Recursive Estimation of Non-gravitational Perturbations from Satellite Observations

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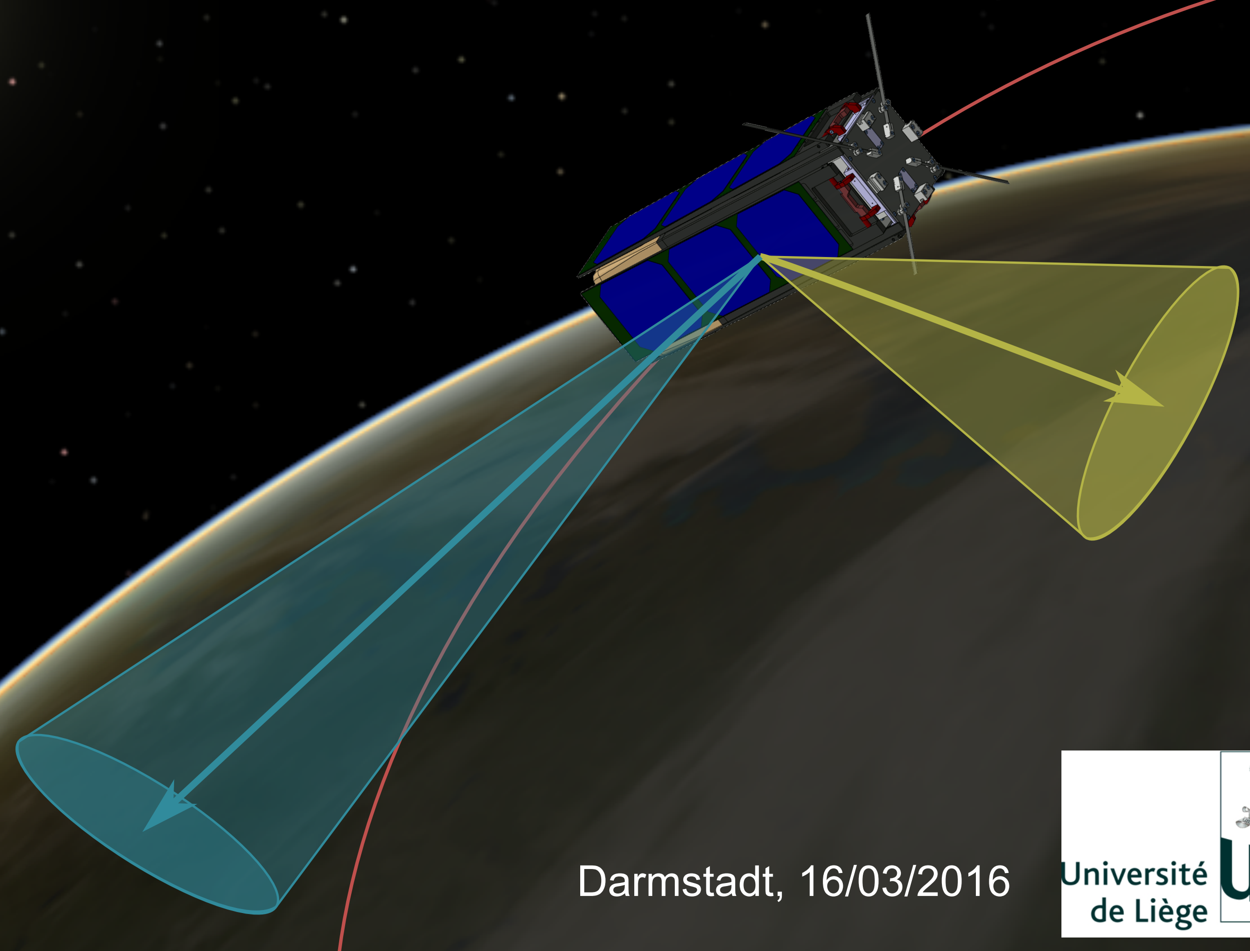

Darmstadt, 16/03/2016

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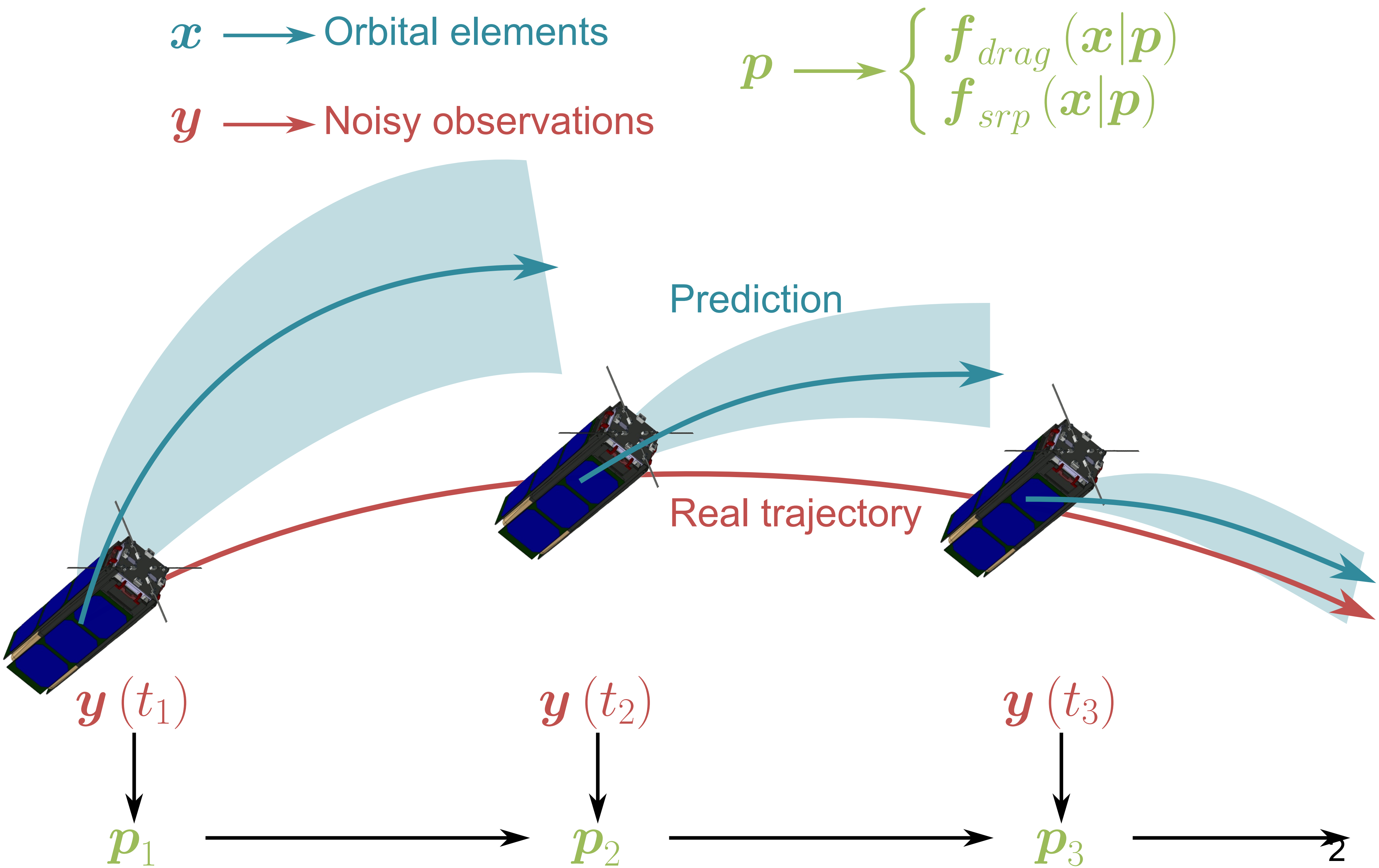
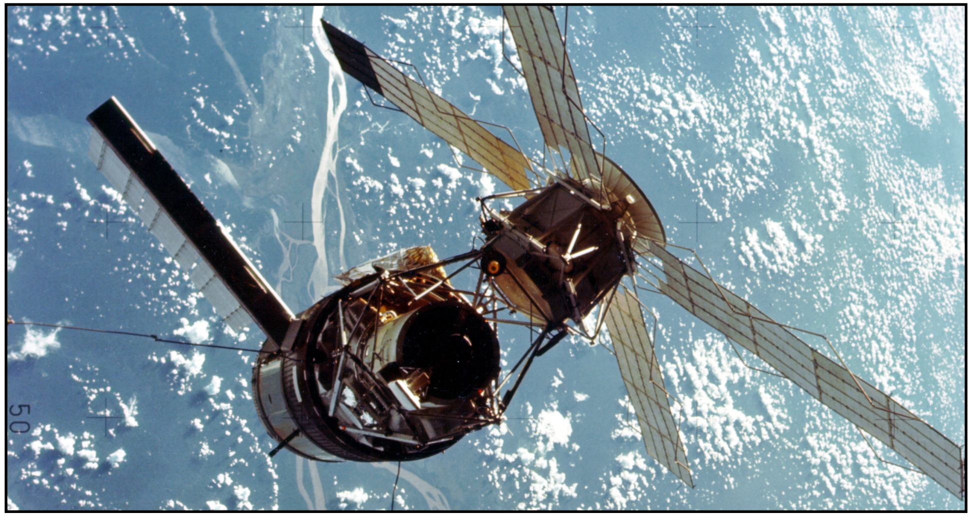
Inferring non-gravitational forces from observations

