6th International Conference on Astrodynamics Tools and Techniques

POLITECNICO MILANO 1863

INNOVATIVE METHOD FOR THE COMPUTATION OF SAFETY RE-ENTRY AREA BASED ON THE PROBABILITY OF UNCERTAINTIES IN THE INPUT PARAMETERS

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Outline

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Objective of the study

Destructive re-entry of satellites and space objects:
Rare event casualty caused by impact of a fragment generated by reentry

According to the French Space law, "*the operator responsible of a spacecraft controlled reentry shall identify and compute the impact zones of the spacecraft and its fragments for all controlled reentry on the Earth with a probability respectively of 99% and 99,999% taking into account the uncertainties associated to the parameters of the reentry trajectories*".

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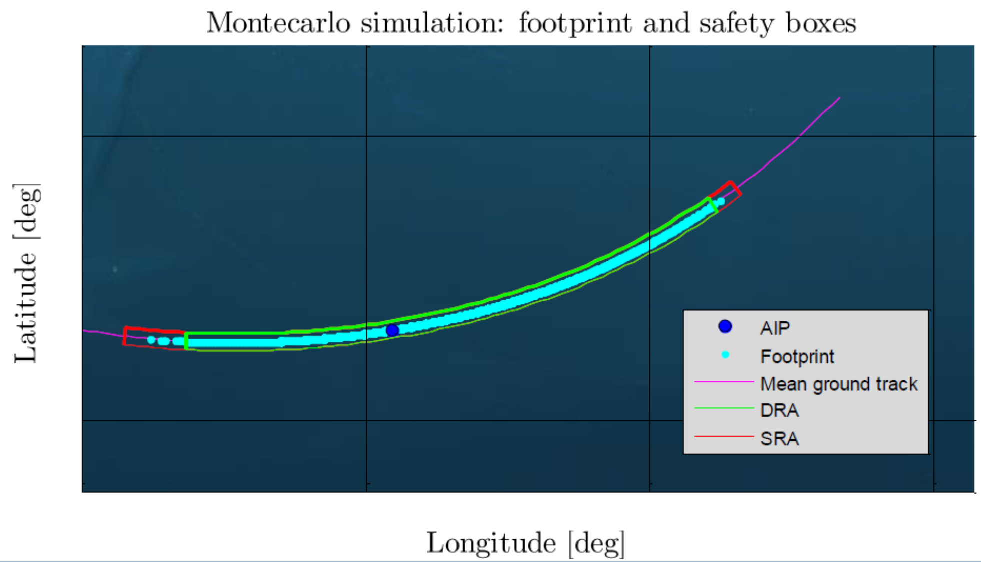
Safety boxes definition

Safety boxes are containment contours on the ground defined such that the probability that a fragment falls outside is a controlled value.

- **Declared Re-entry Area (DRA):** 10^{-2} → The probability that all the fragments fall inside is **> 99%**
- **Safety Re-entry Area (SRA)**: 10^{-5} → The probability that all the fragments fall inside is **> 99.999%**



→ **Engineering design:** the **SRA** shall not extend over inhabited regions and shall not impinge on state territories and territorial waters without the agreement of the relevant authorities

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Preliminary considerations

- **Series of uncertain parameters affect the problem**
  → Relying on a statistical assessment to fulfill the international safety requirements and constrain ground population risk.

- **Extremely low probability of interest (e.g. 10^{-5})**
  → Difficult, slow and inaccurate use of classical statistical techniques.

- **Many State of the Art methods have been developed to deal with similar problems (Morio and Balesdent, 2015*):**
  - Crude Monte Carlo methods
  - Importance sampling techniques
  - Adaptive splitting techniques
  - First and second order reliability methods (FORM/SORM)
  - Extreme value theory:
    i. Bloc Maxima method
    ii. Peak over threshold method

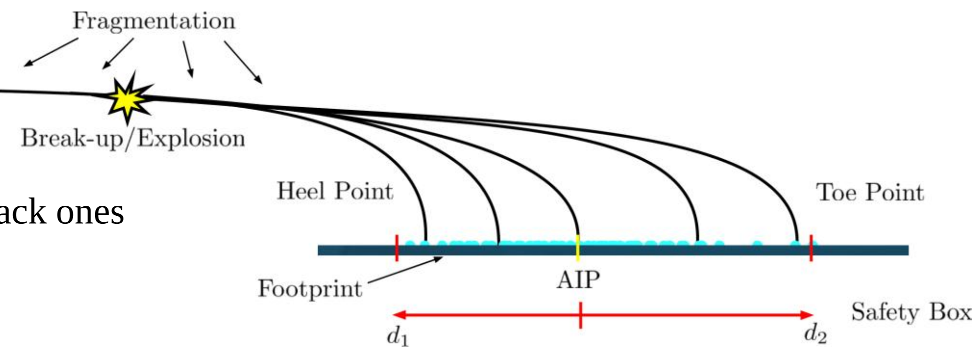
*Jérôme Morio, Mathieu Balesdent, *Estimation of Rare Event Probabilities in Complex Aerospace and Other Systems, A Practical Approach*, Woodhead Publishing, Elsevier.

