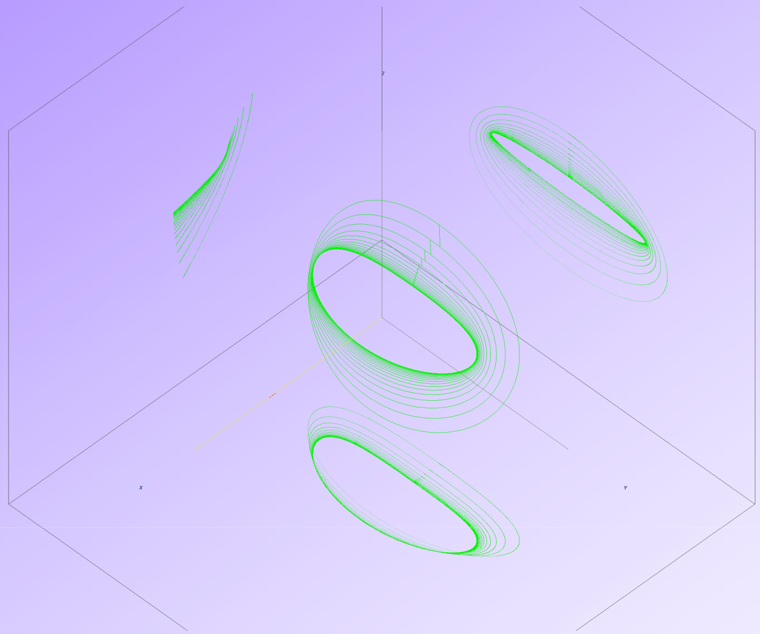
6th ICATT
New Tool for Finding Periodic Halo Orbits: the Solver of a Spacecraft Simulator
(ESPSS -Ecosimpro® European Space Propulsion System Simulation)

Darmstadt, 14th to 17th March 2016



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Summary

- Introduction
- The differential system
- Solution for periodic orbits
- Application to Halo orbits
- Conclusions

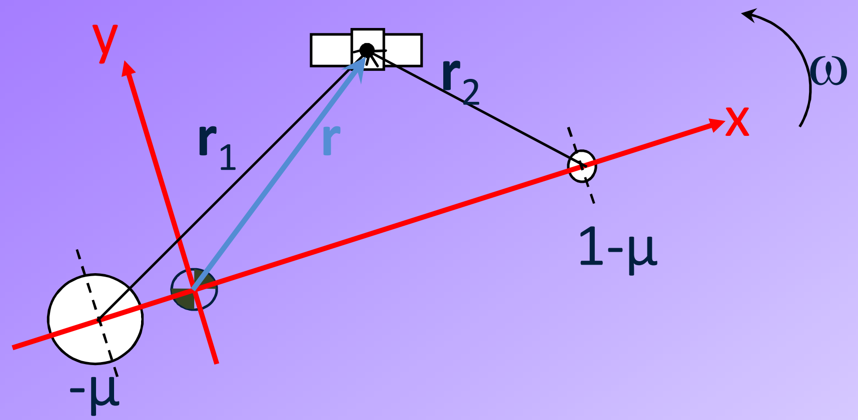
1. New tool periodic Halo, ICATT 6th , 14-17 March 2016, Darmstadt, Germany

Introduction

- The existing ESA developed tool EcosimPro® is a solver of differential algebraic equations
  - It is oriented for system simulations, not for mathematicians, nor for numerical analysts
  - But it can be used as well for solving some problem which are generally solved “only” with some **US tools**
- For engineers, it is sometime needed to assess some orbits
  - But the cost of a sub-contract to do this assessment can be simply out of the scope of normal engineering work
  - In addition time is needed by sub-contractors, and their answers may not be in line with the need (Very low cost)
- Hence, its has been found appropriate to check if a simple EcosimPro model could be used for solving efficiently the question of periodic orbits
  - Because Engineers use preferably EcosimPro, its is not out of the scope of their knowledge
  - But the most difficult part is to get the right equations
  - Unfortunately, this requires some efforts in order to clear the uncertainties in the set of equations
  - In addition high accuracy needed for the numerical resolution can be considered as a showstopper
- The problem will be presneted in simple word
- And the presentation will show the successful Halo orbit solutions