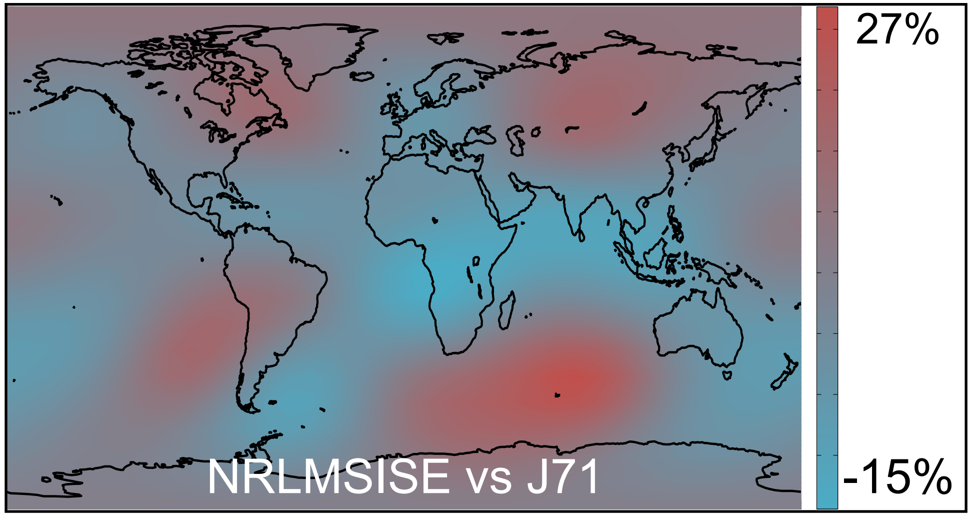


Challenging modeling of non-gravitational forces

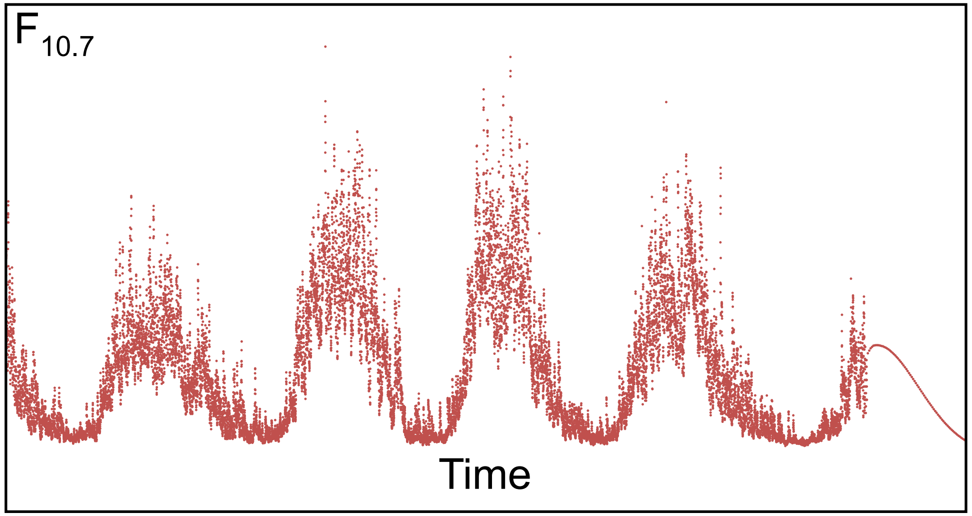
Spacecraft dependent



Biased atmospheric models

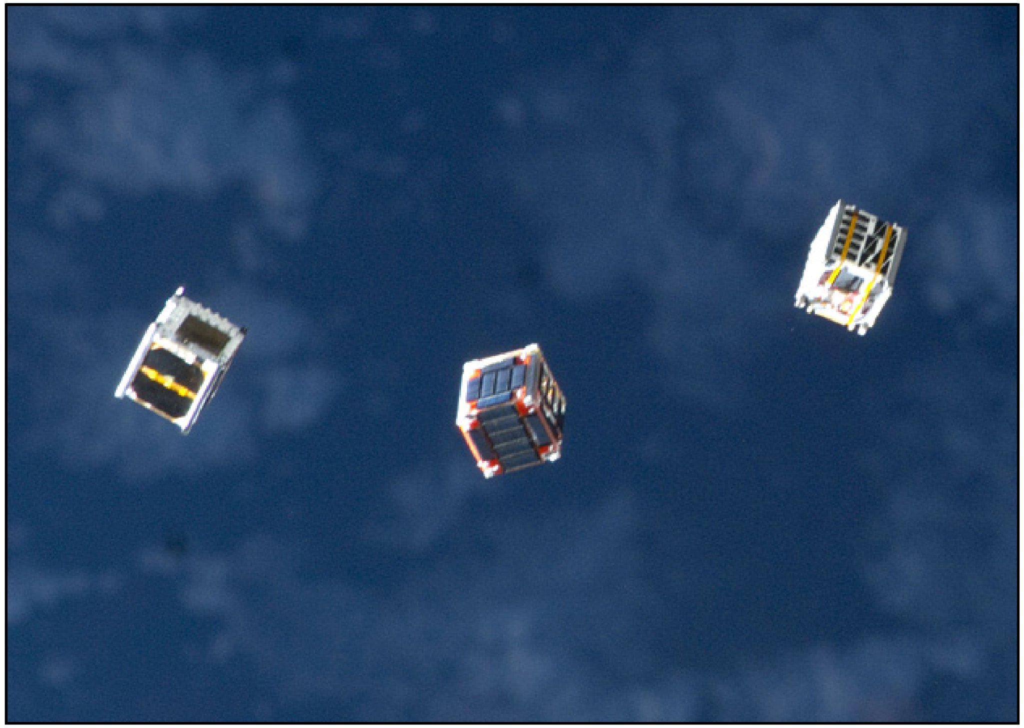


Stochastic space weather

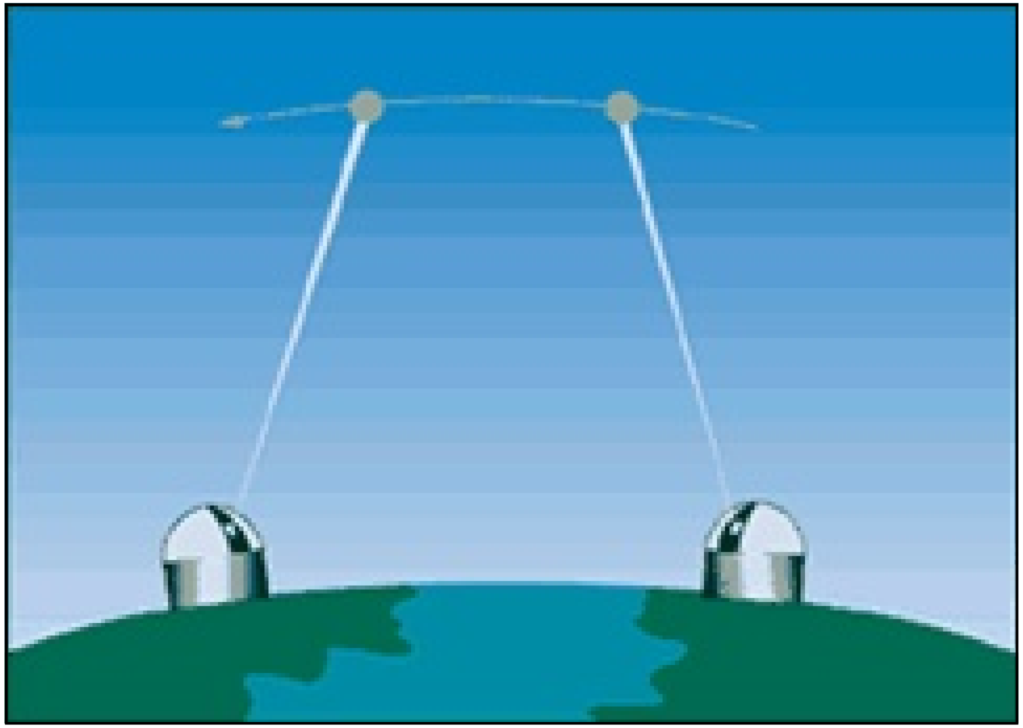


Why not use high-sensitivity accelerometers?

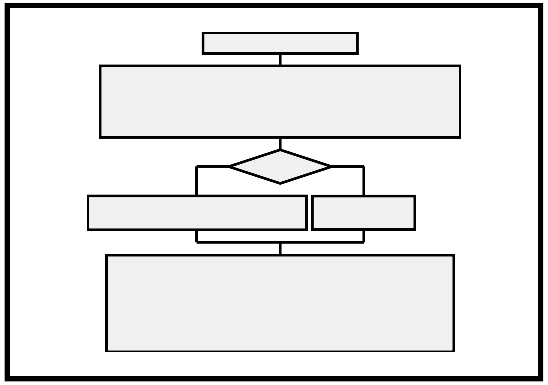
Small satellites



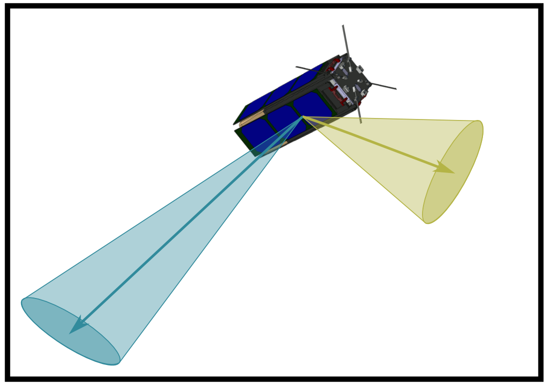
Ground-based estimation



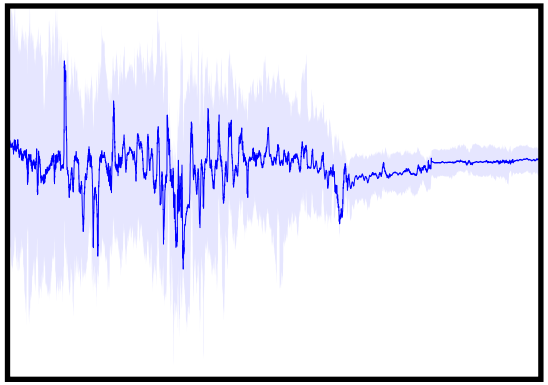
Outline



1. Sequential Monte Carlo for parameter estimation



2. Estimation of non-gravitational perturbations



3. Numerical simulations

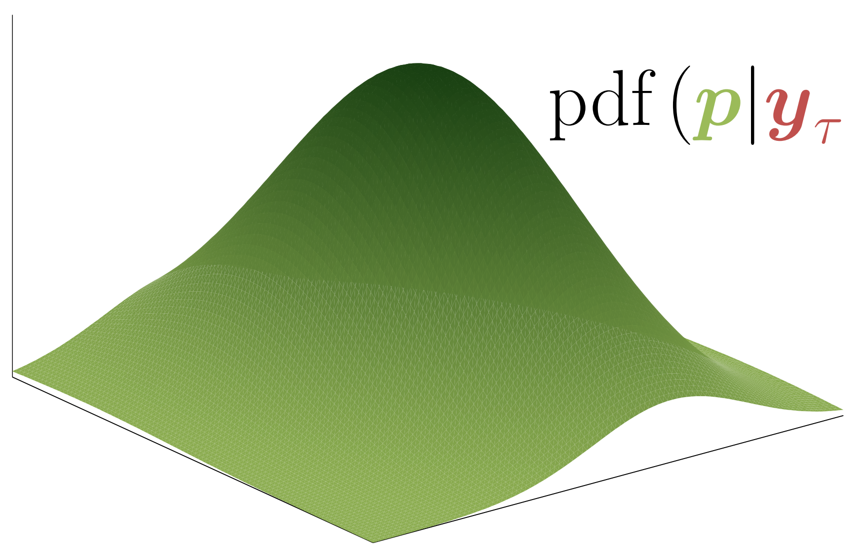
1. Recursive estimation of hidden Markov models

Hidden Markov model

