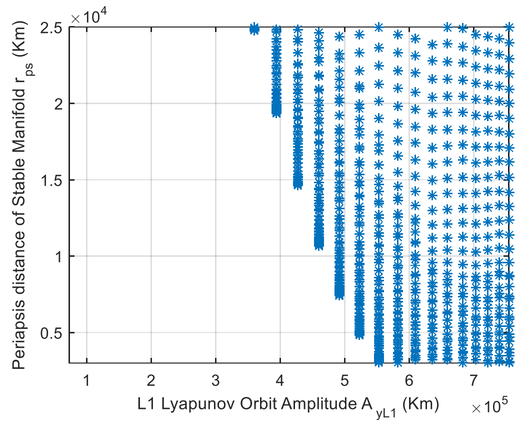


Fig. 2 Correspondence between r_{ps} and A_{yL1}

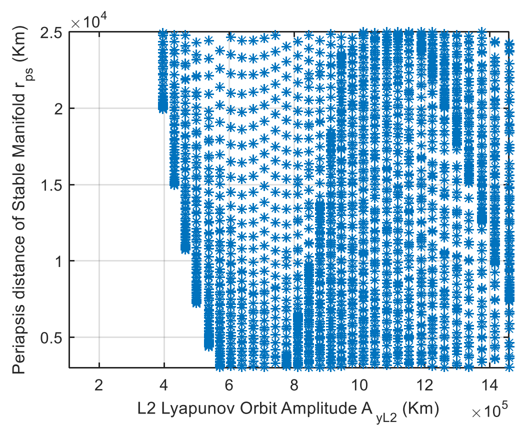


Fig. 3 Correspondence between r_{ps} and A_{yL2}

It is clear to show that not all Lyapunov orbits are suitable for parking orbits. Stable manifolds for small size orbits do not have low periapsis distances, both for L1 and L2 orbits. With the increase of amplitude A_y, the periapsides of stable manifolds are gradually close to the Mars. There is a critical amplitude A_{yc}, in which some branches of natural stable manifold could approach the Mars in less than 200km. For L2 Lyapunov orbits, the critical amplitude is about $A_{yc} = 5.7\times10^5 km$. The critical amplitude for L1 orbit is a little smaller, $A_{yc} = 5.5\times10^5 km$.

Besides the periapsis distance, the periapsis phase angle θ is another important parameter for capture, which determines the approach direction of hyperbolic trajectory. Here we defined the phase angle θ as the angle between X-Axis and Mars-Periapsis line. The phase angle versus periapsis distance for L1 and L2 Lyapunov orbits are shown in Fig. 4 and 5, respectively.

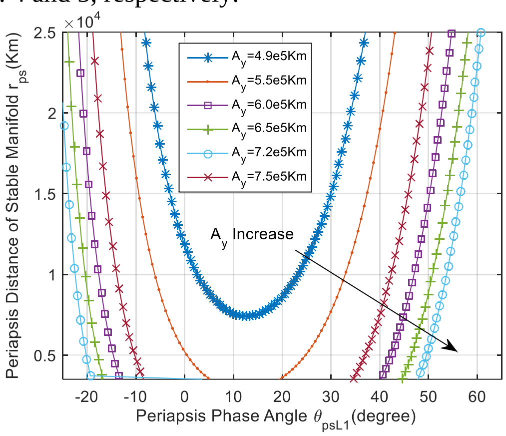


Fig. 4 Relation between θ_{psL1} and r_{ps}

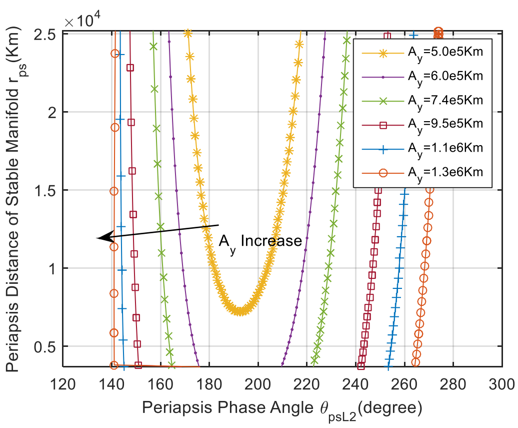


Fig. 5 Relation between θ_{psL2} and r_{ps}

L1 and L2 Lyapunov orbits possess totally different periapsis phase angles. The phase angles for L1 orbits begin at 10° and extend to -20° and 45° for large amplitudes. The phase angles for L2 orbits distribute in the vicinity of $\theta = 190^\circ$. Increasing the orbit amplitude, the phase angles extend to 140° and 260°, which means the periapsis of large amplitude orbits have more extensive distribution in the vicinity of Mars. That could provide more choice for interplanetary transfer.