The differential system

- **Acceleration in the rotating frame with** $\mu = M_{earth}/M_{total}$



$$\ddot{\vec{r}} + \vec{\omega}\times\left(\vec{\omega}\times\vec{r}\right) + 2\vec{\omega}\times\dot{\vec{r}} = -\frac{(1-\mu)}{r_1^3}\vec{r}_1 - \frac{\mu}{r_2^3}\vec{r}_2$$

- **➔ lead to write the system straightforward when the non-dimensional rotation rate** $\omega = 1$
- **System surprisingly so simple**

$$\dot{X} = f(X)$$

$$X = \begin{bmatrix} x \\ y \\ z \\ \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} \quad f(X) = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ x + 2\dot{y} - \frac{(1-\mu)(x+\mu)}{r_1^3} - \mu\frac{(x-(1-\mu))}{r_2^3} \\ y - 2\dot{x} - \frac{(1-\mu)y}{r_1^3} - \mu\frac{y}{r_2^3} \\ -\frac{(1-\mu)z}{r_1^3} - \mu\frac{z}{r_2^3} \end{bmatrix}$$

$$r_1 = \begin{bmatrix} x-(-\mu) \\ y \\ z \end{bmatrix} \quad \left\|r_1\right\| = \sqrt{(x-(-\mu))^2 + y^2 + z^2}$$

$$r_2 = \begin{bmatrix} x-(1-\mu) \\ y \\ z \end{bmatrix} \quad \left\|r_2\right\| = \sqrt{(x-(1-\mu))^2 + y^2 + z^2}$$

Solution for Periodic Orbits

$$X(0)=\begin{bmatrix}x_0\\0\\z_0\\0\\\dot{y}_0\\0\end{bmatrix}\quad X(T\tfrac{1}{2})=\begin{bmatrix}x\\0\\z\\0\\\dot{y}\\0\end{bmatrix}$$

Principles

- **Without deep analysis, simple periodic orbits cannot be unsymmetrical**
  - **In the plane XZ for example ➔After half orbit some state variable are the same**
- **Problem reformulated (x_0 fixed)**
  - **Search problem of the 3 zeros with** $g(Y)=0$
  - **Iteration by Newton Method**

$$Y=\begin{bmatrix}z_0\\\dot{y}_0\\T_{1/2}\end{bmatrix}\quad g(Y)=\begin{bmatrix}y(t)\\\dot{x}(t)\\\dot{z}(t)\end{bmatrix}_{t=T_{1/2}}$$

$$Y_{n+1}=Y_n-\left[\frac{\partial g}{\partial Y_{|Y=Y_n}}\right]^{-1}.g(Y_n)$$

  - **But that *Jacobian* becomes the real problem to solve…**

Solution for Periodic Orbits

Principles

- Search problem of the 3 zeros with $g(Y) = 0$ $\quad Y = \begin{bmatrix} z_0 \\ \dot{y}_0 \\ T_{1/2} \end{bmatrix} \quad g(Y) = \begin{bmatrix} y(t) \\ \dot{x}(t) \\ \dot{z}(t) \end{bmatrix}_{t=T_{1/2}}$
  - But that *Jacobian* becomes the real problem to solve…

$$Y_{n+1} = Y_n - \left[\frac{\partial g}{\partial Y}_{|Y=Y_n} \right]^{-1} .g(Y_n)$$

  - For $\begin{bmatrix} z_0 \\ \dot{y}_0 \end{bmatrix}$ $\frac{\partial g}{\partial Y}_{|Y=Y_n}$ is given by $\dot{M}(t,t_0) = \frac{d}{dt}\left[\frac{\partial X_{|t=t}}{\partial X_{|t=t_0}} \right]$; $M(t_0,t_0) = [Id]$ from $\dot{X} = f(X)$
  - For $\begin{bmatrix} T_{1/2} \end{bmatrix}$ is given by $\frac{dg}{dt}_{|Y=Y_n} = \dot{g}_{|Y=Y_n}$ i.e. the function f in $\dot{X} = f(X)$
  - Finally, a system of 42 variables to integrate into an iterative loop for finding one periodic orbit

