# FAST LOW EARTH ORBIT ACQUISITION PLAN OPTIMISER 

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#### Abstract

This paper describes a tool and the associated mathematical framework that computes an orbit acquisition plan that either minimizes the duration of the orbit acquisition phase or the required Delta-V given the spacecraft characteristics and mission constraints. The automatic algorithms are built upon on a perturbation analysis of the nominal orbit and provide the necessary information to perform preliminary analysis of orbit acquisition phases of Earth observation satellites. Two different strategies are exploited, one based on continuous rate semi-major axis changes and a second based on step-wise semi-major axis changes. The former is suitable for the case when several small manoeuvres can be approximated by a single manoeuvre with a constant altitude change rate and during the early design phases when detailed information about the manoeuvring constrains is not available. The latter produces a more detailed plan that considers the existence of a first and a last manoeuvres with smaller amplitude than the remaining ones. All algorithms take into account Delta-V limits and the existence of periods when manoeuvring is not possible.


Index Terms- Orbit acquisition, phasing, optimization

## 1. INTRODUCTION

Earth observation data is key for an efficient use of land and natural resources, better land and sea monitoring, more informed political decisions, and better understanding of the weather, climate, and land changes. Many Earth observation satellites are operated in Low Earth Orbits (LEO), which provide a good trade-off between revisit time and spatial resolution. Repeat ground track orbits are of particular interest as they allow the acquisition of the same scene at fixed time intervals. Additionally, either for calibration or nominal operation proposes, many satellites are required to overpass an exact location on Earth. For missions comprising spacecraft constellations, accurate orbit phasing is also needed. Hence, the satellite manoeuvres should be carefully planned to control the ground track drift so that the desired longitude at ascending node crossing are achieved.

The work in [1] describes an analytical methodology for modelling the ground track drift of a low-Earth satellite as
well as the semi-major axis and eccentricity change for a given impulsive in-plane Delta-V. The modelling is based on linearisation of ground track shift from one pass to the next. This analytical formulation is implemented computationally and used to select the manoeuvres of the orbit acquisition process.

The conditions required for repeat ground track orbits are analysed in [2]. In this work, an autonomous orbit control system to drive the satellite from any initial condition to the desired repeat ground track is also outlined. Similarly to [1], the control concept is based on the linearised model of the nodal period.

The works in [1] and [2] provide a theoretical framework for the analysis and design of an orbit acquisition phase. However, constraints that limit the manoeuvring strategies and that are inherently present on any mission are not addressed. The different scenarios analysed for the orbit acquisition of MetOp-A are described in [3] as well as the actual manoeuvre executed to achieve the desired orbit position. The ground track was required to be achieved within two weeks. During the Launch and Early Orbit Phase (LEOP) there were two opportunities for performing orbital corrections on the third and last day. Since the out-of-plane manoeuvres have an in-plane component and vice-versa, the last manoeuvre had to be a small in-plane manoeuvre to correct eventual inaccuracies and cross component effects of previous manoeuvres.

This paper describes a tool for rapid orbit acquisition optimisation. Two strategies can be selected: i) constant semimajor axis rate of change and ii) step-wise semi-major axis manoeuvres. In the former, constant Delta-V per time interval is applied to the spacecraft, which is suitable when several small impulsive manoeuvres can be approximated by a continuous manoeuvre and also when there is no detailed information about the manoeuvring capabilities and other constraints. The latter is based on impulsive semi-major axis changes, which allows a more detailed plan, where practical consideration such as the existence of calibration and touchup manoeuvres are taken into account. The tool also allows the analysis of several semi-major axis launch dispersions and launch dates providing for each case the resulting acquisition duration, required Delta-V, and Mean Local Solar Time (MLST) drift. In particular, to insert another spacecraft
within a constellation, a trade-off needs to be made regarding the targeted injection orbit (in terms of semi-major axis, inclination or MLST). Taking into account the spacecraft manoeuvre capabilities, the operational constraints of LEOP, the constraints preparing and implementing manoeuvres, launch date constraints and the agreed (or expected) launch dispersions, the tool identifies the consequences of all the possible scenarios, which can then be used to define the orbital offset to be targeted at launch.

The remainder of this paper is organized as follows. In Section 2 the ground track drift evolution when the orbit semimajor axis and inclination are not nominal is characterised resorting to a perturbation analysis of the orbit. Section 3 describes the algorithms developed for the tool for the minimisation of the orbit acquisition time and of the Delta-V consumption. The tool inputs as well as its outputs are described in Section 4. In Section 5, the tool is used to provide orbit acquisiton plans using the developed algorithms for a realistic scenario. Finally, concluding remarks and directions for future work are outlined in Section 6.

