COUPLED DYNAMICS OF LARGE SPACE STRUCTURES IN LAGRANGIAN POINTS

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ABSTRACT

The topic of the paper is the investigation of Large Space Structures coupled dynamics, whenever located in a Circular Restricted Three-Body Problem (CR3BP) framework. The configuration design/6 DOF dynamics coupling is deeply investigated to, eventually, drive the infrastructure system and operational design. Because of the wideness of possible practical applications in the incoming decade, the Earth Moon Lagrangian points system is here considered. The paper firstly shows the natural periodic orbit-attitude solutions, introducing maps to visually identify the regions where those solutions exist, under the CR3BP approach. The maps are parametrized over the infrastructure inertia properties, and solutions are classified with respect to the number of attitude rotations per orbit. Solar radiation pressure (SRP) disturbance is part of the model enhancing, assessing its effects on stability regions as a disturbing action on the whole coupled 6DOF dynamics. Effects of flexibility in the large infrastructure are then introduced in the model; linear modal analysis is exploited to assess whether orbital and attitude motions could excite/be excited by small vibrations of flexible structures. Preliminary considerations are deduced from a spring-mass single degree-of-freedom system, and are extended to lumped mass models of given complexity.

***Index Terms*—** Space structures, Lagrangian points, distant retrograde orbits

1. INTRODUCTION

Coupled orbit-attitude dynamical behaviors are of great interest for space applications that include large structures; the torque exerted by gravity gradient, if correctly exploited, may yield a significant contribution in satisfying pointing requirements, even coarse, thus relieving some effort from the spacecraft's attitude control system (ACS). In the restricted two-body problem, the attitude behavior of a rigid body in orbit is a well-known and discussed topic [1, 2]. When the gravity gradient torque is investigated in a multi-attractor gravitational environment, an interesting dynamical structure arises, thanks to the coupling between the the torques exerted by all the attracting bodies, even if of small amplitude. Early investigations of Kane [3] and Robinson [4] and more recent ones [5, 6, 7] analyzed the attitude stability of a rigid spacecraft, with the assumption of steady position in a Lagrangian point, within the Earth-Moon CR3BP. The coupling with the orbital motion is investigated by Guzzetti and Howell [8, 9], considering planar orbits and providing a first class of orbit-attitude periodic solutions; other works [10, 11] are devoted to the coupled orbit-attitude dynamics in the Earth-Moon CR3BP, providing results on attitude perturbations and stable/unstable motions triggered by the orbit.

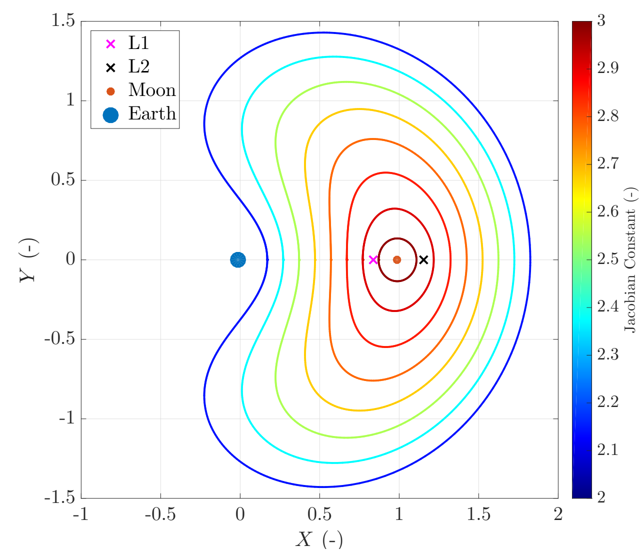


Fig. 1: Distant retrograde orbits

The study presents a tool for the analysis of coupled orbit-attitude behaviors in the Earth-Moon system, focusing on Distant Retrograde Orbits (DRO), depicted in Figure 1. This class of orbits is chosen for the following reasons:

- They possess an high degree of stability [12], allowing to assume that the orbital path remains unperturbed, at least for a preliminary analysis, providing greater significance to periodic solutions; such solutions are in fact obtained assuming the orbit to remain the nominal one, and this hypothesis is more reasonable the stabler the orbit is.
- Encircling both Lagrangian points L1 and L2, they supply a proving ground to investigate dynamical structures based on local equilibria (e.g. [4, 7]).