Solution Periodic Orbits: EcosimPro practical approach

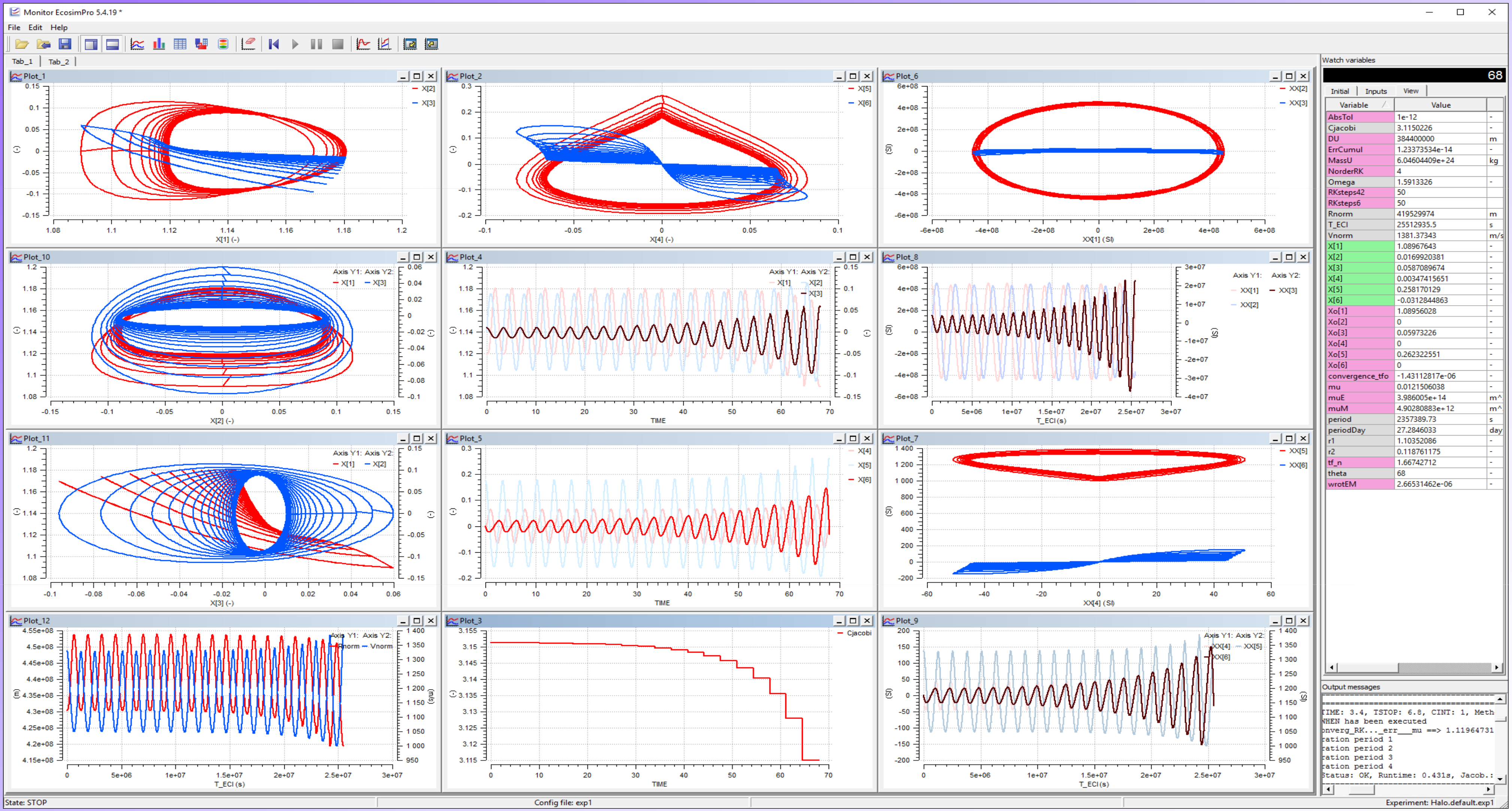
- Numbering the variables of the problem $\dot{X} = f(X)$ with index 1 to 6 and numbering the variable t to index 7
- $\frac{\partial g}{\partial Y}\Big|_{Y=Y_n}$ = $\left[Col.3\,5\ of\ rows\ 2\,4\,6\ of\ M(t,t_0)\right]\ \left[rows\ 2\,4\,6\ of\ f(X)\right]$
  - It was found that it was better to iterate on x_0 instead of z_0
  - ➔ just replace index 3 by 1 in above

