



# Extended Tisserand graph and multiple lunar swing-by design with Sun perturbation

Daniel García Yárnoz, Chit Hong Yam,  
**Stefano Campagnola**, Yasuhiro Kawakatsu



6<sup>th</sup> ICATT. Darmstadt, Germany

# Outline

- Intro
- Tisserand-Poincaré graph and extension
- Lunar transfer database
- Application examples
  - EQUULEUS
  - DESTINY
- Conclusions



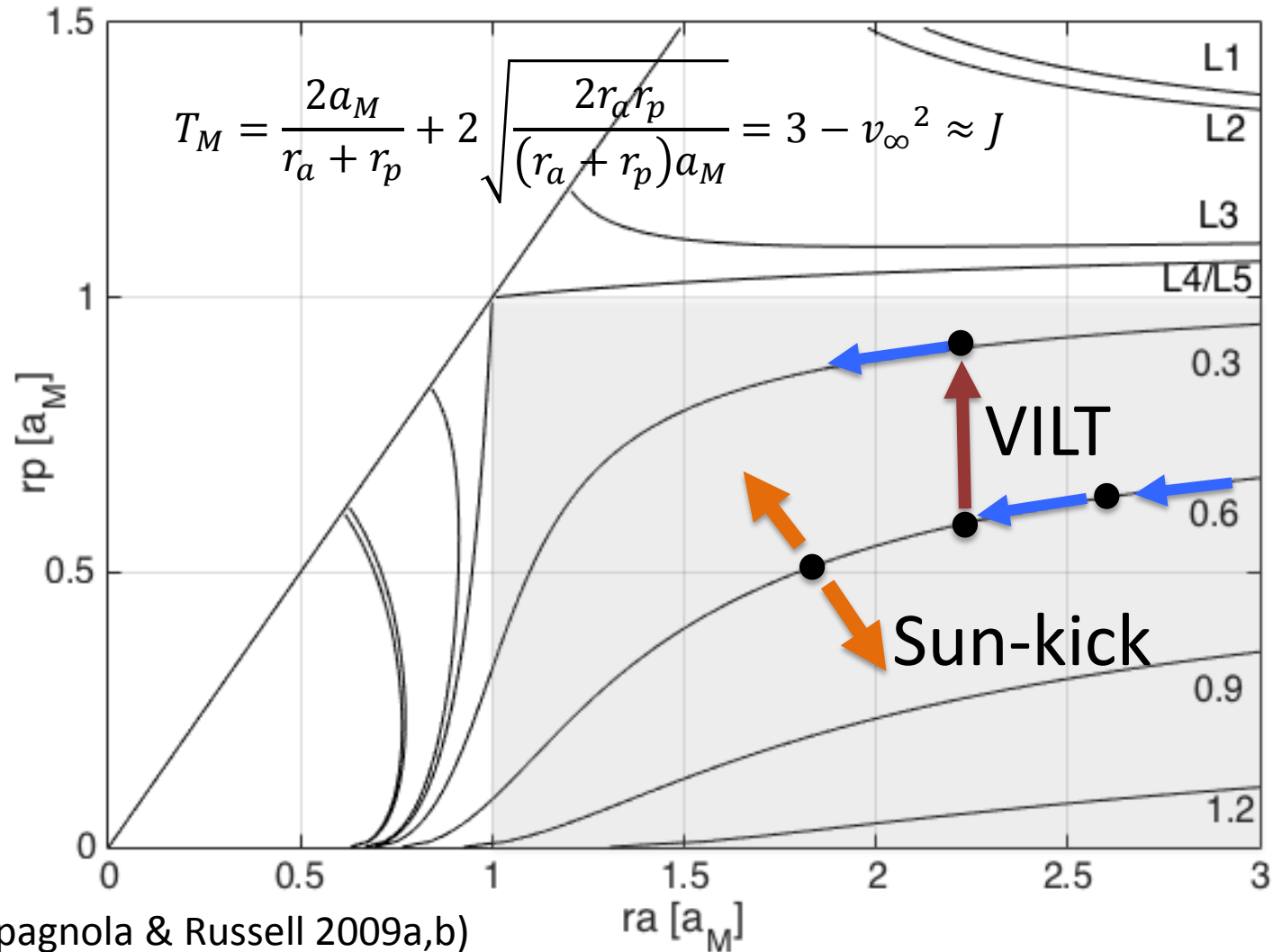
<http://destiny.isas.jaxa.jp/>

# Background and rationale

- New trends in small missions:
  - Small sats / CubeSats beyond LEO, beyond cislunar space?
- Launch / escape strategies
  - Piggy-back / secondary payloads
  - Spiralling from LEO
  - Limited choice of date and target orbits conditions
- **Moon: single, massive, close to ecliptic, high potential**
  - Pump up/down
  - Phasing
  - Transfers



