Low Thrust Trajectory Optimization for Autonomous Asteroid Rendezvous Missions

Anne Schattel, Mitja Echim, Matthias Knauer, Christof Büskens
Optimization and Optimal Control, University of Bremen

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Challenges of Trajectory Optimization for Deep Space Missions

- Huge time scales
- Small control variables (low thrust)
- Scaling issues (e.g. Newton vs. AU)

➢ Highly precise and robust optimization method necessary
Outline

• Challenges of Trajectory Optimization for Deep Space Missions
• Mathematical Background
• Numerical Results
  – Electrically powered (continuous) propulsion system
  – Comparism to chemical (impulsive) thrusters
• Summary and Outlook
Optimal Control Problem (OCP)

How do thrust and attitude have to be controlled to get the system fast and with low fuel consumption from a start point to an orbit without overloading?
Optimal Control Problem (OCP)

\[
\min_{u(t)} \quad \int_0^{t_f} g(x(t), u(t)) \, dt
\]

s.t. \quad \dot{x}(t) = f(x(t), u(t))

\quad x(0) = x_0

\quad \Psi(x(t_f)) = 0

\quad C(x(t), u(t)) \leq 0
Non-linear Optimization Problem (NLP)

- **OCP** → transcription techniques (direct approach) → **NLP**

\[
\begin{align*}
\min_z & \quad F(z) \\
\text{s.t.} & \quad G_i(z) = 0, \quad i = 1, \ldots, M_e \\
& \quad G_i(z) \leq 0, \quad i = M_e + 1, \ldots, M
\end{align*}
\]

- Transcription: full discretization of states and controls
WORHP

- “We Optimize Really Huge Problems”
- Finite-dimensional non-linear optimization software
- Combining SQP and IP methods
WORHP

- “We Optimize Really Huge Problems”
- Finite-dimensional non-linear optimization software
- Combining SQP and IP methods
  - Efficient derivative approximation
  - Considers sparsity of derivative matrices
  - Especially efficient for solving high-dimensional problems like those resulting from discretization of OCPs

- Software library TransWORHP used for transcription
Problem Formulation

Dynamic system:

\[
\dot{x} := \begin{pmatrix}
\dot{p}_{sc} \\
\ddot{p}_{sc} \\
\dot{m}_{sc}
\end{pmatrix} = \left( \sum_{i \in I} \mu_i \cdot \frac{\dot{p}_{sc}}{r_i^3} + \frac{T}{m_{sc}} \right)
\]

- \( p_{sc} \) - position vector of spacecraft
- \( \mu_i, i \in \{sun, mars, jupiter, saturn\} \) - gravitational constant of celestial body
- \( r_i \) - direction vector between spacecraft and body
- \( T \) - thrust vector
- \( m_{sc} \) - spacecraft ‘s recent mass
Impulsive Thrust Optimization

- Thrust: constant control over certain period of time
- Three thrust commands
- Two non-thrust phases in between
- Connecting conditions
Objective Function

- Impulsive thrust: $$F = t_f w - m_f (1 - w), w \in [0, 1]$$
- Low thrust: $$F = t_f w + x_{n,7} (1 - w), w \in [0, 1]$$

- Optimization criterions for competitive mission objectives
  - Flight time
  - Energy consumption

- Spacecraft data
  - Impulsive: ISP 318 sec, max. thrust 440 N, min. thrust 340 N
  - Low thrust: ISP 4000 sec, max. thrust 0.154 N
Low Thrust

![Graphs showing parking orbit, asteroid orbit, and optimal trajectory for two different values of w: 0.2 and 0.8. The graphs depict three-dimensional space with axes x, y, and z in AU.]
Impulsive Thrust

- Parking orbit
- Asteroid orbit
- Optimal trajectory ($w = 0.2$)

- Parking orbit
- Asteroid orbit
- Optimal trajectory ($w = 0.8$)
Low vs. Impulsive Thrust
# Low vs. Impulsive Thrust

<table>
<thead>
<tr>
<th>Thrust</th>
<th>low</th>
<th>impulsive</th>
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<tbody>
<tr>
<td>w</td>
<td>0.2</td>
<td>0.2</td>
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<tr>
<td>Time (d)</td>
<td>1289</td>
<td>308</td>
</tr>
<tr>
<td>Fuel (kg)</td>
<td>149</td>
<td>936</td>
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<table>
<thead>
<tr>
<th>w</th>
<th>0.8</th>
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<tbody>
<tr>
<td>Time (d)</td>
<td>840</td>
<td>88</td>
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<tr>
<td>Fuel (kg)</td>
<td>214</td>
<td>1431</td>
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</table>
Planets Influence

- Parking orbit
- Asteroid orbit
- Optimal trajectory with planets
- Optimal trajectory without planets
# Planets Influence

<table>
<thead>
<tr>
<th>Planets</th>
<th>Thrust</th>
<th>low</th>
<th>impulsive</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>w/</td>
<td>w/o</td>
<td>w/</td>
</tr>
<tr>
<td>Time (d)</td>
<td>1289.23</td>
<td>1319.47</td>
<td>307.53</td>
</tr>
<tr>
<td>Fuel (kg)</td>
<td>148.66</td>
<td>147.38</td>
<td>935.88</td>
</tr>
</tbody>
</table>
Summary and Outlook

• Optimization provides very different trajectories dependent on thrust type and mission objective
• Foundation for autonomous decision making during deep space missions
• Applications like deep sea navigation or autonomous driving
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- Perturbation and parametric sensitivity analysis
- Real-time optimal control
- Multi-node techniques using high-order integration methods
Discussion

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Anne Schattel, Optimization and Optimal Control
Center for Industrial Mathematics (ZeTeM), University of Bremen, 28359 Bremen, Germany
phone: +49-(0)421-218-63867, e-mail: ascha@math.uni-bremen.de