Input-Output formulation

Goal of the design:

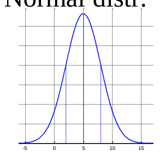
Along track boundaries identification

Cross track boundaries are small with respect to along track ones (deviation of +/-100 km from the ground track)

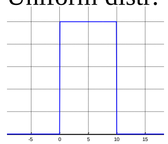


Input statistics

Normal distr.



Uniform distr.



Set of inputs: $\mathbf{X}$

Explosion altitude

ΔV Manoeuvre

Satellite mass

Ballistic coefficient

Thrust

Atmosphere density

...

Transfer function:

Re-entry dynamic model

$$f: \mathbb{R}^n \longrightarrow \mathbb{R}$$

$$f(\boldsymbol{\mu}) = 0$$

Impact point

Output: Y

Along track distance from AIP

- Negative → HEEL point
- Positive → TOE point

A priori statistically modelled using physical considerations and engineering judgment.

Not known a priori. Could be numerically built only.

Natural formulation of the problem and issues

Given $Y = f(\mathbf{X})$ and the probability level of interest $\alpha = 10^{-5}$

find $d_1 < 0$ and $d_2 > 0$ such that

$$1 - P(d_1 < Y < d_2) \leq \alpha$$

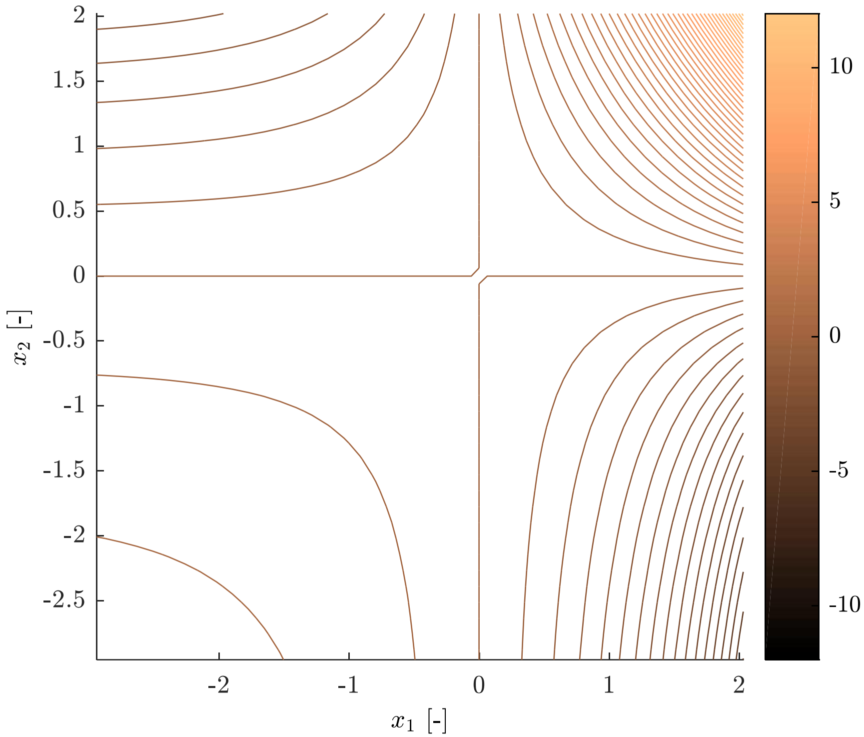
→ Analysis of the contour surfaces of the transfer function f identifying those two that satisfy the probability condition.