$$\boldsymbol{X}_{\tau+1} \sim f\left(\boldsymbol{x}_{\tau+1} \mid \boldsymbol{x}_{\tau}, \ldots, \boldsymbol{x}_{\tau-m}, \boldsymbol{p}\right)$$

$$\boldsymbol{Y}_{\tau} \sim g\left(\boldsymbol{y}_{\tau} \mid \boldsymbol{x}_{\tau}, \boldsymbol{p}\right)$$

Posterior distribution



$$\mathrm{pdf}\left(\boldsymbol{p} | \boldsymbol{y}_{\tau}, \boldsymbol{y}_{\tau-1}, \ldots, \boldsymbol{y}_{0}\right)$$

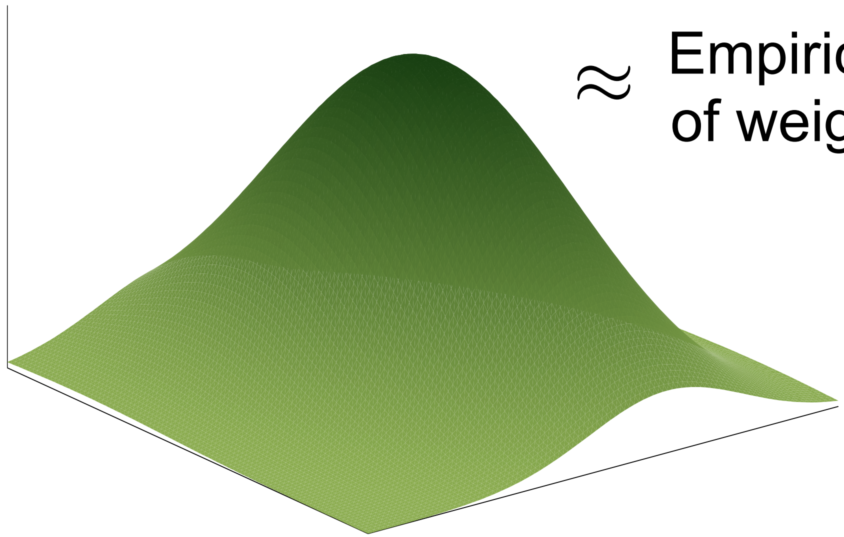
1. Sequential Monte Carlo to estimate the posterior

Hidden Markov model

$$\boldsymbol{X}_{\tau+1} \sim f\left(\boldsymbol{x}_{\tau+1} \mid \boldsymbol{x}_{\tau}, \ldots, \boldsymbol{x}_{\tau-m}, \boldsymbol{p}\right)$$
$$\boldsymbol{Y}_{\tau} \sim g\left(\boldsymbol{y}_{\tau} \mid \boldsymbol{x}_{\tau}, \boldsymbol{p}\right)$$

