Assessing Orbit Determination Requirement With Unscented Transformation:
Case Study Of A Lunar CubeSat Mission

Dr. Sun Hur-Diaz
Dr. Ravishankar Mathur

6th International Conference on Astrodynamics Tools and Techniques (ICATT)
14-17 March 2016

Emergent Space Technologies, Inc.
6411 Ivy Lane, Suite 303 • Greenbelt MD, 20770 • - 301 345-1535 • FAX - 301 345-1553
http://www.emergentspace.com

Outline

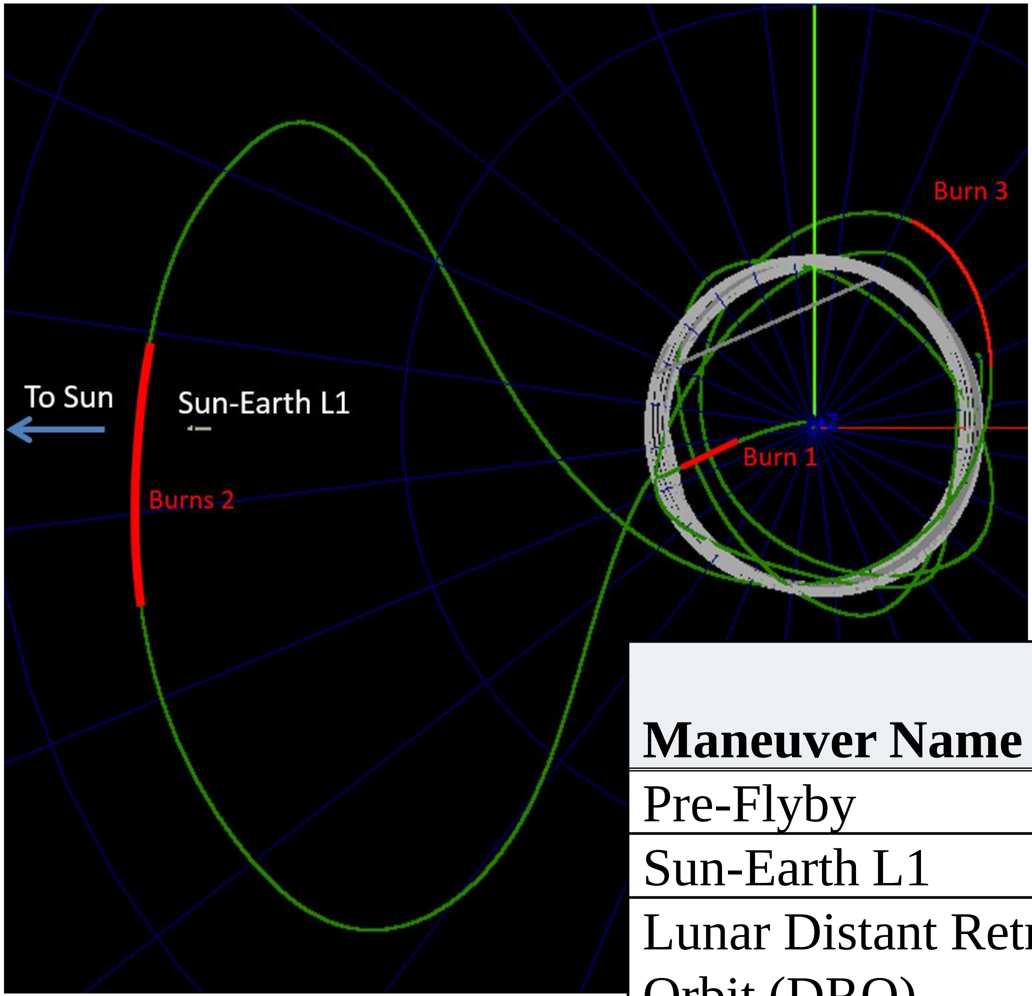
- Motivation –
  - Effect of OD error on delta-V budget of a Lunar CubeSat Mission
- Methods
  - Monte Carlo
  - Linear Covariance
  - Unscented Transformation
- Simulation and Results

CubeQuest Challenge

- One of NASA’s Centennial Challenges
- Winning 6U CubeSats in the competition are offered a launch on the Exploration Mission (EM) 1 mission as secondary payloads

- All such CubeSats will be disposed into a high-energy trajectory that will fly by the Moon
- Most CubeSate will use some form of a low-thrust propulsion system to achieve lunar orbit
- In order to determine the OD strategy for such a mission, the OD accuracy requirement needs to be understood