## 2. PERTURBATION ANALYSIS OF THE NOMINAL ORBIT

Typically, following the launch of a LEO spacecraft, the initial orbit injection errors need to be corrected so that the nominal semi-major axis, eccentricity and inclination are reached. If no phasing or specific ground track are required, the spacecraft is directly manoeuvred to the targeted semi-major axis, eccentricity and inclination. On the other hand, if phasing or a specific ground track are needed, the orbital manoeuvres should be performed at appropriate time instants to achieve the desired orbital drift.

### 2.1. Orbit drift of a perturbed orbit

The equatorial distance between two consecutive (in time) ascending nodes is given by

$$
\begin{equation*}
\lambda_{\text {one rev }}=R_{\oplus}\left(\omega_{\oplus}-\dot{\Omega}\right) P_{\Omega} \tag{1}
\end{equation*}
$$

where $R_{\oplus}$ is the Earth's radius, $\omega_{\oplus}$ is the Earth's rotational velocity, $\Omega$ is right ascension of the ascending node (angle from the vernal equinox to the ascending node), and $P_{\Omega}$ is the nodal period.

The dominant motion of $\Omega$ is caused by $J_{2}$, which represents the Earth's oblateness and it is described by

$$
\begin{equation*}
\dot{\Omega}_{J_{2}}=-\frac{3}{2} \sqrt{\frac{\mu}{a^{7}}} \frac{R_{\oplus}}{\left(1-e^{2}\right)^{2}} \cos (i) \tag{2}
\end{equation*}
$$

where $\mu$ denotes the gravitational constant, $a$ is the semimajor axis of the satellite's orbit, $e$ is the orbit eccentricity, and $i$ is the orbit inclination. By assuming that $\dot{\Omega} \approx \dot{\Omega}_{J_{2}}$, the


Fig. 1: Evolution of the orbital position with a semi-major axis above or below the nominal.
nodal period of an orbit is approximately given by [4]

$$
\begin{equation*}
P_{\Omega}=2 \pi \sqrt{\frac{a^{3}}{\mu}}\left(1+\frac{3 J_{2} R_{\oplus}^{2}}{2 a^{2}}\left(3-4 \sin ^{2}(i)\right)\right) . \tag{3}
\end{equation*}
$$

By using (3), we conclude that the nodal period depends mainly on the semi-major axis, $a$. Thus, assuming that the orbit inclination is nominal, by performing in-plane manoeuvres, the semi-major axis can be controlled to achieve the desired ground track. For the same propellant, inclination changes induced by out-of-plane manoeuvres have a much smaller effect on the nodal period than the semi-major axis changes induced by in-plane manoeuvres. Nevertheless, the design orbit acquisition phase needs to take into account the orbit inclination. We resort to a perturbation analysis of the nominal orbit to study the consequences to the ground track acquisition of small variations of $a$ and of $i$. Since this is a first order perturbation analysis, only the $J_{2}$ zonal harmonic of Earth's gravity field is considered. The mathematical derivation follows mostly the work in [4].

Using (1), we have that a specific ground track or orbital phase can be achieved by reducing or increasing the nodal period so that the ascending node drifts eastwards of westwards, respectively, as illustrated in Fig. 1.

Assuming small variations of $a$ and $i(e$ is assumed to be nominal, and thus not included in this analysis), (1) can be approximated by

$$
\begin{align*}
& \lambda_{\text {one rev }}(a+\Delta a, i+\Delta i) \approx \\
& \lambda_{\text {one rev }}(a, i)+\frac{\partial \lambda_{\text {one rev }}(a, i)}{\partial a} \Delta a+\frac{\partial \lambda_{\text {one rev }}(a, i)}{\partial i} \Delta i . \tag{4}
\end{align*}
$$

The partial derivative of $\lambda_{\text {one rev }}$ with respect to $a$ is given by

$$
\begin{equation*}
\frac{\partial \lambda_{\text {one rev }}(a, i)}{\partial a}=R_{\oplus}\left(\omega_{\oplus}-\dot{\Omega}\right) \frac{\partial P_{\Omega}}{\partial a}-R_{\oplus} P_{\Omega} \frac{\partial \dot{\Omega}}{\partial a} \tag{5}
\end{equation*}
$$

where the partial derivative of (3) yields

$$
\begin{equation*}
\frac{\partial P_{\Omega}}{\partial a} \approx \frac{21}{4} \sqrt{\frac{\mu}{a^{9}}} \frac{R_{\oplus}^{2} J_{2}}{\left(1-e^{2}\right)^{2}} \cos (i) \tag{6}
\end{equation*}
$$

and, ignoring higher order zonal harmonics, from (2), we obtain

$$
\begin{equation*}
\frac{\partial \dot{\Omega}}{\partial a} \approx 3 \pi \sqrt{\frac{\mu}{a}}\left(1+\frac{J_{2}}{2}\left(\frac{R_{\oplus}^{2}}{a}\right)^{2}\left(3-4 \sin ^{2}(i)\right)\right) . \tag{7}
\end{equation*}
$$