The periapsis distances of unstable manifolds are also discussed with different amplitudes. The results for L1 and L2 are shown in Fig. 6 and 7, respectively. As illustrated, the orbits with amplitude larger than A_{yc} could cover the periapsis distance from low Mars orbit ($3589km$) to very high orbit ($300000km$), which meets requirement for most of mission orbits. Therefore, the constrains of final mission orbits would not affect the parking orbit selection. It only determines the initial outbound position on parking orbits.

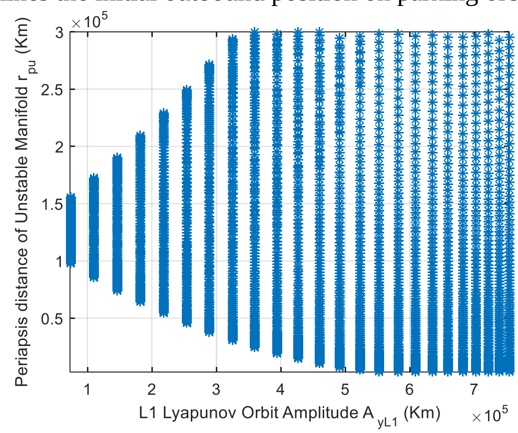


Fig. 6 Correspondence between r_{pu} and A_{yL1}

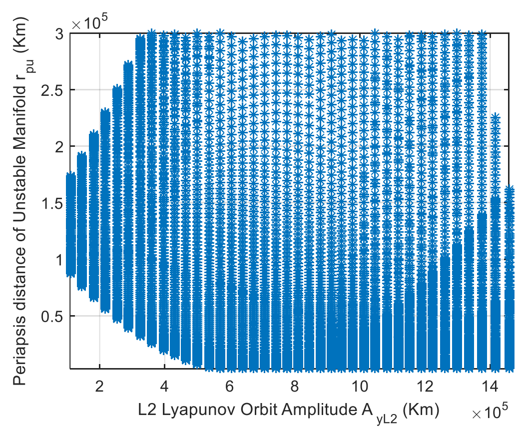


Fig. 7 Correspondence between r_{pu} and A_{yL2}

Based on the analysis above, the candidate parking orbits for indirect capture are from $A_y = 5.5\times10^5 km$ to $A_y = 7.5\times10^5 km$ for L1 Lyapunov orbit and from $A_y = 5.7\times10^5 km$ to $A_y = 1.5\times10^6 km$ for L2 Lyapunov orbit and. Further selection will decide by final state of interplanetary trajectories.

2. Selection in spatial orbits

The spatial parking orbits are also investigated. Two kinds of periodic orbit about libration points are studied. The vertical Lyapunov orbits and halo orbits. For vertical Lyapunov orbits, due to its large amplitude in Z axis, it is found the stable manifolds cannot close to Mars for both L1 and L2 points. Hence, vertical Lyapunov orbits cannot be applied for indirect capture. Here, we pay attention to discuss the halo orbits.

The halo orbits are created based on three-stage analytical and numerical correction. The L1 north Halo orbits with amplitude $A_z = 2.7\times10^4 km$ to $A_z = 6.6\times10^4 km$ and L2 south Halo orbits with amplitude $A_z = 3.7\times10^4 km$ to $A_z = 6.5\times10^5 km$ are studied. The same produces above are followed to investigate the candidate parking orbits.

The periapsis distances of stable manifolds are shown in Fig. 8 for L1 halo orbits and Fig. 9 for L2 halo orbits.

