Comparison of the Orekit DSST Short-Periodic Motion Model with the GTDS DSST and the F77 DSST Standalone Models

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rev 2

Outline

- What is a Semi-analytical Satellite Theory?
- Algorithm details for the Semi-analytical Theory
- Estimation Processes using the Semi-analytical Theory
- Open Source Software for Space Situational Awareness (OS4A)
- Current Versions of the DSST
- Orekit Open Source Flight Dynamics Library DSST Testing
- Conclusions and Future Work
- Acknowledgements

What is a Semi-Analytical Satellite Theory?

- **Cowell Equations of Motion**

$$\vec{x}'' = -\frac{\mu}{r^3}\vec{x} + \vec{Q}(\vec{x}, \vec{x}', t)$$

- **Semi-Analytical Satellite Theory (<u>Averaging Method</u>)**
  - **Equations of Motion for the Mean Elements (including phase)**

$$\frac{d\bar{a}_i}{dt} = \sum_{j=1}^{N} \varepsilon^j A_{i,j}(\vec{\bar{a}}) + O(\varepsilon^{N+1}) \qquad (\mathrm{i} = 1,2,\ldots,5)$$

  - **Analytical Expressions for the Short-Periodic Motion**

$$a_i = \bar{a}_i + \sum_{j=1}^{N} \varepsilon^j \eta_{i,j}(\vec{\bar{a}}, \bar{\lambda}) + O(\varepsilon^{N+1}) \qquad (\mathrm{i} = 1,2,\ldots5)$$

What is a Semi-Analytical Satellite Theory? -- Short-Period Motion Formulas

- **Zonal Harmonics (closed form)**

$$\Delta a_i = C_{i,0} + S_{i,0}(L-\lambda) + \sum_{k=1}^{2N+1}\left[C_{i,k}\cos(kL) + S_{i,k}\sin(kL)\right]$$

- **Tesseral m-Dailies (closed form)**

$$\Delta a_i = \sum_{k=1}^{M}\left[C_{i,k}\cos(k\theta) + S_{i,k}\sin(k\theta)\right]$$

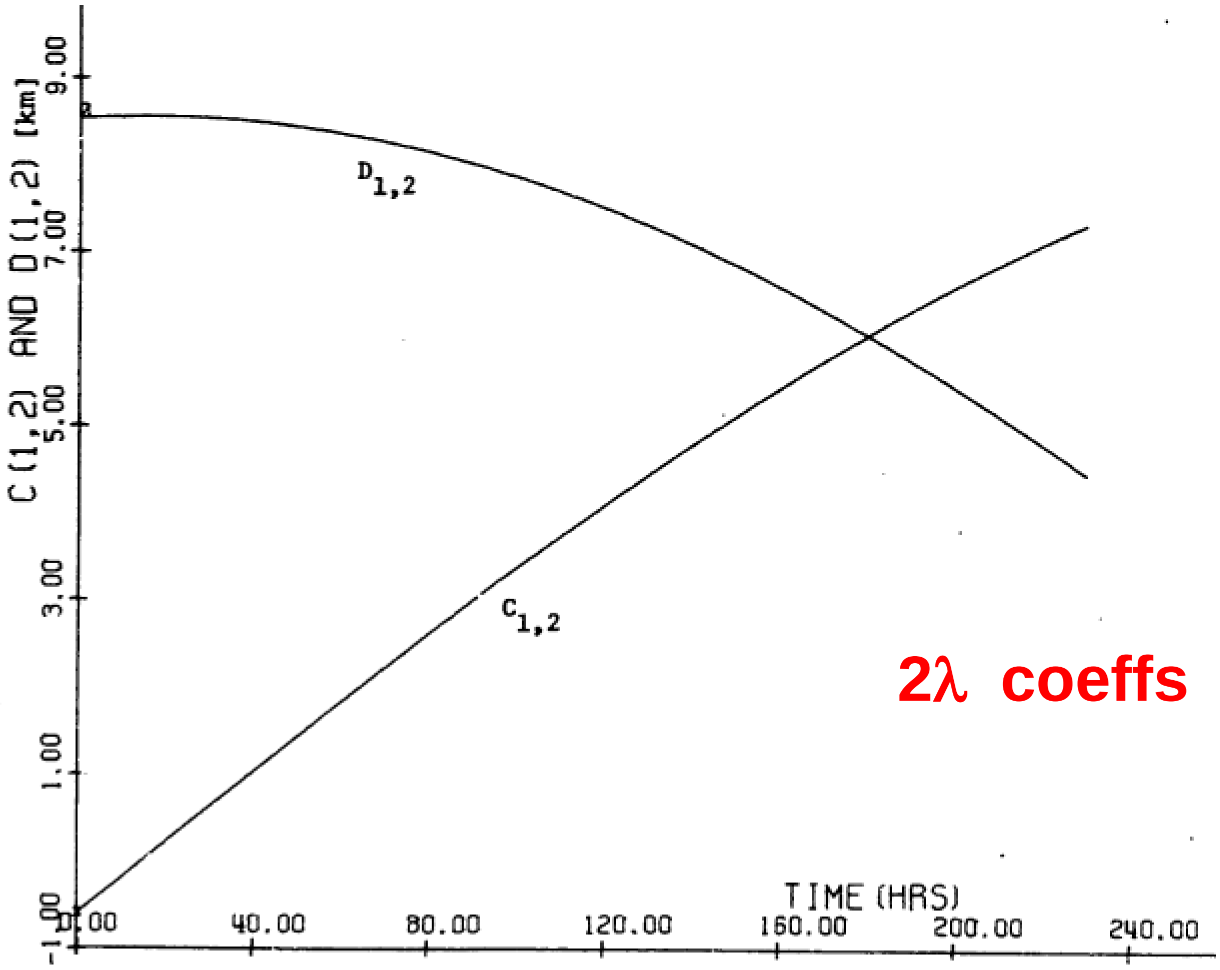
- **Tesseral Linear Combination Terms**

$$\Delta a_i = \sum_{k}\sum_{t}\left[C_{i,t,k}\cos(t\lambda - k\theta) + S_{i,t,k}\sin(t\lambda - k\theta)\right]$$

- **Lunar-Solar Point Masses (closed form)**

$$\Delta a_i = C_{i,0} + \sum_{k=1}^{N+1}\left[C_{i,k}\cos(kF) + S_{i,k}\sin(kF)\right]$$

Zonal Harmonic Semi-major Axis Short-Periodic Coefficients over 10 days (LEO)



2λ coeffs

What is a Semi-Analytical Satellite Theory? – partial derivatives

- **Partial derivatives of osculating equinoctial elements w.r.t. epoch elements and dynamics parameters**

$$G = \left[\left\{ \frac{\partial \underline{a}^{*}(t)}{\partial \underline{\overline{a}}_{0}} \right\} \quad \left\{ \frac{\partial \underline{a}^{*}(t)}{\partial \underline{c}} \right\} \right]$$

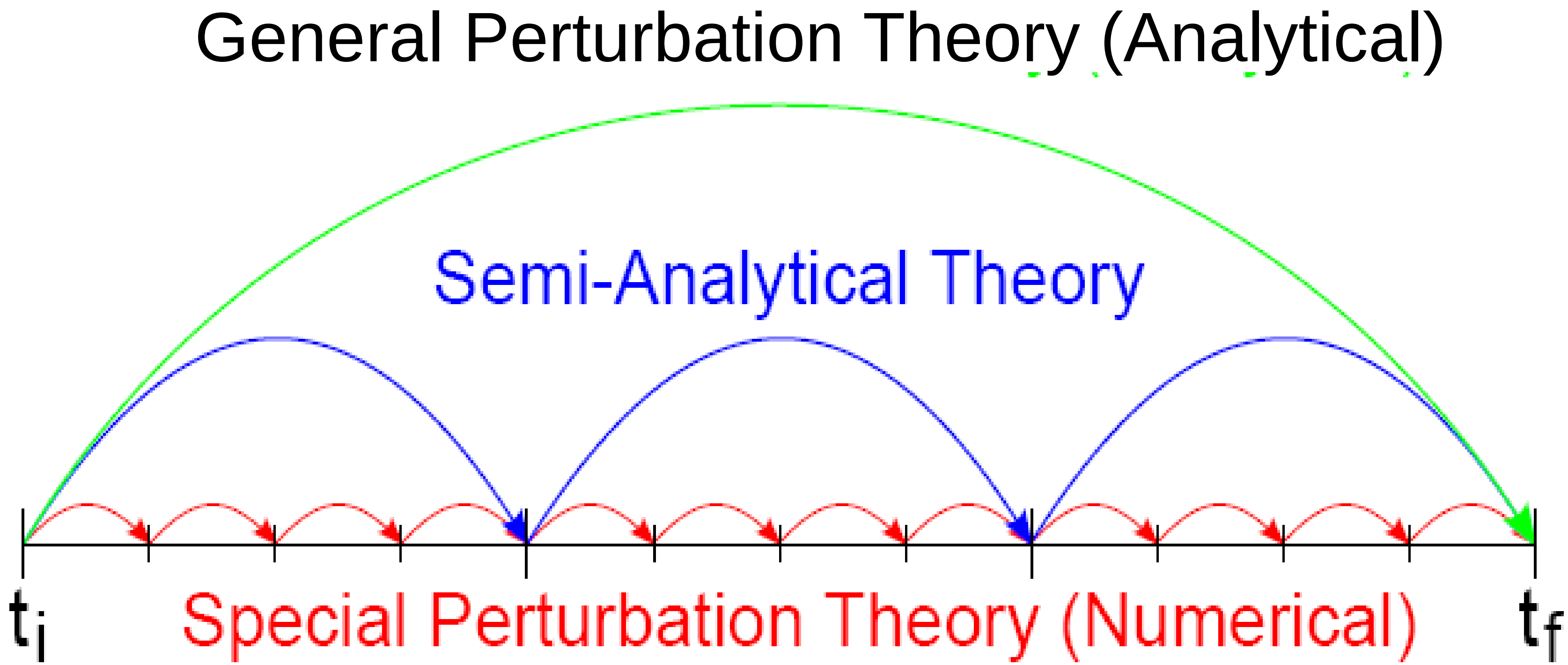
- **Expansion of the G matrix**

$$G = \left[I + B_1 \right] \left[B_2 \quad B_3 \right] + \left[0 \quad B_4 \right]$$

- **Equations of motion**

$$\frac{d}{dt} B_2 = A B_2 \text{ with } \left[B_2 \right]_{t_0} = I \qquad \frac{d}{dt} B_3 = A B_3 + \mathrm{D} \text{ with } \left[B_3 \right]_{t_0} = [0]_{6x(l-6)}$$

Satellite Theory Integration Grids



Ref. J. F. San Juan (5th ICATT, 2012)