- Loop on z_0 given values of each Halo
  - Loop on the 3 init conditions: solve the problem of 3 zeros
    - Integrate the 42 equation system
  - Iterate until zeros are found
- Further plots on the monitor the current Halo

$$\begin{pmatrix} x_0 = 1.12 \\ 0 \\ z_0 = 0.01\ldots \\ 0 \\ \dot{y}_0 = 0.17 \\ 0 \\ T_{1/2} = 1.7 \end{pmatrix}$$

Application to Halo orbits

Powerful plots under Monitor of EcosimPro



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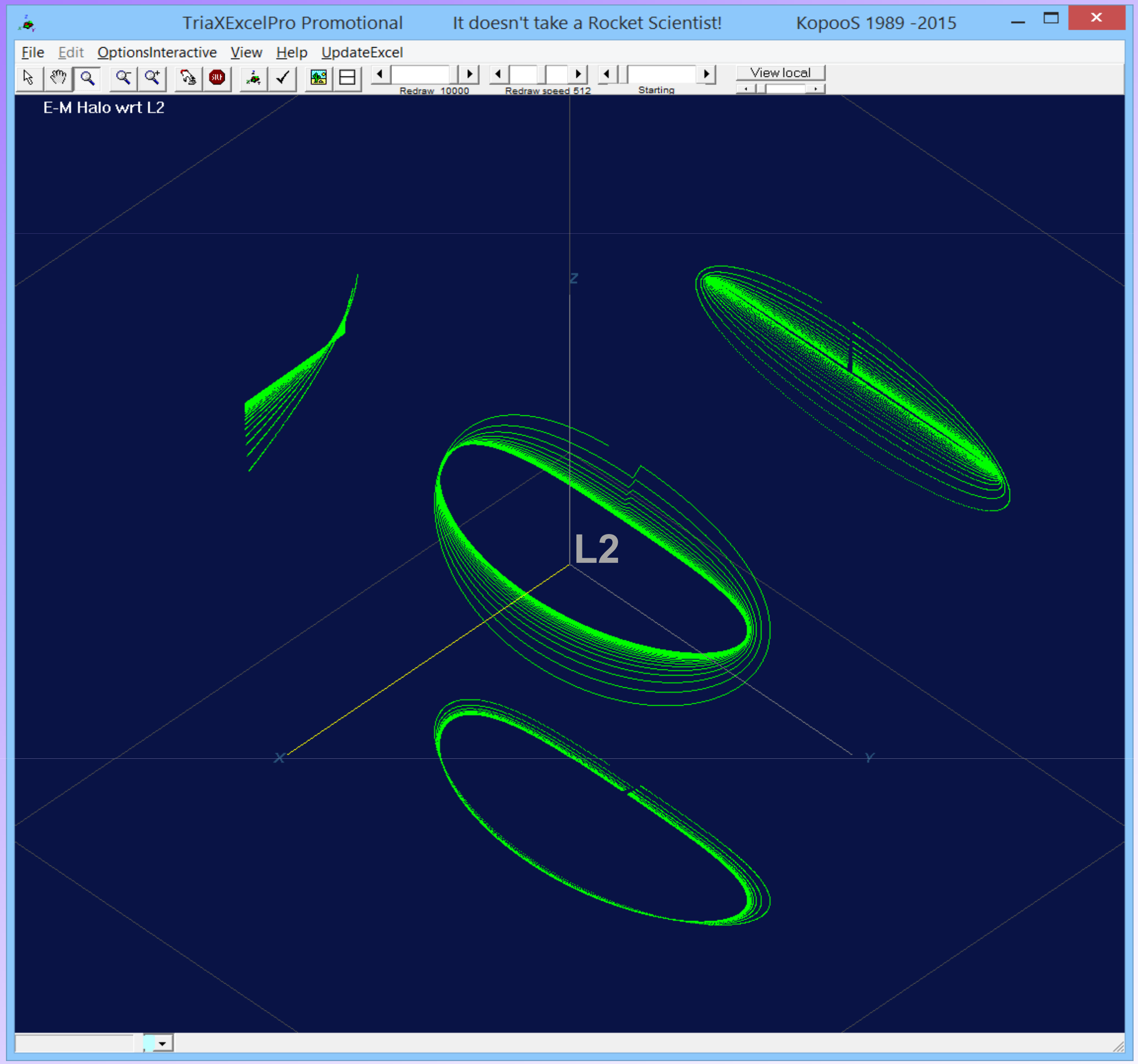
Application to Halo orbits