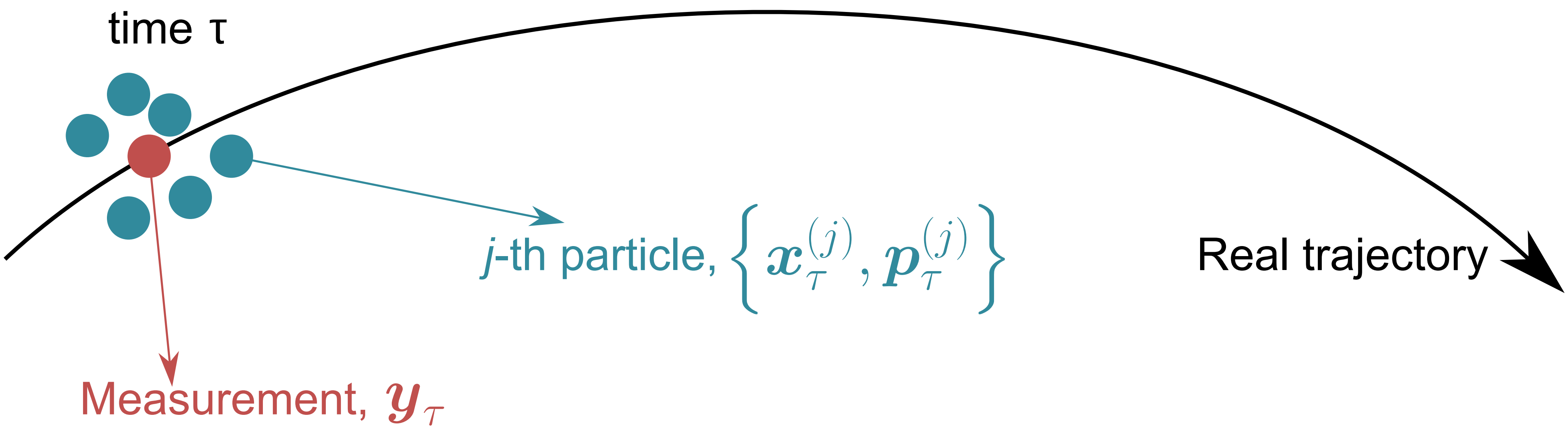
Particle filter

Posterior distribution

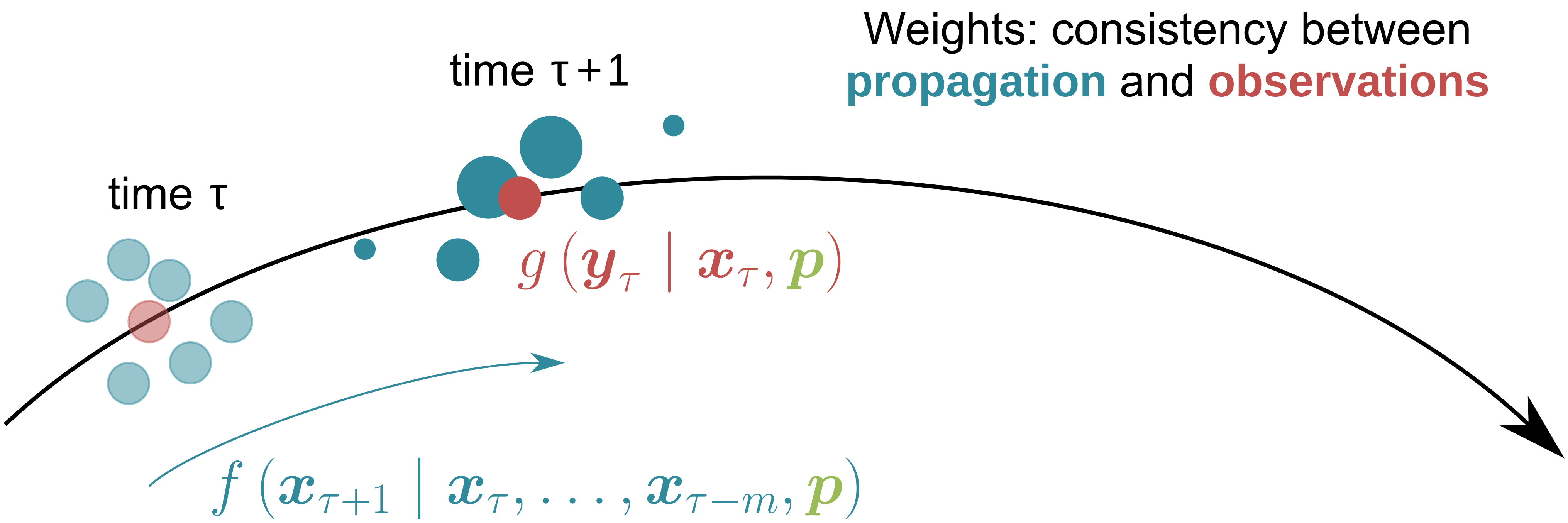


$\approx$ Empirical distribution of weighted samples

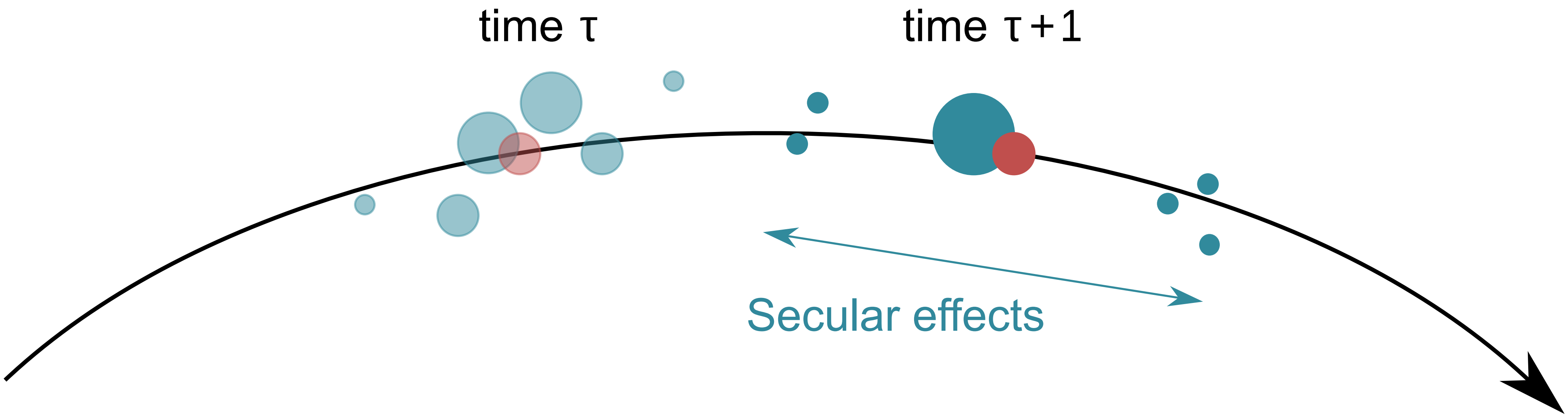
1. Each particle has a set of parameters and states