- NASA's Asteroid Redirect Mission [13] will likely exploit a DRO, and a periodic orbit-attitude solution might benefit both the asteroid boulder and the manned segment, providing a passive attitude partial stabilization.

The paper is organized as follows: Section 2 presents the algorithm used to seek for coupled orbit-attitude solutions, detailing their benefits and their possible exploitation; the solution space is visually mapped, in order to provide the user with a tool suitable for preliminary analyses and assessments. Section 3 introduces the effects of solar radiation pressure (SRP), showing sample results and suggesting guidelines for design. Section 4 drops the rigid body assumption, and investigates the effects of flexible parts on the attitude motion; the flexibility is approached with a linear, lumped-parameters model, which well describes the coupled behavior when the structural natural frequencies are far from the attitude-orbital motions characteristic angular pulsations. Conclusions are drawn in Section 5, together with remarks on possible future studies and model improvements.

2. PERIODIC ORBIT-ATTITUDE SOLUTIONS

The coupled orbit-attitude dynamics opens a wide range of operational possibilities for space structures in the Earth-Moon system. The combined gravity-gradient torque of the Earth and the Moon is a significant source of perturbation for attitude dynamics; its inclusion in the model, and its exploitation for natural periodic motion, may then provide a significant relief to the attitude control system of a LSS.

2.1. Model and assumptions

Figure 2 depicts the reference frames used for the analysis. The Circular Restricted Three-Body Problem (CR3BP) framework is used [14]; equations of motion (EoM) are written in the synodic frame XY, which rotates with the Earth-Moon relative motion's angular velocity. EoM are normalized, so that the universal gravitation constant is the unity, and the problem is governed by a single non-dimensional parameter μ, i.e. the ratio between the Moon mass and the total system mass.

Limiting the analysis to the planar case, the orbit dynamics is described by the classical CR3BP set of equations [15, 14]

$$\ddot{x} - 2\dot{y} - x = -\frac{1-\mu}{r_1^3}(x+\mu) - \frac{\mu}{r_2^3}(x-1+\mu) \tag{1}$$

$$\ddot{y} + 2\dot{x} - y = -\frac{1-\mu}{r_1^3}y - \frac{\mu}{r_2^3}y \tag{2}$$

Considering the principal body axis frame $x_b y_b$, rigid body attitude dynamics with gravity-gradient torque [6] is

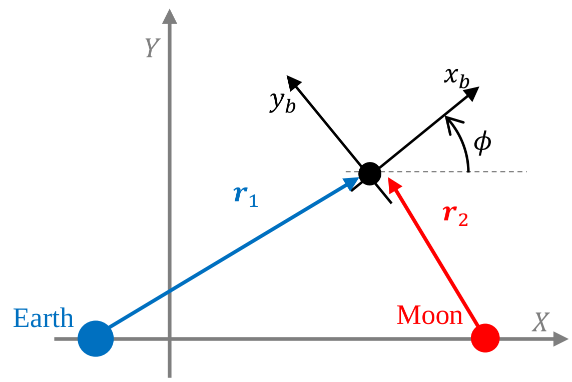


Fig. 2: Reference frames

described by equation (3)

$$\dot{\omega}_z = \frac{I_y - I_x}{I_z}\left(\frac{3(1-\mu)}{r_1^3}e_2 e_1 + \frac{3\mu}{r_2^3}l_2 l_1\right) \tag{3}$$

where ω_z is the spacecraft's angular velocity about the z axis, e_1, e_2 and l_1, l_2 are, respectively, the Earth and the Moon direction cosines in body frame, and I_x, I_y, I_z are the spacecraft's principal inertia moments. The rotation angle of the body is

$$\dot{\phi} = \omega_z \tag{4}$$

The orbital path is assumed not to be perturbed, while its influence on the attitude dynamics is exerted through the gravity-gradient torque, varying in magnitude along the orbit. Since the investigation is limited to the planar motion, two parameters are sufficient for the rigid-body analysis:

1. The angular velocity ω_z about the out-of-plane axis; being the sole quantity necessary to describe the body's rotation, the subscript will be dropped from now on to lighten the notation;

2. The inertia coefficient K_z, defined in equation (5)

$$K_z = \frac{I_y - I_x}{I_z} \tag{5}$$

which governs the gravity-gradient torque effect.

In the following, we will describe periodic orbit-attitude solutions, defined as a dynamical structure where both the orbital and the attitude motion are periodic, under the combined gravitational forces and torques due to the Earth and the Moon. Thanks to their appealing features for future missions, DROs and their associated periodic solutions are investigated in this work.

2.2. Results

This Section presents the methodology to obtain a map of the periodic orbit-attitude solutions. Given the inertia ratio K_z, the aim of the investigation is to find the initial angular velocity ω_0 necessary to establish such periodic motion. The initial instant where such angular velocity must be applied is defined as the X-axis crossing with positive velocity along Y-axis, i.e. the X-crossing point between the Earth and the Moon. Periodic solutions are obtained as follows:

1. A DRO is obtained with standard correction techniques, or from a database;
2. The initial condition is set at the aforementioned positive X-crossing point; the x_b axis is assumed to be initially aligned with the synodic X axis;
3. The attitude dynamics is propagated for different values of initial angular velocity ω_0;
4. The final values of angular velocity ω_f and body rotation ϕ_f after an entire orbit are gathered;
5. A periodic attitude behavior is identified when both ω_f and ϕ_f are null (within a tolerance of 10^{-6} non dimensional units).

Other approaches [8, 9] are based on differential correction techniques, and directly find both the orbital and the attitude initial state of the periodic solution. The present method, differently, is focused on searching attitude periodicity superposed to a given, fixed orbit.

For each DRO, multiple values of ω_0 guarantee a periodic rotational motion; each value corresponds to a family of solutions, where the spacecraft carries out a given number N of revolution per orbit. This number may be positive (i.e. the rigid body revolves N times about its z_b axis along one orbit), zero (the spacecraft oscillates, but the net number of body revolution per orbit is null), or negative (the rigid body carries out N revolutions per orbit with negative angular velocity). All the angular velocities and rotation angles are referred to the synodic frame, while the actual numerical integration keeps into account the latter's rotation.

Figure 3 portrays a map of the results: for each DRO, characterized by its period T (horizontal axis), multiple periodic orbit-attitude solutions are obtained. They are grouped into families, represented with different trait lines (bold, dash-dot, dots, dashed), following the previous definition. The vertical axis indicates the non-dimensional angular velocity ω_0, referred to the synodic frame, that is needed at the initial instant to establish the periodicity of the attitude motion. Note that this body angular velocity does not remain constant along the orbits, but undergoes short-period oscillations; as a consequence, the number N represents the overall number of body revolutions, even though they are not carried out uniformly. Figure 4 depicts the angular rotation and velocity time histories of a sample solution.

The parametrization of the solution space with respect to K_z allows a further degree of operational flexibility. A periodic solution may in fact be sought for a space station, whose