# Tisserand-Poincare graph



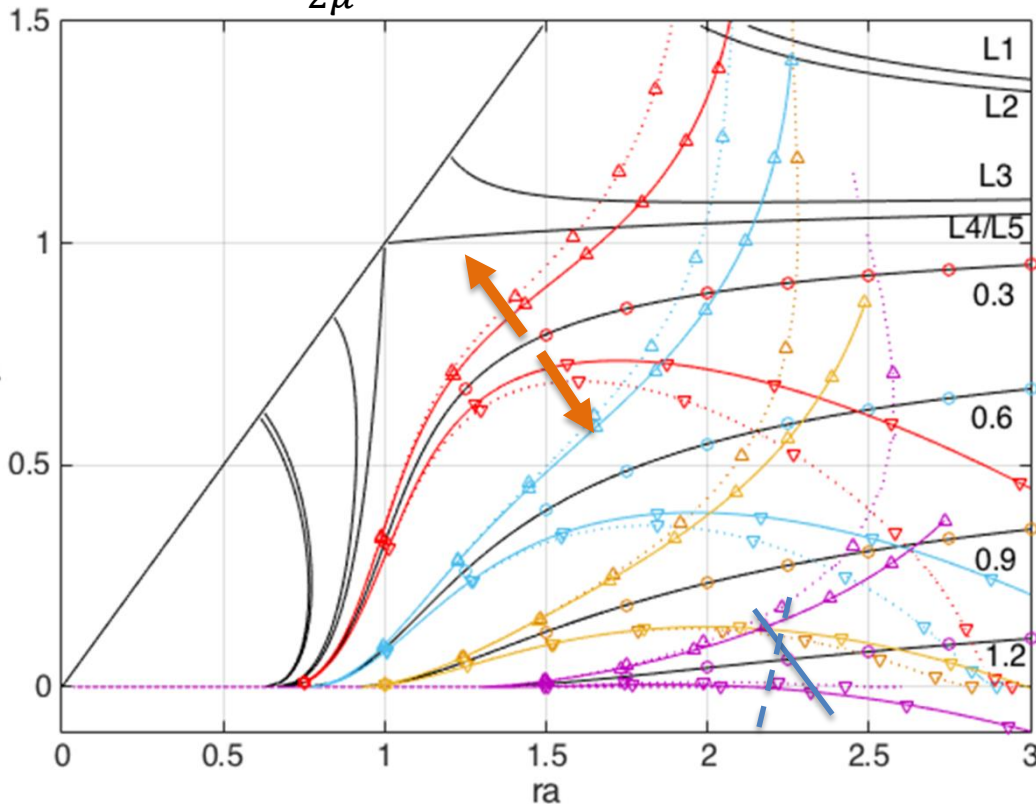
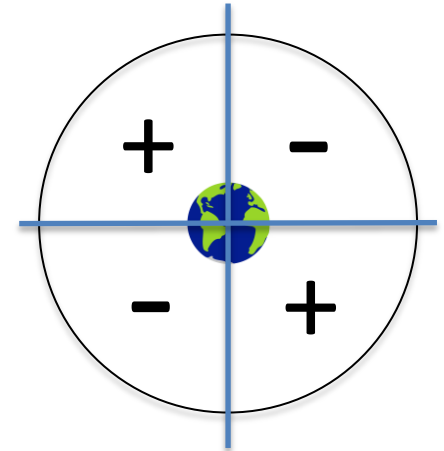
(Campagnola & Russell 2009a,b)

# Extension with Sun perturbation

- Analytical approximation (continuous)

$\Delta a \approx 0 \rightarrow$  Lines of slope -1

$\Delta e = 15 \frac{\pi}{2\mu} a^3 e \sqrt{1 - e^2} \sin 2(\omega_p - \theta) \rightarrow$  quadrants



- Numerical propagation (dashed)

- Similar slope for low vel or low apocentres
- Greater deviations achievable

- Issues: geometry dependant, multiple revs, deviations at  $\uparrow r_a, v$

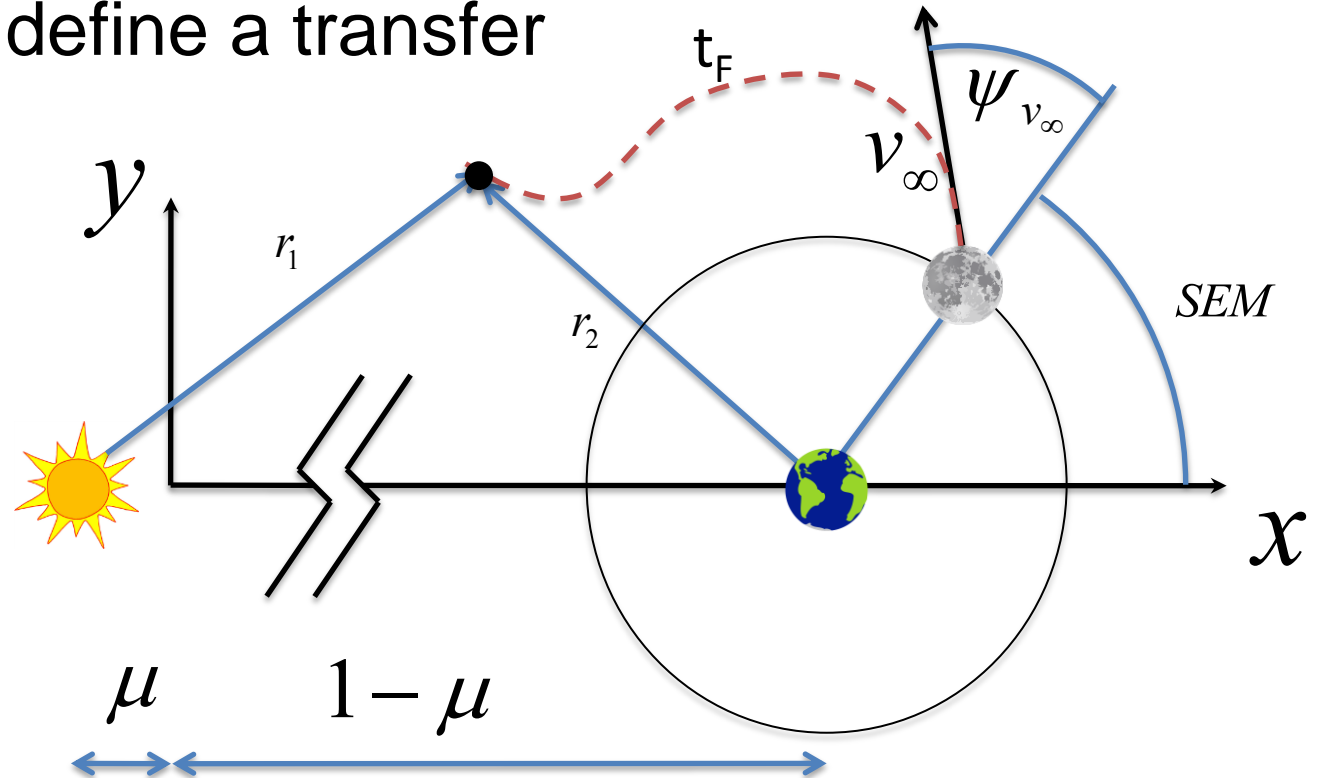
# Introducing: The “Mooning” Database

- Planar CR3BP
  - Neglect  $e_E, e_M, i_M \rightarrow$  planar, pseudo epoch-free transfers
  - Transfers repeat every Moon period
- Families generated with continuation method
- Database stored for quick and easy access
- Generation of first guesses for full-model optimisation
  - “Substitute” Lambert arc calculations
  - Usual methods used in multiple gravity assists can be build on top (branch and pruning, genetic algorithms...)
- Extend to multiple revolutions + additional families