The partial derivative of $\lambda_{\text {one rev }}$ with respect to $i$ is given by

$$
\begin{equation*}
\frac{\partial \lambda_{\text {one rev }}(a, i)}{\partial a}=R_{\oplus}\left(\omega_{\oplus}-\dot{\Omega}\right) \frac{\partial P_{\Omega}}{\partial i}-R_{\oplus} P_{\Omega} \frac{\partial \dot{\Omega}}{\partial i} \tag{8}
\end{equation*}
$$

where the partial derivative of (3) yields

$$
\begin{equation*}
\frac{\partial P_{\Omega}}{\partial i} \approx-12 \pi \sqrt{\frac{a^{7}}{\mu}} R_{\oplus}^{2} \sin (2 i) \tag{9}
\end{equation*}
$$

and, ignoring higher order zonal harmonics, by using (2), we obtain

$$
\begin{equation*}
\left.\frac{\partial \dot{\Omega}}{\partial i} \approx \frac{3}{2} \sqrt{\frac{\mu}{a^{7}}} \frac{R_{\oplus}^{2} J_{2}}{\left(1-e^{2}\right)^{2}} \sin (i)\right) . \tag{10}
\end{equation*}
$$

Expressing continuously in time the equatorial drift between the nominal and the perturbed orbits, we obtain

$$
\begin{align*}
& \Delta \lambda(t)= \\
& =\int_{t_{0}}^{t} \frac{\left.\lambda_{\text {one rev }}(a+\Delta a(\tau)), i+\Delta i(\tau)\right)-\lambda_{\text {one rev }}(a, i)}{P_{\Omega}} d \tau \\
& =\frac{1}{P_{\Omega}}\left(k_{a} \int_{t_{0}}^{t} \Delta a(\tau) d \tau+k_{i} \int_{t_{0}}^{t} \Delta i(\tau) d \tau\right) \tag{11}
\end{align*}
$$

where $k_{a}=\frac{\partial \lambda_{\text {one rev }}(a, i)}{\partial a}$ and $k_{i}=\frac{\partial \lambda_{\text {one rev }}(a, i)}{\partial i}$. As mentioned above, in general, changing $\Delta a$ is more efficient than modifying $\Delta i$. For this reason, for sake of simplicity, let us assume that $\Delta i=0$. Hence,

$$
\begin{equation*}
\Delta \lambda_{\Delta i=0}(t)=\frac{k_{a}}{P_{\Omega}} \int_{t_{0}}^{t} \Delta a(\tau) d \tau \tag{12}
\end{equation*}
$$

From (12), we conclude that the ground track drift is closely associated with the area given by the integration of the semimajor axis time evolution, i.e. with $A=\int_{t_{0}}^{t} \Delta a(\tau) d \tau$.

### 2.2. Sun-synchronous and repeat ground track orbits

Despite the above derivation being valid for any LEO, the tool is specially targeted at Sun-synchronous orbits with a repeat ground track. A spacecraft with a repeat ground track orbit passes over the exact same location on the earth surface at fixed time intervals. This is important to guarantee ground stations visibility and for monitoring of the evolution of terrain over time (e.g. shoreline, land-coverage, and landchange). In repeat ground track orbits, we have

$$
\begin{equation*}
\lambda_{\text {one rev }}=\frac{k_{\text {days }}}{k_{\text {ref }}} 2 \pi R_{\oplus} \tag{13}
\end{equation*}
$$

where $k_{r e v}$ denotes the number of different ascending nodes, and $k_{\text {days }}$ denotes the number of days necessary to complete a full cycle.

Sun-synchronous orbits are near circular with altitudes, mostly, between 600 km and 800 km . In these orbits, the orientation of the orbital plane with respect to the Sun is approximately constant and the satellite observes a scene on ground always with the same illumination conditions. This has several advantages for Earth observation. For passive imaging satellites which rely on the light reflected by the Earth, orbits with Sun incidence angle different from 90 deg are advantageous since, the height of terrain or other feature can be computed from its shadow. On the other hand, radar satellites can be placed on dawn/dusk orbits, so that they receive solar power during mostly of its orbit, which maximizes the active time of the instruments. The MLST is used to characterize the Sun lightning conditions. The MLST of an equator crossing (ascending or descending node) at longitude $L$ (expressed in degrees) is given by [5]

$$
\begin{equation*}
M L S T=U T+L \frac{24}{360} \text { (hours) } \tag{14}
\end{equation*}
$$

where $U T$ is the universal time based on the Earth's rotation expressed in hours. Consequently, $U T$ is the $M L S T$ at 0 deg longitude. This time is constant in Sun-synchronous orbits. The constant orientation of the orbit plane with respect to the direction of the Sun is achieved by a judicious selection of the orbital parameters, in particular, of semi-major axis, eccentricity, and inclination, so that the perturbation effect due to the Earth oblateness results in a rotation of the right ascension of the ascending node $\Omega$.