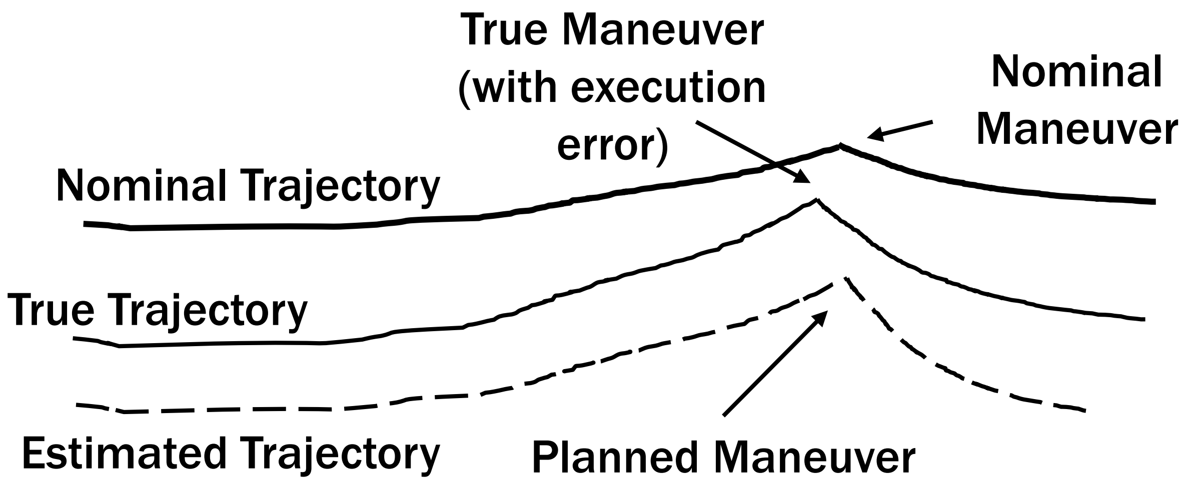
Nominal Lunar Transfer Trajectory



Maneuver Name	Delta-V (m/s)	Duration (days)
Pre-Flyby	14.2	2.3
Sun-Earth L1	215.9	35.0
Lunar Distant Retrograde Orbit (DRO)	43.2	7.0

Maneuver Change from OD Error

- There are many sources contributing to changes in the nominal maneuver



- In this paper, we focus on the OD error contribution to the maneuver change

$$\delta(\Delta V)_{OD\ Error} = \text{Planned Maneuver} - \text{Nominal Maneuver}$$

Maneuver Function

- **Nonlinear function utilizing NASA GSFC's open source General Mission Analysis Tool (GMAT)**

$$u = f(x)$$

- **Goal is to determine the maneuver variation due to OD state estimate uncertainty**

$$P_{uu} = E\left[(u - \bar{u})(u - \bar{u})^T\right]$$

Monte Carlo (MC) Method

- Considered most accurate if large number of cases are simulated
- Perform N cases of OD simulation
- Compute the i-th maneuver corresponding to each OD solution
- Compute the covariance of the resulting N planned maneuvers

$$\bar{u}(i) = \frac{\sum_{j=1}^{N} u_j(i)}{N}$$

$$P_{uu}(i) = \frac{\sum_{j=1}^{N} (u(i) - \bar{u}(i))(u(i) - \bar{u}(i))^T}{N}$$

Linear Covariance (LC) Method

- Linear transformation of the state estimate uncertainty into the variation in the *i*-th maneuver

$$P_{xx} \rightarrow P_{uu}(i)$$

- Requires computing the sensitivity matrix (Jacobian) of the maneuver relative to the OD state estimate

$$P_{uu}(i) = \frac{\partial(u_i)}{\partial(\hat{x})} P_{xx} \left(\frac{\partial(u_i)}{\partial(\hat{x})} \right)^T$$

- The maneuver function "Black Box" is highly nonlinear with no analytic expression so numerical difference:

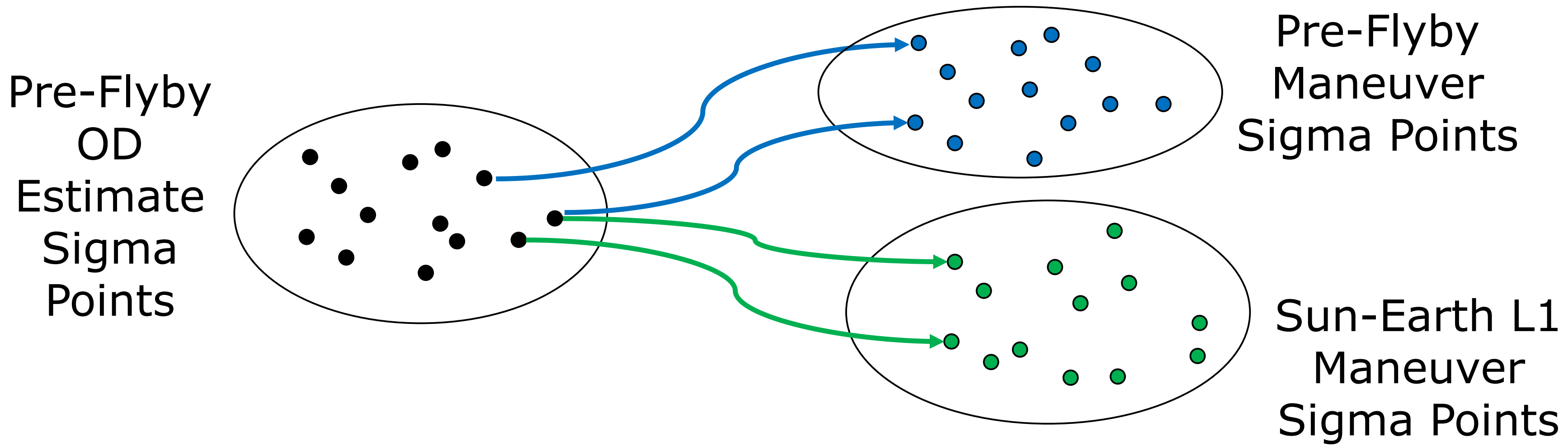
$$\frac{\partial(u_i)}{\partial(\hat{x})}(:, j) \cong \frac{u(i, j) - u(i)_{NOM}}{\hat{x}_j - \hat{x}_{j\,NOM}}$$

Unscented Transformation (UT)

- $2L+1$ **sigma points are generated that statistically represent the OD estimates, where** L **is the number of states**

$$\chi = \left[\bar{x} \quad \bar{x} \pm \left(\sqrt{(L+\lambda)P_j} \right) \right] \quad \text{for} \quad j = \{1, \ldots, L\}$$

$$\lambda = \alpha^2 (L+\kappa) - L$$



UT Maneuver Variation

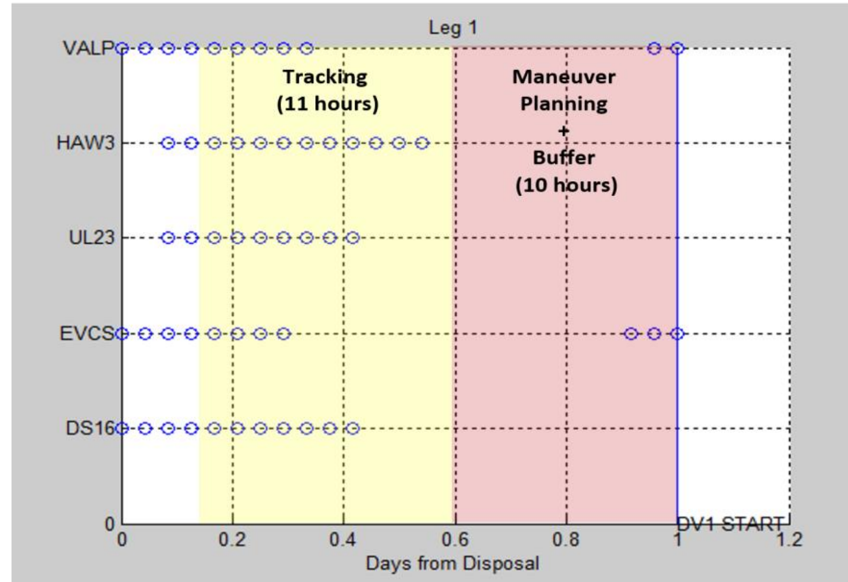
- Weights associated with the OD estimate sigma points are applied to the maneuver sigma points to form the maneuver variation

$$W_0^m = \frac{\lambda}{L+\lambda} \qquad W_0^c = \frac{\lambda}{L+\lambda} + (1-\alpha^2+\beta) \qquad W_k^m = W_k^c = \frac{1}{2(L+\lambda)}$$