2 main issues:

1) Infinite number of feasible solutions (1 inequality, two unknowns d_1 and d_2)
2) Contour surfaces of f not known and cannot be numerically built due to computational time limitations (n-dimensional numerical propagator)

$$f(x_1, x_2) = (e^{x_1} - 1)(e^{x_2/2} - 1)$$

Contour lines illustration



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Issues facing

1) Infinite number of feasible solutions

→ *Engineering objective:* safety box design to minimize the distance between the two values d_1 and d_2 → d_1^{Opt} and d_2^{Opt}

2) Contour surfaces of f not known and cannot be numerically built

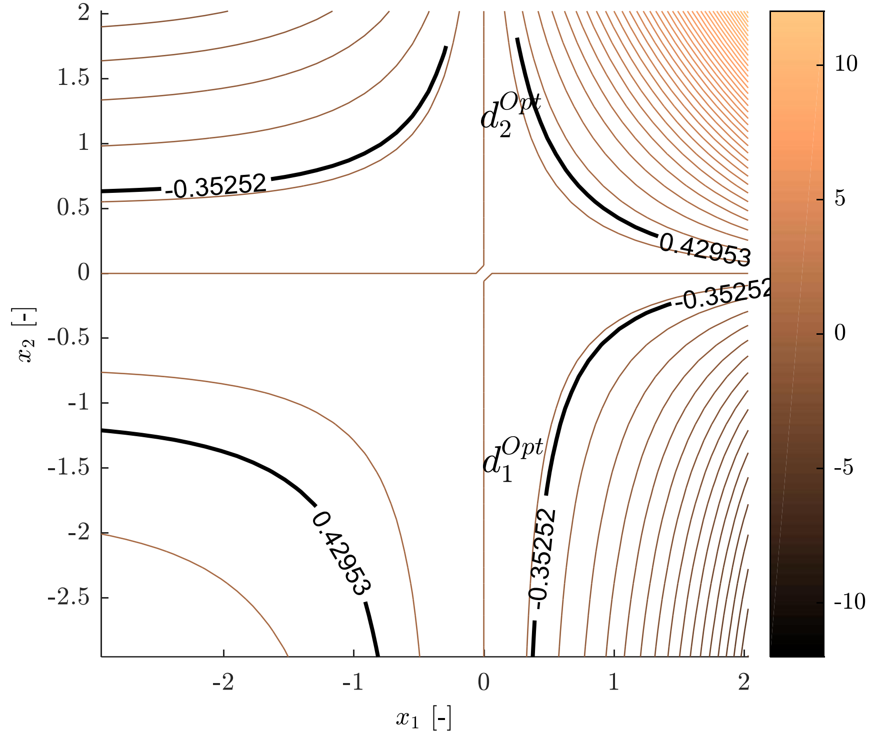
→ *Two possible approaches:*

i. **State of the Art Monte Carlo based:** creating a cloud of outputs (footprint) by sampling all over the input domain and post-processing this output statistics to get a *probabilistic* information (safety boxes)

ii. **Inputs' statistics approach:** approximated solution of an ***alternative*** formulation of the problem

$$f(x_1, x_2) = (e^{x_1} - 1)(e^{x_2/2} - 1)$$

Contour lines



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Alternative formulation of the problem: studying the input space

Given $Y = f(\mathbf{X})$, $\alpha = 10^{-5}$ and introducing

$$p = \mathrm{pdf}_{\mathrm{X}}\,(\mathbf{X})$$

find $d_1 < 0$ and $d_2 > 0$ such that

$$1 - \int_{\Omega} p(\mathbf{x})\, \mathrm{d}\mathbf{x} \leq \alpha$$

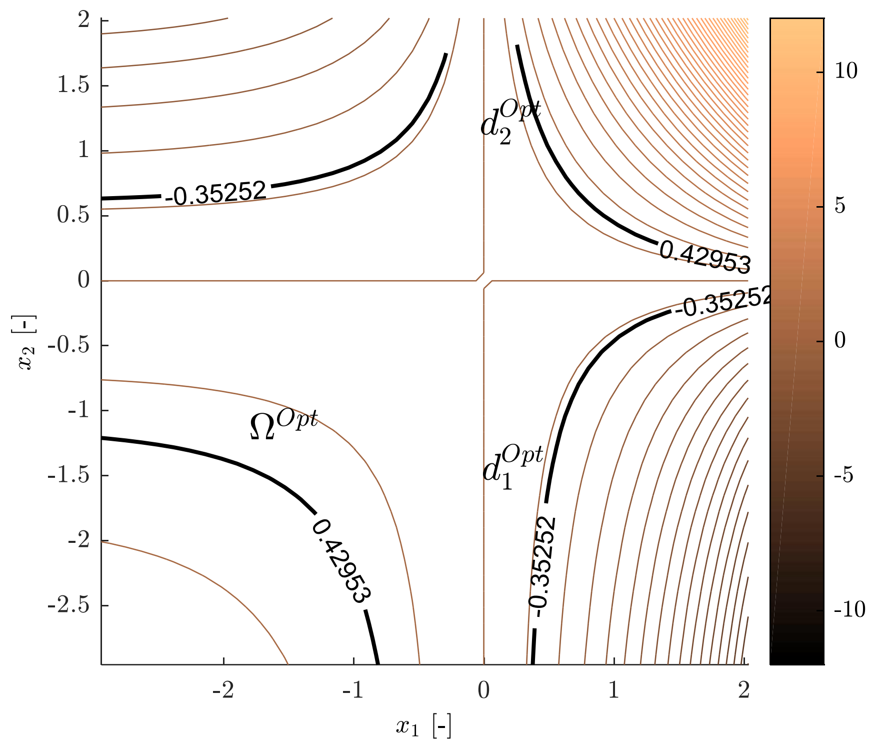
Where $\Omega = \{\mathbf{X} \in \mathbb{R}^n\colon\ d_1 < f(\mathbf{X}) < d_2\}$