### 2.3. Computing the targeted drift

Let the reference orbit be characterized by nominal $a, i$, and $e$, and in which $\lambda_{r e f}\left(t_{r e f}\right)$ is the equatorial position of the ascending node at instant $t_{\text {ref }}$. The mean time evolution of $\lambda_{r e f}$ is given by
$\lambda_{r e f}(t)=R_{\oplus}\left(\omega_{\oplus}-\dot{\Omega}\right)\left(t-t_{r e f}\right)+\lambda_{r e f}\left(t_{r e f}\right) \bmod 2 \pi R_{\oplus}$,
where $a \bmod b$ denotes the modulo operation, i.e. the remainder after division of $a$ by $b$. Also, consider that, the equatorial position of the ascending node of the injection orbit at time $t_{0}$ is given by $\lambda_{0}$. Then, the equatorial distance between the injection orbit ascending node to the reference orbit ascending node is given by

$$
\begin{equation*}
\Delta \lambda_{0}=\lambda_{0}-\lambda_{r e f}\left(t_{0}\right) \tag{16}
\end{equation*}
$$

Figure 2 depicts the initial equatorial drift with respect to the reference orbit. The figure also show the orbital nodes of a repeat ground track orbit with $k_{\text {days }}=10$.


Fig. 2: Ground track and orbital plane representation of the orbital nodes and injection orbit.

To achieve the desired ground track, the spacecraft can drift to any node, then the target drift is given by

$$
\begin{equation*}
\Delta \lambda_{\text {target }}=n \frac{2 \pi R_{\oplus}}{k_{\text {rev }}}-\Delta \lambda_{0}, \quad n \in \mathbb{Z} \tag{17}
\end{equation*}
$$

On the other hand, if the spacecraft needs to be in a specific node, the target drift must satisfy

$$
\begin{equation*}
\Delta \lambda_{\text {target }}=n \frac{2 \pi R_{\oplus} k_{\text {days }}}{k_{\text {rev }}}-\Delta \lambda_{0}, \quad n \in \mathbb{Z} \tag{18}
\end{equation*}
$$

The problem of ground track acquisition can then be posed as the optimal selection of orbital manoeuvres that satisfy

$$
\begin{equation*}
\int_{t_{0}}^{t_{e n d}} \Delta a(\tau) d \tau=\frac{P_{\Omega}}{k_{a}} \Delta \lambda_{\text {target }} \tag{19}
\end{equation*}
$$

while minimizing the either acquisition time $t_{\text {end }}$ or the DeltaV consumption. The optimal selection of in-plane manoeuvres is subject to several constraints that can be due to operations, planning, platform limits, or instrument safety. For instance, it is usual to have a period without manoeuvres after launch, and the first and last manoeuvres are normally smaller than the others, to allow thruster calibration and to minimize effects of thruster misperformance, respectively. However, these constraints vary from mission to mission. Thus, a systematic analysis of the impact of different design options calls for automatic algorithms that produce suitable orbit acquisition solutions while taking into account the mission constraints.

## 3. ALGORITHMS

In this section, the algorithms developed for the tool are described. Two distinct approaches are pursued, one based on constant rate semi-major axis changes and another based on step-wise semi-major axis changes. The algorithms are specially target at Sun-synchronous repeat ground track orbits. Thus, the targeted drift $\Delta \lambda_{\text {target }}$ is computed using (17) or (18). As inputs, the algorithms require:

- nominal orbit ( $a, i, e, k_{r e v}, k_{\text {days }}$ );
- longitude of ascending node crossing of the reference orbit and associated epoch;
- longitude of ascending node crossing of the injection orbit and associated epoch;
- period of time without orbit acquisition manoeuvres (since the epoch of the injection ascending node);
- Delta-V budget for orbit acquisition;
- initial semi-major axis, inclination, MLST difference of the injection orbit with respect to the targeted;
- days when manoeuvring is not possible due to scheduling or other constraints;
- targeted node (e.g. for phasing).

Additionally, the algorithms based semi-major axis changes with constant rate require the maximum Delta-V per day and algorithms based on step-wise semi-major axis manoeuvres require the maximum and minimum amplitudes of each manoeuvre (a different maximum and minimum can be specified for the first, last and intermediate manoeuvres).

