LOW THRUST TRAJECTORY OPTIMIZATION FOR AUTONOMOUS ASTEROID RENDEZVOUS MISSIONS

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ABSTRACT

The demand for deep space missions and the desire to investigate small space objects like asteroids and comets is increasing constantly. Such exploration missions open up the possibility to gain further scientific knowledge about the origin of our solar system as well as to find and eventually mine rare earth elements that are narrow on Earth or even to discover novel resources. To realize such missions, trajectory planning and optimization are of utmost importance. When flying further into outer space, the spatial distances become enormously huge while fuel is very limited and signal runtimes are increasing rapidly. These conditions impose an unmanned and autonomously working spacecraft. The mathematical field of optimization and optimal control provides the foundation for autonomous decisions and facilitates more safety and minimal resource consume. Solutions may additionally be transferred to other, earth-bound applications like e.g. deep sea navigation and autonomous driving with minor additional expenses. The aspects investigated in the present paper focus on the specific challenges of guidance and control regarding the cruise and approach phase of a spacecraft starting in a parking orbit around the Sun and reaching for an asteroid in the main belt. The underlying optimal control problems are solved using so called direct methods also known as transcription techniques. Those transform an infinite-dimensional optimal control problem (OCP) into a finite-dimensional non-linear optimization problem (NLP) via discretization methods. The resulting high dimensional non-linear optimization problems can be solved efficiently by special methods like sequential quadratic programming (SQP) or interior point methods (IP). For solving the problems introduced in this paper the NLP solver WORHP, which stands for 'We Optimize Really Huge Problems', is used, a software routine combining SQP at an outer level and IP to solve underlying quadratic sub problems. Within this paper the transcription is performed using the robust method of full discretization. The trajectory optimization and optimal control problems are modeled and solved using low thrust electric propulsion on the one hand and chemical propulsion on the other hand for comparison. The movement of the spacecraft is described through ordinary differential equations (ODE) considering the gravitational influences of the Sun and the planets Mars, Jupiter and Saturn as well as the different thrust commands. Competitive mission aims like short flight times and low energy consumption can be provided with a weighting factor within the optimization process. The varying challenges of the two propulsion types are analyzed and comparative solutions and results introduced. Several mission trajectories are compared, optimizing with different weighting factors for energy cost and flight time duration, in order to investigate the different possibilities of an asteroid rendezvous mission. The results show the huge gain of trajectory optimization as input for on-board autonomous decision making during deep space missions as well as the great increase in possibilities for flight maneuvers by providing solutions for changing and contradictory mission objectives. Furthermore, trajectory optimization can be used to analyze the potentials of different propulsion systems beforehand.

***Index Terms*—** guidance, low thrust, non-linear optimization, optimal control, trajectory planning

1. INTRODUCTION

Invading deeper into space and exploring the structure of asteroids, which exist since the emergence of our universe, may reveal great insight into the early history of our solar system and help understand its formation and evolution. Additionally, finite resources on earth keep to be wasted, and the finding of other, extraterrestrial sources for these materials or even of new ones becomes of increasing interest. The latest deep space explorations have shown that rare earth elements as well as materials like iron, nickel, cobalt, methane, water, and ammonia can be found in many asteroids [1]. In addition to bringing goods back to earth, some of them, like water or ice, could be converted into fuel and help expand deep space missions or could be used as oxygen and drink water supply for space stations, thus providing necessaries for the expan-

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sion of space infrastructure. Therefore, there is a great interest in deep space explorer missions that provide the opportunity to obtain detailed knowledge about the composition of asteroids. Due to the large spatial distances and flight times, such missions have to be performed unmanned and autonomously.

This paper describes the solution of a low thrust trajectory optimization problem, starting at a parking orbit at 2.8 AU around the Sun and aiming at an asteroid in the main belt. For comparison, we formulate and solve an analogous optimization problem regarding an impulsive thrust trajectory. In contrast to applications in other disciplines, long time frames and badly scaled problems have to be considered. The system dynamics incorporate gravitational perturbations from the Sun as well as the planets Mars, Jupiter, and Saturn. Furthermore, the method we present can be expanded for other significant model forces like the gravitational influence of the asteroid, which becomes of interest during near-asteroid operation phases of a mission, without much effort. We implement high-precision methods of large sparse non-linear optimization in conjunction with a transcription method using full discretization to approximate the ordinary differential equations, providing a highly robust method. The algorithms we present aim at the support of an autonomous optimal trajectory planning scenario for a space mission, but could easily be adapted for a wide range of non-linear dynamic systems concerning on-earth applications like deep sea explorations or autonomous driving.