And even better in 3D



Conclusions

- The paper has presented in simple words the mathematical problem of finding some Halo orbits
  - and the method implemented to solve it within the EcosimPro environment.
- The major advantages of the EcosimPro approach used successfully is to benefit of a real simulation framework based on models and on experiments
  - no mixing between the inputs\outputs needs and the real problem being to be solved.
- Hence the full model can be clearly and explicitly described
  - while the results coming from the experiments can be extensively assessed and analysed with simple EcosimPro monitor outputs.

Thanks for your attention

Questions?

Ackowledgments

- The research leading to these results is a KopooS funding

A taste of the EcosimPro model

•LISTING OF THE EXPERIMENT

```
/*---------------------------------------------------------------------------------
LIBRARY: MY_SAT
COMPONENT: Halo
PARTITION: default
EXPERIMENT: exp1
TEMPLATE: TRANSIENT
---------------------------------------------------------------------------------*/
EXPERIMENT exp1 ON Halo.default
DECLS
   REAL T_Halo
   STRING Filnam="Rep"
   INTEGER nbHalo=1
OBJECTS
INIT
BOUNDS
   MY_SAT.AbsTolM12 = 1e-012
   MY_SAT.NbSteps2000 = 50
   MY_SAT.NorderRK85 =4 -- 5
BODY
   GuessZ3notX1_o=1
   Xo1= 1.12 -- x_o
   Xo3=0.01 -- zo FIXED NOW
   Xo5= 0.17--ydot_o
   Thalfperiod_o=1.7-- t
   NloopNewtonHalo=15
   T_Halo=2*Thalfperiod_o
   nbHalo=20
   Filnam="Halo20.rpt"
   -- creates an ASCII file with the results in table format
   REPORT_TABLE(Filnam, " *X[*] *XX[*] *PHI* Cj* A* G* Halo* *12 *00 *85 *U *RK Om* R* T *
Teco* V* conv* mu* per* r* wrot*] dE* L* ")
   DEBUG_LEVEL= 1
   IMETHOD= DASSL
   setStopWhenBadOperation(FALSE)
   REL_ERROR = MY_SAT.AbsTolM12
   ABS_ERROR = REL_ERROR
   TOLERANCE =REL_ERROR    REPORT_MODE=IS_STEP
   TIME = 0
   FOR (i IN 1, nbHalo)
      FlagSearchPeriodicOrbit=TRUE
      INTEG_TO(TIME+T_Halo,1)
      -- Case of series of Halo orbits (evolution of z)
      IF i!=nbHalo THEN --change but not for the last one to keep all results of the last case
         Xo[3]=Xo[3]+i*Xo[3]*0.01
      END IF
   END FOR
END EXPERIMENT
```