### 3.1. Continuous semi-major axis manoeuvres

This approach only requires knowledge of the Delta-V per time interval that can be used for orbital manoeuvres. It is specially suitable when several small manoeuvres can be approximated by a constant semi-major axis rate of change. Moreover, it can also be valuable in early mission phases when detailed information about the manoeuvring constraints is not available.

### 3.1.1. Constraints and assumptions

In addition to the Delta-V budget for the orbit acquisition phase, the main assumption taken in this approach is that the orbital manoeuvres can be approximated by a constant semimajor axis rate of change, i.e.

$$
\begin{equation*}
\frac{d}{d t} \Delta a(t)=\Delta \dot{a}=\text { constant } . \tag{20}
\end{equation*}
$$

### 3.1.2. Minimizing Delta-V consumption

The minimum Delta-V consumption is achieved when the orbit acquisition is performed exclusively by taking advantage of the initial injection error, i.e. $\Delta a\left(t_{0}\right)$. The typical time evolution of $\Delta a(t)$ is illustrated in Fig. 3. In general, there is a period of time after launch before actual the orbit acquisition phase. In Fig. 3, this period corresponds to the interval between $t_{0}$ and $t_{1}$. The manoeuvring time necessary to achieve the nominal semi-major axis corresponds to $t_{3}-t_{2}$ in Fig. 3 and it satisfies

$$
\begin{equation*}
t_{3}-t_{2}=\frac{\Delta a\left(t_{0}\right)}{\Delta \dot{a}} \tag{21}
\end{equation*}
$$



Fig. 3: Orbit acquisition using constant $\Delta a$ rate of change and natural drift.

The total drift is given by

$$
\begin{equation*}
\Delta \lambda_{t o t a l}=\Delta \lambda_{\left[t_{0}, t_{1}\right]}+\Delta \lambda_{\left[t_{1}, t_{2}\right]}+\Delta \lambda_{\left[t_{2}, t_{3}\right]}, \tag{22}
\end{equation*}
$$

where

$$
\begin{align*}
& \Delta \lambda_{\left[t_{0}, t_{1}\right]}=\frac{k_{a}}{P_{\Omega}}\left(t_{1}-t_{0}\right) \Delta a\left(t_{0}\right)  \tag{23}\\
& \Delta \lambda_{\left[t_{1}, t_{2}\right]}=\frac{k_{a}}{P_{\Omega}}\left(t_{2}-t_{1}\right) \Delta a\left(t_{0}\right) \tag{24}
\end{align*}
$$

and

$$
\begin{equation*}
\Delta \lambda_{\left[t_{2}, t_{3}\right]}=\frac{k_{a}}{P_{\Omega}} \frac{t_{3}-t_{2}}{2} \Delta a\left(t_{0}\right) \tag{25}
\end{equation*}
$$

After taking into account the drift during the periods $\left[t_{0}, t_{1}\right]$ and $\left[t_{2}, t_{3}\right]\left(\Delta \lambda_{\left[t_{0}, t_{1}\right]}\right.$ and $\Delta \lambda_{\left[t_{2}, t_{3}\right]}$, respectively), the algorithm computes the next suitable node (westwards, if $\Delta a\left(t_{0}\right)>0$, or eastwards, if $\left.\Delta a\left(t_{0}\right)<0\right)$ to determine the target drift $\Delta \lambda_{\text {target }}$. The interval $t_{2}-t_{1}$ necessary to achieve the targeted drift is then given by

$$
\begin{equation*}
t_{2}-t_{1}=\frac{P_{\Omega}}{k_{a}} \frac{\Delta \lambda_{\text {target }}-\Delta \lambda_{\left[t_{0}, t_{1}\right]}-\Delta \lambda_{\left[t_{2}, t_{3}\right]}}{\Delta a\left(t_{0}\right)} \tag{26}
\end{equation*}
$$

If manoeuvring is not possible in certain periods, the interval $\left[t_{1}, t_{2}\right]$ is found iteratively. After the minimum drift being calculated, $\Delta \lambda_{\text {target }}$ is obtained based on the next node suitable node (eastwards if $\Delta a\left(t_{0}\right)<0$ and westwards if $\left.\Delta a\left(t_{0}\right)>0\right)$. Then $t_{2}$ is increased until $\Delta \lambda_{\text {target }}$ is reached. Note that, if the period between $t_{2}$ and $t_{3}$ coincides with the period when manoeuvring is not possible, (25) is no longer valid. In such circumstances, $\Delta \lambda_{\text {total }}$ is no longer given by (22) and it needs to de adapted appropriately. This case is illustrated in Fig. 4.