$$\bar{u}_i = \sum_{k=0}^{2L} W_k^m u_{i,k}$$

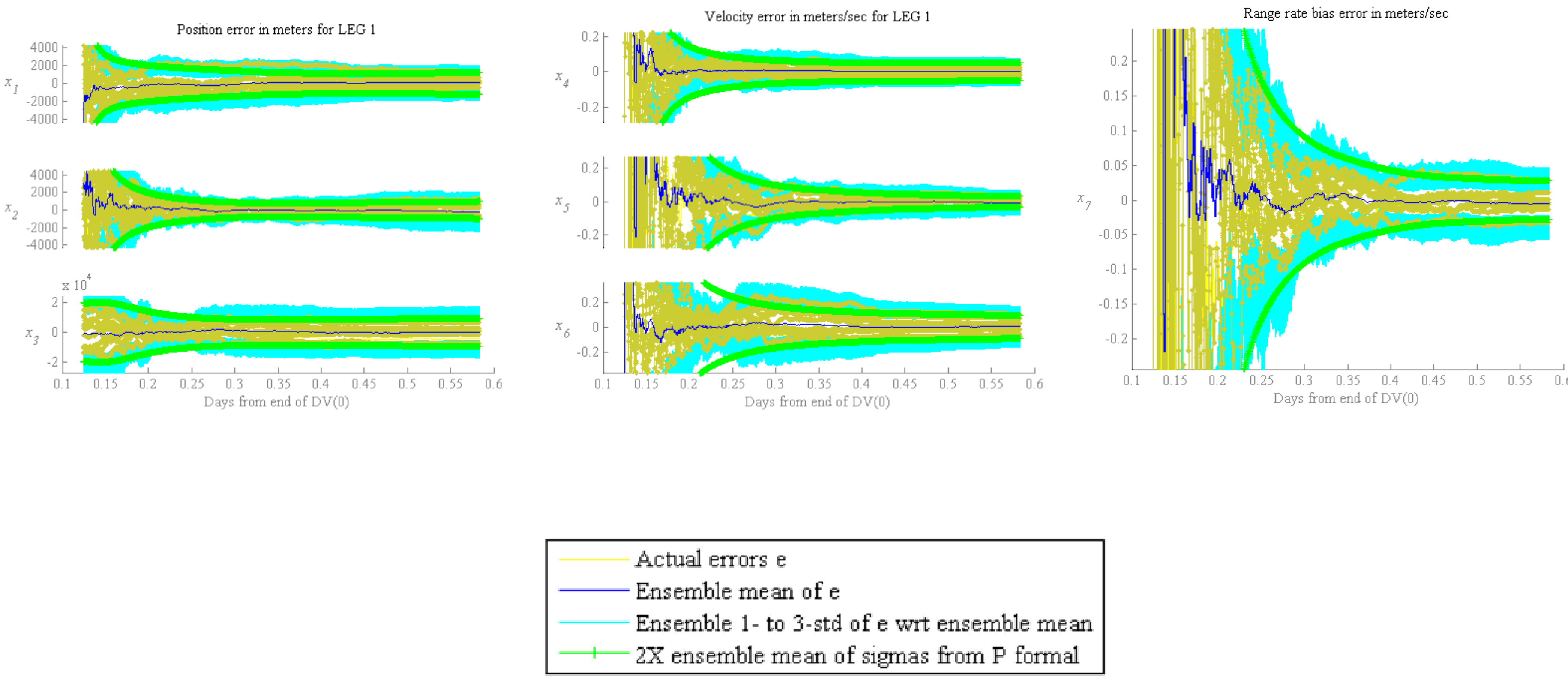
$$(P_{uu})_i = \sum_{k=0}^{2L} W_k^c \left(u_{i,k} - \bar{u}_i\right)\left(u_{i,k} - \bar{u}_i\right)^T$$

Pre-Flyby Orbit Determination Setup



- 5 US ground stations
- One-way range rate measurements – 15 mm/s (1σ)
- Range rate bias – 1 km/s
- 11 hours of tracking @ 1 minute intervals
- Extended Kalman Filter - solve for position, velocity, range rate bias
    - NASA GSFC's OD Tool Box (*estseq* function)