•LISTING OF THE MODEL

```
COMPONENT Halo
DATA
   REAL Xo1=0.99197555537727 UNITS "DU" "xo"
   REAL Xo3=-0.00191718187218 UNITS "DU" "zo"
   REAL Xo5=-0.01102950210737 UNITS "DU/TU" "vyo"
   REAL Thalfperiod_o=1.52776735363559 UNITS "TU" "half period for periodic orbit, initial guess"
   INTEGER NloopNewtonHalo=0 UNITS "-" " 0 --no convergence-- else up to 14 is enough for convergennce"
   INTEGER GuessZ3notX1_o=3 UNITS "-" " flag=3 for xo fixed and zo guess ==>find a Lyapunov plan; flag=1 for zo fixed and xo guess ==>find Halo from a Lyapunov plan with some small zo "
   --REAL RunCode=2 UNITS "-" "code=0: J.D. Mireles James 1 Nick Truesdale 2: Earth Moon L2, 10: J.D. Mireles James L2 from Lyapunov, etc..."
DECLS
   BOOLEAN FlagSearchPeriodicOrbit=TRUE --directive for new search of periodic orbits
   CONST INTEGER LDIM=6
   INTEGER NorderRK,NbSteps, RKsteps42,RKsteps6,
GuessZ3notX1,Function_ODE_IVP --info
   INTEGER i462[3]={4,6,2}
   INTEGER i357[3]={3,5,7}
   REAL X[LDIM] UNITS "-" --position then velocity in barycentric rotating frame addim
   REAL theta UNITS "-"
   REAL T_ECI,period UNITS "s"
   REAL periodDay UNITS "day"
   REAL r1,r2,Omega,Cjacobi UNITS "-"
   EXPL REAL wrotEM3D[3] , wrotEMCrossXXrot[3] UNITS "-" --dim
   EXPL REAL XX[6], XXrot[3] UNITS "SI" --dim
   EXPL REAL Rnorm UNITS "m"
   EXPL REAL Vnorm UNITS "m/s"
   DISCR REAL Xf_n[LDIM] UNITS "-" --point then velocity in barycentric rotating frame addim
   DISCR REAL dX6_dt[LDIM] UNITS "-" --velocity then acceleration in barycentric rotating frame addim
   DISCR REAL Xo_n[7+10], Xo[7] UNITS "-" -- 6+added more rowse for compact information data
   DISCR REAL PHI[6,7] UNITS "-"
   DISCR REAL DF[3,3],D[3,3],XSo[3], XSo_star[3] ,Xff[3],ErrCumul UNITS "-"
   DISCR REAL muE,muS,muM UNITS "m^3/s^2"
   DISCR REAL dEM,AU,DU UNITS "m"
   DISCR REAL MassU UNITS "kg"
   DISCR REAL wrotEM UNITS "-"
   DISCR REAL mu UNITS "-"
   DISCR REAL G = 6.67384E-11 UNITS "m^3/(kg.s^2)"--+- 0.00080 m^3.kg^-1.s^-2
   DISCR REAL convergence_tfo UNITS "-"
   DISCR REAL to_n,tf_n,Thalfperiod UNITS "-"
   DISCR REAL AbsTol UNITS "-"
   DISCR REAL L1, L2, L3 UNITS "DU" --for info
INIT
   FOR (i IN 1,6)
     Xo[i] = 0
   END FOR
   GuessZ3notX1=GuessZ3notX1_o
   muE = 1*3.986005E14
   muS = 328902.82113001*3.986005E14--; % was Relative to earth
   muM = 0.0123000569113856 *3.986005E14
   mu=muM/(muE+muM)
   dEM=384400e3
   Xo[1]=Xo1 --GuessZ3notX1=3 --guess Z User to choose or default =3
   Xo[3]=Xo3
   Xo[5]=Xo5
   Thalfperiod=Thalfperiod_o
   DU=dEM
  MassU=(muE+muM)/G
   wrotEM=sqrt(G*MassU/DU**3)
   --for info here only because mu in known and allow computation of L1 L2 L2
   L1=findLagrangePoints(0.83, mu)-- init value not too far from the wanted roots
   L2=findLagrangePoints(1.15 , mu)
   L3=findLagrangePoints(-1.0, mu)
   PRINT (" for information: L1,L2,L3 in DistanceUnits,earthMoon= $L1 $L2 $L3 ")