Equinoctial Element Definition and Equinoctial Coordinate Frame

$$a = a$$

$$h = e \sin(\omega + I\Omega)$$

$$k = e \cos(\omega + I\Omega)$$

$$p = \tan^{I}\left(\frac{i}{2}\right) \sin \Omega$$

$$q = \tan^{I}\left(\frac{i}{2}\right) \cos \Omega$$

$$\lambda = \ell + \omega + I\Omega$$

I = +1 for Direct Equinoctial Frame

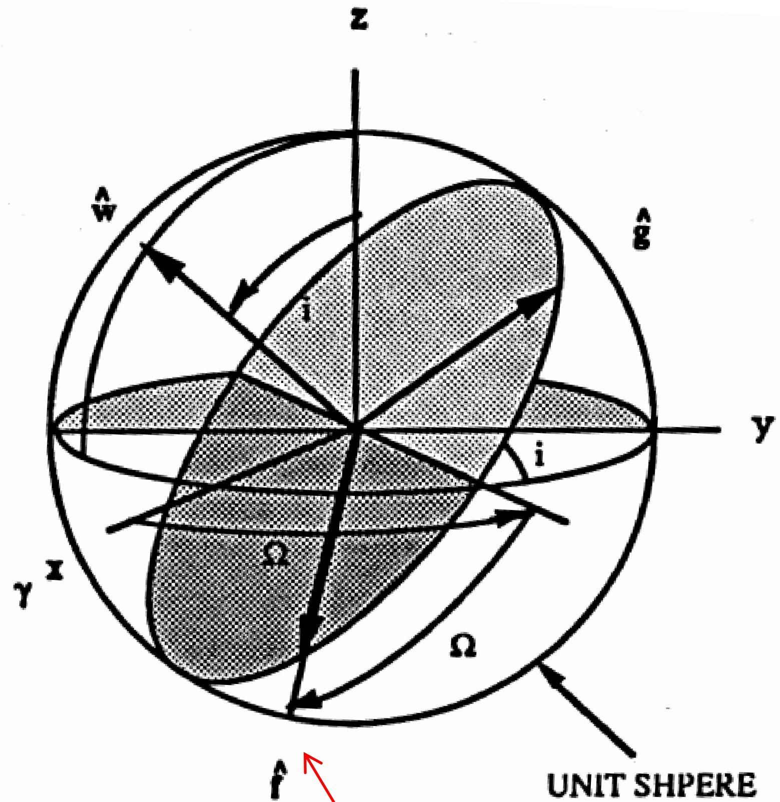


Figure A.1: Direct Equinoctial Coordinate Frame

'Departure Point' in the astronomical literature

Expansion of the Geopotential in Equinoctial Elements

- **Expression in spherical harmonics relative to the ECI frame**
  - radial distance, latitude, and longitude
- **Rotation of the spherical harmonics to the equinoctial orbital frame**
  - Jacobi polynomials replace the Kaula inclination functions
  - stable recursion formulas are available in the mathematical literature
- **Products of radius to a power times sin or cos of true longitude (and multiples) are expanded as a Fourier series in the mean longitude**
  - Modified Hansen coefficients replace the Kaula eccentricity functions
  - stable recursion formulas due to Hansen (1855) were rediscovered around 1978 in the astronomical literature
  - Jozef Van Der Ha translated the Hansen manuscript from German to English (1978)

Rotation of the Spherical Harmonics

- Generating function

$$P_{nm}(\sin\phi)e^{jm\alpha} = \sum_{r=-n}^{n} \frac{(n-r)!}{(n-m)!} P_{n,r}(0) S_{2n}^{(m,r)}(p,q) e^{jrL}$$

- where (for m $\geq$ 0)

Ref. Courant & Hilbert

$$S_{2n}^{(m,r)}(p,q) = (1+p^2+q^2)^r (p-jq)^{m-r} P_{n+r}^{(m-r,-m-r)}(\gamma)$$

r $\leq$ -m

$$S_{2n}^{(m,r)}(p,q) = \frac{(n+m)!(n-m)!}{(n+r)!(n-r)!}(1+p^2+q^2)^{-m}(p-jq)^{m-r} P_{n-m}^{(m-r,r+m)}(\gamma)$$

-m $\leq$ r $\leq$ m

$$S_{2n}^{(m,r)}(p,q) = (-1)^{m-r}(1+p^2+q^2)^{-r}(p+jq)^{r-m} P_{n-r}^{(r-m,r+m)}(\gamma)$$

r $\geq$ m

- and $\gamma = \cos i$

Hansen Coefficients

- **Generating Function**

$$(r/a)^n e^{isf} = \sum_{t=-\infty}^{t=+\infty} X_t^{n,s} e^{itM}$$

- **Recursion Relation employed in the Tesseral Resonance (and in the Tesseral Linear Combination Short-Periodic Motion)**

$$n[(n-2)^2 - m^2]\cos^2\varphi X_t^{n-4,m} = n(n-2)(2n-3)X_t^{n-3,m}$$
$$-(n-1)[n(n-2) + 2tm\cos\varphi]X_t^{n-2,m} + t^2(n-2)X_t^{n,m}$$

- **Special Case (t=0; zonal harmonics)**

$$n[(n-2)^2 - m^2]\cos^2\varphi X_0^{n-4,m} = n(n-2)(2n-3)X_0^{n-3,m}$$
$$-(n-1)[n(n-2)]X_0^{n-2,m}$$

Modified Newcomb Operator Expansion

- **Standard Expansion**

$$X_t^{n,s} = e^{|t-s|} \sum_{i=0}^{\infty} X_{i+a,i+b}^{n,s} e^{2i}$$

Newcomb Operator

- **New Expansion**

$$X_t^{n,s} = (1-e^2)^{n+3/2} e^{|t-s|} \sum_{i=0}^{\infty} Y_{i+a,i+b}^{n,s} e^{2i}$$

$$a = \frac{|t-s| + t - s}{2}$$

$$b = \frac{|t-s| - (t-s)}{2}$$

Convergence for the set of Hansen coefficients $X_t^{n,s}$ at e = 0.7 with relative accuracy 1.0D-5

n	s	t	N	Y
-3	0	1	17	6
-3	2	1	9	9
-4	-1	1	20	7
-4	1	1	20	7
-5	0	2	22	6
-5	2	2	23	7
-6	-1	1	25	6
-6	1	1	26	5
-6	-1	2	25	7
-6	1	2	25	7

Newcomb Operator Usage

- Build the Newcomb operator file
  - Separate utility from the DSST
  - Originally, files specific to low, medium, and high eccentricity cases were constructed
  - More recently, plentiful data storage allows the usage of larger, general purpose Newcomb operator file
- As part of the initialization, the DSST reads in the portion of Newcomb operator file necessary to support the specific orbital case
  - Tesseral resonance
  - Tesseral linear combination short-periodic motion
- The Newcomb operators are then employed in the DSST to initialize the Hansen Coefficients as required
  - Issue: repetitive calculations for nearly the same eccentricities

Short-Periodic Coefficient Interpolator Errors

Points	Step (days)	Zonal Errors*		
		ΔC_a (m)	ΔC_p (m)	ΔC_λ (m)
2	.25 .5 1.	.86 3.5 14.0	.064 .26 1.0	.76 3.1 12.
3	.25 .5 1.	.012 .098 .78	0 .004 .029	.011 .036 .69
4	.25 .5 1.	0 .002 .015	0 0 .004	0 .002 .021

***These errors are the position errors caused by errors in the interpolated coefficients For the 2 lambda short-periodics. The stepsize is the interval between successive interpolation points.**

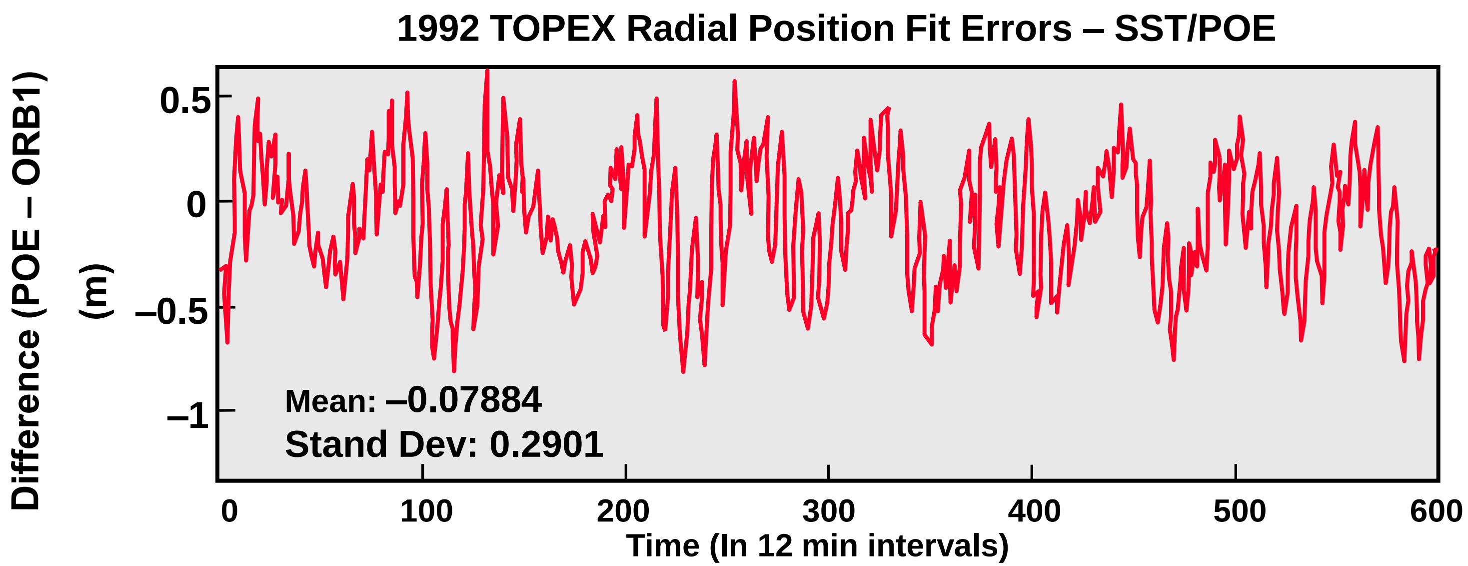
Semi-analytical Satellite Theory Estimation Algorithms