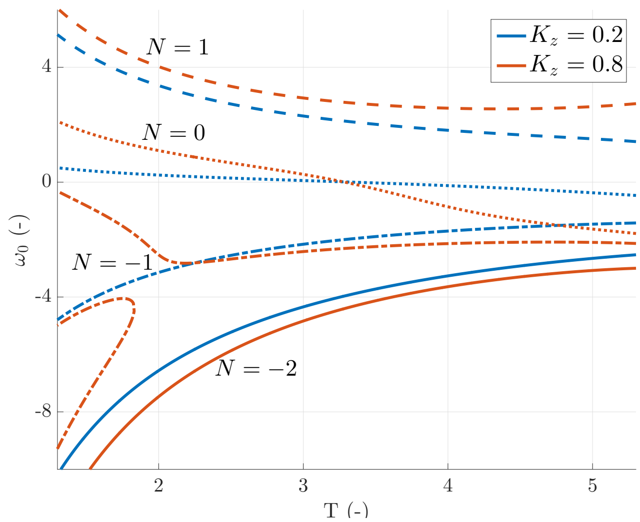


Fig. 3: Periodicity maps for DROs

attitude motion would then remain naturally periodic (or at least bounded, as a response to small perturbations); if such station is approached by a vehicle which subsequently docks, or if some modules are to be relocated in different positions, its inertia properties will change after the operations; the coefficient K_z changes accordingly. It will be then possible to find the new parameters needed to preserve periodicity in the dynamical orbit-attitude behavior, with two main possibilities:

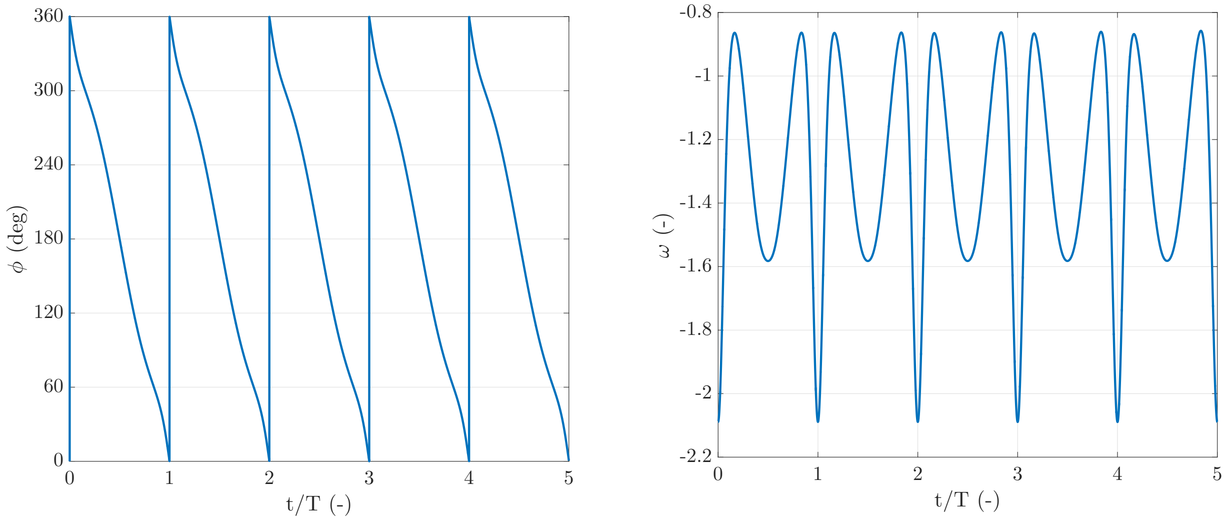
- If the mean angular velocity is to be maintained, the orbit can be changed, shifting towards a different DRO that allows the same ω_0 of the previous solution;
- If the orbit should be kept the same, a new value of ω_0 ought to be identified, performing a spin-up (or spin-down) maneuver at the beginning of the new orbital period.

2.3. Extension to other orbits

The presented technique may be exploited to find periodic orbit-attitude solutions for multiple classes of planar orbits in the CR3BP framework; DRO are presented due to their appealing features and their possible exploitation for near-future mission, but no loss of generality is involved in the usage of the tool.

Periodicity maps, like the sample of Figure 3, can be generated for potentially any type of planar periodic orbit, starting from a reference catalog as [16] or including the orbit identification into the code.