Main issues still there:

1) Problem not well posed: infinite possible choices of Ω
   → looking for Ω^{Opt} such that $d_2^{Opt} - d_1^{Opt}$ is minimum, i.e. smallest possible safety box
2) Contour surfaces of f not known
   → Approximating Ω^{Opt} using conservative considerations: ***the Inputs' Statistics method***

$$f(x_1, x_2) = (e^{x_1} - 1)(e^{x_2/2} - 1)$$

Contour lines illustration



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Inputs' Statistics method: goal

In a nutshell:

Being $\tilde{\mathcal{E}}$ the contour surface of the PDF enclosing a probability equal to $1-\alpha$, then $\tilde{\Omega}$ is the region identified by contour surfaces of the transfer function f corresponding to the thresholds $\tilde{d}_1$ and $\tilde{d}_2$ being the minimum and maximum cases which may occur inside $\tilde{\mathcal{E}}$. $\tilde{d}_1$ and $\tilde{d}_2$ are the safety box dimensions.

Goal of the method:

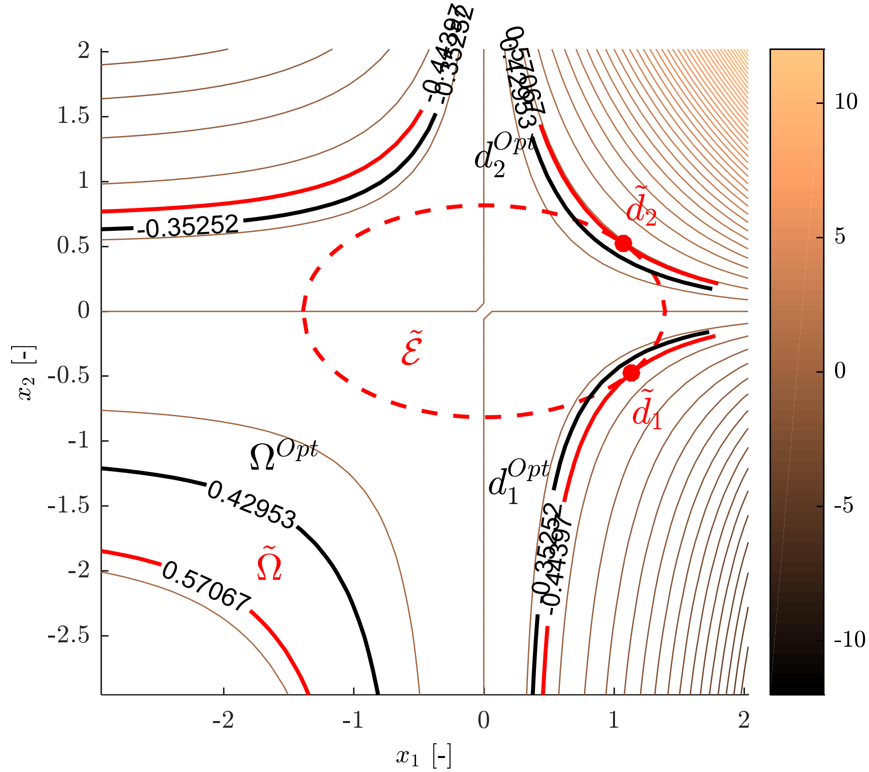
Find $\tilde{d}_1 < 0$ and $\tilde{d}_2 > 0$ such that

$$1 - \int_{\tilde{\Omega}} p(\mathbf{x})\, d\mathbf{x} \leq \alpha$$

Where $\tilde{\Omega} = \{\mathbf{X} \in \mathbb{R}^n : \tilde{d}_1 < f(\mathbf{X}) < \tilde{d}_2 .\} \cong \Omega^{\mathrm{Opt}}$

$$f(x_1, x_2) = (e^{x_1} - 1)(e^{x_2/2} - 1)$$

Contour lines illustration



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Inputs' Statistics method: solution