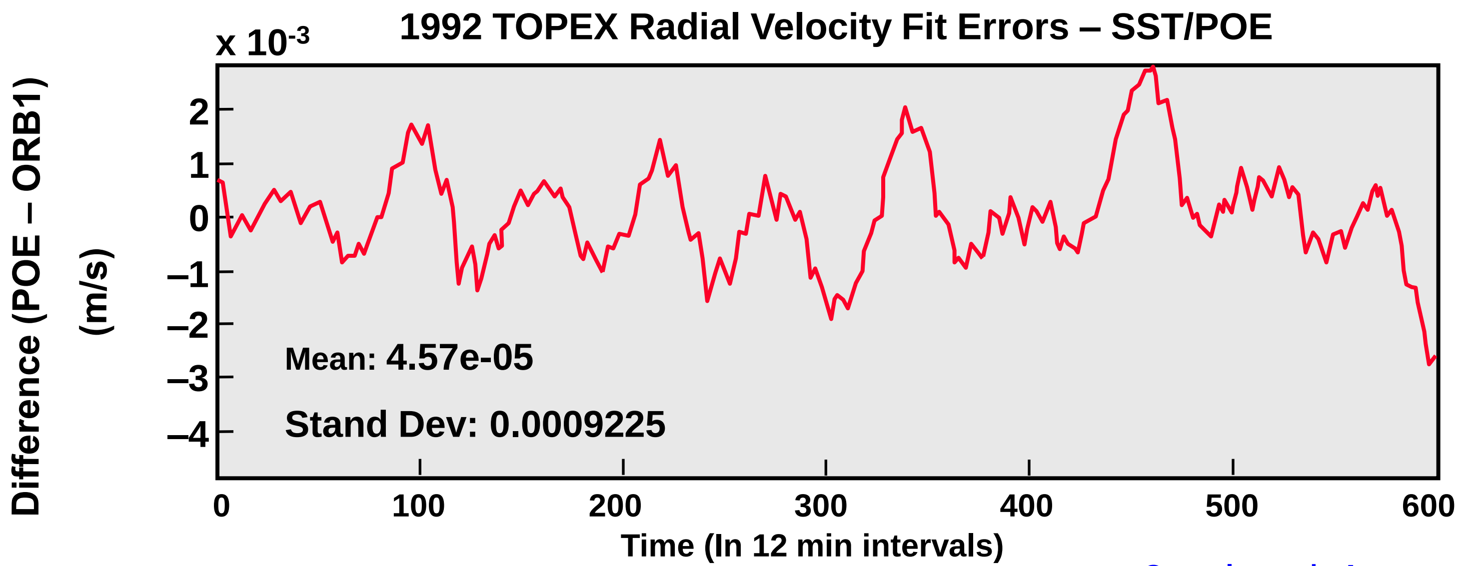
- **Several estimation algorithms have been built for estimating the Mean Elements directly from the tracking data**
  - **Conventional Weighted Least Squares**
  - **Extended Semi-analytical Kalman Filter (ESKF)**
  - **Square-Root Information Filter (SRIF)**
  - **Backward-Smoothing Extended Semi-analytical Kalman Filter (BSESKF)**
- **The nonsingular mean equinoctial elements have linearity properties that are very desirable for orbit estimation and realistic uncertainty propagation**

Semianalytical Satellite Theory Modeling of the TOPEX Orbit

- Dynamics
  - Mean Element Equations of Motion
    - 50 x 50 Geopotential (truncated JGM 2)
    - Lunar-Solar Point Masses
    - Atmosphere Drag (Jacchia-Roberts)
    - Solar Radiation Pressure
    - Solid Earth Tides (solar & lunar terms)
  - Short Periodic Terms
    - Zonals, Tesseral M-dailies, Tesseral Linear Combinations
    - J2 / Tesseral M-daily Coupling
  - Integration Coordinate System – Mean of J2000.0
- Solve-for Vector
  - Mean Equinoctial Elements
  - Solar Radiation Pressure Coefficient
  - Drag Coefficient

Least Squares Fit of SST Theory to TOPEX External Reference Orbit – Radial Fit Errors





Current Open Source Software Space Situational Awareness (OS4A) Demonstration Projects

- The DSST Standalone is included on the Web-Site ASTRODY prototype, which provides a friendly web interface for DSST. The web site was established at the Universidad de La Rioja, Spain. This prototype has now evolved into a stable platform based on the Drupal open source content management system

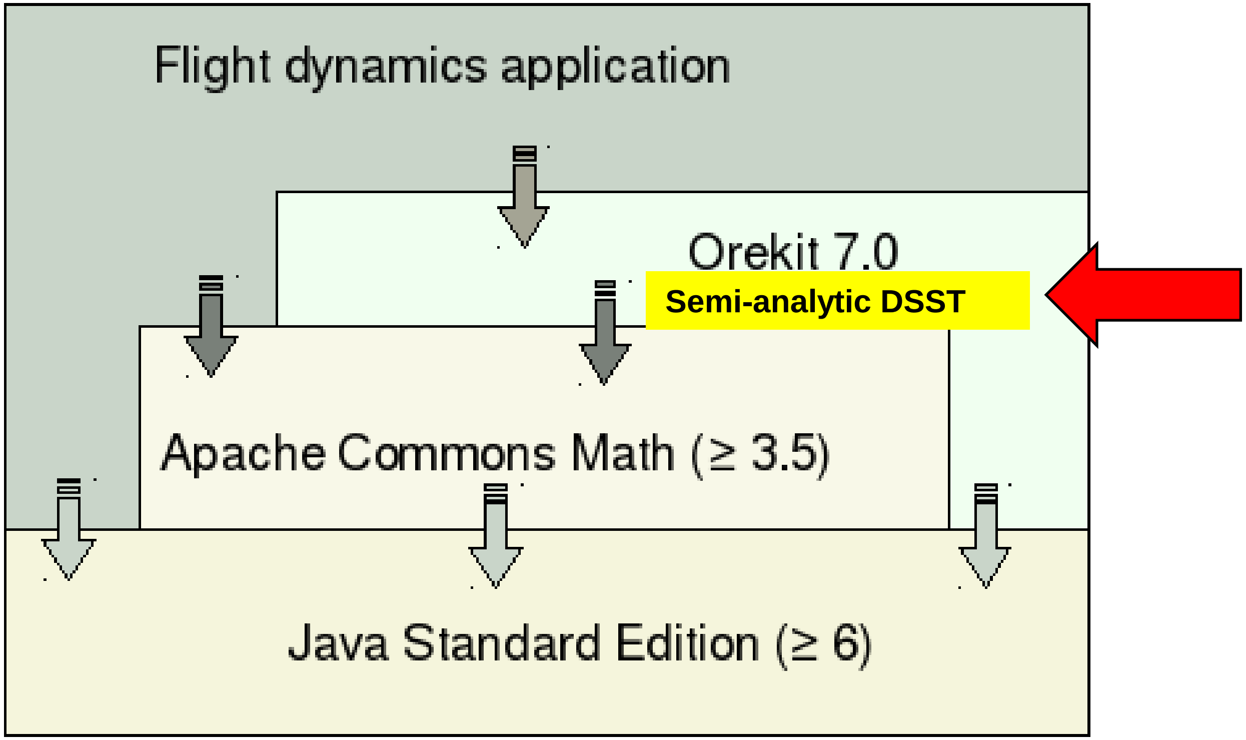
  The url of tastrodyWebTool is http://tastrody.unirioja.es/.

- The DSST is implemented as an orbit propagator in object-oriented java as part of Orekit open source library for space flight dynamics.
  - Validation of the Orekit DSST Short-periodic model is in progress
- The accuracy and computation time characteristics of the DSST Standalone relative to the requirements of space object catalog maintenance are being evaluated in the Space Situational Awareness (SSA) Group at the DLR GSOC
  - Demonstration of Standalone Orbit Determination Program

Current Versions of the DSST

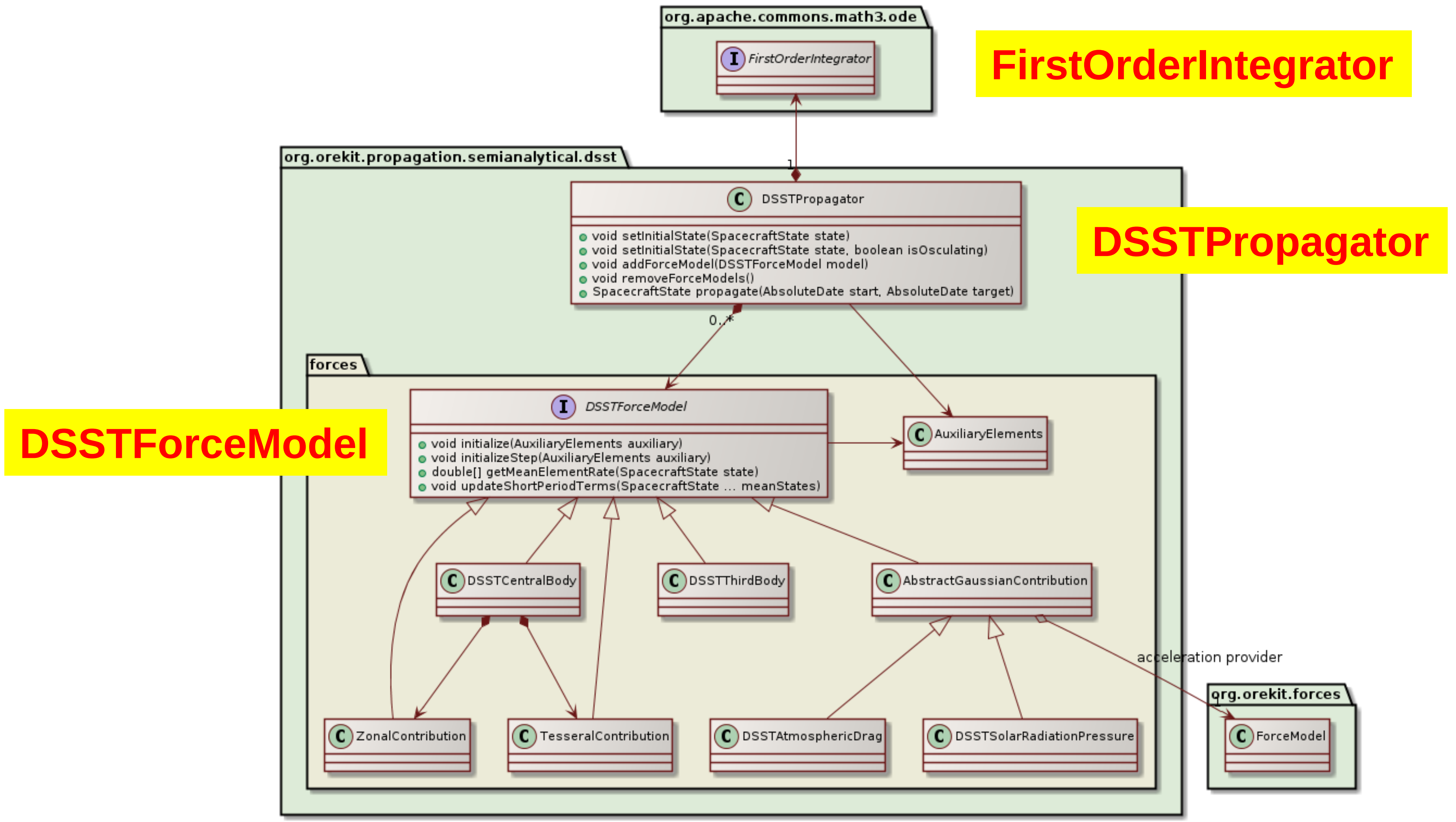
- **Linux Ubuntu GTDS**
  - **Intel and G Fortran compilers**
- **Linux Ubuntu F77 DSST Standalone**
  - **Intel compiler**
- **F77 DSST Standalone (Prof. Juan Felix San Juan, Univ. Rioja, Spain)**
  - **Intel compiler, WebDSST**
- **Linux SUSE F77 DSST Standalone (Srinivas Setty, Germany)**
  - **Intel compiler, SSA research at DLR/GSOC**
- **Orekit Open Source Flight Dynamics Library -- java DSST (Luc Maisonobe, France)**