(Lantoine & McElrath 2014)

# Synodic frame and transfer parameters

- 4 variables define a transfer

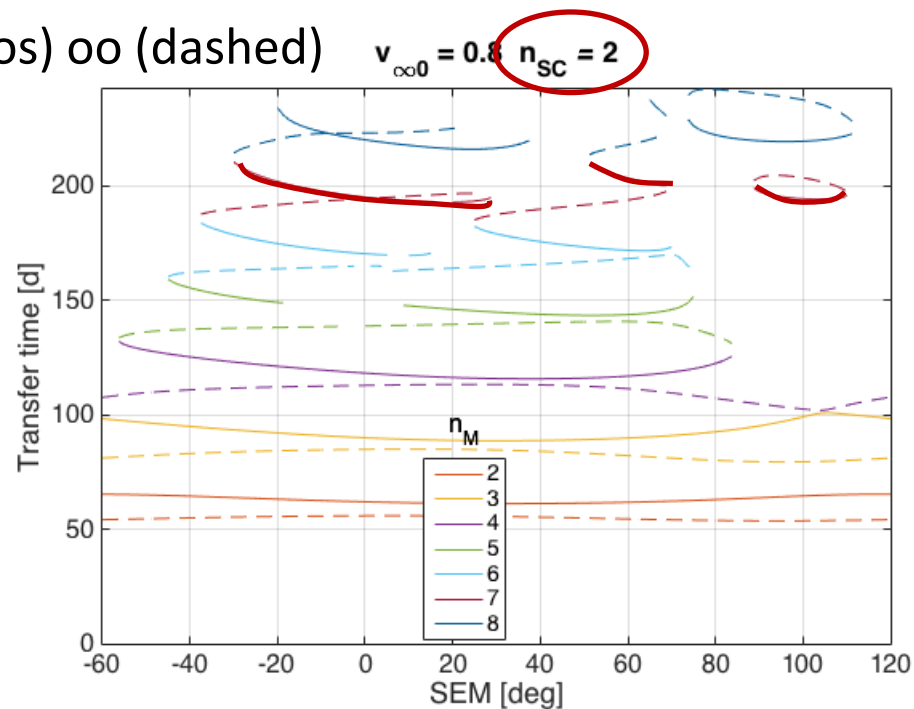
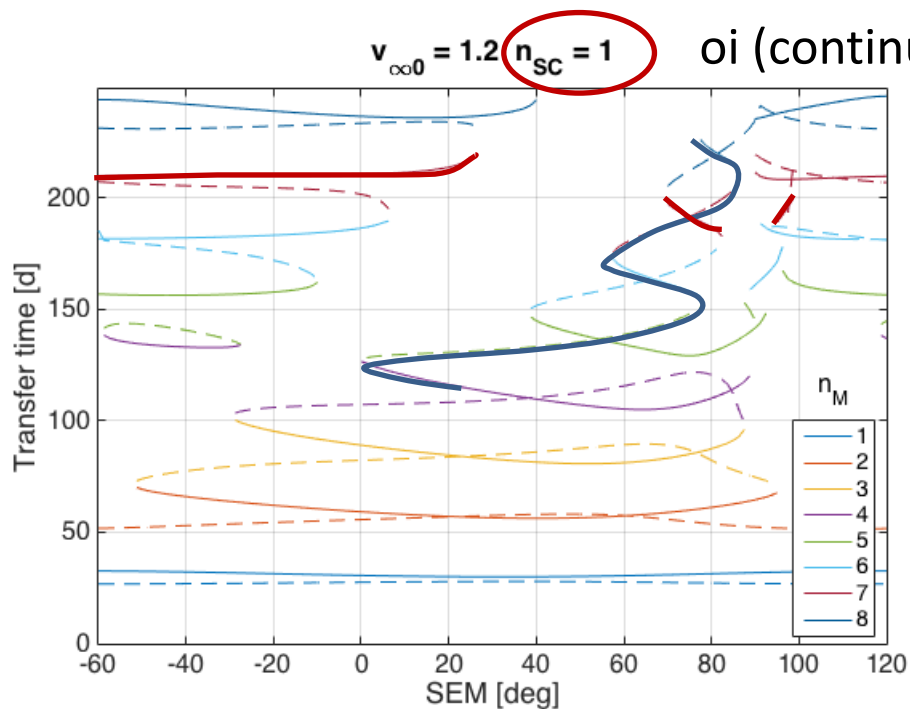


Database of transfers stored:

$$\underbrace{SEM, v_\infty, \Psi_{v_\infty}}_{\text{initial}}, t_F, \underbrace{SEM, v_\infty, \Psi_{v_\infty}}_{\text{final}}, r_{2min}$$