### 3.1.3. Minimizing duration of the orbit acquisition phase

The minimum time necessary to achieve a specific node, i.e. a specific $\Delta \lambda_{\text {target }}$, is accomplished by taking fully advantage of the manoeuvring capabilities of the spacecraft to raise or lower the orbit as quickly as possible. A typical semi-major axis profile which minimizes the duration of the orbit acquisition is depicted in Fig. 5. The plateu between $t_{2}$ and $t_{3}$ is due to Delta- V budget limitations.


Fig. 4: Orbit acquisition using constant $\Delta a$ rate of change and natural drift when manoeuvring is not possible for a time period.


Fig. 5: Orbit acquisition with minimum duration using constant rate of change of $\Delta a$. The plateu between $t_{2}$ and $t_{3}$ is due to Delta-V budget limitations.

If there is no period when manoeuvring is not possible, the strategy to achieve the desired drift can determined analytically. The minimum absolute ground track drift is given

$$
\begin{equation*}
\Delta \lambda_{\min }=\Delta \lambda_{t_{1}-t_{0}}+\Delta \lambda_{t_{4}-t_{5}} \tag{27}
\end{equation*}
$$

If $\Delta \lambda_{\text {target }}<\Delta \lambda_{\text {min }}$, the solution is to raise the orbit the maximum allowed by then Delta- $V$ budget and then lower it as quickly as allowed by the spacecraft and ground operations.

The maximum semi-major axis variation is given by [4]

$$
\begin{equation*}
\bar{\Delta} a=2 \sqrt{\frac{a^{3}}{\mu}} \bar{\Delta} v \tag{28}
\end{equation*}
$$

where $\mu$ is and $\bar{\Delta} v$ is the Delta-V budget for orbit acquisition. Let us assume that $\bar{\Delta} a>\left|\Delta a\left(t_{0}\right)\right|$, otherwise there would not be enough available Delta-V to manoeuvre the spacecraft to its nominal orbit. Then, the maximum semi-major axis with respect to the nominal satisfies

$$
\begin{equation*}
\Delta a_{\max }=\frac{\bar{\Delta} a+\Delta a\left(t_{0}\right)}{2} \tag{29}
\end{equation*}
$$

and the minimum semi-major axis with respect to the nominal satisfies

$$
\begin{equation*}
\Delta a_{\min }=-\frac{\bar{\Delta} a-\Delta a\left(t_{0}\right)}{2} . \tag{30}
\end{equation*}
$$

The time instants $t_{2}, t_{3}$, can be easily obtained resorting to a similar reasoning as the one used in Section 3.1.2.

It is straightforward to conclude that, the furthest away that $\Delta \lambda_{\text {target }}$ is from $\Delta \lambda_{\text {min }}$, the longer it takes to reach. Thus, the node that minimises the acquisition time can be found iteratively using a bisection method adapted for discrete functions.

If there are periods when manoeuvring is not possible, the time to achieve a certain node ( $\Delta \lambda_{\text {target }}$ ) needs to be found iteratively. To that end, we define a continuous bijective function $f(\alpha)=\Delta \lambda$. The absolute value of $\alpha$ is the corresponds to the time when the semi-major axis reaches its nominal value subtracted of $t_{1}-t_{2}+t_{4}-t_{3}$ and the time associated with the periods when manoeuvring is not possible. If $\alpha>0, \Delta a(t)$ first raises and then lowers to zero, whereas if $\alpha<0, \Delta a(t)$ first lowers and afterwards raises to zero. Using this function, one obtains the time for each specific node. Then, using a similar method as in the case without constraints on the manoeuvring periods, one can find the optimal target node.

### 3.2. Stepwise semi-major axis manoeuvres

In the previous two algorithms, the semi-major axis changes are approximated by a constant rate of change. In reality, each individual manoeuvre has a short duration in time. The set of algorithms proposed in this section explore the idea of stepwise semi-major axis changes, this is also an approximation but is closer to the reality and allows to take into account more constraints such as the necessity of a calibration manoeuvre and a small last manoeuvre to reduce the associated uncertainty (soft touch-up manoeuvre).

### 3.2.1. Constraints and assumptions

The algorithms proposed in this section assume that the inplane manoeuvres can be approximated by step-wise semimajor axis changes not taking into account eccentricity variations. It also assumes that the inclination is nominal. As main parameters, the algorithms use the maximum and minimum amplitude of the fist and last manoeuvres, the maximum amplitude of the intermediate manoeuvres, and the frequency of the manoeuvres, i.e. what is the minimum period between $\Delta a$ changes. The user can also set a period of time after launch without orbit acquisition manoeuvres, the DeltaV budget available for those manoeuvres, and periods of time when manoeuvring is not possible.