The results we present were generated within the context of the project “KaNaRiA” [2], which develops key technologies for the realization of a cognitive autonomous deep space navigation on the example of an asteroid mining mission, including the cruise phase and approach as well as the asteroid rendezvous, landing, and surface exploration. Here, dynamic decisions in complex situations are required. To verify and test all methods, an interactive, real-time capable simulation system using virtual reality for visualization and allowing for various variations in mission parameter configurations is developed. For the best of our knowledge, so fare no autonomous on board planning of optimal trajectories has been performed during a space mission. Recently, all trajectories are preplanned from Earth.

2. MATHEMATICAL BASIS

An optimal control problem is of the form

$$\min_{x,u} \; F(x,u,t) := g(x(t_f),t_f) + \int_0^{t_f} f_0(x(t),u(t),t)dt \tag{1}$$

$$\begin{aligned} s.t. \qquad \dot{x}(t) &= f(x(t),u(t),t) \\ x(0) &= x_0 \\ \Psi(x_0, x(t_f)) &= 0 \\ C(x(t),u(t),t) &\leq 0. \end{aligned}$$

Herein, x describes the state of the system, and f defines the dynamic system, which can be influenced via the control u. Further, g is a continuously differentiable, and f_0 a continuously and with respect to x and u continuously partial differentiable function. The control u has to be chosen in such a way that the constraints C as well as the initial and final conditions Ψ are fulfilled while minimizing the objective functional F. Often, optimal control problems are intended to guide an initial state $x(t_0)$ on a trajectory into a final state $x(t_f)$. Both, the initial and the final state, can be left open or be defined, which is specified by the boundary conditions Ψ. Additionally, the state and the control of the system can be constrained in each time point by C. A solution is called feasible, if it fulfills the system and meets the boundary conditions as well as the state constraints and if the control respects a given range.

To solve Eq. (1), there exist two techniques, so called indirect and direct methods [3]. The former are based on the evaluation of the necessary optimality conditions and generally lead to a multipoint boundary value problem that has to be solved numerically. These methods need in-depth knowledge of optimal control theory and therefore they are not suitable for complex, application oriented optimal control problems. In this work, we choose a direct approach, which is based on the discretization of the infinite dimensional optimal control problem (Eq. (1)). For this transcription, and therefore to solve the ODE system, in the presented algorithms we implemented the robust method of full discretization. Herein, all states and controls are calculated for a chosen number of discrete time points. The resulting high-dimensional optimization problems of the form

$$\begin{aligned} \min_{z} \quad & F(z) \\ s.t. \quad & G_i(z) = 0, \; i = 1, ..., M_e \\ & G_i(z) \leq 0, \; i = M_{e+1}, ..., M \end{aligned} \tag{2}$$

are solved by the NLP solver WORHP [4], which provides high-precision methods of mathematical non-linear optimization using sequential quadratic programming and interior point methods. By using the sparsity of the derivative matrices (Hessian, Jacobian, Gradient), it is especially efficient for solving large-scale problems like those resulting from the discretization of optimal control problems. Thus problems with more than one billion optimization variables and two billion constraints can be solved. For the transcription, we use the software library TransWORHP, which provides implicit integration methods. The derivatives and derivative structures of the defining functions can be provided by the user or be determined automatically. As many practical problems run in stages, TransWORHP supports the declaration and the connection of multiple phases. This can be used to optimize trajectories with changes in the dynamical behavior or so called bilevel optimal control problems, where one trajectory is optimized subject to the existence of auxiliary trajectories.

2.1. Orbit Mechanics

The fundamentals of orbit mechanics can be found in [5]. The orbit of a celestial body in the gravity field of a central body is defined uniquely via six independent parameters. The eccentricity ϵ and the semi-major axis a determine the orbit's form, the inclination i, the longitude of the ascending node Ω, and the argument of periapsis ω define its position in space relative to a reference system, e.g. the ecliptic, and the time-dependent parameter mean anomaly M describes the actual position of the body on the orbit at a specific time. Starting with a mean anomaly M_0 for a certain orbit, the position after a time t can be calculated via

$$M = M_0 + \sqrt{\frac{\mu_c}{a^3}} \cdot t, \tag{3}$$

with μ_c being the gravitational constant of the central body, and a being the semi-major axis of the orbiting body.