The extension to three-dimensional orbits presents some issues, not addressed in this paper. The fully 3D dynamics does not allow for a clear and simple visualization of the results, since all the three components of the angular velocity



(a) Rotation angle

(b) Angular velocity

Fig. 4: Sample periodic solution $K_z = 0.8$, $T = 4.85$

vector are involved; furthermore, the single coefficient K_z is no longer sufficient for a complete description of the spacecraft's inertia properties, but at least another parameter is necessary. A preliminary investigation of a Halo orbit-attitude periodic solution is presented in [17].

3. EFFECTS OF SOLAR RADIATION PRESSURE

Solar radiation pressure (SRP) may be a significant disturbance for LSS in the Earth-Moon system, acting both on the orbital path (acceleration) and on attitude dynamics (torque). In particular, large surfaces exposed to solar radiation (e.g. solar arrays) are able to produce a substantial torque component with magnitude even greater than the gravity-gradient torque, unless they are symmetric with respect to the spacecraft's center of mass. The present work is focused on the torque arising from SRP, while the orbital perturbation is assumed to be compensated by some sort of station-keeping system; is also worth noting that DROs, being a highly stable family, do not undergo significant path variation before a few orbital periods, easing the station-keeping strategy.

3.1. Model and assumption

The force dF acting on an infinitesimal surface dA, due to the radiation incoming from the sun, can be divided into three components [18, 1]:

- The specularly reflected radiation (subscript s)

$$dF_s = -2\frac{W_0}{c_0}C_s(\hat{s}\cdot\hat{n})^2\hat{n}\,dA \tag{6}$$

- The diffusively reflected radiation (subscript d)

$$dF_d = -\frac{W_0}{c_0}C_d\left(\frac{2}{3}(\hat{s}\cdot\hat{n})\hat{n} + (\hat{s}\cdot\hat{n})\hat{s}\right)dA \tag{7}$$

- The absorbed radiation (subscript a)

$$dF_a = -\frac{W_0}{c_0}C_a(\hat{s}\cdot\hat{n})\hat{s}\,dA \tag{8}$$

where W_0 is the solar constant (1361 W/m^2), c_0 the speed of light in vacuum, $\hat{n}$ the unit vector normal to the surface and $\hat{s}$ the sun direction, using the spacecraft as basepoint. For an opaque surface, the three coefficients C_s, C_a, C_d are linked by equation (9)

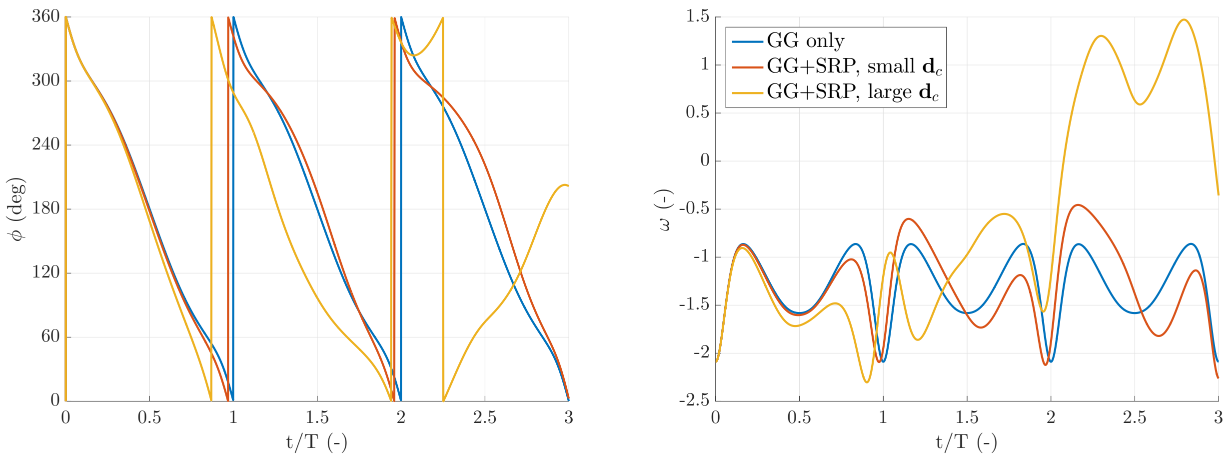
$$C_s + C_a + C_d = 1 \tag{9}$$

In order to provide a simple tool for preliminary analyses, with the aim of assessing the effect of SRP on the attitude behavior of a spacecraft in design phase, a single-surface model is employed, assuming all the irradiated area A to be lumped into a plate. The total SRP acceleration results in