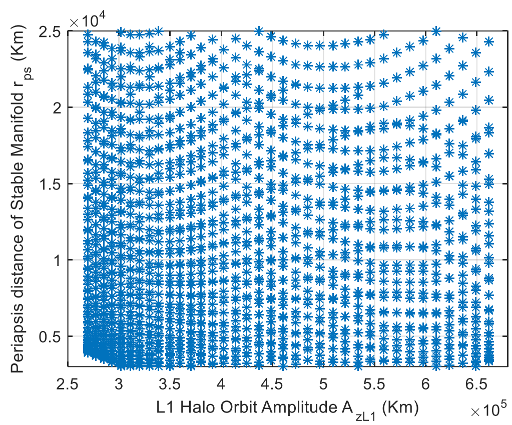


Fig. 8 Correspondence between r_{ps} and A_{zL1}

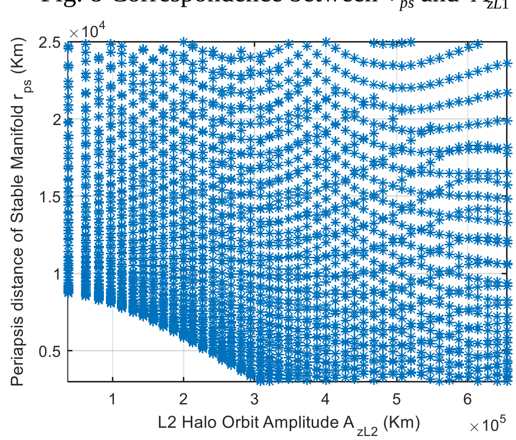


Fig. 9 Correspondence between r_{ps} and A_{zL2}

As shown in Fig. 8 and 9, the halo orbits from two libration points have similar critical amplitude $A_{zc} = 2.9\times10^5 km$.

The periapsis states are also investigated in spatial situation. The traditional Keplerian parameter cannot present the position of periapsis clearly. Therefore, we use the periapsis phase angle θ and the spatial angel β to describe the periapsis. The periapsis phase angle θ is modified as the angle between X axis and the projector of Mars-periapsis line in XY plane. The spatial angle β represents the angle between Mars-periapsis line and XY plane. Another inclination angle i is used to describe the velocity direction at periapsis. Three angles are shown in Fig. 10.

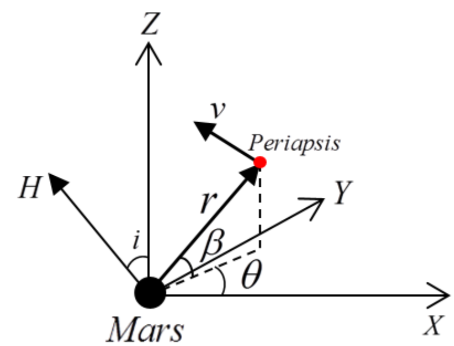


Fig. 10 Periapsis state angles

The periapsis states for L1 stable manifolds is shown in Fig. 11. The periapsis for L1 north halo orbit all locate below the XY plane. With the increase of amplitude A_z, the periapsis of manifold gradually apart from the XY plane. The absolute value of spatial angle increases from about 17° at $A_z = 2.9\times10^5 km$ to more than 40° at $A_z = 6.6\times10^5 km$. The orbital inclination also increases with amplitude A_z. Similar to the planar situation, the phase angles of manifold still near 10°. But the angles barely change as the size of Halo orbit increases. If we substitute the north family to south family. The phase angle