To model the movement of celestial bodies on their orbits, Kepler's equation, which is based on Kepler's laws of planetary motion, is needed, that states

$$M = E - \epsilon \cdot \sin E. \tag{4}$$

To solve Kepler's equation, $E \in [0, 2\pi]$ has to be determined for $M \in [0, \pi]$, $\epsilon \in [0, 1]$. Besides Kepler's solution, there exists a number of ways to solve this equation, e.g. iterative methods like Newton's method and Halley's method. We use the latter, in which the root of the function

$$f(x) = x - \epsilon \sin x - M = 0 \tag{5}$$

has to be found. This leads to the iteration

$$x_{i+1} = x_i - \frac{x_i - \epsilon \sin x_i - M}{1 - \epsilon \cos x_i}. \tag{6}$$

Since Kepler's equation needs to be solved very often during optimization, a lower number of iterations is desirable. In [6], an initial estimation of

$$x_0 = M + \frac{\epsilon \sin M}{1 - \sin(M + \epsilon) + \sin M} \tag{7}$$

is given, which reduces the average number of iterations the most and which we use for the results presented in Sect. 4.

2.2. Hohmann Transfer Orbit

There exist several ways to bring a spacecraft from one orbit to another. The Hohmann transfer orbit [5] is a relative simple way to calculate an orbit transfer, but it is only applicable under strong constraints. I.e., the two orbits must be circular and lie in the same plane, but may differ in their radius, and it is not possible to define side constraints like the gravitational influence of a further body. A tangential impulse is applied in the perihelion of the first orbit, which brings the spacecraft on an elliptical transfer orbit. With another tangential impulse in the aphelion of the ellipse, the second orbit is reached. By using these two thrusts, this transfer is energy minimal.

3. PROBLEM FORMULATION

In the following, we define the dynamic system and the objective function according to Sect. 2 for our concrete problem formulation. The parking orbit, from which the spacecraft trajectories start, is located within the asteroid main belt at 2.8 AU around the Sun.

The dynamic system of the optimal control problem (Eq. (1)) describes the movement of the spacecraft due to gravitational influences of the Sun, Mars, Jupiter, and Saturn as well as the thrust commands through the ordinary differential equations

$$\dot{x} := \begin{pmatrix} \dot{p}_{sc} \\ \ddot{p}_{sc} \\ \dot{m}_{sc} \end{pmatrix} = \begin{pmatrix} \dot{p}_{sc} \\ \sum_{i \in I} \mu_i \frac{r_i}{\|r_i\|_2^3} + \frac{T}{m_{sc}} \\ -\frac{\|T\|}{g_0 I_{sp}} \end{pmatrix}. \tag{8}$$

In the ODE system, p_{sc} describes the position vector of the spacecraft, μ_i, $i \in \{sun, mars, jupiter, saturn\}$ the gravitational constant of the according celestial body, and r_i the direction vector between spacecraft and body, $T = u(t) = (u_0(t)\ u_1(t)\ u_2(t))^T$ defines the thrust vector, m_{sc} the spacecraft's recent mass, I_{sp} its specific impulse in seconds, and g_0 the gravitational constant of Earth. For stability reasons within the optimization, we define an eighth equation

$$\dot{x}_7 = \|T\|^2. \tag{9}$$

We define the thrust vector T in a spacecraft fixed reference frame. The main direction d_0 of the thrust vector is diametrically opposed to the main thruster and therefore corresponds to the direction of the spacecraft's velocity vector $\dot{p}_{sc}$. The second direction d_1 is perpendicular to the recent orbital plane of the spacecraft, spanned by the velocity vector $\dot{p}_{sc}$ and the position vector p_{sc}, and the third direction d_2 lies in the orbital plane, perpendicular to the first two directions:

$$d_0 = \frac{\dot{p}_{sc}}{\|\dot{p}_{sc}\|}, \quad d_1 = \frac{\frac{p_{sc}}{\|p_{sc}\|} \times d_0}{\left\| \frac{p_{sc}}{\|p_{sc}\|} \times d_0 \right\|}, \quad d_2 = d_0 \times d_1 \tag{10}$$

We define the objective function as

$$F = w t_f - m_f (1 - w), \tag{11}$$

wherein t_f is the total flight time and m_f the spacecraft's final mass, which considers the competitive overall mission objectives to minimize the fuel consumption and flight duration during transfer via the weighting factor $w \in [0, 1]$. Note that the energy demand is considered in terms of fuel consumption, as these two are equivalent or at least proportional for most of the propulsion systems.