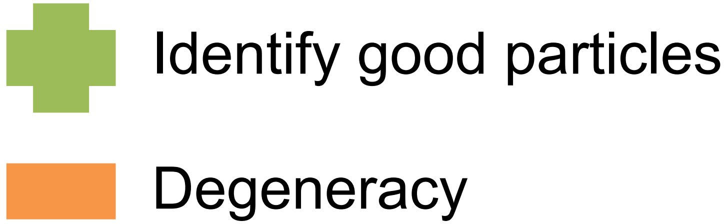
1. Integrate, don't differentiate



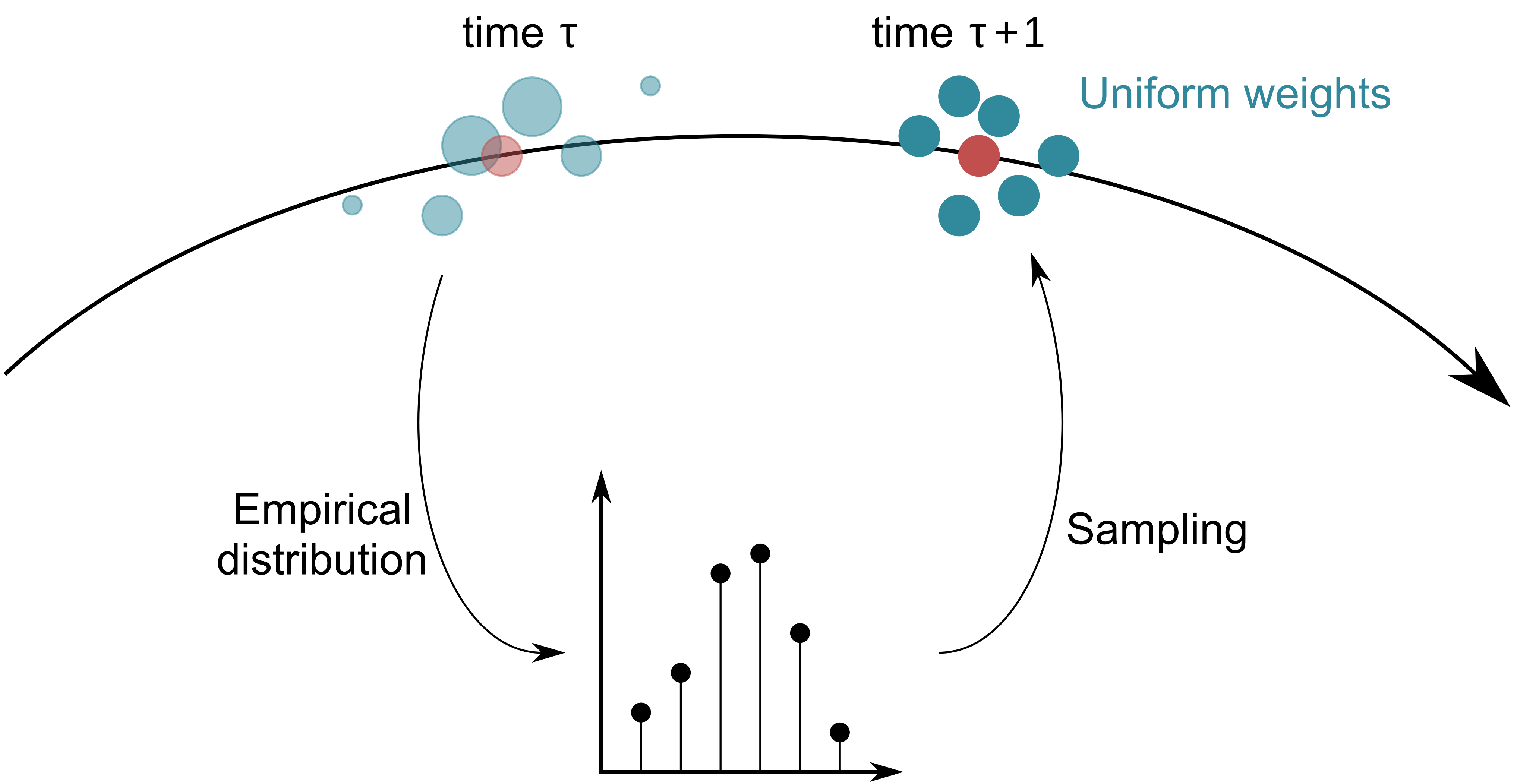
1. Good particles are identified via recursive updates



Multiple updates:



1. Resampling prevents degeneracy of the weights



2. How does it apply to satellite force estimation?

$\boldsymbol{x}$ → Averaged Orbital elements

$\boldsymbol{y}$ → Noisy observations

$\boldsymbol{p}$ → Drag/reflectivity coefficient, density, ...

Measurement noise (GPS, contact transformation)

$$g\left(\boldsymbol{y}_\tau \mid \boldsymbol{x}_\tau, \boldsymbol{p}\right)$$

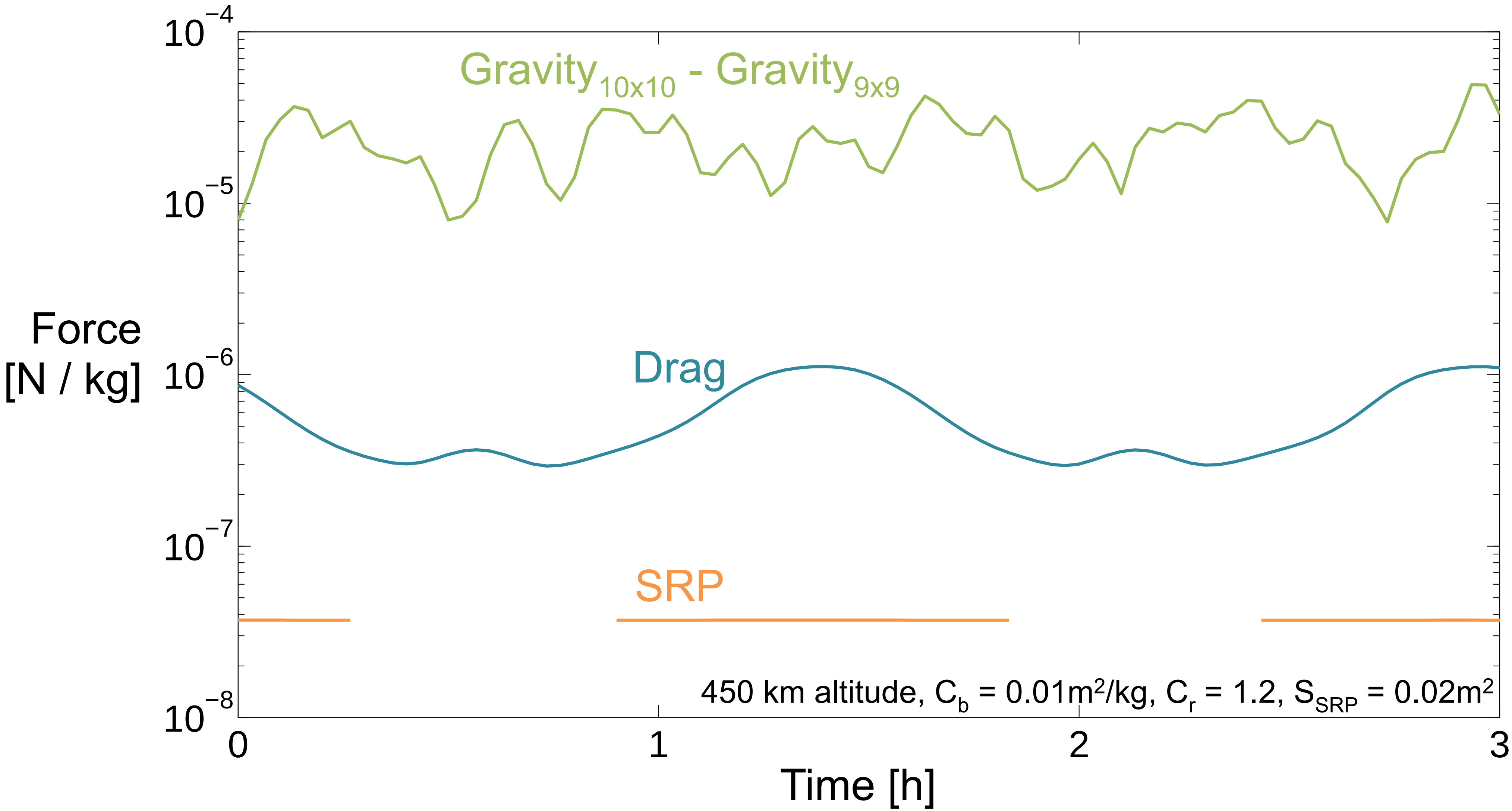
Parametric force model

$$\begin{cases} \boldsymbol{f}_{drag}\left(\boldsymbol{x}|\boldsymbol{p}\right) \\ \boldsymbol{f}_{srp}\left(\boldsymbol{x}|\boldsymbol{p}\right) \end{cases}$$

Orbital propagator & process noise

$$f\left(\boldsymbol{x}_{\tau+1} \mid \boldsymbol{x}_\tau, \ldots, \boldsymbol{x}_{\tau-m}, \boldsymbol{p}\right)$$