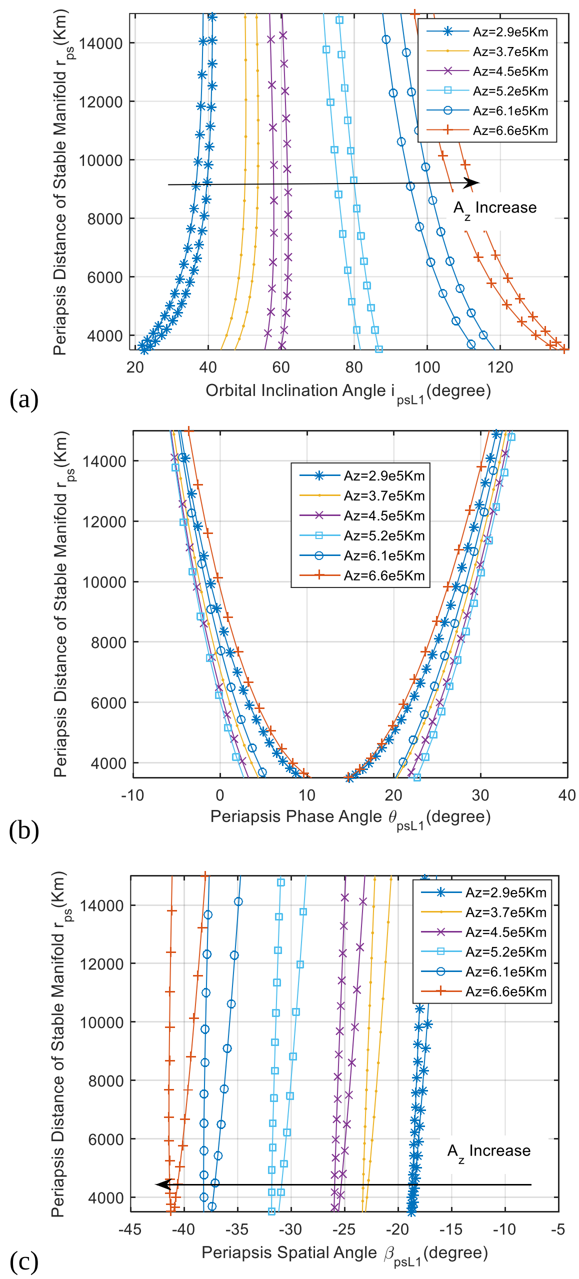


Fig.11 the periapsis state for L1 stable manifolds (a) orbital inclination angle (b) periapsis phase angle (c) periapsis spatial angle.

and inclination do not changes. The periapsides of manifolds locate above the XY plane with the same value of spatial angles.

The periapsis states for L2 stable manifolds is shown in Fig. 12. Periapsides for L2 stable manifolds locate in totally different area. With the increase of amplitude, the orbital inclination and spatial angle both increase.

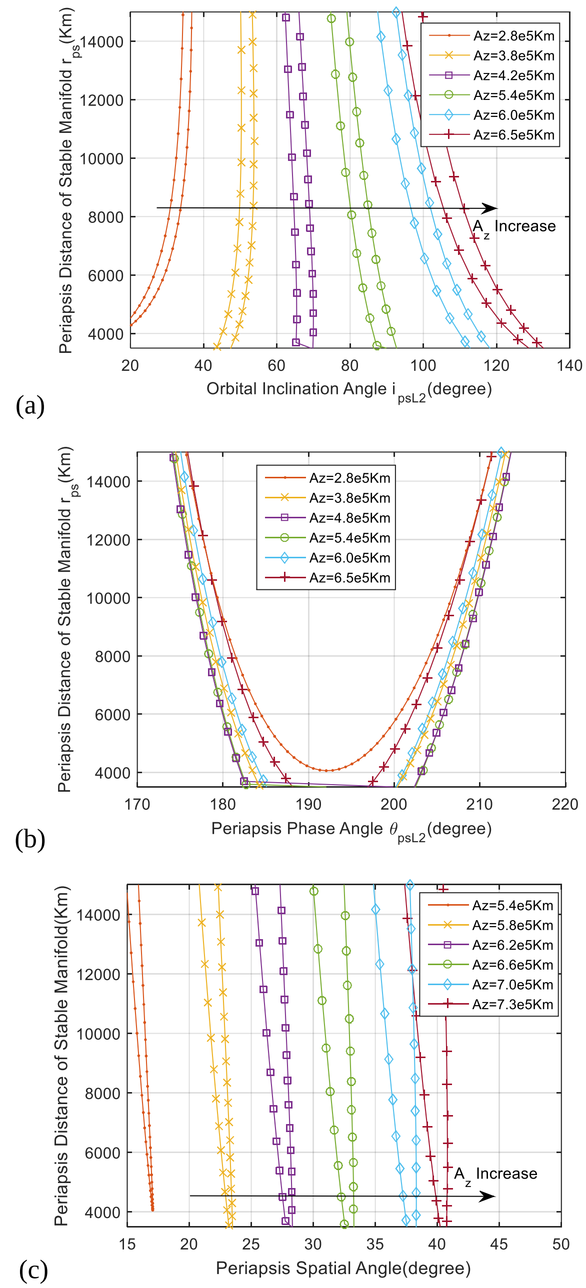


Fig.12 the periapsis state for L2 stable manifolds (a) orbital inclination angle (b) periapsis phase angle (c) periapsis spatial angle.

Similar to planar situation. All candidate halo orbits could satisfy the terminal state constrains. Therefore, the candidate Halo orbits for indirect capture are from $A_z = 2.9\times10^5 km$ and $A_z = 6.6\times10^5 km$ for L1 points and $A_z = 2.9\times10^5 km$ to $A_z = 6.5\times10^5 km$ for L2 points.