$$\begin{aligned}\mathbf{a}_{SRP} = -\frac{W_0}{c_0}\frac{A}{m}(\hat{s}\cdot\hat{n})\Big[&(1-C_s)\hat{s}\\ &+\left(2C_s|\hat{s}\cdot\hat{n}| + \frac{2}{3}(1-C_s-C_a)\right)\hat{n}\Big]\end{aligned} \tag{10}$$

Assuming to know the position of the center of pressure $\mathbf{d}_c$, in principal body axes, the torque about the z axis reads

$$T_{SRP} = \mathbf{d}_c \times m\mathbf{a}_{SRP} \tag{11}$$



(a) Rotation angle

(b) Angular velocity

Fig. 5: Sample periodic solution with SRP torque perturbation

3.2. Results

The SRP torque is a major source of disturbance for the attitude motion of LSS in the Earth-Moon system. As example, for a structure with 1000 m^2 of surface and a weight of 500 metric tons, a difference of 30 cm between the position of the center of radiation pressure and the barycenter provokes a SRP torque of the same order of magnitude of gravity gradient torque, i.e. $\simeq 1 \times 10^{-3}$ Nm. This small value is actually able to highly disturb the spacecraft's attitude dynamics, and especially for long-term mission may not be acceptable (e.g. for momentum wheels operational life and fuel quantity for desaturation maneuvers).

Small asymmetry of the surface with respect to the barycenter does not hinder the periodic solutions obtained with gravity gradient only. Figure 5 depicts how the previous sample periodic solution (Figure 4) is affected by the perturbation of SRP torque, assuming a displacement of 2 and 10 cm of the center of pressure and a structure with $A = 1000\, m^2$, $m = 500 \times 10^3\, kg$, $C_a = 0.4$ and $C_s = 0.6$.

3.3. Highlights

It is noted that, even for preliminary analyses, SRP torque is not negligible, since it may lead to large deviations from the nominal attitude. At this stage, the study may proceed following two different paths:

- The perturbation can be faced with an active control system, designing a sequence of maneuvers to keep the nominal periodic solution obtained with gravity gradient only; excess angular momentum may be stored and regularly dumped with proper wheels and thrusters.
- The periodic solution search might be enhanced with the implementation of SRP into the model, looking for periodic orbit-attitude dynamical structures that include SRP torque; the acceleration component may be included as well, investigating the deviations from the nominal orbital path.

4. FLEXIBLE SPACECRAFT ANALYSIS

The last step in the analysis is the assessment of the flexibility of the spacecraft and its effect on attitude motion. The previously presented solutions and methods are in fact based on a rigid body assumption, while low-frequency structural modes might hinder and perturb such motion. The effects of flexibility on the spacecraft attitude are investigated, neglecting at this preliminary stage interactions with the orbital path; further efforts may be directed to the extension of the model to consider the full translational and rotational dynamics.

4.1. Model and assumptions

Figure 6 depicts the lumped-parameter model used for the analysis: a rigid body, with moment of inertia $\bar{I}_z$, is free to rotate in a XY frame centered in its barycenter, with angular velocity ω. The angle ϕ indicates the rotation of the x_b body axis with respect to the X axis. Attached to this rigid part there are N spring-mass systems, with stiffness k_i and mass m_i intended to model a flexible part of the spacecraft. Each i-th spring is located at a point P_i, with known and fixed coordinates in body frame $\bar{x}_i, \bar{y}_i$. The angle α_i, known in body frame, indicates the vibration direction; it might be assumed as a further degree of freedom for more complex parts to be modeled, but for the present preliminary analysis it is assumed to be fixed.