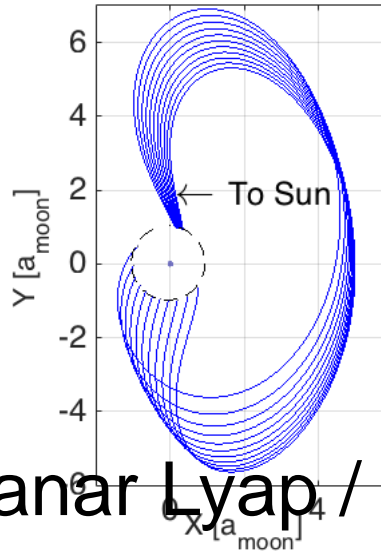
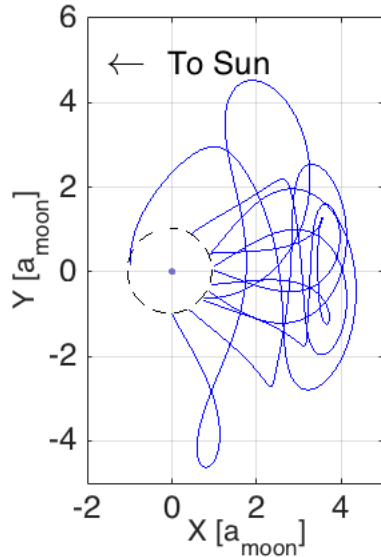
# “Mooning” Database Structure

- Four types: *oo*, *oi*, *ii*, *io*
  - Outgoing:  $\cos(\Psi_{v_\infty}) > 0$
  - Incoming:  $\cos(\Psi_{v_\infty}) < 0$
- Families branch off and reconnect (loops, helix)
- Example connections *oi-oo*: tangent to Moon orbit

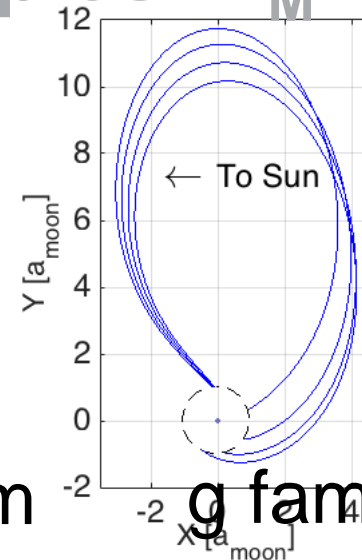




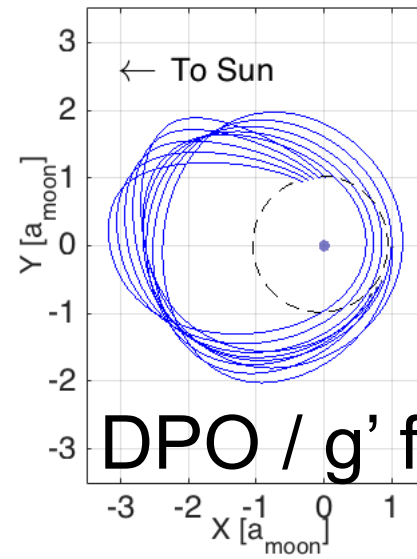
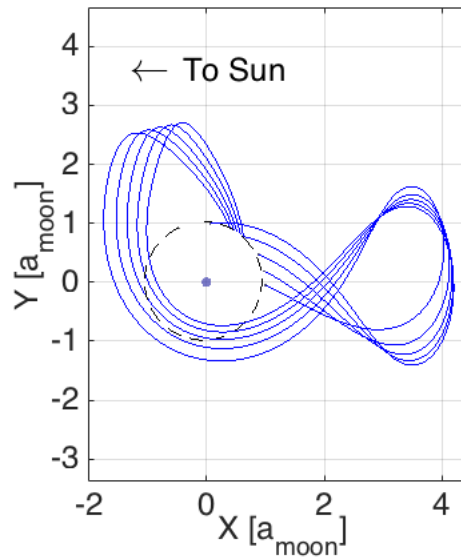
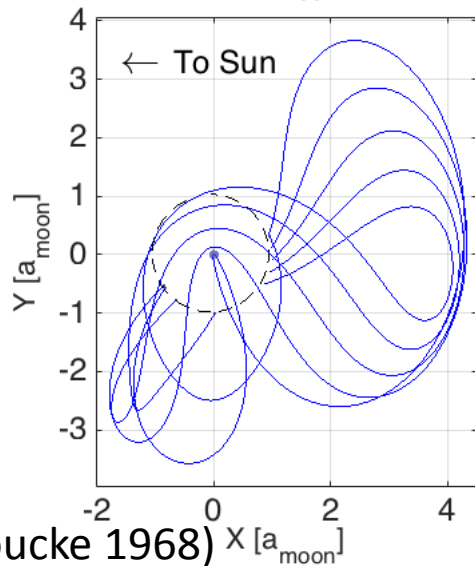
# Database sub-families examples: $n_M=7$



Planar Lyap / a fam



$n_{SC}=1$



$n_{SC}=2$

DPO / g' fam

(Broucke 1968)

# Applications

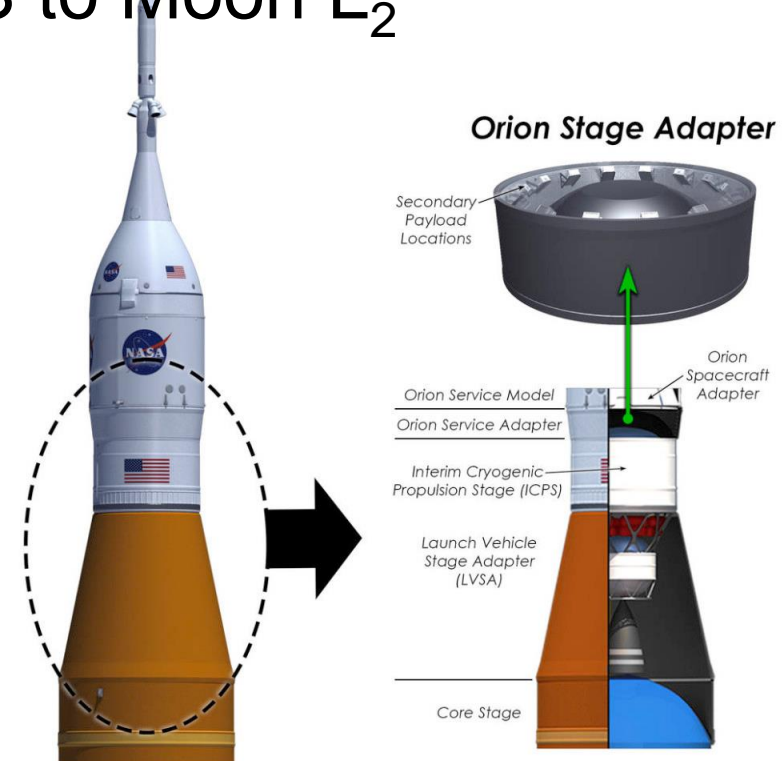
Two opposite problems:

- EQUULEUS: decrease velocity (+phasing)
- DESTINY: increase velocity (+phasing)



# Application 1: EQUULEUS

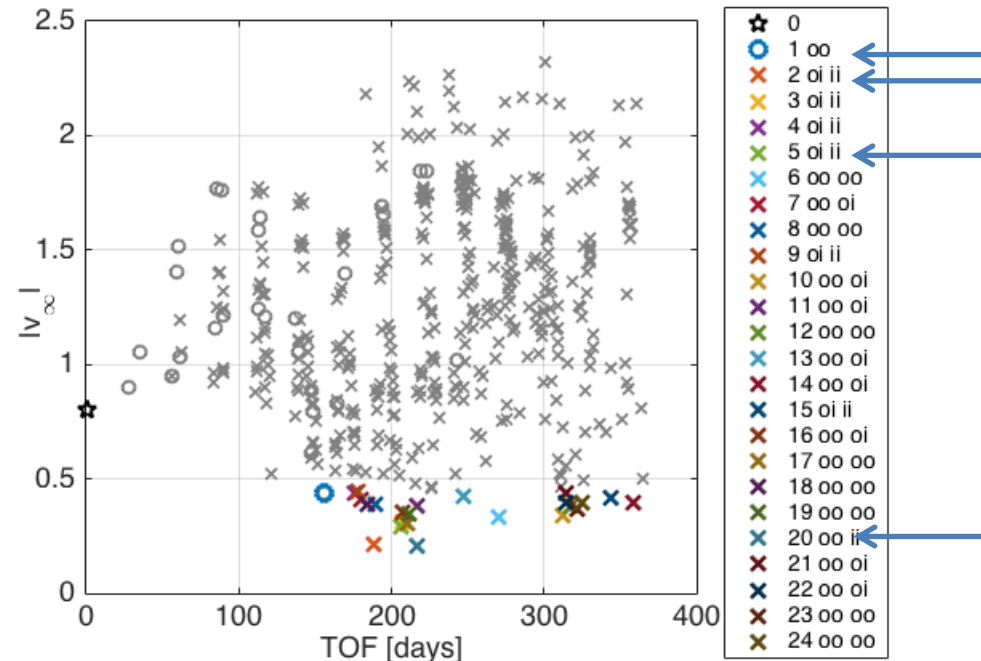
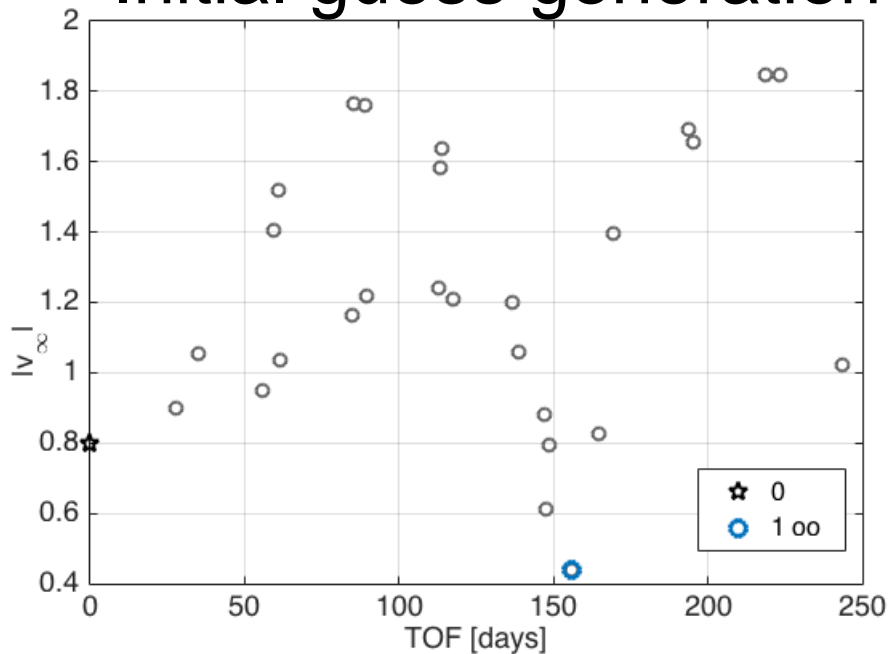
- NASA offering 11-13 Secondary Payload opportunities on SLS EM-1 test launch
- JAXA proposed EQUULEUS to Moon L<sub>2</sub>
- 6U CubeSat: study plasmasphere, navigation and control WSB
- Launch “moon-bound”:  
 $v_{\infty} = 0.8 \text{ km/s}$
- Reduction of velocity to libration orbit