10-Case Monte Carlo Simulation



OD Estimate

- Formal covariance matrix obtained from the Pre-Flyby OD

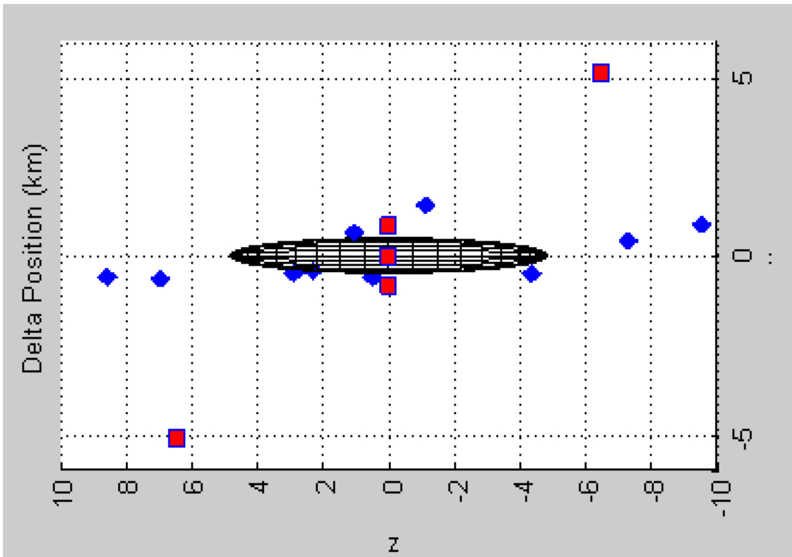
$$P_{xx}(1) = \begin{bmatrix} 0.4 & .07 & 0.6 & 9\times10^{-6} & 3\times10^{-6} & 7\times10^{-6} \\ & 0.2 & -1 & -3\times10^{-6} & 7\times10^{-6} & -1\times10^{-5} \\ & & 20 & 1\times10^{-4} & -4\times10^{-5} & 2\times10^{-4} \\ & & & 6\times10^{-10} & -1\times10^{-10} & 9\times10^{-10} \\ & & \cdot & & & -3\times10^{-10} \\ & \cdot & & & 2\times10^{-10} & \\ \cdot & & & & & 2\times10^{-9} \end{bmatrix}$$