4. NUMERICAL RESULTS

At first, we define some spacecraft data and optimization features. The spacecraft's start mass for each transfer is 3000 kg, its low mass 1500 kg. To calculate optimal trajectories using an electrically powered propulsion system, the I_{sp} of the spacecraft is set to 4000 seconds with a maximal thrust of 0.154 Newton. By contrast, for using impulsive thrust the I_{sp} is set to 318 seconds with a thrust magnitude limited to 340 to 440 Newton.

Within the optimization, we calculate the orbital elements of the destination asteroid by analytically solving Kepler's equation (Eq. (4)). The boundary condition is to meet the position and velocity of the asteroid within a certain range sufficient for a cruise phase. We provide the analytical derivatives of the objective function, the ODE system, the constraints, and the boundary conditions. Hereby, the numerical problems of finite differences can be avoided, which usually produce numerical inaccuracies that spread within the optimization process. Since WORHP produces quadratic problems based on the Hessian matrix of the Lagrangian function, the second derivative is needed as well. Here, we use an estimation by finite differences. Additionally, we give the structure of all derivative matrices within the implementation.

As we use iterative methods, it is necessary to supply an initial guess for the problem formulation, in this case the NLP variables as well as the grid distribution. The quality of the initial estimation has a big influence on the convergence velocity as well as the quality of the solution. Since orbit transfers are known to often have many locally optimal solutions, the solution is strongly dependent on the initial estimation chosen for the optimization. To give an estimate of the spacecraft's position, velocity, and mass at each discrete time point for low thrust trajectories is a very complex and challenging task. Here, we use a very simple approximation by assuming zero thrust accelerations, where the mean anomaly $\tilde{M}_i$ for the i-th discrete time point, with $i \in \{0, ..., dis-1\}$, is calculated according to Eq. (3) via

$$\tilde{M}_i = M_0 + \sqrt{\frac{\mu_c}{a^3}} \cdot \frac{i}{dis-1} \cdot t_f, \tag{12}$$

where t_f is the total flight time. The robustness of the full discretization provides the advantage that this simple approximation is sufficient as initial guess. For the grid of discrete time points, we choose an equidistant distribution.

4.1. Low Thrust Trajectories

We compare solution trajectories aiming at the same asteroid using different weighting factors w within the objective function Eq. (11). The destination is asteroid 42937 (1999 TU28) with $a = 2.737\,AU$, $\epsilon = 0.076$, $i = 1.669°$, $\Omega = 354.562°$, $\omega = 137.914°$, and $M = 9.292°$.[1] For the discretization of the optimal control problem, a grid point number of 81 points is chosen. Figs. 1 to 4 illustrate four optimal trajectories under low thrust using the weighting factors $w = 0.2$,

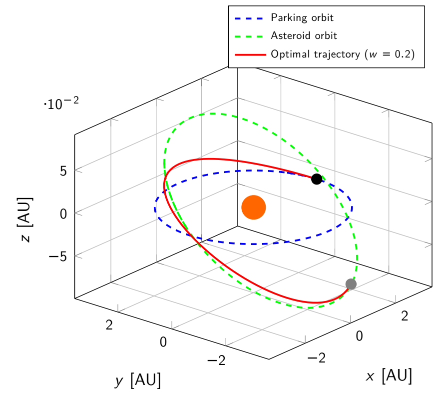

Fig. 1. Optimal trajectory using low thrust for weighting factor $w = 0.2$ according to Eq. (11), defining priority between fuel- and time-optimality. Black dot denotes start point on parking orbit, gray dot denotes final position of asteroid. Note the different scaling of the z-axis for illustration purposes.

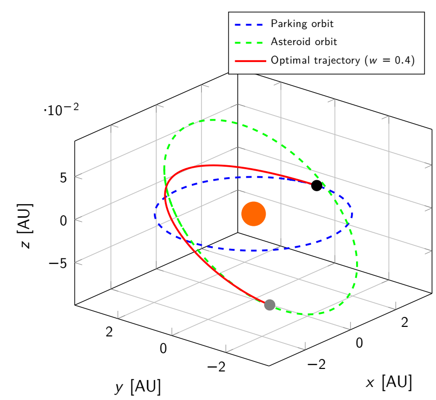

Fig. 2. Optimal trajectory using low thrust with weighting factor $w = 0.4$ according to Eq. (11). (Detailed description see Fig. 1.)