Inclusion of DSST in Orekit

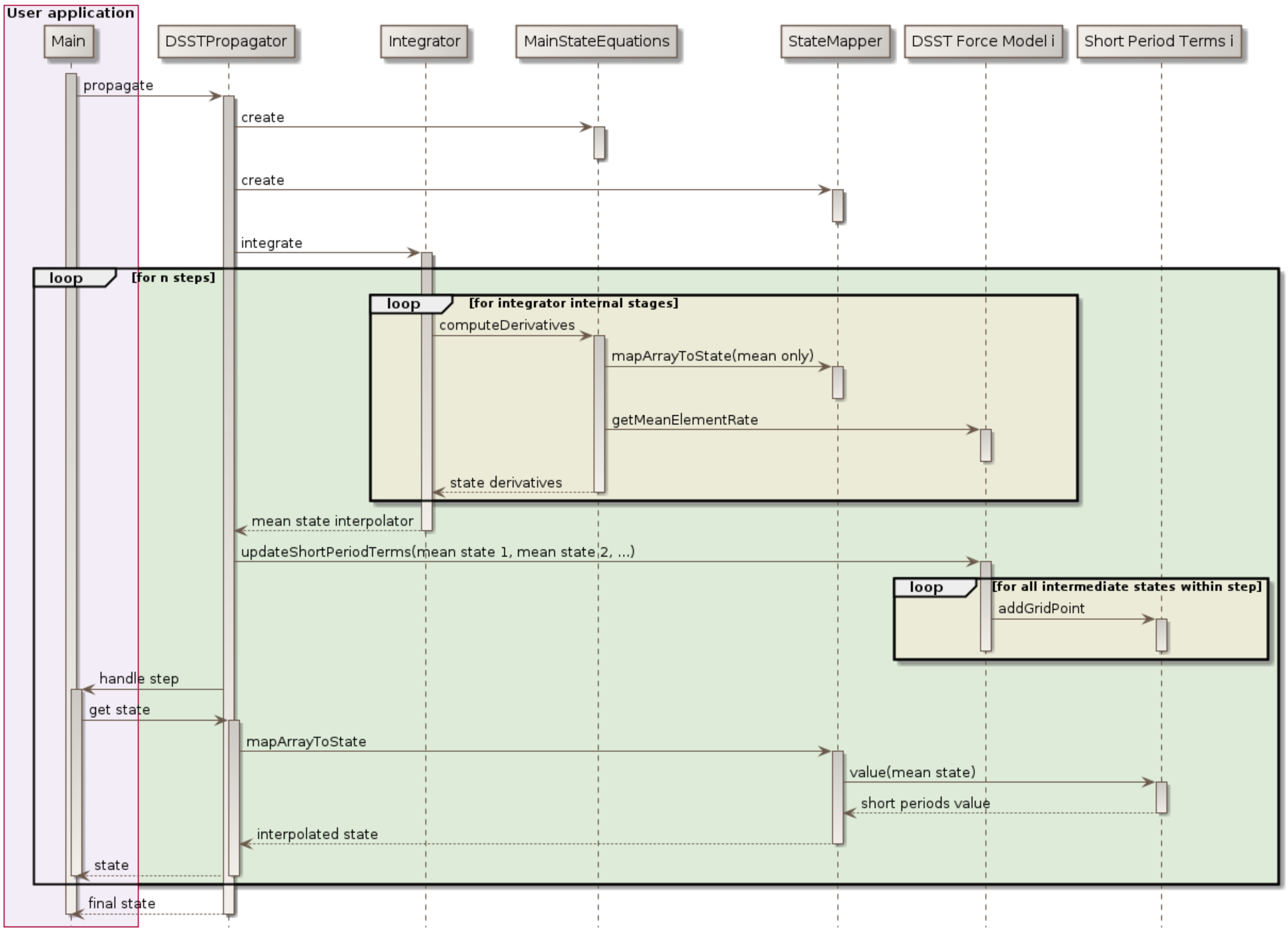


- **DSST Propagator Class and DSST Propagation Sequence Diagrams**
- **Orekit java DSST functional distribution vs. F77 DSST**

Orekit DSST Propagator Class Diagram



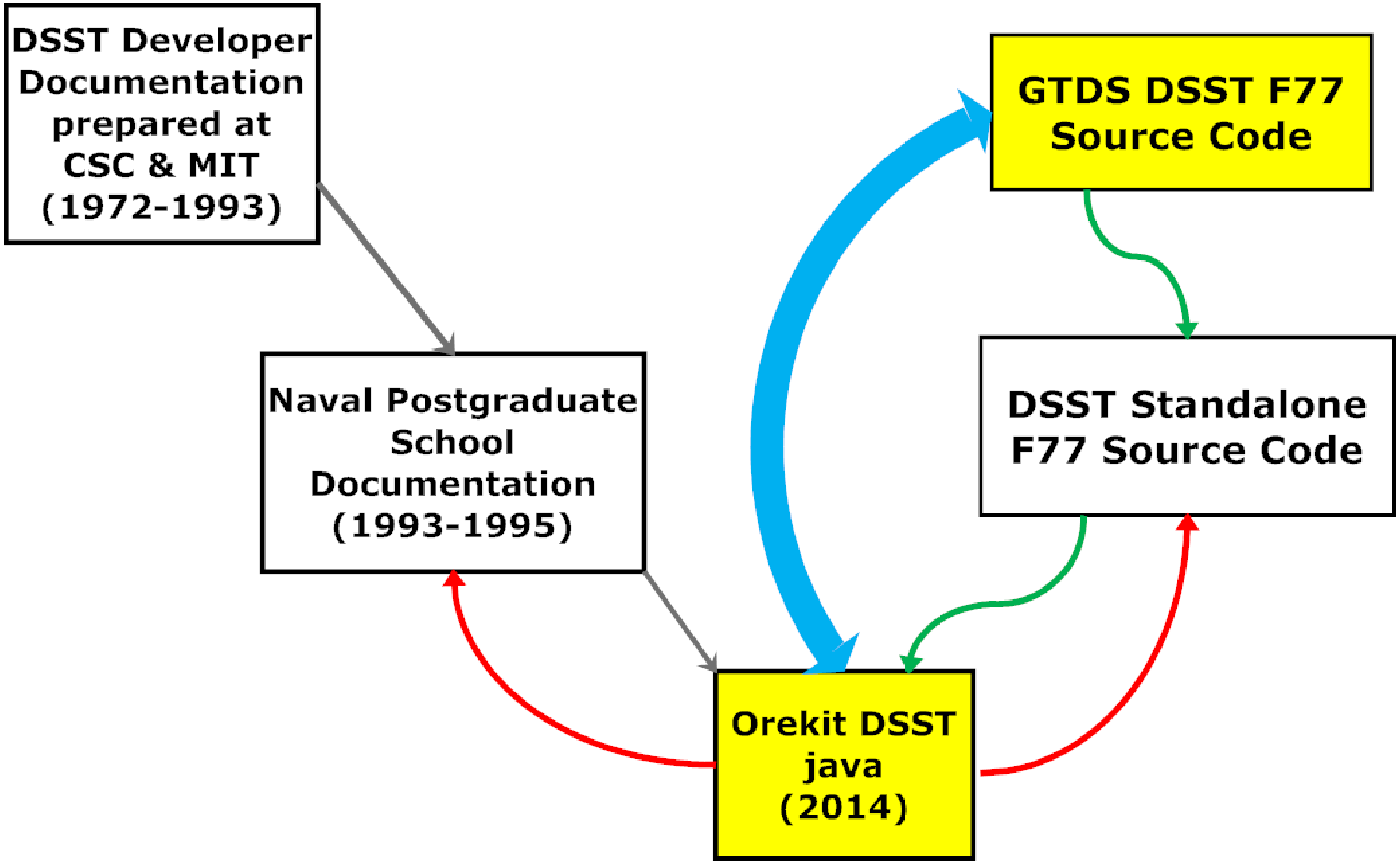
DSST Propagation Sequence Diagram



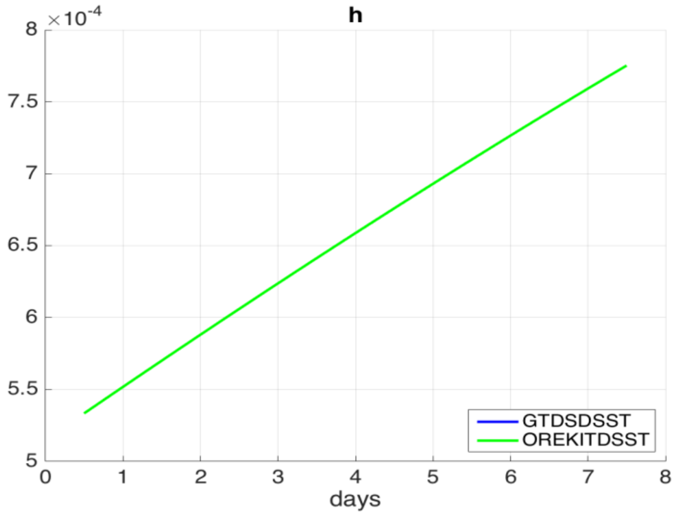
Modifications to Orekit DSST to support test of short periodics (included in Orekit 7.1 release)

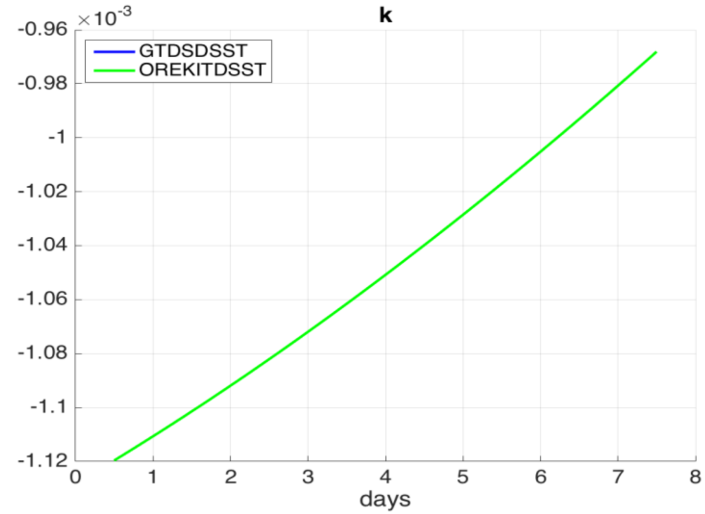
- **Allow DSST standalone validation program to use different orbit types (osculating vs. mean) for input and output**
- **Added capability to output the short periodic coefficients with the spacecraft state**
- **fixed a difficult interpolation issue for short periodic coefficients, that prevented proper generation of ephemerides**
- **re-designed short periodic coefficient interpolation, allowing finer customization (number of points, gap between points)**
- **insure interpolated coefficients in one step rely only on mean parameters in the same step(i.e. try to avoid interpolation Runge phenomena near step boundaries)**
- **added the capability to configure the maximum degree and order for short periodic terms to values smaller than the ones used for the mean elements, which are directly inherited from the loaded gravity field settings**