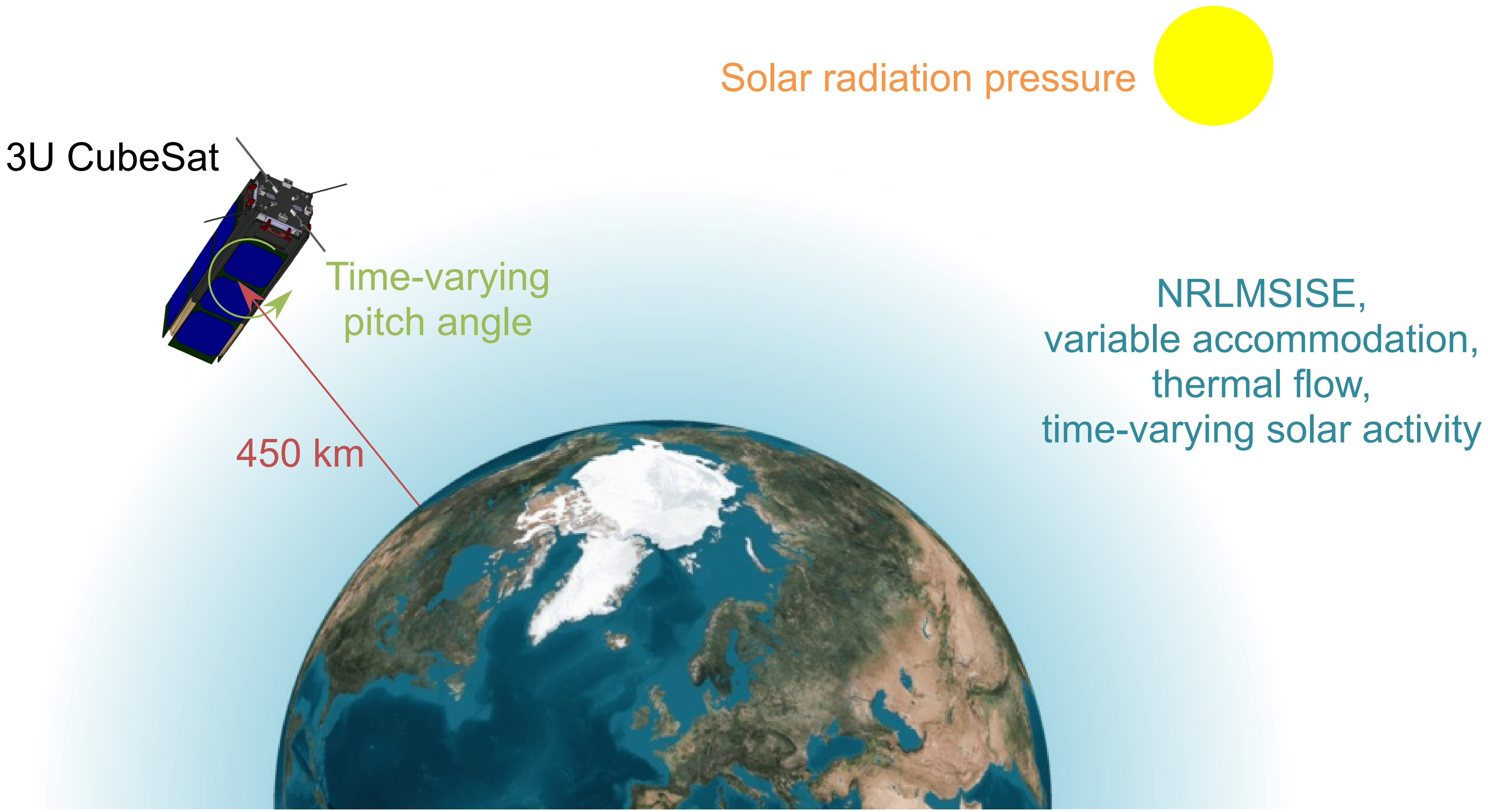
2. Why averaged elements?



- Robustness to unmodeled dynamics
- Analytical & semi-analytical techniques

- Measurement noise

3. Simulations in a high-fidelity environment

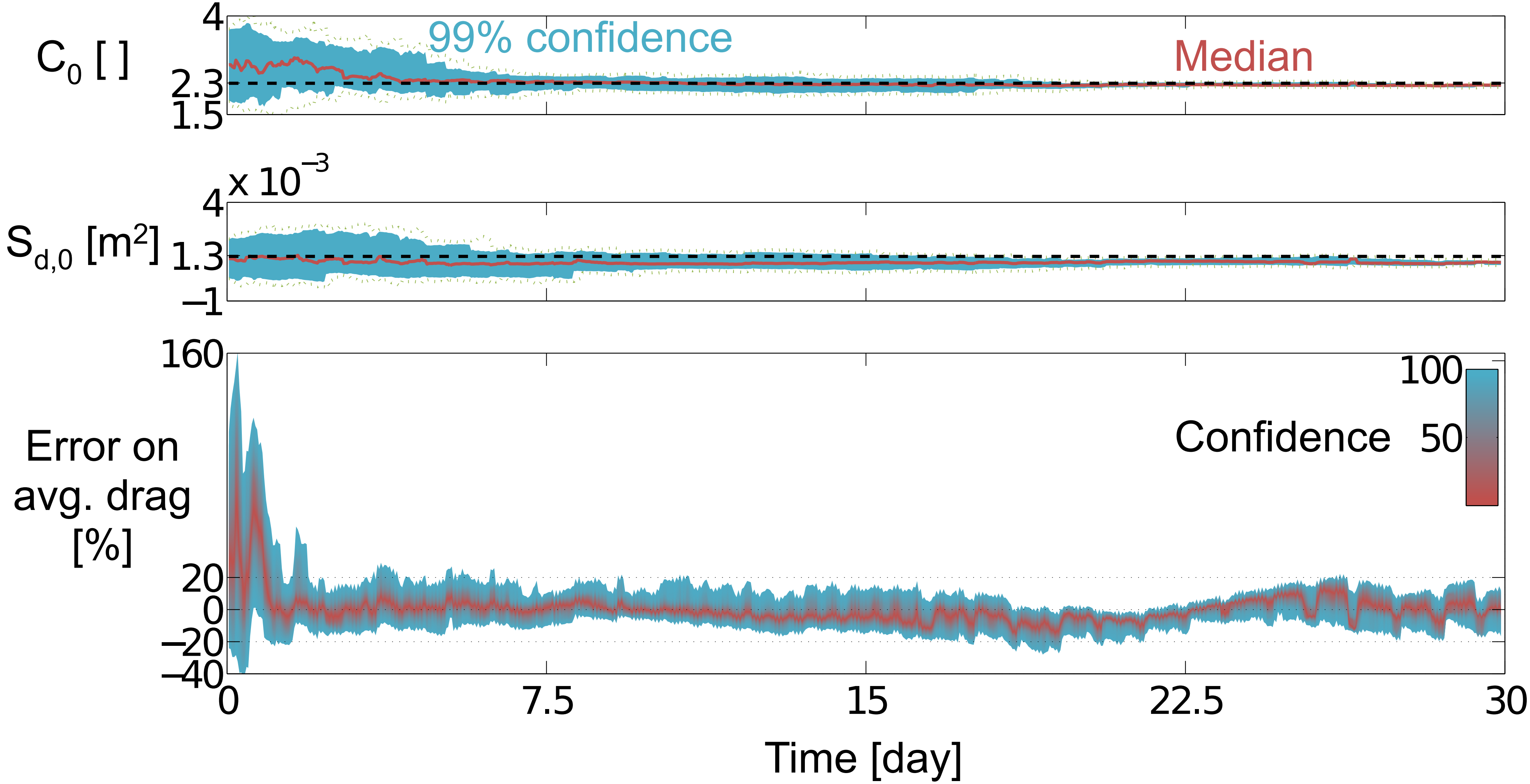


10x10 gravitational harmonics
Third-body (Sun and Moon)

