Assessing Orbit Determination Requirement With Unscented Transformation: Case Study Of A Lunar CubeSat Mission

Dr. Sun Hur-Diaz
Dr. Ravishankar Mathur

6th International Conference on Astrodynamics Tools and Techniques (ICATT)
14-17 March 2016
Outline

- Motivation –
  - Effect of OD error on delta-V budget of a Lunar CubeSat Mission
- Methods
  - Monte Carlo
  - Linear Covariance
  - Unscented Transformation
- Simulation and Results
CubeQuest Challenge

- One of NASA’s Centennial Challenges
- Winning 6U CubeSats in the competition are offered a launch on the Exploration Mission (EM) 1 mission as secondary payloads
- All such CubeSats will be disposed into a high-energy trajectory that will fly by the Moon
- Most CubeSats will use some form of a low-thrust propulsion system to achieve lunar orbit
- In order to determine the OD strategy for such a mission, the OD accuracy requirement needs to be understood
**Nominal Lunar Transfer Trajectory**

<table>
<thead>
<tr>
<th>Maneuver Name</th>
<th>Delta-V (m/s)</th>
<th>Duration (days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-Flyby</td>
<td>14.2</td>
<td>2.3</td>
</tr>
<tr>
<td>Sun-Earth L1</td>
<td>215.9</td>
<td>35.0</td>
</tr>
<tr>
<td>Lunar Distant Retrograde Orbit (DRO)</td>
<td>43.2</td>
<td>7.0</td>
</tr>
</tbody>
</table>
Maneuver Change from OD Error

- There are many sources contributing to changes in the nominal maneuver

\[ \delta(\Delta V)_{OD\ Error} = \text{Planned Maneuver} - \text{Nominal Maneuver} \]

- In this paper, we focus on the OD error contribution to the maneuver change
Maneuver Function

- Nonlinear function utilizing NASA GSFC’s open source General Mission Analysis Tool (GMAT)

- Goal is to determine the maneuver variation due to OD state estimate uncertainty

\[
P_{uuu} = E[(u - \bar{u})(u - \bar{u})^T]
\]
Monte Carlo (MC) Method

- Considered most accurate if large number of cases are simulated
- Perform $N$ cases of OD simulation
- Compute the $i$-th maneuver corresponding to each OD solution
- Compute the covariance of the resulting $N$ planned maneuvers

$$\bar{u}(i) = \frac{\sum_{j=1}^{N} u_j(i)}{N}$$

$$P_{uu}(i) = \frac{\sum_{j=1}^{N} (u(i) - \bar{u}(i))(u(i) - \bar{u}(i))^T}{N}$$
Linear Covariance (LC) Method

- Linear transformation of the state estimate uncertainty into the variation in the $i$-th maneuver

$$P_{xx} \rightarrow P_{uu}(i)$$

- Requires computing the sensitivity matrix (Jacobian) of the maneuver relative to the OD state estimate

$$P_{uu}(i) = \frac{\partial (u_i)}{\partial (\hat{x})} P_{xx} \left( \frac{\partial (u_i)}{\partial (\hat{x})} \right)^T$$

- The maneuver function “Black Box” is highly nonlinear with no analytic expression so numerical difference:

$$\frac{\partial (u_i)}{\partial (\hat{x})} (\cdot, j) \approx \frac{u(i, j) - u(i)_{NOM}}{\hat{x}_j - \hat{x}_{j_{NOM}}}$$
Unscented Transformation (UT)

- 2L+1 sigma points are generated that statistically represent the OD estimates, where L is the number of states.

\[
\chi = \left[ \bar{x}, \bar{x} \pm \left( \sqrt{(L + \lambda)P_j} \right) \right] \quad \text{for } j = \{1, \ldots, L\}
\]

\[
\lambda = \alpha^2 (L + \kappa) - L
\]
UT Maneuver Variation

- Weights associated with the OD estimate sigma points are applied to the maneuver sigma points to form the maneuver variation

\[
W_0^m = \frac{\lambda}{L + \lambda} \quad W_0^c = \frac{\lambda}{L + \lambda} + (1 - \alpha^2 + \beta) \quad W_k^m = W_k^c = \frac{1}{2(L + \lambda)}
\]

\[
\bar{u}_i = \sum_{k=0}^{2L} W_k^m u_{i,k}
\]

\[
(P_{uu})_i = \sum_{k=0}^{2L} W_k^c (u_{i,k} - \bar{u}_i)(u_{i,k} - \bar{u}_i)^T
\]
Pre-Flyby Orbit Determination Setup