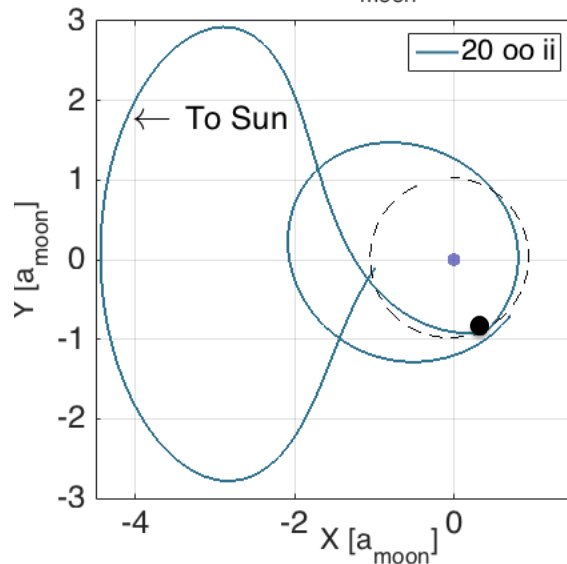
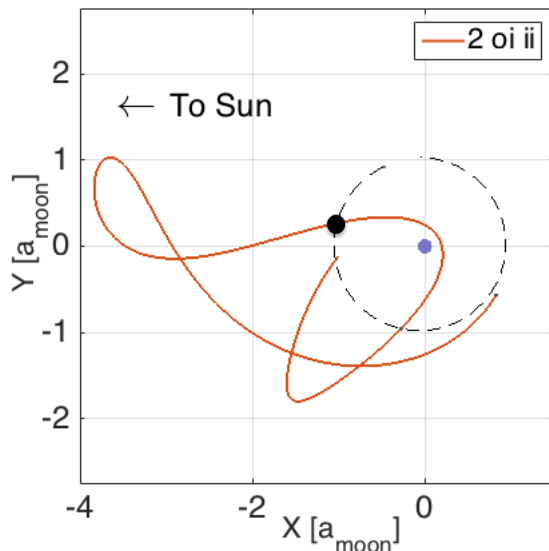
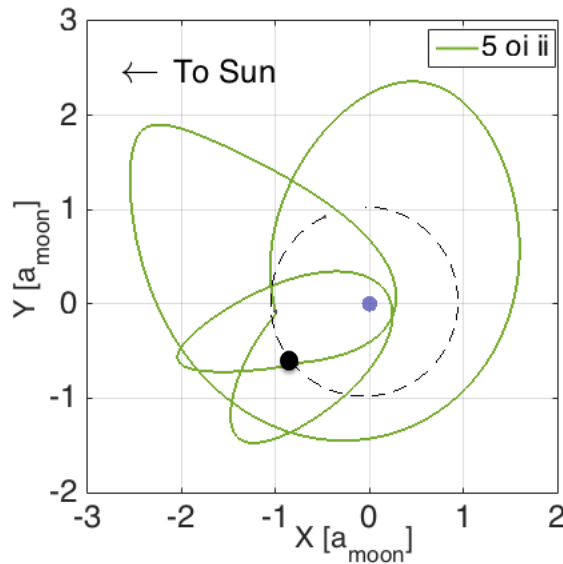
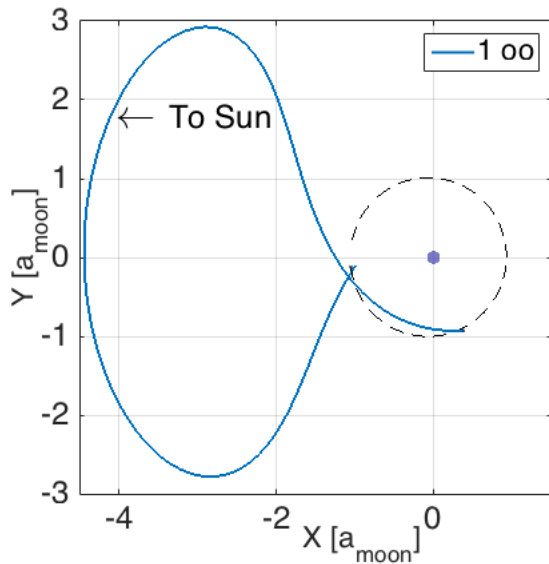
<http://www.nasa.gov/content/exploration-mission-1-secondary-payloads>

# Application 1: EQUULEUS results

- One leg transfers: 25, one with  $v_\infty < 0.45$
- Two leg transfers: 468, 24 with  $v_\infty < 0.45$
- Branch and pruning can be recursively applied
- Initial guess generation



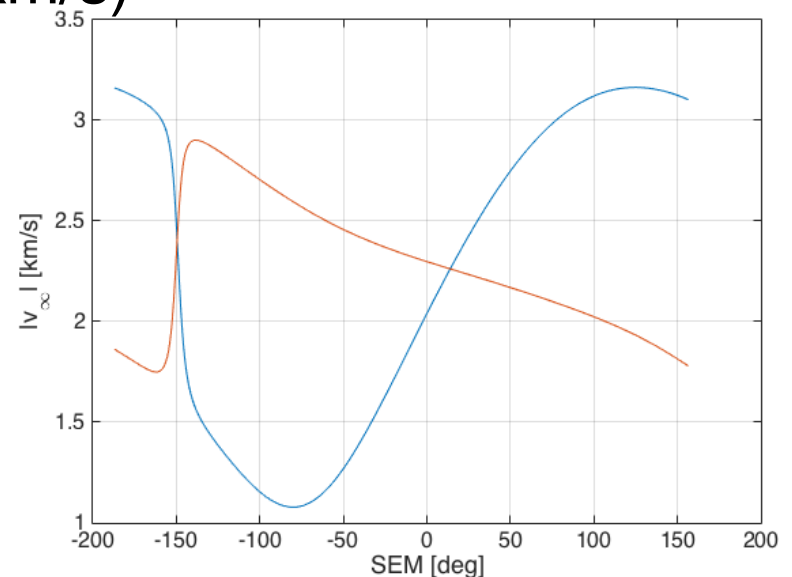
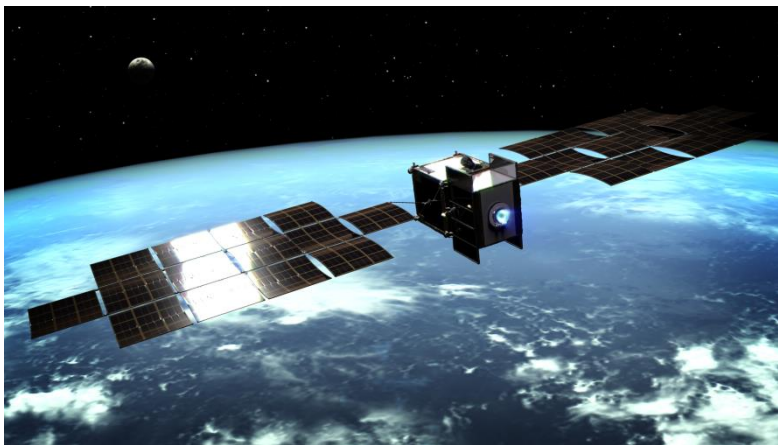
# EQUULEUS selected transfers