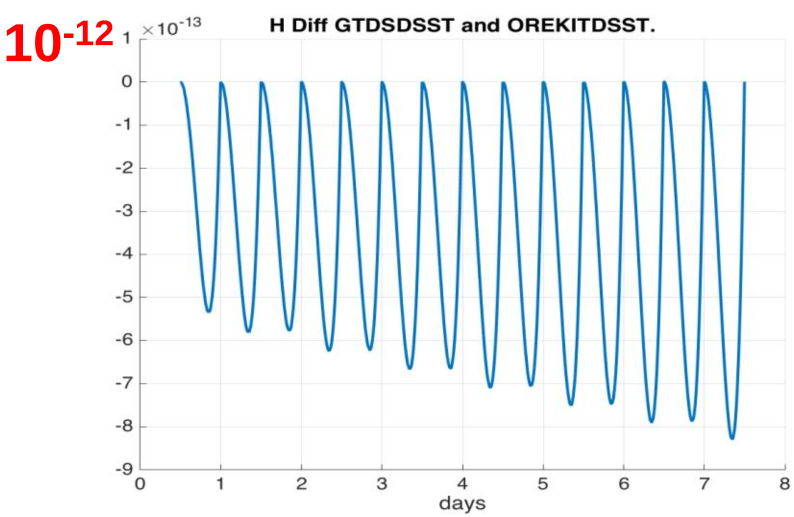
Data Flow from the Fortran 77 DSST to the java Orekit DSST Design

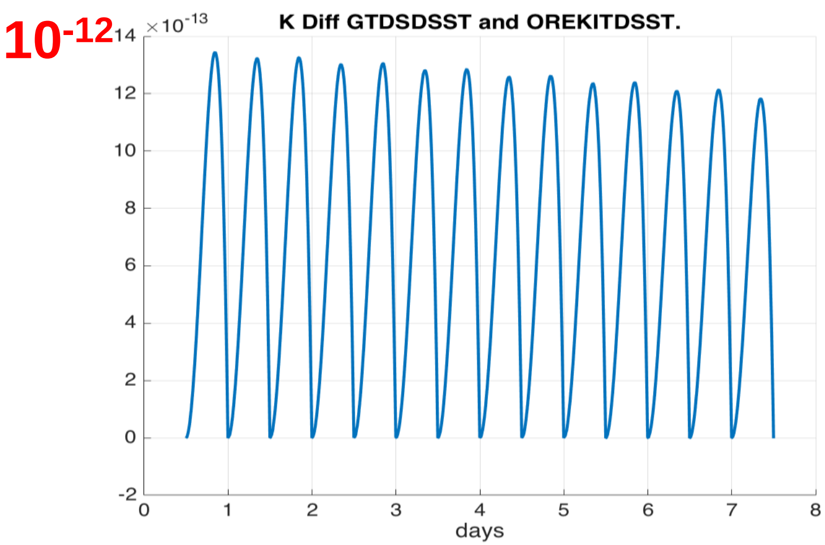


Mean Equinoctial Element h and k Time Histories and Differences (LEO Sun Sync) – Orekit, GTDS(J2)



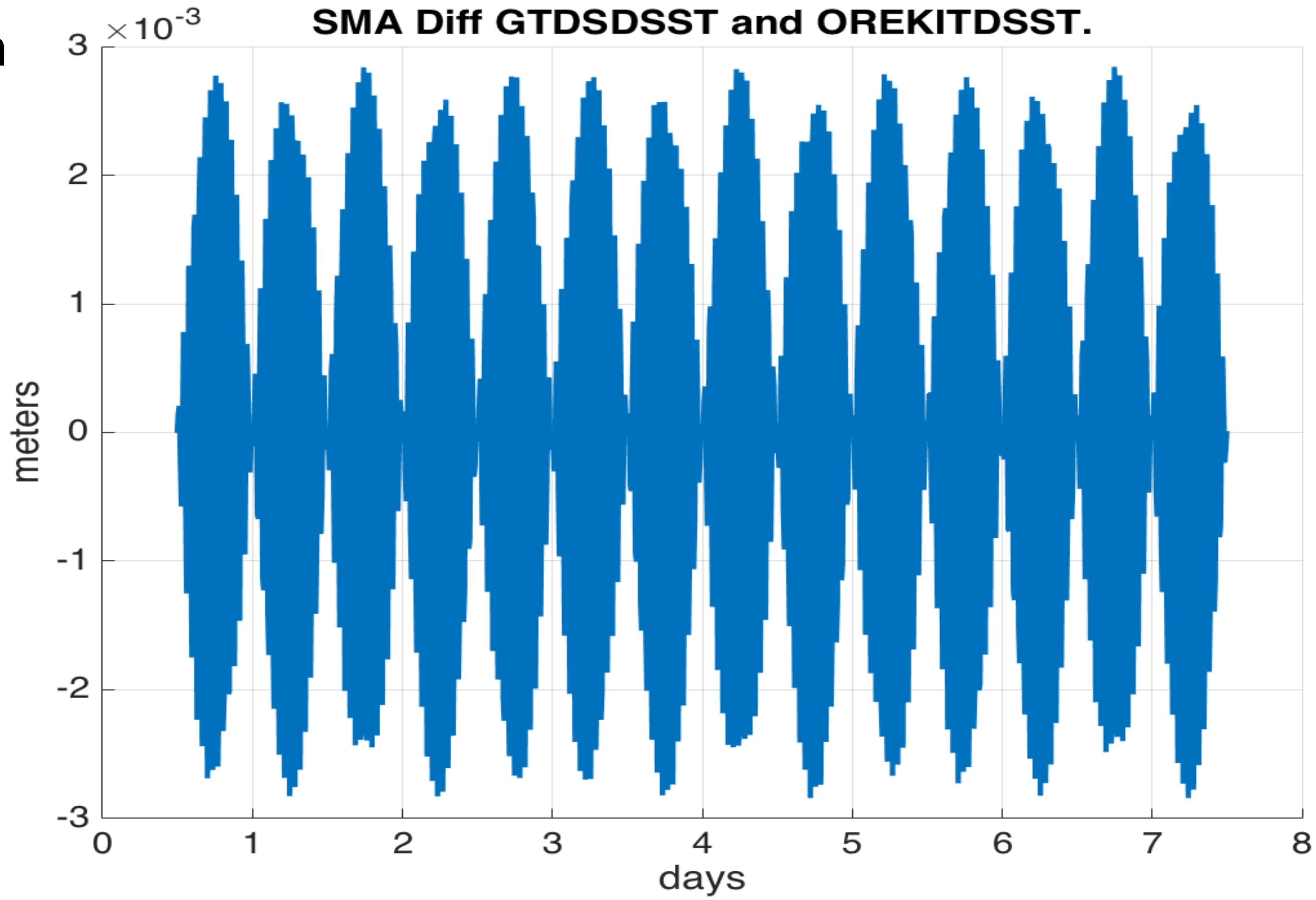




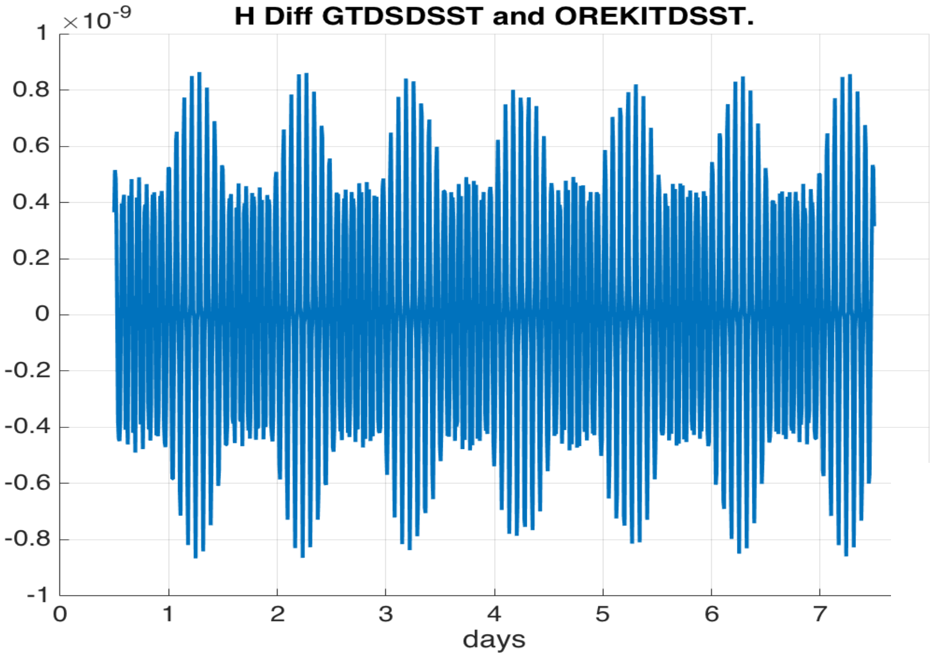


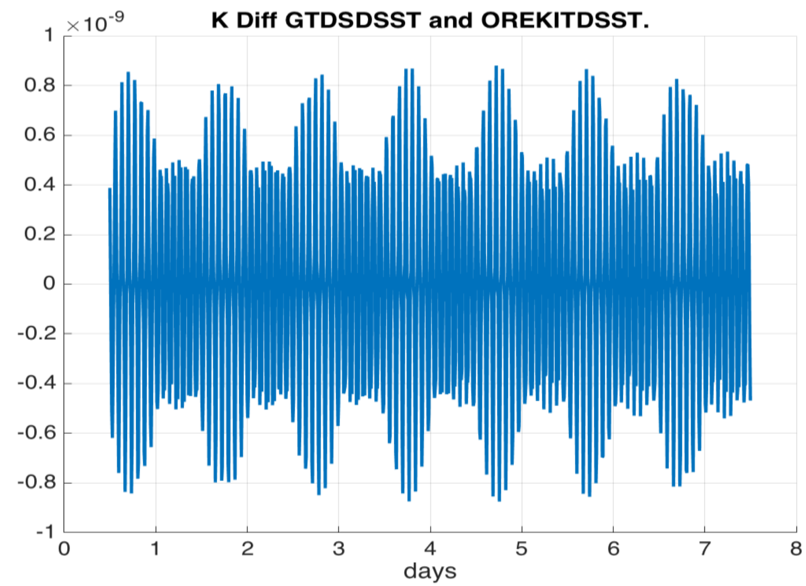
Osculating Semi-Major Axis Time Differences – Orekit, GTDS (J2 thru J36)

3 mm



Osculating Equinoctial Element h and k Differences – Orekit, GTDS (J2 thru J36)