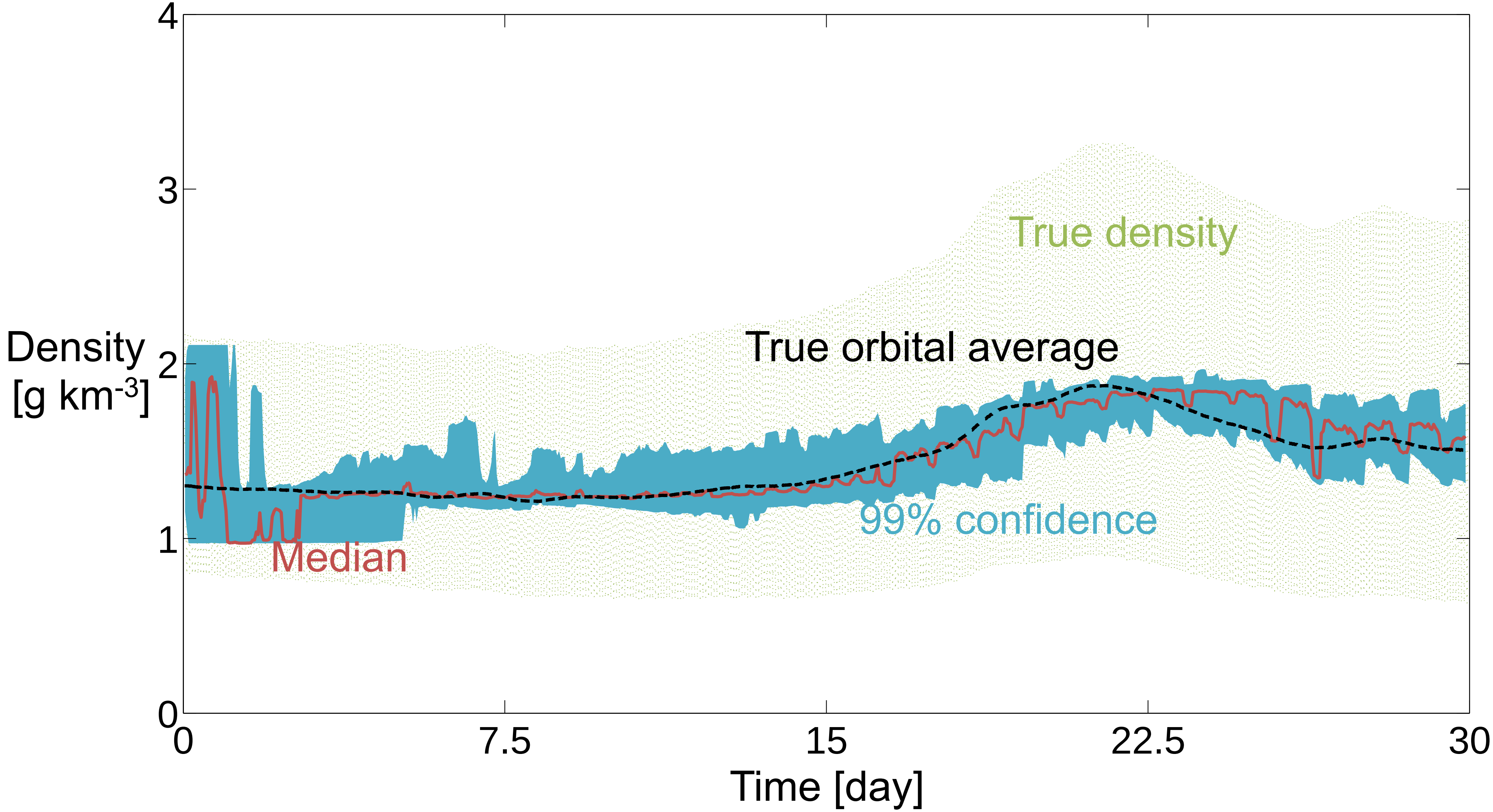
3. Comparing two implementations of the filter

	Analytical	Numerical
Modeled perturbations	J_2 + averaged drag	J_{4x4}, drag, SRP
Force model		
Ballistic coefficient	$\frac{(S_d(t) + \boldsymbol{S_{d,0}})}{m}\boldsymbol{C_d}$	$\frac{(S_d(t) + \boldsymbol{S_{d,0}})}{m}\boldsymbol{C_d}$
Density	$\bar{\rho}_0$	$\bar{\rho}_0 \sum_i (\boldsymbol{c_i} \cos(i\ L) + \boldsymbol{s_i} \sin(i\ L))$
Reflectivity coefficient	n/a	$\boldsymbol{C_r}$
Time step	1 hour	3 min
Filter order, *m*	4	1

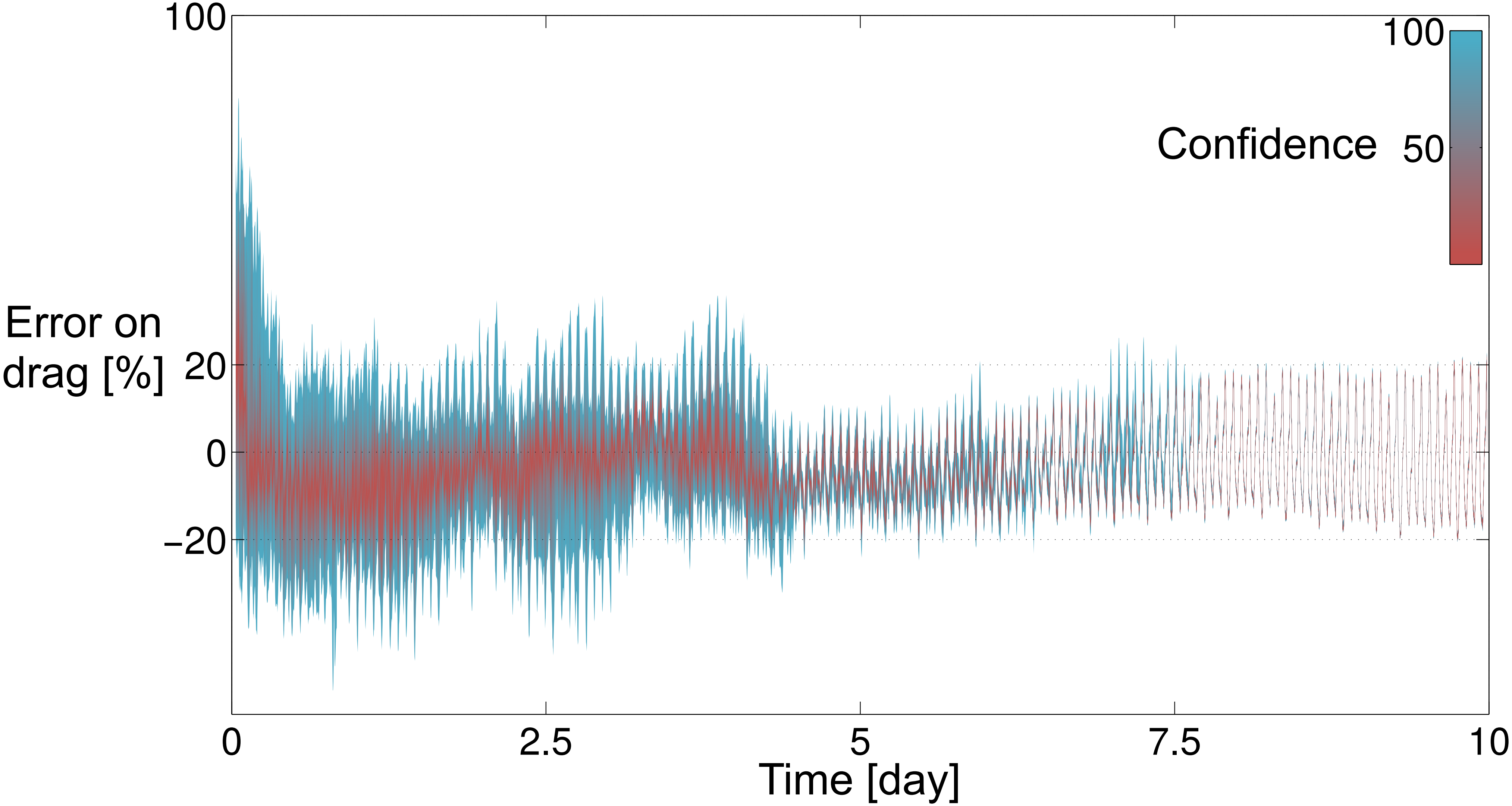
3. Estimation using the analytical propagator