- 1 leg oo
- 2 legs  $v_\infty < 0.3$ 
  - oi ii ( $n_{SC}=1$ )
  - oi ii ( $n_{SC}=1$ )
  - oo ii (low fb height: unfeasible?)
- Tangent transfers: lower velocity

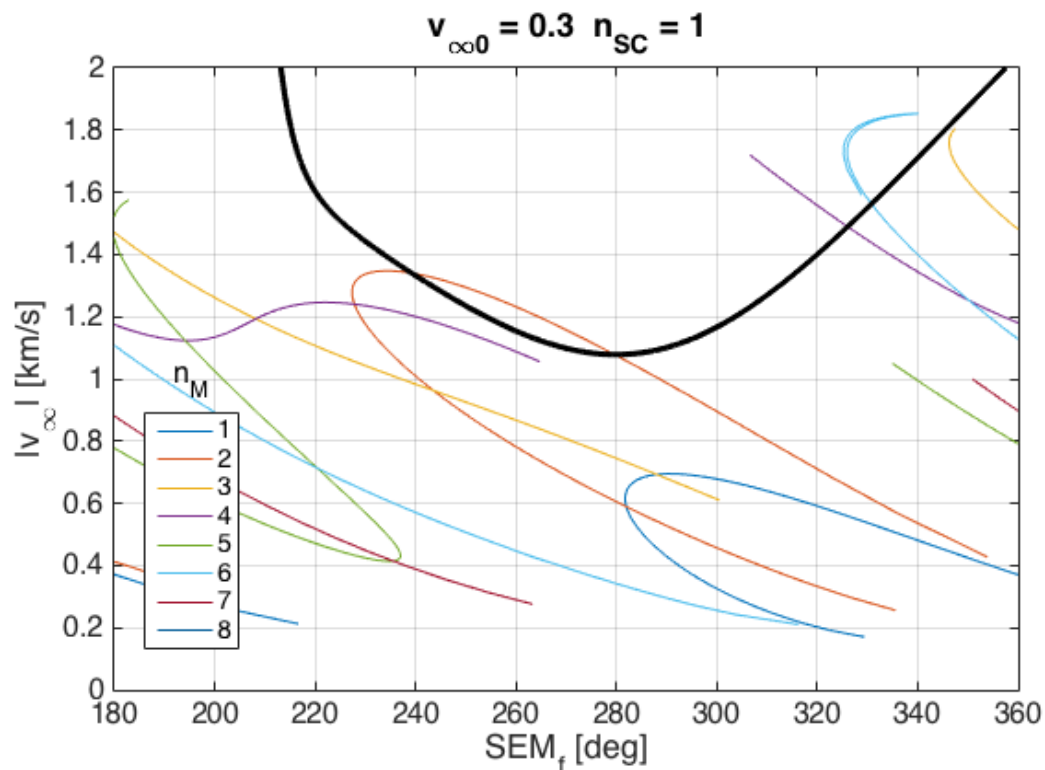
# Application 2: DESTINY

- Interplanetary low-thrust traj. to asteroid Phaethon
- Escape from Earth-Moon system at least 1.5 km/s
  - Connect with spiral up with multiple flybys
  - Last swing-by velocity wrt Moon 2.08 km/s
  - SEM-v<sub>∞</sub> feasible pairs (1-3 km/s)
- Database search