   --Eco Normal Init of the derivatives
FOR (i IN 1,6)
   X[i]=Xo[i]
 END FOR
 Xo[7]= Thalfperiod --variable added
 i357[1]=GuessZ3notX1
DISCRETE
 WHEN FlagSearchPeriodicOrbit THEN -- this is like a program to be run before starting integratons by EcosimPro depending on the directive FlagSearchPeriodicOrbit .
   --Inputs : Xo[i] (including Xo[7]= Thalfperiod), NloopNewtonHalo , mu OUT: X[i] initialized by Xo which is set to the last converged Xo_n[i] (for a good starting guess for other periodic orbits)
   --Iteration on the suited IVP fulfilling the goal (with xo fixed (index 1) )
   -- goal: after a half_period vx,vz and y shall be all null (index 4,6,2) with free variables to guess: initial values of zo, vyo, half_period (index 3,5 and variable tf_n)
   FlagSearchPeriodicOrbit=FALSE --clear the condition for running this routine
   to_n=0 --never modified here
   FOR (i IN 1,7)
     Xo_n[i]=Xo[i] --here we work with IVP Xo_n (including Thalfperiod) because Xo is never modified inside the next loop
   END FOR
   --@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@
   FOR (k IN 1,NloopNewtonHalo)
     --call ODE integration for the final state Xf_n from the given IVP Xo_n to see how good are the guesses and process the iterations
     AbsTol=AbsTolM12--1E-12
     NbSteps=NbSteps2000
     NorderRK=NorderRK85
     Function_ODE_IVP=LDIM
     tf_n=Xo_n[7] -- tf is a condition final for the ODE but it is as Thalfperiod an initial condition for the process of finding a periodic solution by convergence Newton
     ODE113 (LDIM, to_n, tf_n, Xo_n, Xf_n, NorderRK, AbsTol, NbSteps, mu, Function_ODE_IVP, RKsteps6 )--out Xf_n
     --Zero search by Newton method iterations
     FOR (i IN 1,3)
       XSo[i]=Xo_n[i357[i]]
     END FOR
     FOR (i IN 1,3)--Array with the 3 components results of ODE integration to be nullified by converging the IVP XSo to XSo_star
       Xff[i]=Xf_n[i462[i]] -- i462[3]={4,6,2} i357[3]={3,5,7}
     END FOR
     --Jacobian at current final point tf_n=Xo_n[7] wrt IVP initial Xo_n given for to_n -- IT INCLUDES THE ODE113 SIZE 42
     STMatrixCR3BP ( to_n, tf_n , Xo_n, PHI, mu , RKsteps42)-- out PHI = d FF / d xx = d xxdot_i / d xx_j
     --derivative of X6 wrt time at final point, needed for getting the time derivatives to fill the matrix DF (dFF/dxx)
     Function_ODE_IVP_6( 6, Xf_n, dX6_dt, mu )
     FOR (i IN 1,6)--extended PHI last column added with time derivatives d FF / d t = d xxdot_i / d t in column 7
       PHI[i,7] = dX6_dt[i]
     END FOR
     -- dFF/dxx Full derivative of XXf (to be nullified) wrt XXo (selected state variables and time) i462[3]={4,6,2} i357[3]={3,5,7}
   FOR (i IN 1,3)
     FOR (j IN 1,3)
       DF[i,j] =PHI[i462[i],i357[j]] -- i462[3]={4,6,2} i357[3]={3,5,7}
     END FOR
   END FOR
   InvMatrix( 3,DF, D , ErrCumul)
   --XSo_star The next solution guess : XSo_star = XSo-inv(dFF/dxx)*Xff