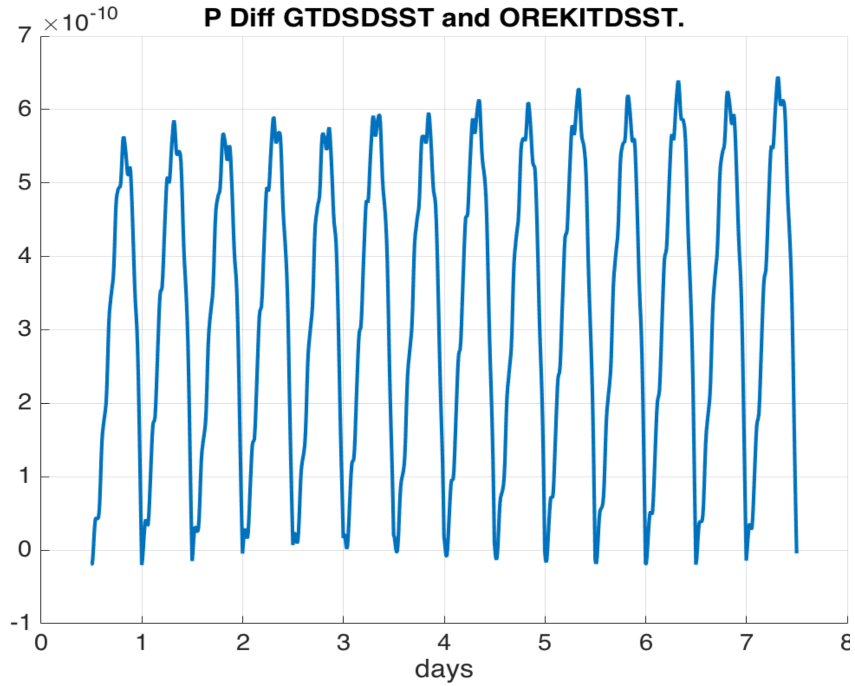


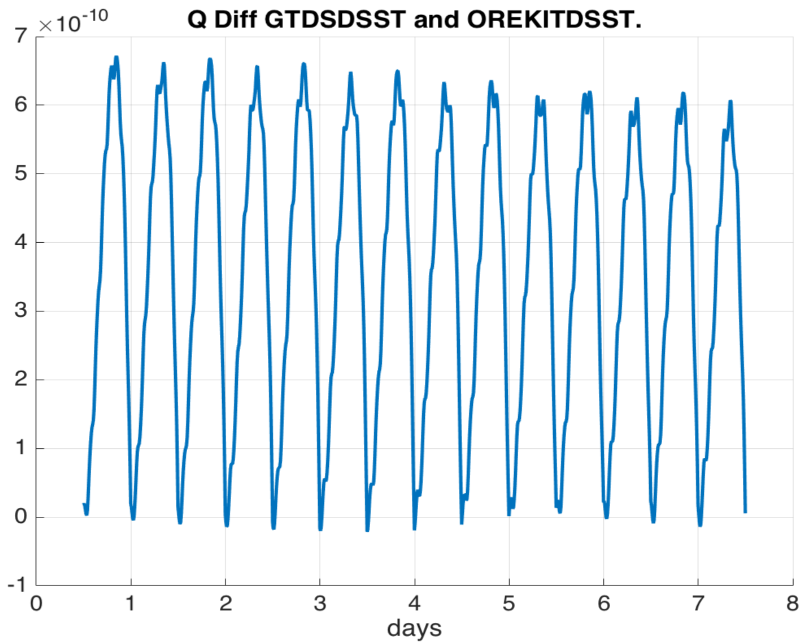


Differences bounded by 10(-9)**

Daily and 12 hour patterns in the residuals

Osculating Equinoctial Element p and q Differences – Orekit, GTDS (J2 thru J36)

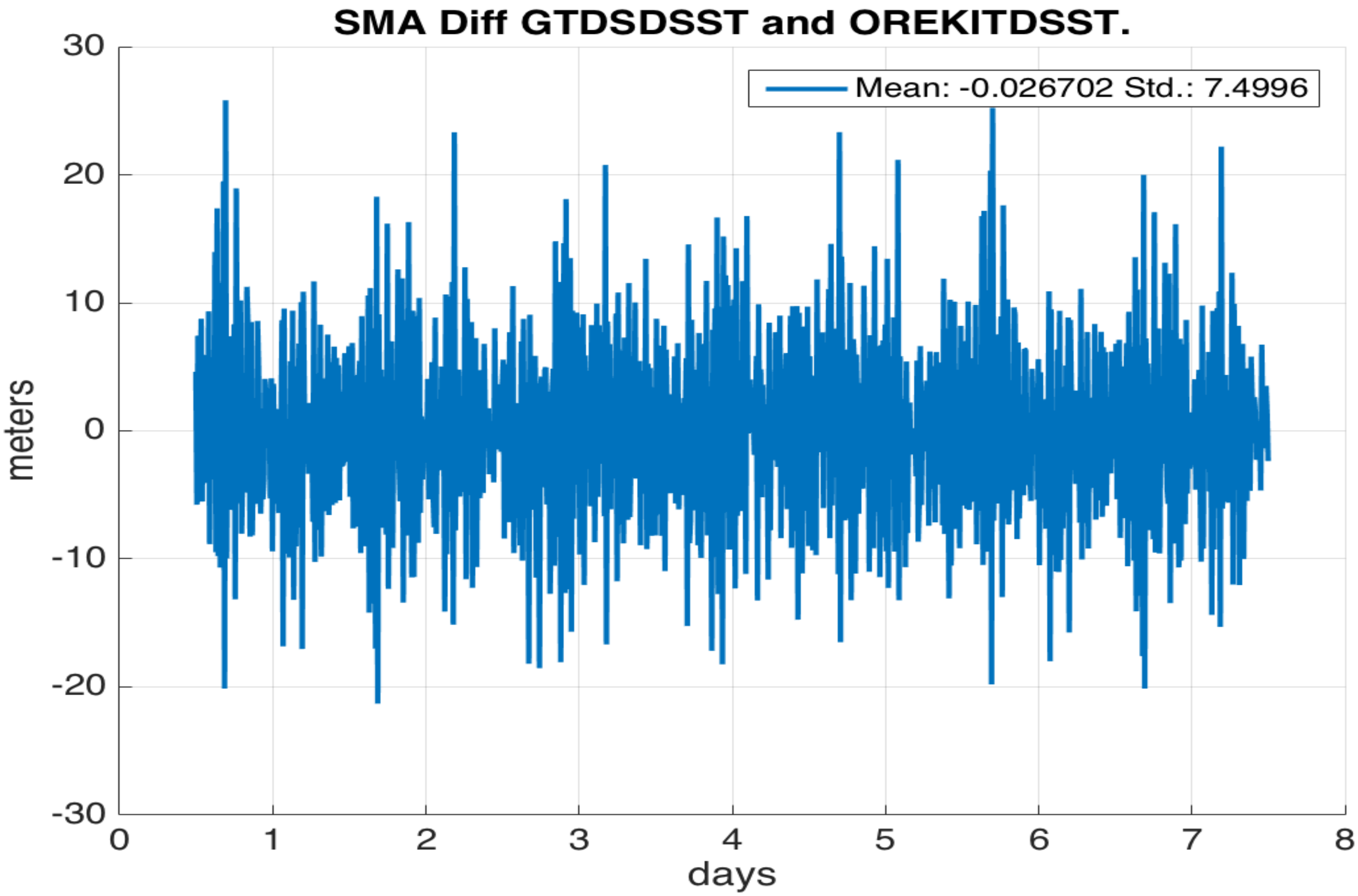




Differences bounded by 10(-9)**

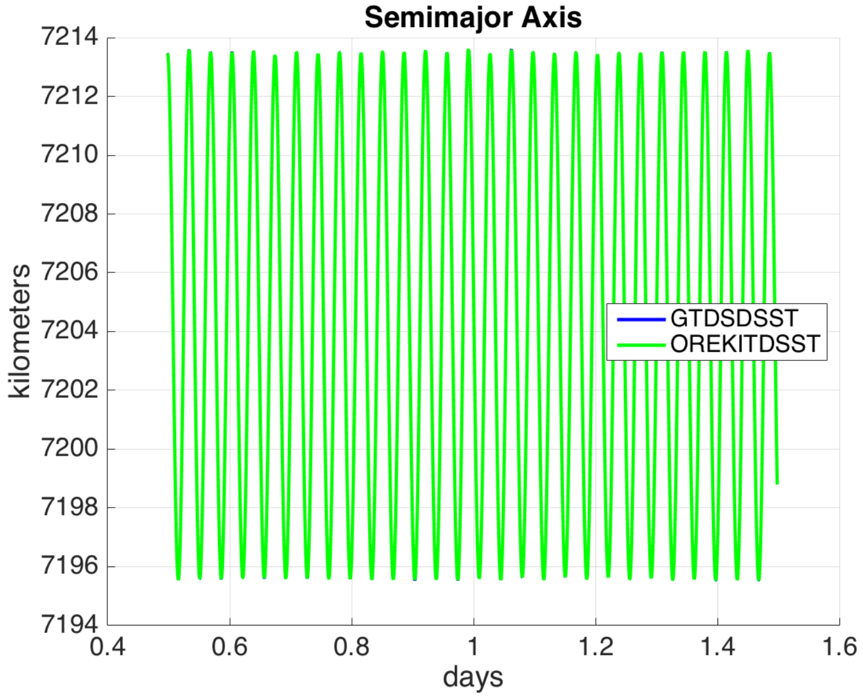
~12 hour pattern in residuals

Osculating Semi-Major Axis Time Histories and Differences – Orekit, GTDS (36 x 36)



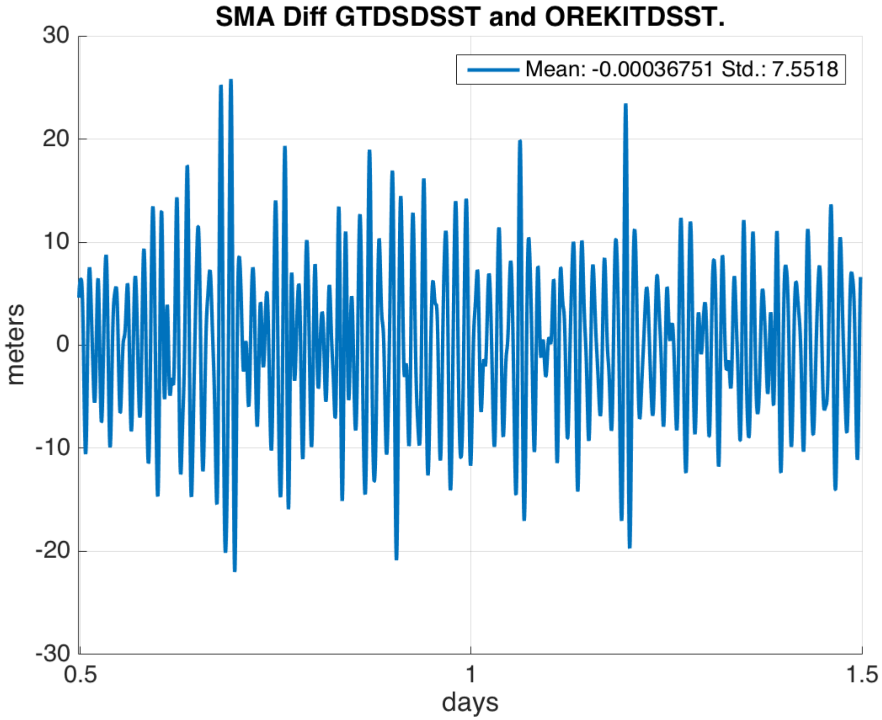
Impact of these errors on position and velocity is small

Osculating Semi-Major Axis Time Histories and Differences – Orekit, GTDS (36 x 36) (1 day arc)