5. SIMULATION RESULTS AND DISCUSSION

In this part, the efficiency of indirect capture strategy is evaluated under different conditions and compared with direct capture.

The interplanetary trajectory is simplified as hyperbolic trajectory with different excess velocity and different types of mission orbits are considered.

For direct capture method, periapsis altitude of hyperbolic trajectory is chosen directly as the periapsis of mission orbits. The velocity increment is easy to express as,

$$\Delta v_d = \sqrt{v_\infty^2 + \frac{2\mu}{r_p}} - \sqrt{\frac{(1+e)\mu}{r_p}} \qquad (4)$$

where $r_p = a(1-e)$ is the periapsis distance of mission orbit. a is the semi-axis of mission orbit and e is the eccentricity of mission orbit.

For indirect capture method, the periapsis of hyperbolic trajectory is set equal to that of stable manifold. The periapsis altitude is not strictly restricted to 200km. The natural stable manifold which has periapsis altitude close to 200km are selected without further correction. The first maneuver Δv_1 is

$$\Delta v_1 = \sqrt{v_\infty^2 + \frac{2\mu}{r_{ps}}} - v_{ps} \qquad (5)$$

On the other hand, the natural unstable manifold which has the periapsis distance close to mission orbit is chosen as the initial guess. A differential correction process is used to correct the distance error. The perturbation velocity is noted as Δv_2.

Finally, the third velocity increment Δv_3 is implement at the periapsis of unstable manifolds. It can be written as,

$$\Delta v_3 = v_{pu} - \sqrt{\frac{(1+e)\mu}{r_p}} \qquad (6)$$

where the velocity of unstable manifold at periapsis is v_{pu}. The total velocity for indirect capture is $\Delta v = \Delta v_1 + \Delta v_2 + \Delta v_3$. The total transfer time T includes the stable transfer time T_s, parking time T_p and unstable transfer time T_u, that is $T = T_s + T_p + T_u$.

In this paper, we focus on evaluate the efficiency of indirect capture. Therefore, the periapsis states have not been considered except the periapsis distance. There are multiple choices for parking orbits. However, numerical simulations show that different parking orbits cost nearly the same velocity so long as the periapsis distances are identical.

Three kinds of mission orbits are chosen (1) 200km circular orbit (2) 800×60000 elliptic orbit (3) 20000km circular orbit. For each orbit, different excess velocity is investigated, $v_\infty = 1.88 km/s$, $v_\infty = 2.09 km/s$ and $v_\infty = 3.39 km/s$. Table 1-3 shows the detailed information.

Table 1 Cost for different capture strategies (200km circular orbit)

v_∞ (km/s)	Direct Capture Δv_d (km/s)	Indirect capture	
		Δv (km/s)	T (day)
1.88	1.780	1.779	775.37
2.09	1.859	1.858	
3.39	2.492	2.487	

In first situation, the L2 planar Lyapunov orbit with amplitude $A_y = 5.7\times 10^5 km$ is chosen. The stable manifolds with periapsis distance $r_{ps} = 3576 km$ is chosen. The periapsis velocity is $v_{ps} = 4.8828 km/s$. Stable transfer cost 294.41 days. The unstable transfer time is about. Spacecraft will stay in parking orbit for 186.58days to insert into unstable manifold, the insert perturbation is only 1m/s. The velocity at unstable manifold's periapsis is $v_{pu} = 4.8732 km/s$. The transfer requires about 294.38 days. As shown in Table.1, the cost for direct and indirect capture is similar at low altitude orbit. Though indirect capture cost long transfer time, it provides a chance to explore the space environment in the vicinity of Mars and Lagrange points without extra velocity increment.

Table 2 Cost for different capture strategies (800*60000km orbit)

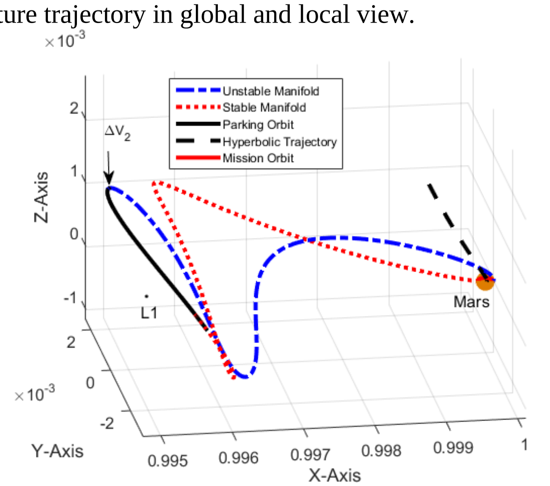
v_∞ (km/s)	Direct Capture Δv_d (km/s)	Indirect capture		$\Delta v_d - \Delta v$ (km/s)
		Δv (km/s)	T (day)	
1.88	0.518	0.493	696.85	0.025
2.09	0.602	0.572		0.030
3.39	1.272	1.205		0.067