### 3.2.2. Minimizing Delta-V consumption

This algorithm starts by computing the absolute minimum drift, which corresponds to the drift when the orbit is driven to the nominal as quickly as allowed by the spacecraft capabilities. From that, it computes the targeted drift, which corresponds to the next node satisfying (17) or (18) (depending if the aim is to achieve any node or a specific one). The solution
is a set of step-wise $\Delta a$ changes that need to satisfy the manoeuvring constraints as illustrated in Fig. 6. Then, to reach


Fig. 6: Step-wise manoeuvres yielding minimum ground track drift.
the target drift firstly $\Delta a$ at each time is adjusted (Fig. 7). In case, it is not possible to reach the targeted drift only by


Fig. 7: Semi-major axis at each time adjusted to reach the target drift.
changing $\Delta a$, the drift duration needs to be also increased.


Fig. 8: Duration of the drift adjusted to reach the target drift.

The manoeuvring sizing and scheduling taking into account periods without manoeuvres is a non convex optimization problems. Thus, heuristics were devised that provide a suitable solution. The first step is to increase the duration of the drift time associated with the semi-major axis more distant to the nominal one because that is the one which minimises the duration of the acquisition phase (greatest drift per time unit). If one or more conflicts with periods when manoeuvring is not possible occur, the drift time is reduced so that we have no conflicts. The drift time of the second $\Delta a$ steps is then increased until the desired node is reached. If one or more conflicts are detected, the drift time is reduced to avoid then. This process is then repeated for all $\Delta a$ step until the desired node is reached. If after the adjustment of
the last $\Delta a$ step, the target drift has not been reached yet, the size of all manoeuvres are adjusted until the last manoeuvre is performed at a moment when manoeuvring is possible.

### 3.2.3. Minimizing duration of the orbit acquisition phase

This is the most complex case, because both the optimal node and the optimal number of $\Delta a$ steps are not known. The developed algorithm is based on a divide to conquer strategy. Similarly to the algorithm presented in Section 3.2.2, we use a derivative-free method to find the final node that locally minimizes the orbit acquisition duration and, for each final node, another derivative-free method is used to find the optimal number of $\Delta a$ steps. In order to use this approach, an algorithm was devised that computes the sizing and schedule of the manoeuvres for each pair final node and number of $\Delta a$ steps. This function follows a similar approach as in the previous algorithm but it allows that the drift rate is firstly increased before being stopped. A manoeuvring plan using this algorithm is depicted in Fig 9.


Fig. 9: Orbit acquisition with minimum duration using stepwise $\Delta a$ manoeuvres.

To address the case when manoeuvring is not possible, for each pair final node and number of $\Delta a$ steps, a heuristic process was developed. This heuristic process consists in firstly adjust the $\Delta a$ steps to achieve the target drift. When that is not possible, the drift time of the $\Delta a$ step that are furthest from the nominal is increased until the targeted node is reached. If a conflict is detected with the periods when manoeuvring is not possible, the drift of that step is truncated, the duration of the drift of the following step is increased until the targeted node is achieved. If again a conflict occurs, the duration of drift of the following step is truncated and the process is repeated with the subsequent $\Delta a$ steps until the node is reached and there is no conflict.

## 4. INTERFACE

As inputs the tool requires information about the nominal orbit, injection orbit, and mission constraints. The inputs regarding the nominal orbit are: semi-major axis, eccentricity, inclination, repeat cycle, cycle length, reference longitude at ascending node and corresponding epoch. The inputs from the injection orbit are: initial semi-major axis relative
to the nominal, initial inclination relative to the nominal, initial MLST relative to the nominal, and longitude at ascending node of the injection orbit and corresponding epoch. Mission constraints necessary are: the period of time the spacecraft will be drifting before the orbit acquisition manoeuvres, maximum Delta-V, maximum Delta-V per day (for the algorithms that rely on constant change rate of the semi-major axis), maximum and minimum sizes of the calibration and touch-up manoeuvres, and maximum size of the remaining manoeuvres (for the algorithms based on step-wise semi-major axis changes). The user can also specify the final targeted node (e.g. for phasing) or let the tool select the optimal one.

The tool has two modules: i) a single orbit acquisition plan optimiser, and ii) a parametric analyser. The former gives the user a detailed orbit acquisition plan based on the inputs, providing the sizing and scheduling of the manoeuvres that allow to achieve the desired ground track. Information regarding MLST drift, total duration of the acquisition phase, and Delta-V consumption is also provided. The single orbit acquisition plan optimiser also allows to manually modify the schedule and size of each manoeuvres and to add or remove manoeuvres, once the orbit acquisition plan has been computed. The current algorithms do not take into account inclination and MLST correcting manoeuvres. Thus, these also need to be inserted manually. The parametric analyser runs the algorithm for several different conditions (initial relative semi-major axis, launch date, and selected algorithms) and presents the user key information to evaluate each case, such as the final target node, the Delta-V consumption, and the duration of the orbit acquisition phase.