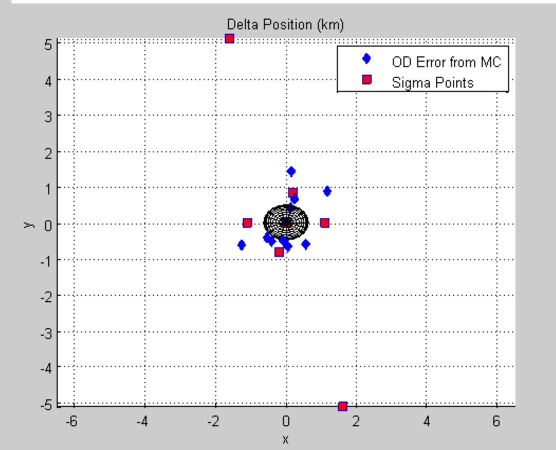
- 1-σ uncertainties

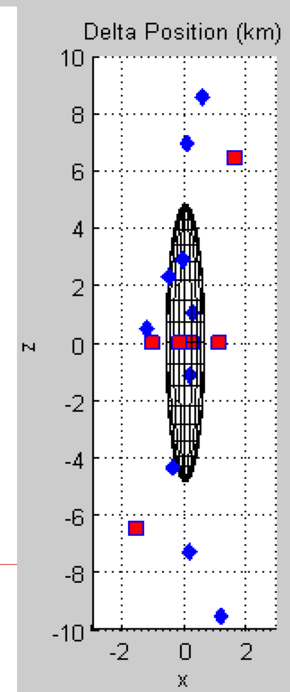
δx = [0.63   0.49   4.9] km

δv = [25   15   47] mm/s

Position Sigma Points & OD Estimates



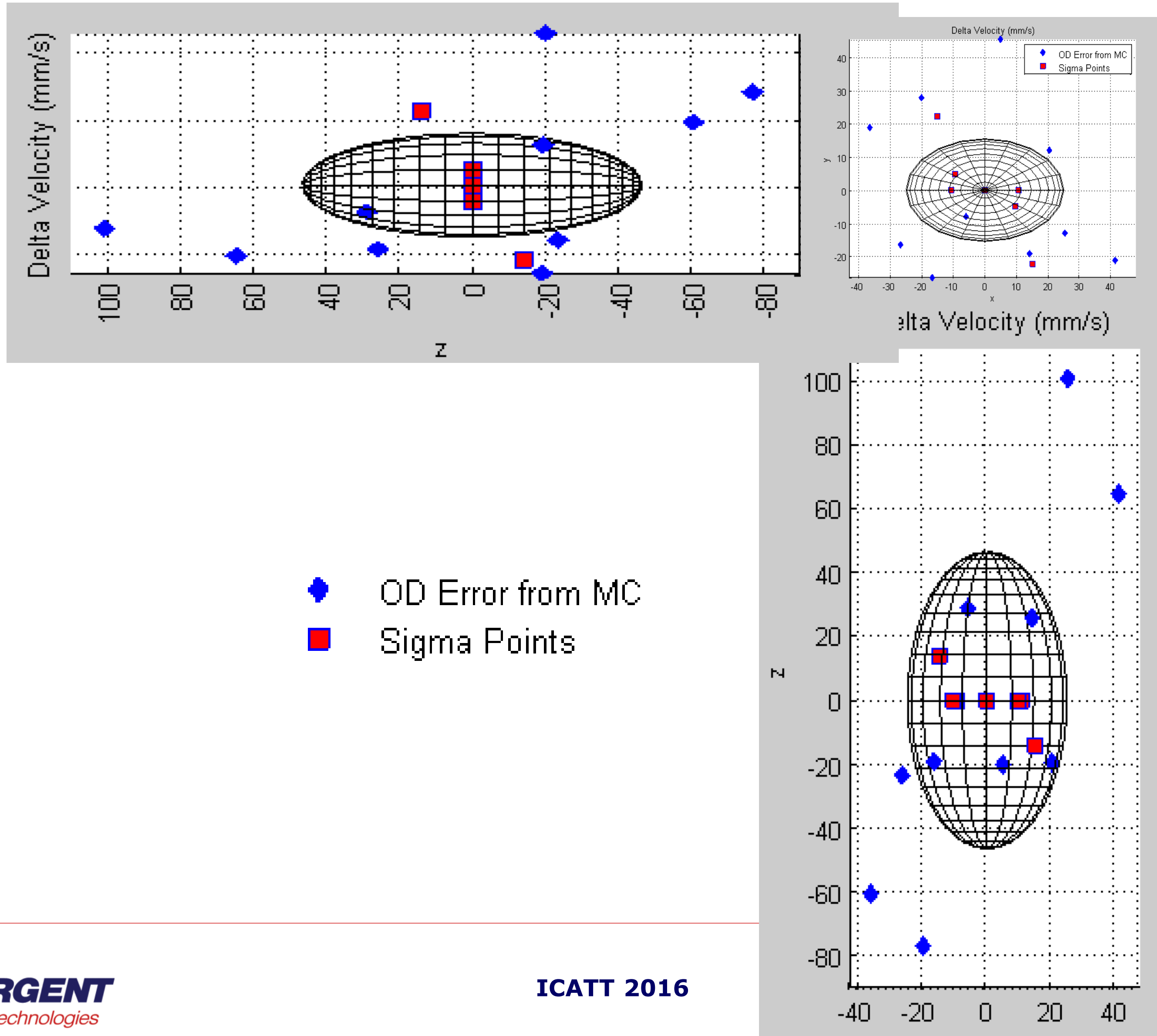




OD Error from MC

Sigma Points

Velocity Sigma Points & OD Estimates



Simulation Results

Effects of Pre-Flyby OD Uncertainties on the Transfer Maneuvers

Pre-Flyby Maneuver Variations (m/s)

Method	δ(ΔVx)	δ(ΔVy)	δ(ΔVz)	|δΔV|
MC	0.74	0.04	0.18	0.76
LC	0.66	0.05	0.17	0.68
UT	0.73	0.08	0.17	0.75

Sun-Earth L1 Maneuver Variations (m/s)

Method	δ(ΔVx)	δ(ΔVy)	δ(ΔVz)	|δΔV|
MC	0.36	0.94	0.68	1.22
LC	0.32	0.85	0.62	1.10
UT	0.36	0.93	0.68	1.21

- If there is room in the ΔV budget, a lower OD accuracy may be acceptable with a possibility of relaxing the tracking schedule or the number of ground stations to reduce operational cost
- NOTE: A similar analysis should be performed for OD done before the other transfer maneuvers

Conclusions

- Three methods of determining the impact of OD accuracy on the delta-V were presented
- The Unscented Transformation method was shown to match the (10-case) Monte Carlo method better than the Linear Covariance method by about 10%
- The Unscented Transformation is an alternative to the Monte Carlo method for assessing OD requirements for a space mission
  - A larger number of Monte Carlo cases are required to really prove this out

Thank you!