$w = 0.4$, $w = 0.6$, and $w = 0.8$, which increase the weight of time-optimality while decreasing that of fuel-optimality. We observe that the total mission duration is reduced constantly, while there is a faster change in the inclination i of the solution trajectory. The latter causes a higher fuel consumption, while ensuring a more rapid adoption of the asteroid's orbital course. In Tab. (1), the results for the four different weighting factors w are compared in values. It can easily be seen that the flight time is reduced constantly, while the fuel consumption is constantly increased. On the whole, changing w from 0.2 to 0.8, the flight time decreases from 1289 to 840 days, a reduction of 449 days or 34,8%, while the fuel that is needed increases from 149 to 214 kg, an increment of 65 kg or 43.6%.

[1] Taken from the Jet Propulsion Laboratory (JPL) Small-Body Database, URL: http://ssd.jpl.nasa.gov/sbdb.cgi.

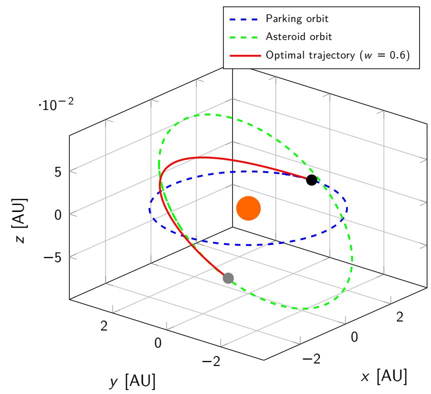

Fig. 3. Optimal trajectory using low thrust with weighting factor $w = 0.6$ according to Eq. (11). (Detailed description see Fig. 1.)

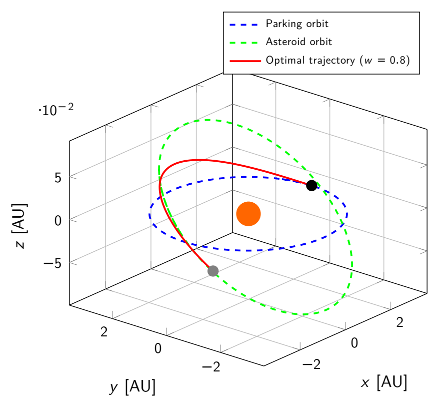

Fig. 4. Optimal trajectory using low thrust with weighting factor $w = 0.8$ according to Eq. (11). (Detailed description see Fig. 1.)

Table 1. Comparison of flight time in days and fuel consumption in kg using low thrust for different weighting factors w according to Eq. (11) (see Figs. 1 – 4).

w	0.2	0.4	0.6	0.8
flight time (d)	1289	981	883	840
fuel consumption (kg)	149	173	193	214

4.2. Comparison to Impulsive Thrust Trajectories

In accordance to the restrictions of a Hohmann transfer orbit described in Sect. 2.2, we developed an application-adapted model for impulsive thrust optimization. This implementation allows for a transfer calculation between elliptical orbits with deviating inclinations while considering further influences like the gravitational influences of the planets Mars, Jupiter, and Saturn. To integrate the discrete instantaneous impulsive thrust commands in the continuous problem formulation more appropriately, we model an impulsive thrust command as continuous but constant in relation to the spacecraft fixed reference frame (Eq. (10)) over a certain period of time. For a cruise phase, a maximum of three impulsive thrust commands may be applied, one at the beginning of a transfer trajectory, one at the end and one at an optimized time point in between. If a command is not needed to achieve an optimal solution, the time frame of the respective phase is set to zero during optimization. The three commands are sufficient regarding the long time frame of the flight without greater perturbation forces. Additionally, it takes longer to perform a multiple burn transfer [7]. The first thrust control is restricted by boundary conditions, i.e., the initial state of the spacecraft, the last one is restricted by the final conditions. The second thrust control has no further constraints and therefore allows for enough freedom in thrusting. The phases have to be connected by additional constraints at the phase boundaries to ensure continuity in the state across the phase boundaries. For each of these phases the system's dynamics are applied.