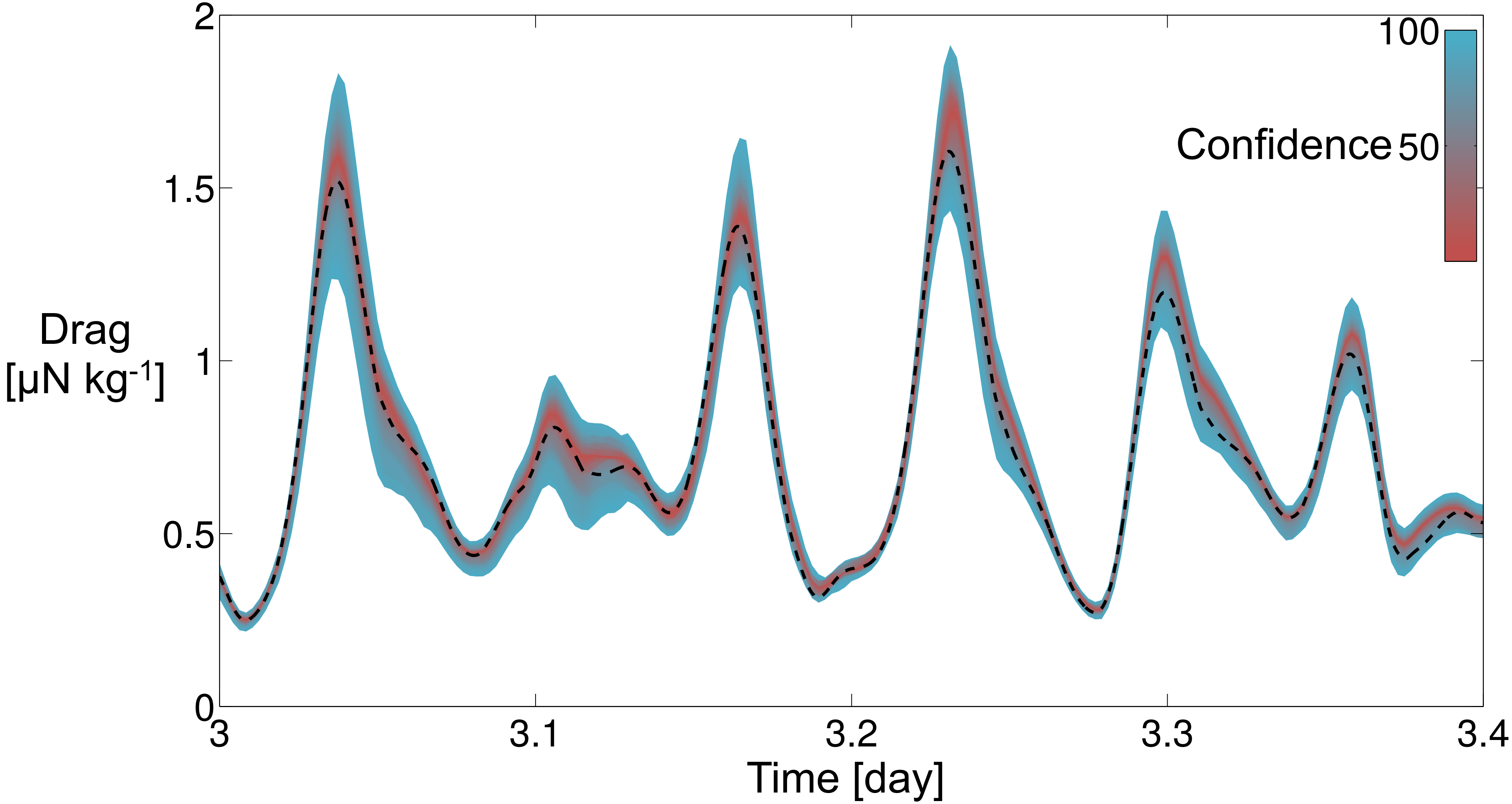
3. Solar activity variations are automatically detected



3. Estimation using the numerical propagator



3. Numerical propagators yield more flexibility



Conclusion

Particle filter enables extreme **flexibility** (nonlinear & non-Gaussian model)

Averaged elements are used to mitigate mis-modeled dynamics

Computational burden can be reduced by using **analytical techniques**

Good estimation from **sparse & fairly inaccurate measurements**

Adequate **tuning of filter's parameter** is mandatory
(recommendations are available in the paper)

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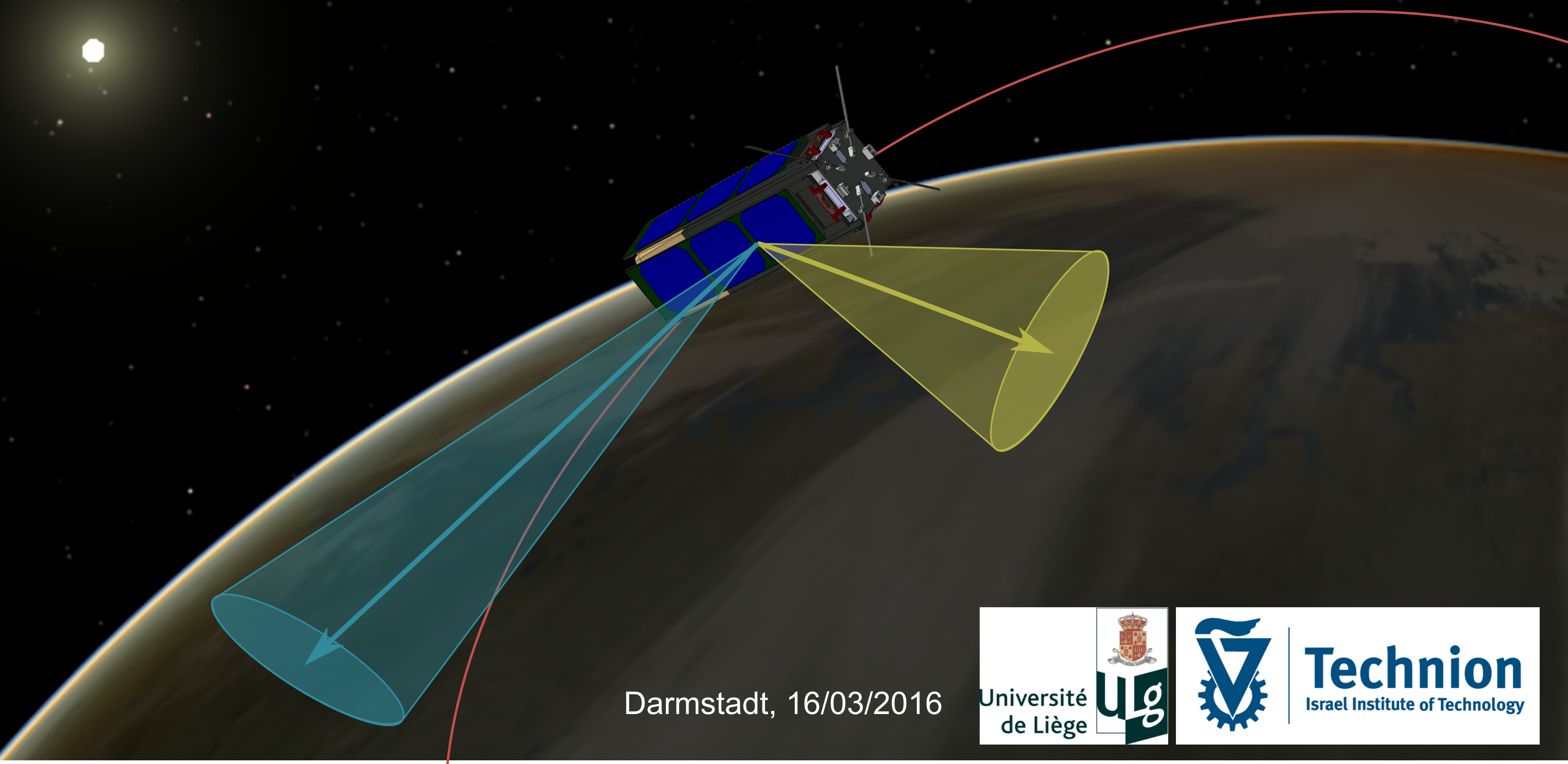

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