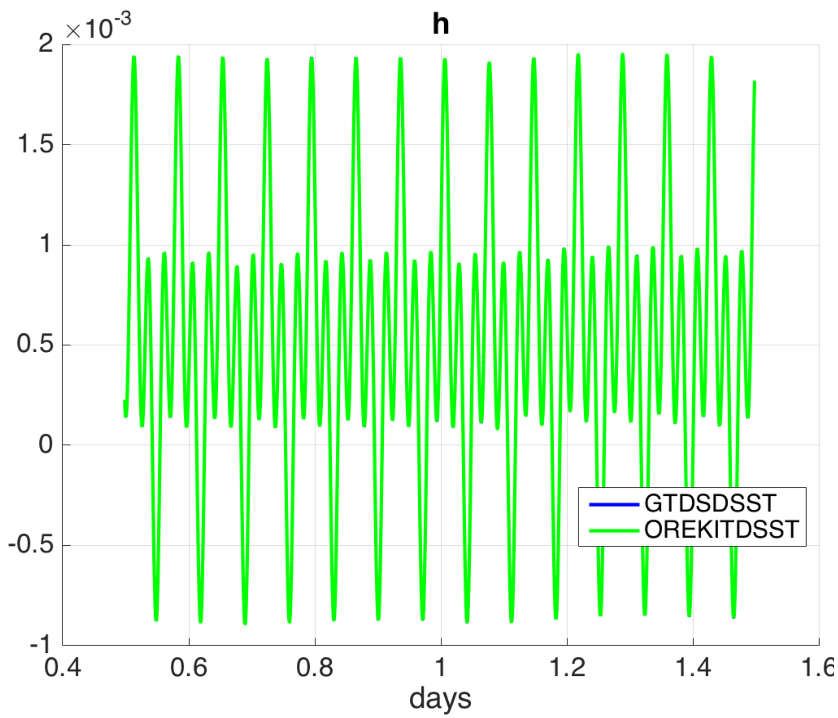


Better picture of the short periodic motion: 29 or 30 cycles per day

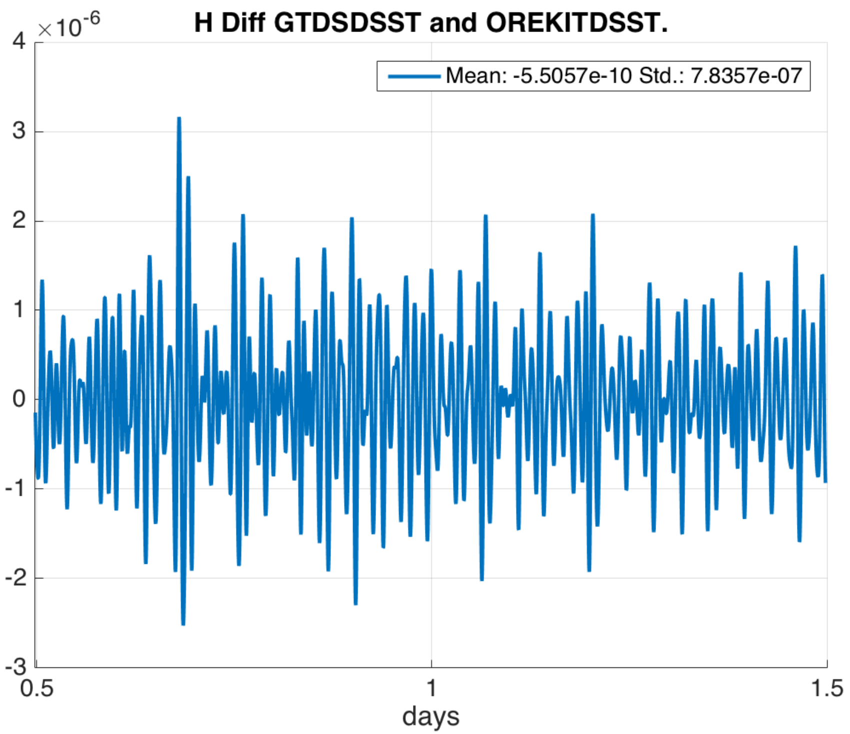
The difference has a higher frequency structure



Osculating Equinoctial Element h Time Histories and Differences – Orekit, GTDS (36 x 36) (1 day arc)



Better picture of the short periodic motion



Higher frequency structure (similar to semi-major axis) still apparent

GTDS Short-Periodic Tesseral Linear Combination (SPTESSLC) Keyword

- **Set the central body high frequency tesseral harmonic options for the Fourier coefficients in the short periodic expansions of the DSST**

- **Maximum degree of the central body high frequency tesseral field**
- **Maximum order of the central body high frequency tesseral field**
- **Maximum d'Alembert characteristic (maximum power of the eccentricity outside the Hansen coefficients)**
- **Maximum power of e**2 in the power series expansion for the Hansen coefficients**
- **Minimum frequency in mean longitude in the central body high frequency tesseral field**
- **Maximum frequency in mean longitude in the central body high frequency tesseral field**

Conclusions

- Reviewed the DSST
- OS4A demonstration tasks
- Orekit and Orekit DSST Testing
  - Zonal, tesseral, and lunar-solar short-periodic models
  - Need for more precise control of the force model options in the Orekit DSST vs. GTDS DSST test

Future Work (1 of 2)

- **Further testing of the current short-periodic models in Orekit DSST**
  - Employ the Grid technique developed by Srinivas Setty
  - Add further capabilities to tailor the Orekit DSST short-periodic models
  - More detailed test of Orekit-unique numerical integration and interpolation
  - Eccentric orbits
- **Extend the short-periodic models in Orekit DSST**
  - J2-squared terms, J2 secular-tesseral m-daily coupling terms, WTD
- **Include partial derivatives in Orekit DSST**
- **Develop a java Orekit orbit determination application using DSST**

Future Work (2 of 2)

- **Revision of the DSST to employ normalized geopotential coefficients**
- **Hansen coefficient computational approach**
- **Procedures for applying complex spacecraft geometrical and material models in the DSST non-conservative forces**
- **DSST for arbitrary central body – planets, natural satellites, asteroids**
- **Recursive estimation techniques employing the DSST Standalone**
- **Develop approaches for applying heterogeneous parallel computing to Astrodynamics software**

Acknowledgements

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- ***Srinivas Setty, Dr. Oliver Montenbruck and Dr. Hauke Fiedler, DLR/GSOC, Wessling, Germany***
- ***Original Developers of DSST: Wayne McClain, Leo Early, Dr. Ron Proulx, Dr. Mark Slutsky, Dr. David Carter, Rick Metzinger, all at the Draper Laboratory***
- ***Mr. Zachary Folcik, MIT***
- ***Prof Don Danielson at Naval Postgraduate School for his effort to generate a unified document for DSST***
- ***The several MIT Aeronautics and Astronautics Department graduate students who participated in the development, test, evolution, and application of DSST***

Backup Charts

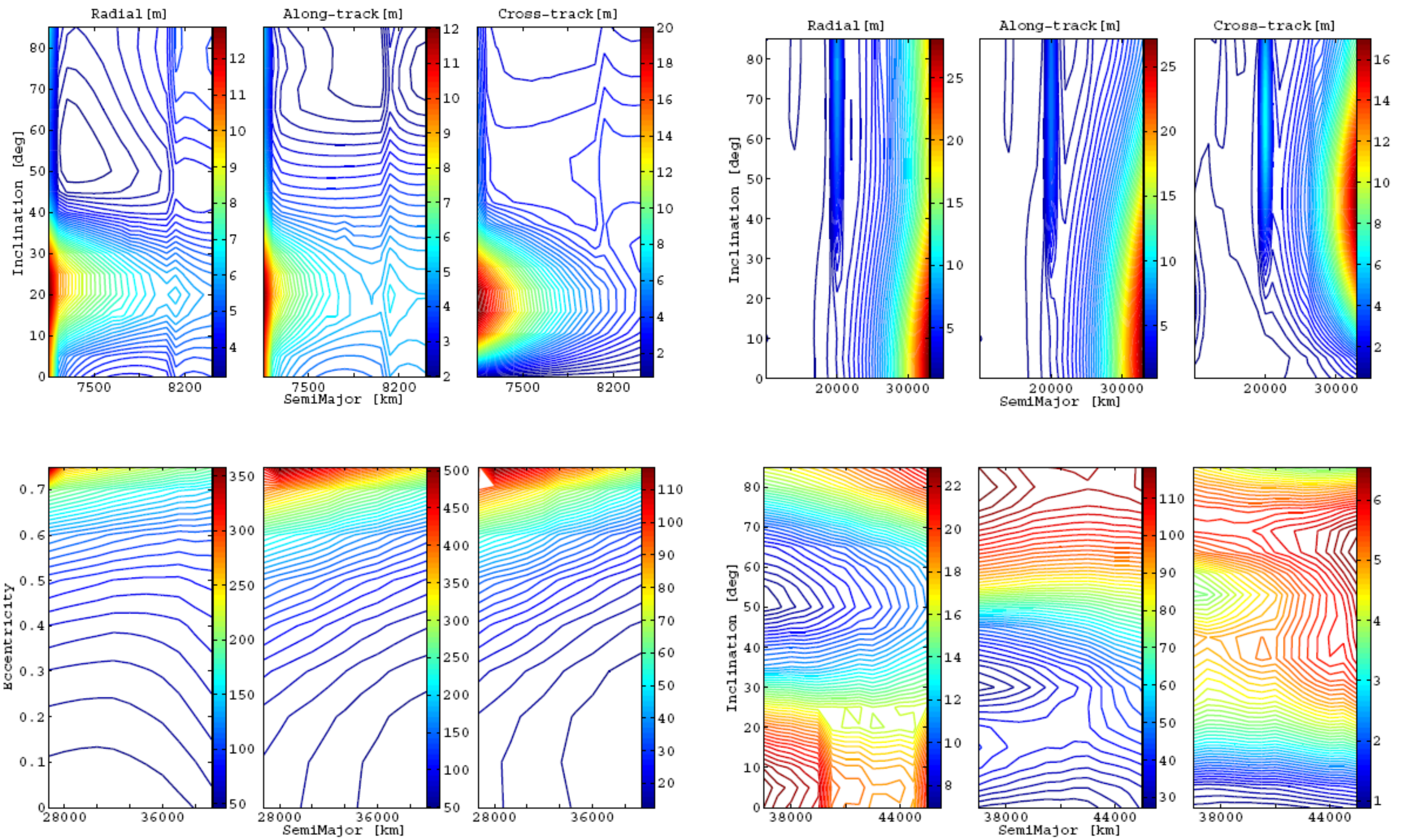
Charles Stark ‘Doc’ Draper (1901 – 1987)

- ‘Father’ of inertial navigation now used in aircraft, ships, land vehicles, space vehicles, and civil applications.
- Established a teaching lab in the late 1930s for instrument engineering as part of the MIT Aero Department
- Evolved to be the MIT Instrumentation Laboratory, which later became the Charles Stark Draper Laboratory
- Design of the Apollo Guidance Computer which controlled the navigation and guidance of the Apollo Lunar Excursion Module which landed on the Moon's surface.
- Draper Laboratory celebrated its 80th Anniversary in 2013

DSST Analysis Workstation

- **Intel Core i7 Multi-core Processor with NVIDIA GPU**
- **Ubuntu Server Distribution 12.10 (64-bit)**
- **Linux Intel Fortran 77, Fortran 90, GNU Fortran compilers**
- **Linux R&D GTDS program including DSST**
- **Linux DSST Standalone**
- **Linux TRAMP (in progress)**
- **Version control**
- **Orekit java Flight Dynamics Library**
- **Java, Eclipse IDE**
- **Matlab**
- **Symbolic Algebra tool - maxima**
- ***TeamViewer – online meetings, remote support***