- 5 US ground stations
- One-way range rate measurements – 15 mm/s (1σ)
- Range rate bias – 1 km/s
- 11 hours of tracking @ 1 minute intervals
- Extended Kalman Filter - solve for position, velocity, range rate bias
  - NASA GSFC’s OD Tool Box (estseq function)
10-Case Monte Carlo Simulation

Position error in meters for LEG 1

Velocity error in meters/sec for LEG 1

Range rate bias error in meters/sec

- Actual errors $e$
- Ensemble mean of $e$
- Ensemble 1- to 3-std of $e$ wrt ensemble mean
- $2\times$ ensemble mean of sigma from P formal
**OD Estimate**

- **Formal covariance matrix obtained from the Pre-Flyby OD**

\[
P_{xx}(1) = \begin{bmatrix}
0.4 & 0.07 & 0.6 & 9 \times 10^{-6} & 3 \times 10^{-6} & 7 \times 10^{-6} \\
0.2 & -1 & -3 \times 10^{-6} & 7 \times 10^{-6} & -1 \times 10^{-5} \\
20 & 1 \times 10^{-4} & -4 \times 10^{-5} & 2 \times 10^{-4} \\
6 \times 10^{-10} & -1 \times 10^{-10} & 9 \times 10^{-10} \\
& & & 2 \times 10^{-10} & -3 \times 10^{-10} \\
& & & & 2 \times 10^{-9}
\end{bmatrix}
\]

- **1-σ uncertainties**

\[
\delta x = [0.63 \ 0.49 \ 4.9] \text{ km}
\]
\[
\delta v = [25 \ 15 \ 47] \text{ mm/s}
\]
Position Sigma Points & OD Estimates

OD Error from MC

Sigma Points

ICATT 2016
Velocity Sigma Points & OD Estimates

- OD Error from MC
- Sigma Points
Simulation Results

Effects of Pre-Flyby OD Uncertainties on the Transfer Maneuvers

Pre-Flyby Maneuver Variations (m/s)

| Method | \(\delta(\Delta V_x)\) | \(\delta(\Delta V_y)\) | \(\delta(\Delta V_z)\) | \(|\delta \Delta V|\) |
|--------|-----------------|-----------------|-----------------|-----------------|
| MC     | 0.74            | 0.04            | 0.18            | 0.76            |
| LC     | 0.66            | 0.05            | 0.17            | 0.68            |
| UT     | 0.73            | 0.08            | 0.17            | 0.75            |

Sun-Earth L1 Maneuver Variations (m/s)

| Method | \(\delta(\Delta V_x)\) | \(\delta(\Delta V_y)\) | \(\delta(\Delta V_z)\) | \(|\delta \Delta V|\) |
|--------|-----------------|-----------------|-----------------|-----------------|
| MC     | 0.36            | 0.94            | 0.68            | 1.22            |
| LC     | 0.32            | 0.85            | 0.62            | 1.10            |
| UT     | 0.36            | 0.93            | 0.68            | 1.21            |

- If there is room in the \(\Delta V\) budget, a lower OD accuracy may be acceptable with a possibility of relaxing the tracking schedule or the number of ground stations to reduce operational cost.
- NOTE: A similar analysis should be performed for OD done before the other transfer maneuvers.
Conclusions

- Three methods of determining the impact of OD accuracy on the delta-V were presented.
- The Unscented Transformation method was shown to match the (10-case) Monte Carlo method better than the Linear Covariance method by about 10%.
- The Unscented Transformation is an alternative to the Monte Carlo method for assessing OD requirements for a space mission.
  - A larger number of Monte Carlo cases are required to really prove this out.

Thank you!