The interface of the single orbit acquisition plan optimiser is depicted in Fig. 10.


Fig. 10: Interface with the single orbit acquisition plan optimiser where the user can input some of the orbital characteristics and mission constraints.

The parametric analyser interface is depicted in Fig. 11. In this interface, the user can select the algorithms that should be run, an interval of initial relative semi-major axis, and an interval of launch dates.

## 5. EXAMPLE

Let us consider the following scenario. We wish to design an orbit acquisition plan for a spacecraft on a Sun-synchronous


Fig. 11: Interface with the parametric analyser.
orbit, with repeat cycle of 27 days and cycle length of 385 orbits. No phasing is necessary, thus, any node can be targeted. The initial relative semi-major is $\Delta a\left(t_{0}\right)=3000 \mathrm{~m}$, the inclination is nominal and manoeuvring is possible 7 days after launch. The equatorial distance to the closest node is 1653 m .

Assuming that the spacecraft can provide a constant Delta-V of $1 \mathrm{~m} / \mathrm{s}$ per day and that the algorithm that minimises the Delta-V consumption using constant rate manoeuvres is selected, the computed acquisition plan has a duration of 9.09 days and a Delta-V consumption of $1.56 \mathrm{~m} / \mathrm{s}$. The relative semi-major axis evolution is depicted in Fig 12. On


Fig. 12: Semi-major axis profile that minimises the orbit acquisition duration using constant Delta-V manoeuvres.
the other hand, the algorithm that minimizes the acquisition duration reaches the target node in 8.49 days with a Delta-V consumption of $3.49 \mathrm{~m} / \mathrm{s}$. The semi-major axis evolution of this plan is shown in Fig 12.

To assess the results using the step-wise algorithms, let us now assume that more detailed information is available. That it is possible to perform one manoeuvre per day, that the maximum Delta-V of the fist and last manoeuvres is 0.3 $\mathrm{m} / \mathrm{s}$, the minimum Delta- V of the first and last manoeuvres is $0.1 \mathrm{~m} / \mathrm{s}$, and the maximum Delta- V of the intermediate manoeuvres is $1 \mathrm{~m} / \mathrm{s}$. The algorithm that minimises the Delta- V consumption resorting to step-wise manoeuvres computes an acquisition plan with a duration of 9.38 days and a Delta-V consumption of $1.56 \mathrm{~m} / \mathrm{s}$. The relative semi-major axis evo-


Fig. 13: Semi-major axis profile that minimises the Delta-V consumption using constant Delta-V manoeuvres.
lution for this case is depicted in Fig 14. The algorithm that


Fig. 14: Semi-major axis profile that minimises the orbit acquisition duration using step-wise manoeuvres.
minimizes the orbit acquisition duration provides the solution if we wish to reach the desired ground track as quickly as possible. In this case the final node is reached in 9.02 days with a Delta-V consumption of $2.16 \mathrm{~m} / \mathrm{s}$. The semi-major axis evolution of this plan is shown in Fig 14. The duration of the plan


Fig. 15: Semi-major axis profile that minimises the Delta-V consumption using step-wise manoeuvres.
is slightly reduced but the Delta-V consumption is necessarily higher.

## 6. CONCLUSIONS

This paper presented a tool and the the algorithms that automatically compute optimised orbit acquisition plans. The user can select the algorithm that minimizes the duration of the orbit acquisition phase or the required Delta-V given the spacecraft characteristics and mission constraints. A perturbation analysis of the nominal orbit is described and used as the basis for the proposed solutions.

Two different strategies were exploited, one based on continuous rate semi-major axis changes and a second based on step-wise semi-major axis changes. The former is suitable for the case when several small manoeuvres can be approximated by a single manoeuvre with constant rate and during the early design phases when detailed information about the manoeuvring constrains is not available. The latter produces a more detailed plan that considers the existence of a first and a last manoeuvres with smaller amplitude than the remaining ones. All algorithms take into account Delta-V limits and the existence of periods when manoeuvring is not possible.

The tool is not intended to produce the final orbit acquisition plan, but to rather provide an initial suitable strategy which can be used by the flight dynamics team as the base to more detailed plan. To that end, the tool allows to manually modify the schedule and size of each manoeuvres and also to add or remove manoeuvres. The developed algorithms do not take into account inclination and MLST correcting manoeuvres. These also need to be inserted manually. By performing a parametric analysis considering several initial dispersions and launch dates, the tool is also suitable to provide information about how the mission constraints influence the acquisition plan during the early design phase of a mission.

Future work will focus on the improvement of the interface with the used and the automatic sizing and scheduling of out-of-plain manoeuvres for inclination correction.

## 7. REFERENCES

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