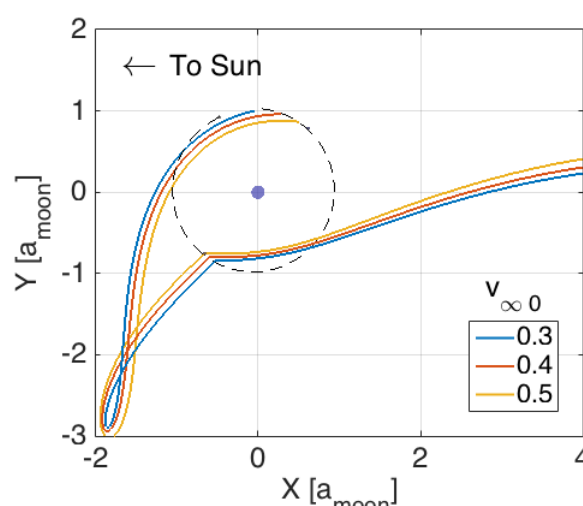
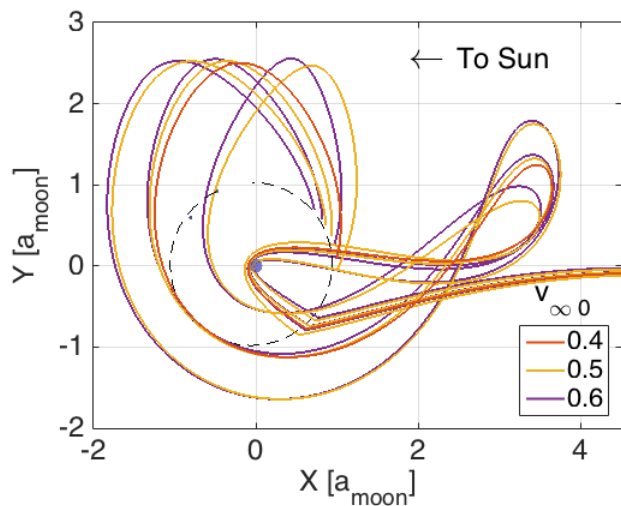
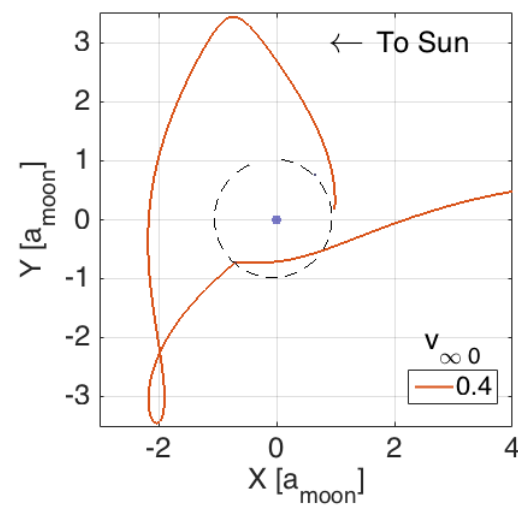
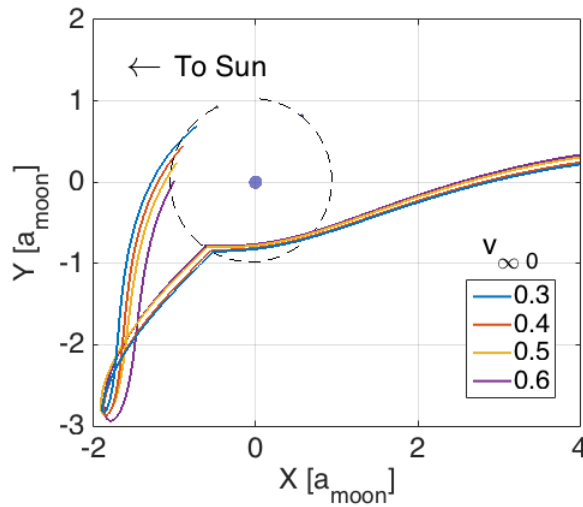
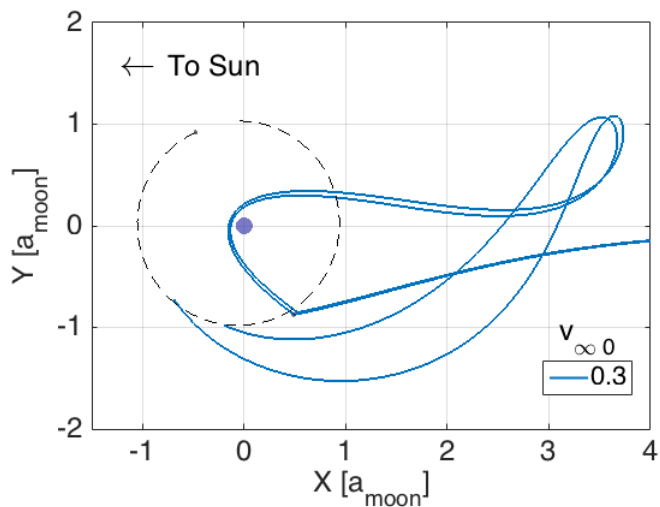


# DESTINY: Final leg search

- Assume last swing-by velocity  $< 2$  km/s
- SEM-vinf pairs intersection with families 0.3-0.6 km/s
- Check rotation vinf latitude feasible
- Multiple solutions



# DESTINY: transfer options

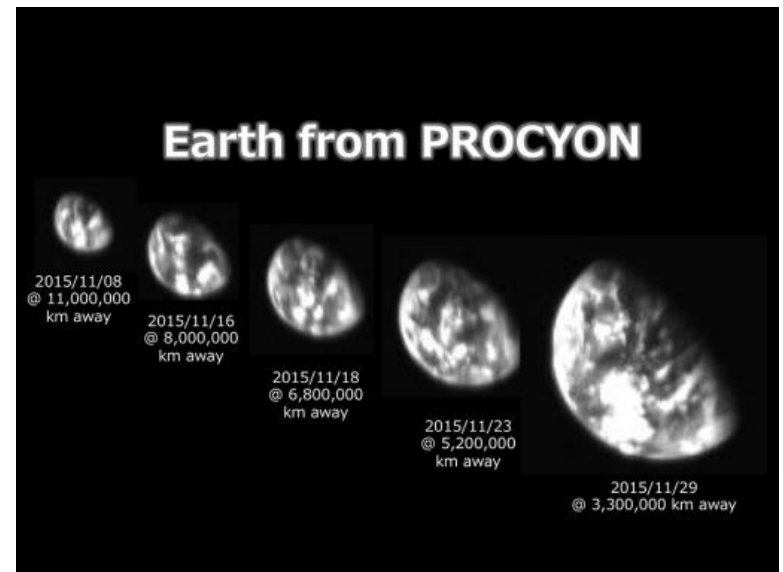


- Vel: 0.3-0.6 km/s
- Diverse shapes
- Multiple options
- Recursively backwards...



# Conclusions and future work

- Small missions will benefit of trajectory design with one/multiple lunar swing-bys
- T-P graph allows estimation of reachable regions
- Database of trajectories built → easy generation of initial guesses
- Application examples
- Ongoing / future work:
  - Extension of database
  - Regularized eq. of motion
  - Pseudo-arclength continuation
  - Include  $\pi$  transfers (3D)



Kawakatsu Lab - 川勝研究室

<http://kawakatsu.isas.jaxa.jp/>

Daniel.Garcia@ac.jaxa.jp