In Table 2, we choose an elliptic orbit, which is more common in Mars exploration. The L2 Halo orbit with amplitude $A_z = 4.6\times 10^5 km$ is selected. The periapsis distance for stable manifolds is $r_{ps} = 3594 km$, the total capture time is 696.85 days includes 107.45 days parking time. As the periapsis of mission orbit increases, the indirect capture shows its advantages when compared with direct capture. Moreover, indirect capture saves more fuel when the excess velocity v_∞ increases.

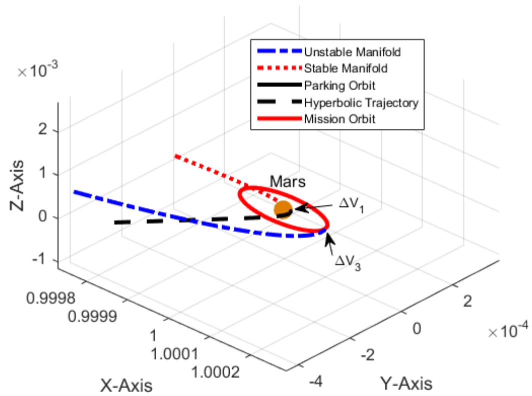
Table 3 Cost for different capture strategies (20000km circular orbit)

v_∞ (km/s)	Direct capture Δv_d (km/s)	Indirect capture		$\Delta v_d - \Delta v$ (km/s)
		Δv (km/s)	T (day)	
1.88	1.329	0.897	691.03	0.432
2.09	1.481	0.976		0.505
3.39	2.540	1.609		0.931

In third situation, a high altitude orbit is investigated, which is similar to Mars geostationary orbit. Such orbits might be utilized for Mars navigation. The L1 halo orbit with amplitude $A_z = 3.4\times10^5 km$ is used as parking orbit. It is clearly seen from Table 3 that indirect capture could save more than 30% velocity in such high orbit. The efficiency is even better in higher v_∞. Figure 13 shows the indirect capture trajectory in global and local view.



(a)



(b)

Fig. 13 Indirect capture trajectory to 20000km circular orbit (a) global view (b) in the vicinity of the Mars

Based on the discussion and evaluation, it is concluded that the indirect planetary capture via periodic orbit around Lagrange points could save velocity increment than direct capture. There is no limitation to the capture altitude and excess velocity. Moreover, indirect capture shows better efficiency for high altitude and high v_∞ orbit insertion.

Besides the fuel saving, other advantages of indirect capture are discussed as follow,

(1) Indirect capture takes full advantage of properties of invariant manifolds. The spacecraft required no more maneuver during the manifolds transfer. That provides a good opportunity for space observation and environment exploration.

(2) The parking orbit could achieve transfer to different mission orbits. Therefore, the spacecraft could choose mission orbit after detailed analysis during transfer and parking, which increases the flexibility of transfer.

(3) Compared with direct capture, one large capture maneuver is replaced by two smaller capture maneuvers, which also reduce the gravity loss during capture.

The indirect planetary capture strategy is well suited for exploration mission which is insensitivity to time. It can also be applied to other planets capture and provide reference for future exploration mission.

6. CONCLUTION

In this paper, the indirect planetary capture via periodic orbit is investigated under the background of Mars exploration. The periodic orbit is considered as a park orbit during the capture, which connects with the interplanetary trajectory and mission orbit by stable and unstable manifolds, respectively. The orbit selection for planar and spatial periodic orbit are investigated. The effect of indirect capture is evaluated under different scenarios. The result shows that indirect capture requires less velocity than direct capture. Better efficiency can be found for high altitude and high v_∞ orbit capture. It also has the advantages in flexibility of transfer, extra scientific returns, which may be interested for future planetary exploration missions.

7. ACKNOWLEDGEMENT

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