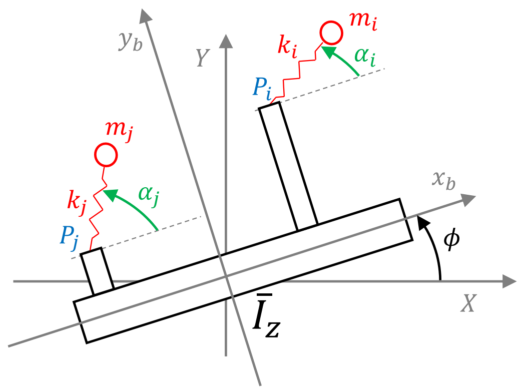


Fig. 6: Flexible spacecraft lumped-parameter model

4.2. Flexible parts response

Calling s_i the elongation of the i-th spring, each flexible component will be governed by the following differential equation:

$$\ddot{s}_i + (\frac{k_i}{m_i} - \omega^2)s_i = \omega^2(\bar{x}_i \cos\alpha_i + \bar{y}_i \sin\alpha_i) \\ - \dot{\omega}(\bar{x}_i \sin\alpha_i + \bar{y}_i \cos\alpha_i) \quad (12)$$

The N EoM that describe the flexible dynamics may be written in matrix form as

$$[M]\ddot{\mathbf{s}} + ([K] - [M]\omega^2)\mathbf{s} = [L_c]\omega^2 - [L_e]\dot{\omega} \quad (13)$$

where $[M]$ and $[K]$ are, respectively, the mass and stiffness matrix of the flexible system; for the case at hand they result diagonal, but the treatment may be extended to non-diagonal models, i.e. where each flexible system's vibration is not totally decoupled from the others. The term $[M]\omega^2$ indicates the gyroscopic softening effect, which apparently reduces the stiffness of the flexible component. The matrices $[L_c]$ and $[L_e]$ denote the centrifugal and Euler accelerations contributions.

Denoting the fundamental frequencies $\Omega_i = \sqrt{k_i/m_i}$, one notes that if $\Omega_i \gg \omega$ each flexible system behaves as a forced mass-spring oscillator, and the contribution of the body angular velocity to the stiffness may be neglected. Furthermore, variations in angular velocity are negligible too, since they will be very slow with respect to the fast dynamics of the oscillator. It is then possible to simplify equation (12) reducing it to

$$\ddot{s}_i + \frac{k_i}{m_i}s_i = \bar{\omega}^2(\bar{x}_i \cos\alpha_i + \bar{y}_i \sin\alpha_i) \quad (14)$$

where the angular velocity ω is assumed to have a fixed, mean value $\bar{\omega}$ and its variations $\dot{\omega}$ are neglected.

4.3. Attitude motion perturbation

The attitude motion is in turn perturbed by the vibrations of the flexible systems. For the model at hand, the rotational motion is described by equation (15)

$$\dot{\omega}\Big[\bar{I}_z + \sum_{i=1}^{N} m_i(s_i^2 + \bar{x}_i^2 + \bar{y}_i^2) + 2m_i s_i(\bar{x}_i \cos\alpha_i + \bar{y}_i \sin\alpha_i) \\ + m_i \dot{s}_i(\bar{x}_i \sin\alpha - \bar{y}_i \cos\alpha)\Big] \\ + \omega \sum_{i=1}^{N} 2m_i s_i \dot{s}_i + 2m_i \dot{s}_i(\bar{x}_i \cos\alpha_i + \bar{y}_i \sin\alpha_i) = \\ M_z - \sum_{i=1}^{N} m_i \ddot{s}_i(\bar{x}_i \sin\alpha - \bar{y}_i \cos\alpha) \quad (15)$$

where M_z is the external torque applied to spacecraft (e.g. gravity gradient torque).