Again, we compare four solution trajectories, aiming at the same asteroid and using the same weighting factors w as defined in Sect. 4.1. For the discretization of the optimal control problem, a grid point number of 5 times 21 equidistant points was chosen. The solutions can be seen in Figs. 5 to 8 and Tab. (2). While we increase the weighting factor w from 0.2 over 0.4 and 0.6 to 0.8, the length of the trajectory is again reduced constantly due to a faster change in the inclination i, leading to a higher consumption of fuel. With 936 kg of fuel and a total flight time of 308 days, the trajectory regarding the weighting factor $w = 0.2$ needs 495 kg or 52.9% of fuel less but 220 days or 71.4% longer than the trajectory for $w = 0.8$.

When we now look at Figs. 1 to 4 compared against Figs. 5 to 8, the most obvious difference is the much longer transfer duration using low thrust. On the other hand, when we compare Tab. (1) to Tab. (2), it is immediately noticeable that the low thrust trajectories need fare less fuel and therefore mass, the most valuable and expensive good in deep space. In this example, the mostly time optimal low thrust trajectory ($w = 0.8$) needs 752 days longer, but whole 1217 kg less fuel, while the mostly energy optimal trajectory ($w = 0.2$) using impulsive thrust still needs 787 kg of fuel more than using low thrust. It has to be taken into account that by optimizing the start point of the spacecraft on the orbit, the long time frames using low thrust may be reduced.

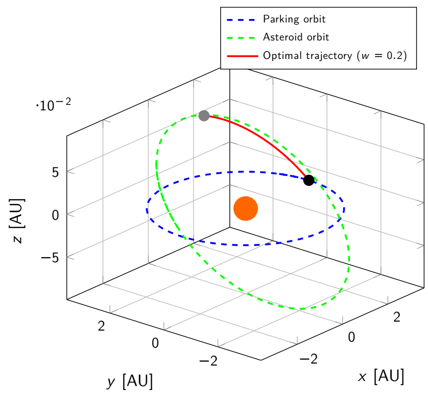

Fig. 5. Optimal trajectory using impulsive thrust for weighting factor $w = 0.2$ according to Eq. (11), defining priority between fuel- and time-optimality. Black dot denotes start point on parking orbit, gray dot denotes final position of asteroid. Note the different scaling of the z-axis for illustration purposes.

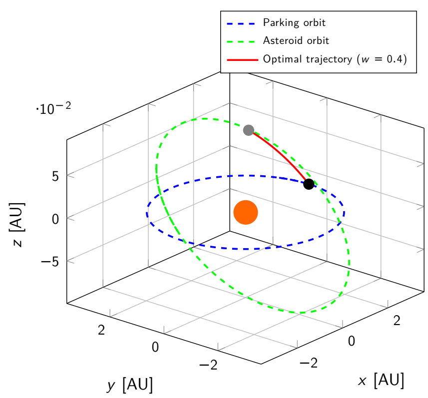

Fig. 6. Optimal trajectory using impulsive thrust with weighting factor $w = 0.4$ according to Eq. (11). (Detailed description see Fig. 5.)

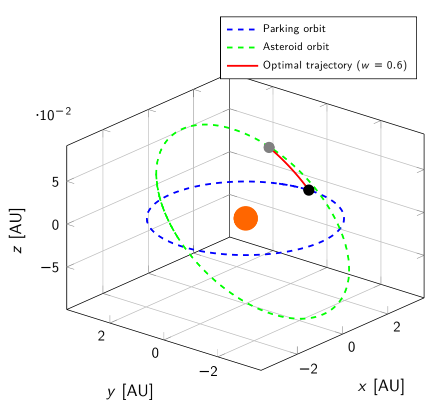

Fig. 7. Optimal trajectory using impulsive thrust with weighting factor $w = 0.6$ according to Eq. (11). (Detailed description see Fig. 5.)

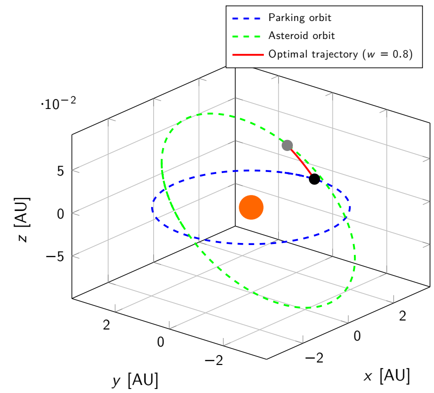

Fig. 8. Optimal trajectory using impulsive thrust with weighting factor $w = 0.8$ according to Eq. (11). (Detailed description see Fig. 5.)