F77 DSST Standalone Orbit Determination Program fit to ODEM SP Orbit



Top Row: LEO and MEO; Bottom Row: HEO and GEO

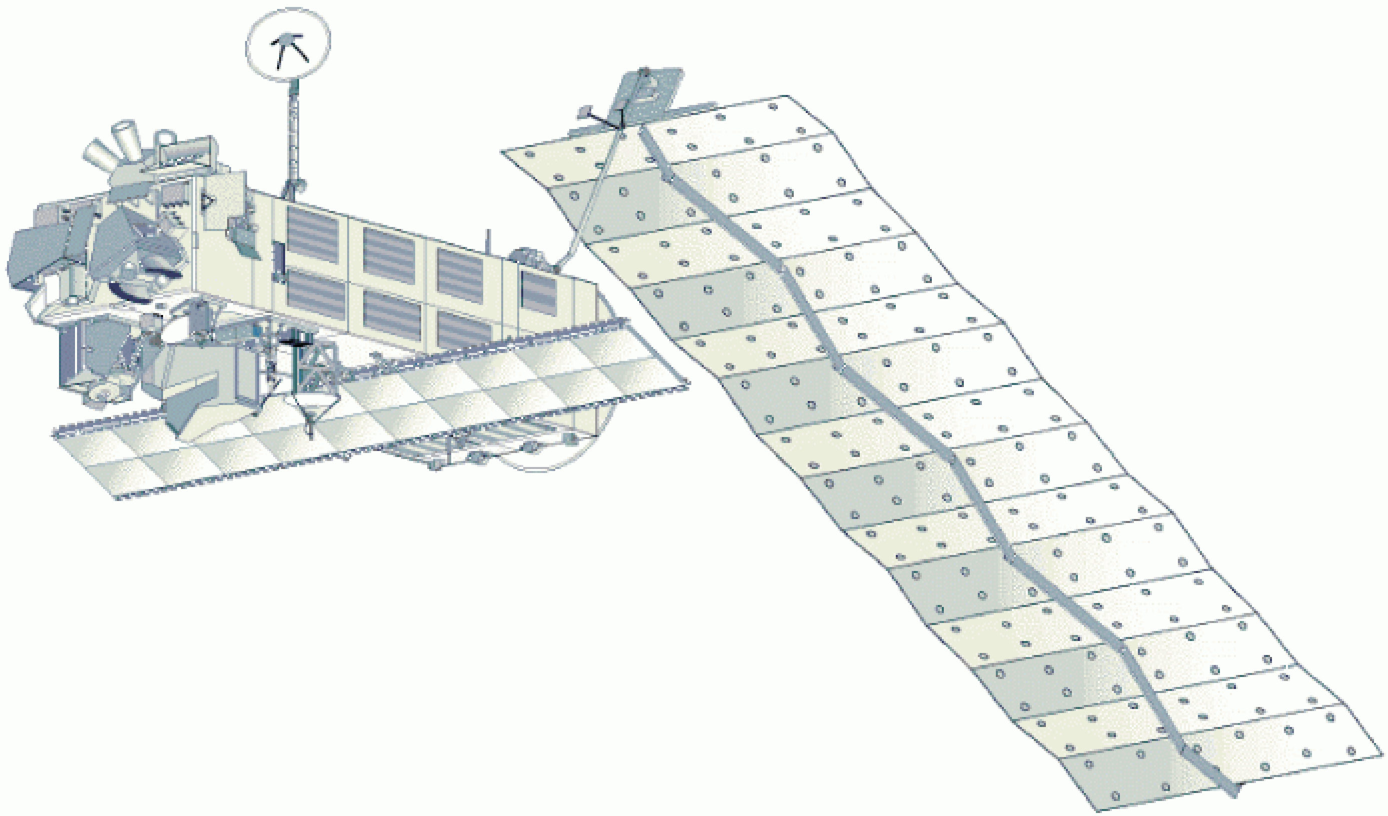
Modern Geopotential Models

Model	Model year	Country (organization)	N	Source data
EGM-96	1996	USA	360	S, G, A
GAO-98	1998	Russia (TsNIIGAiK)	360	S, G, A
GPM-98	1998	Germany	1800	S, G, A
PZ-2002/360	2002	Russia (29 NII MD)	360	S, G, A
EIGEN-CG03C	2005	Germany	360	S (CHAMP, GRACE), G, A
EIGEN-GLO4C	2006	Germany, France	360	C (GRACE, LAGEOS), G, A
GAO-2008	2008	Russia (TsNIIGAiK, 29 NII MD)	360	S (CHAMP, GRACE), G, A
EGM-2008	2008	USA	2160	S (GRACE), G, A
EIGEN-5C	2008	Germany, France	360	S(GRACE, LAGEOS), G, A
GGM-03C	2009	USA	360	S (GRACE), G, A
GOCE-DIR	2010	Germany, France	240	S (GOCE)
GOCE-TIM	2011	Germany, Austria	250	S (GOCE)
GOCO02S	2011	Austria, Germany, and Switzerland	250	S (GOCE, GRACE, CHAMP and oth.)
EIGEN-6C	2011	Germany, France	1420	S (GOCE, GRACE, LAGEOS), G, A
GIF48	2011	USA	360	S (GRACE), G,A
GAO-2012	2012	Russia (TsNIIGAiK)	360	S (GOCE), G, A

Modern Atmosphere Density Models

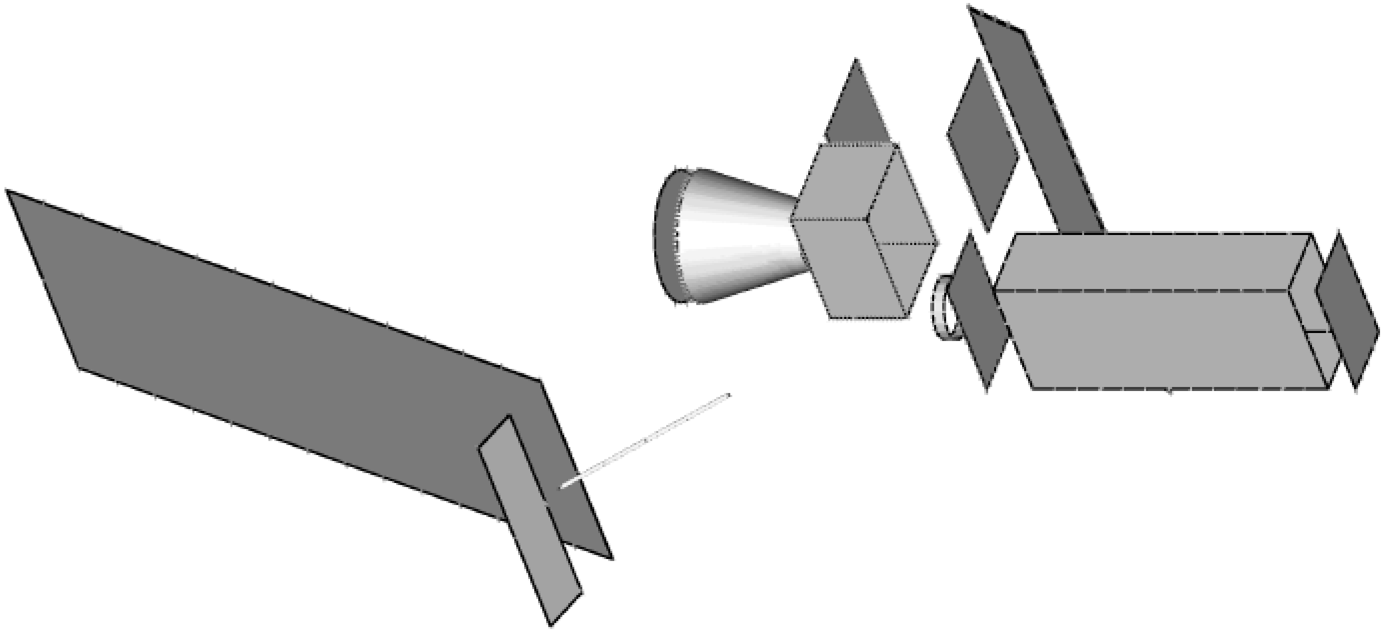
- Jacchia-Bowman 2008
  - New physical parameter inputs
  - Requires new physical model binary file(s)
- Other Jacchia models
  - Supports calibration of GTDS versus other Special Perturbation programs
- DTM-03
  - Developed in France
- GOST-2004
  - Developed in Russia
  - Evolved from manned space flight program models
  - Polynomial models – may be very efficient
  - Requires new physical model binary file(s)

Spacecraft Modeling



View of the ENVISAT satellite.

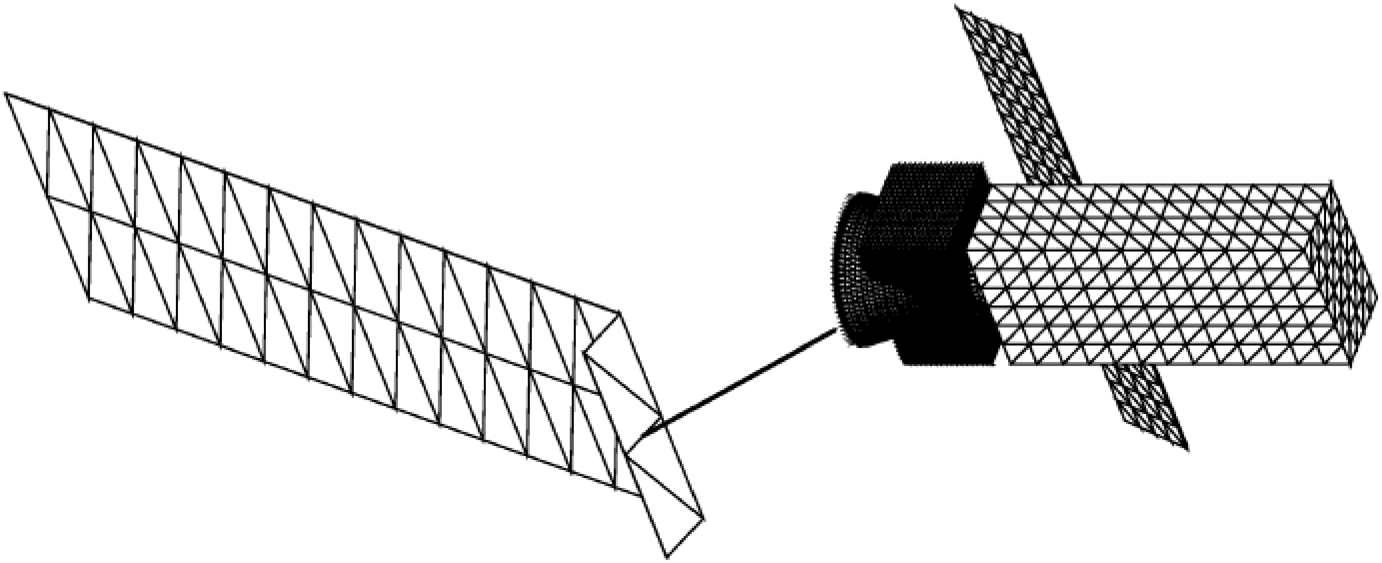
Spacecraft Modeling



Primitives of the ENVISAT model.

Ref. Bent Fritsche, ANGARA program

Spacecraft Modeling



Panel model of the ENVISAT.

Test Particle Monte Carlo Method

General idea