The first term contains all the inertia properties of the full spacecraft, summing the contribution of the rigid and flexible parts; the change of inertia due to structural vibrations might be neglected, since the displacements s_i are expected to be small and yield no significant contribution with respect to the static terms, as proven in the next Section. One may then group the inertia contributions into an overall reduced inertia I_z, i.e. the inertia of the full spacecraft at rest. The second term denotes a coupling between the body angular velocity and both the displacements s_i and velocities $\dot{s}_i$; the last term represents the equivalent torque resulting from the inertia forces of the moving components, depending on their arms with respect to the barycenter.

4.4. Results

Under the assumption of high flexible frequencies in comparison to the attitude motion's ones, equation (14) shows that the flexible response of the system is statically excited by its angular rotation along the orbit, and the small vibrations of the spring-mass systems can be computed in closed form with the simple expression of equation (16), assuming null initial conditions.

$$s_i(t) = \frac{\bar{\omega}^2(\bar{x}_i \cos\alpha_i + \bar{y}_i \sin\alpha_i)}{\Omega_i^2}(1 - \cos\Omega_i t) \quad (16)$$

It is remarked that such vibration is of infinitesimal amplitude, due to assumption that $\Omega_i \gg \omega$. For non-diagonal mass and stiffness matrices, the problem may be generally diagonalized (e.g. using modal coordinates) and expressions analogous to equation (16) will result.

The vibration of the flexible parts, in turn, perturbs the attitude motion according to equation (15); using the analytical, approximate solution of equation (16), and dropping infinitesimal terms, the attitude motion equation may be simplified as

$$\dot{\omega}I_z = M_z - \bar{\omega}^2 \sum_{i=1}^{N} I_i \cos\Omega_i t \quad (17)$$

defining

$$I_i = m_i(\bar{x}_i \cos\alpha_i + \bar{y}_i \sin\alpha_i)(\bar{x}_i \sin\alpha - \bar{y}_i \cos\alpha) \quad (18)$$

with the units of an inertia.

At this point of the analysis, analogy between equations (3) and (18) is evident, and the angular motion of the spacecraft may be described summing the gravity gradient torque and the flexibility contribution, resulting in equation (19)

$$\dot{\omega}_z = K_z \left(\frac{3(1-\mu)}{r_1^3} e_2 e_1 + \frac{3\mu}{r_2^3} l_2 l_1 \right) - \bar{\omega}^2 \sum_{i=1}^{N} \frac{I_i}{I_z} \cos\Omega_i t \quad (19)$$

Note that the mean value $\bar{\omega}$ must be known, and may be computed e.g. from a rigid body solution. The equivalent torque arising from flexible parts is then a short-period contribution, which may be superposed to the free motion. Its magnitude depends on the ratios I_i/I_z, which will be in general small (since I_z inherently includes the contributions of the flexible parts), even though not negligible.

5. CONCLUSIONS AND FINAL REMARKS

The paper presents a tool for preliminary analyses of coupled orbit-attitude dynamics in the CR3BP, with peculiar attention to distant retrograde orbits. The presented technique allows to:

- Obtain periodic orbit-attitude solutions with gravity gradient torque in a multi-body gravitational environment, providing working examples in the Earth-Moon system;
- Analyze solar radiation pressure perturbation, assessing its effect on rigid body motion and, as a consequent step, on orbital path;
- Investigate couplings between structural vibrations of flexible parts and attitude rotation, obtaining preliminary analytical results and guidelines for the analysis.

Throughout the work, the orbital dynamics is assumed unperturbed, while the coupling with the attitude behavior is expressed through the gravity gradient torque; future works may be devoted to the fully coupled dynamics, including second order terms in the gravitational potential [19] and assessing the effect of an extended body on the orbital trajectory. The inclusion of SRP perturbing acceleration might be part of the model enhancing as well, with a more detailed representation of the orbital and attitude dynamics both disturbed by solar radiation; the inclusion of 4th body gravitational attraction (Sun) might be interesting to assess the stability of the mentioned solutions in a more realistic dynamical environment.

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