Table 2. Comparison of flight time in days and fuel consumption in kg using impulsive thrust for different weighting factors w according to Eq. (11) (see Figs. 5 – 8).

w	0.2	0.4	0.6	0.8
flight time (d)	308	181	125	88
fuel consumption (kg)	936	1070	1207	1431

4.3. Planets' Gravitational Influence on Transfer Trajectories

Since our trajectory optimization approach provides the means to consider the gravitational influence of planets, we analyze its impact on the previously calculated optimal transfer trajectories using low thrust as well as impulsive thrust. The planets Mars, Jupiter, and Saturn are considered due to their distance-related relevance. The results presented in Figs. 9 and 10 and Tab. (3) show first and foremost that their gravitational influence has a relatively strong impact on the low thrust trajectory, for which in fact only 1.28 kg more fuel are needed, but which duration is shortened by 30.24 days. In contrast, the impact on the impulsive thrust trajectory only resides in a saving of 0.75 kg of fuel and a gain of 0.1 days of flight time. These results are owed to the total transfer

Table 3. Planets' gravitational influence on flight time in days and fuel consumption in kg for low and impulsive thrust (see Figs. 9 – 10).

	flight time (d)	fuel cons. (kg)
low thrust w/ planets	1289.23	148.66
low thrust w/o planets	1319.47	147.38
impulsive w/ planets	307.53	935.88
impulsive w/o planets	307.43	936.63

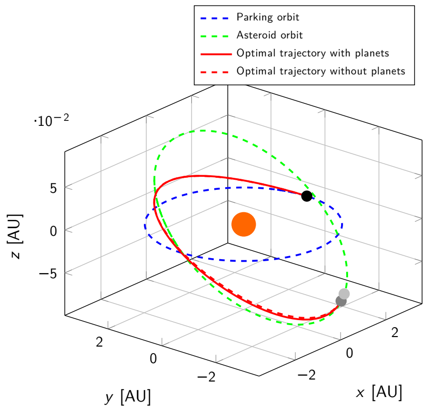

Fig. 9. Influence of planets' gravitation (Mars, Jupiter, Saturn) on optimization results using low thrust with $w = 0.2$ according to Eq. (11). Black dot denotes start point on parking orbit, gray dot final position of asteroid considering planets' gravitational influence, light gray dot final position without planets' gravitational influence. Note the different scaling of the z-axis for illustration purposes.

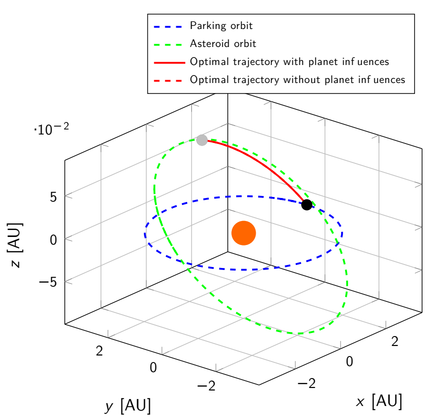

Fig. 10. Influence of planets' gravitation (Mars, Jupiter, Saturn) on optimization results using impulsive thrust with $w = 0.2$ according to Eq. (11). (Detailed description see Fig. 9.)

duration. For low thrust trajectories, which take relatively long to perform, the planets' gravitational influence makes a difference, whereas it is not that relevant for the relatively short flight times of impulsive transfer trajectories.

5. CONCLUSION

In this paper, we presented an optimization approach for low thrust long-distance trajectories. Additionally, we formulated the corresponding optimization problem for impulsive thrust trajectories. Each problem formulation allows for any desired weighting between the mission objectives minimizing the flight time and energy consumption and is able to consider additional conditions like the gravitational influence of planets. First, the solution trajectories for both thrust types showed strong differences according to the chosen objective priorities. This makes it possible to save significant mission time or fuel according to the mission's needs and allows for various and considerably different decisions within an autonomously working spacecraft system during deep space missions. Simultaneously, different and even opposite mission objectives can be considered. Secondly, the trajectory analysis showed the major advantage of using low thrust engines in contrast to impulsive ones for deep space missions. The enormous reduction of fuel demand and therefore payload mass to perform a mission on the one hand crucially reduces its cost, while on the other hand it raises the capabilities for long term space missions. Furthermore, the algorithms we developed may be adapted for earth-bound applications like deep sea navigation or autonomous driving with minor expenses.

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