 FOR (i IN 1,3)--extended PHI with time derivatives
   XSo_star[i]=XSo[i]-SUM (j IN 1,3; D[i,j]*Xff[j])
   END FOR
 --New Xo_n = Xo_n+1 for iterations
   FOR (i IN 1,7)
     Xo_n[i]=Xo[i] --come back to the first init conditions before update of the selected ones
   END FOR
   FOR (i IN 1,3)
     Xo_n[i357[i]]=XSo_star[i]--update the selected ones with better guesses
   END FOR
 -- end for the new Xo_n, ready to go for iterations
     --PRINTa1 (3, XSo_star , "new guess")
     --convergence and for info
     convergence_tfo=XSo_star[3]-XSo[3]
     Xo_n[8]= convergence_tfo --for info only and printing
     Xo_n[9]= NorderRK --for info only and printing
     Xo_n[10]= RKsteps6 --for info only and printing
     Xo_n[11]= RKsteps42 --for info only and printing
     Xo_n[12]= ErrCumul --for info only and printing
     Xo_n[13]= mu --for info only and printing
   END FOR --k
   --@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@
   PRINTa1 (13, Xo_n, "final_Xo_n--_tf_n_converg_RK..._err--_mu")
   FOR (i IN 1,7)--Update Xo from last converged Xo_n, and also memorized for starting other periodic orbit search if any
     Xo[i]=Xo_n[i] --including the time tf_n
   END FOR
   --Update wrt Init: New init conditions for derivative variables for EcosimPro integration: the right one for a periodic orbit
   FOR (i IN 1,6) --only 6 for X
     X[i]=Xo[i]
   END FOR
 END WHEN
CONTINUOUS
  r1=((mu+X[1])**2+X[2]**2+X[3]**2)**(1/2)--distance point to body1
  r2=((mu+X[1]-1)**2+X[2]**2+X[3]**2)**(1/2)--distance point to body2
  EXPAND (i IN 1,3) X[i+3] = X[i]'
  --dynamic f=ma in barycentric rotating frame, see for example J.D. Mireles James and many others
  X[4]'=+X[1]+2*X[5]-(X[1]+mu)*(1-mu)/r1**3-(X[1]+mu-1)*mu/r2**3
  X[5]'=+X[2]-2*X[4]-X[2]*(1-mu)/r1**3-X[2]*mu/r2**3
  X[6]'=-X[3]*(1-mu)/r1**3-X[3]*mu/r2**3
  --for info
  Omega=0.5*(X[1]**2+X[2]**2)+(1-mu)/r1+mu/r2
  Cjacobi=2*Omega-(X[4]**2+X[5]**2+X[6]**2)
  --Geocentric results in ECI with vector XX
  T_ECI=TIME/wrotEM --TIME is addim = 6.28 for 1 period
  period=2*3.14159265358979323846264338327950 /wrotEM
  periodDay=period/86400
  EXPAND (i IN 1,2) wrotEM3D[i]=0 -- only 2 first coordinates
  wrotEM3D[3]=wrotEM -- the 3rd coordinate
  --cross product
  wrotEMCrossXXrot[3]=wrotEM3D[1]*XXrot[2]-wrotEM3D[2]*XXrot[1]
  wrotEMCrossXXrot[1]=wrotEM3D[2]*XXrot[3]-wrotEM3D[3]*XXrot[2]
  wrotEMCrossXXrot[2]=wrotEM3D[3]*XXrot[1]-wrotEM3D[1]*XXrot[3]
  EXPAND_BLOCK (i IN 1,3)
     XXrot[i] = X[i]*DU
     XX[i+3] = X[i+3]*DU*wrotEM+wrotEMCrossXXrot[i]
  END EXPAND_BLOCK
  theta=TIME --wrotEM*T_ECI
  XX[1] = XXrot[1]*cos(theta)-XXrot[2]*sin(theta)
  XX[2] = XXrot[1]*sin(theta)+XXrot[2]*cos(theta)
  XX[3] = XXrot[3]
  -- useful
  Rnorm=sqrt(SUM(i IN 1,3; XX[i]**2))
  Vnorm=sqrt(SUM(i IN 4,6; XX[i]**2))
END COMPONENT
```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