Solution: introduction of the contour surfaces of the PDF rather than of f

Supposing to have only *normal distributed* input variables, then

$$p(\mathbf{x}) = p_{MVN}(\mathbf{x}, \boldsymbol{\mu}, \boldsymbol{\Sigma}) = \frac{1}{\sqrt{|\boldsymbol{\Sigma}|\,(2\pi)^n}} e^{-\frac{1}{2}(\boldsymbol{x}-\boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1}(\boldsymbol{x}-\boldsymbol{\mu})}$$

and its contour surfaces are n-dimensional ellipsoids:

$$\mathcal{E}(\mathrm{t}) = \{\mathbf{X} \in \mathbb{R}^n : (\mathbf{X} - \boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1} (\mathbf{X} - \boldsymbol{\mu}) \leq \mathrm{t}\}$$

Then, compute $\tilde{\mathrm{t}}$ such that

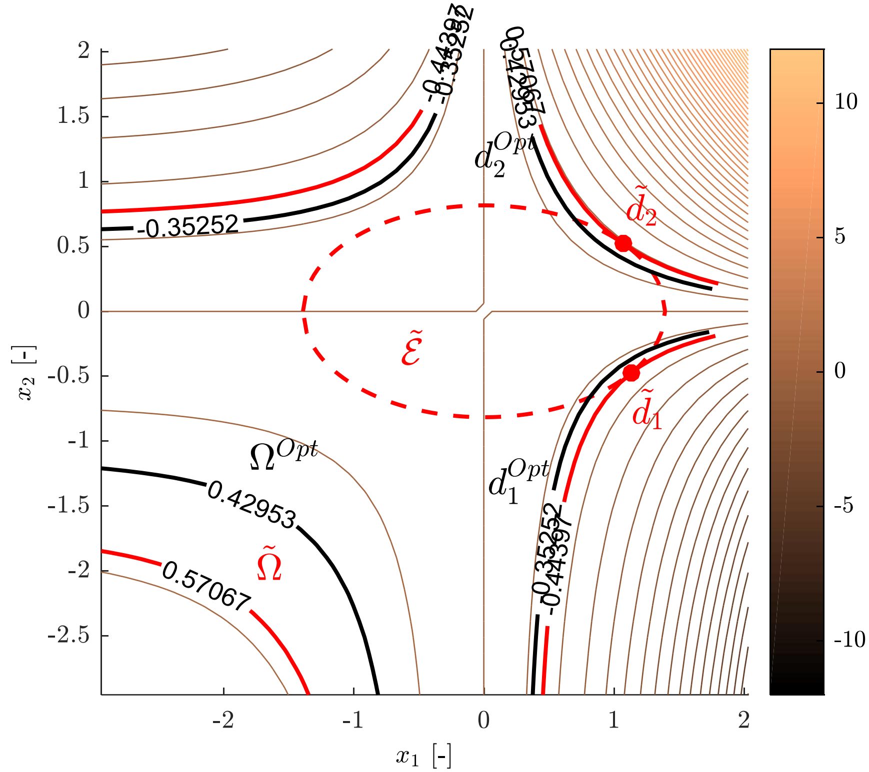
$$1 - \int_{\tilde{\mathcal{E}}(\tilde{\mathrm{t}})} p(\mathbf{x})\, d\mathbf{x} = \alpha$$

then, using an optimization process:

$$\tilde{d}_1 = \min f(\mathbf{X}) \text{ and } \tilde{d}_2 = \max f(\mathbf{X})$$
$$\text{subjected to } \mathbf{X} \in \tilde{\mathcal{E}}$$

$$f(x_1, x_2) = (e^{x_1} - 1)(e^{x_2/2} - 1)$$

Contour lines illustration



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Compliance with the safety requirements

By construction, $\tilde{\Omega}$ includes $\tilde{\mathcal{E}}$, i.e. $\tilde{\mathcal{E}}$ is a subset of $\tilde{\Omega}$:

$$\tilde{\mathcal{E}} \subseteq \tilde{\Omega}$$

then, by definition of $\tilde{\mathcal{E}}$, the solution identified by the Inputs' Statistics method ***always*** satisfies the safety condition:

$$1 - \int_{\tilde{\Omega}} p(\mathbf{x})\, \mathrm{d}\mathbf{x} \le \alpha$$

Since, by definition:

$$1 - \int_{\Omega^{\mathrm{Opt}}} p(\mathbf{x})\, \mathrm{d}\mathbf{x} = \alpha$$

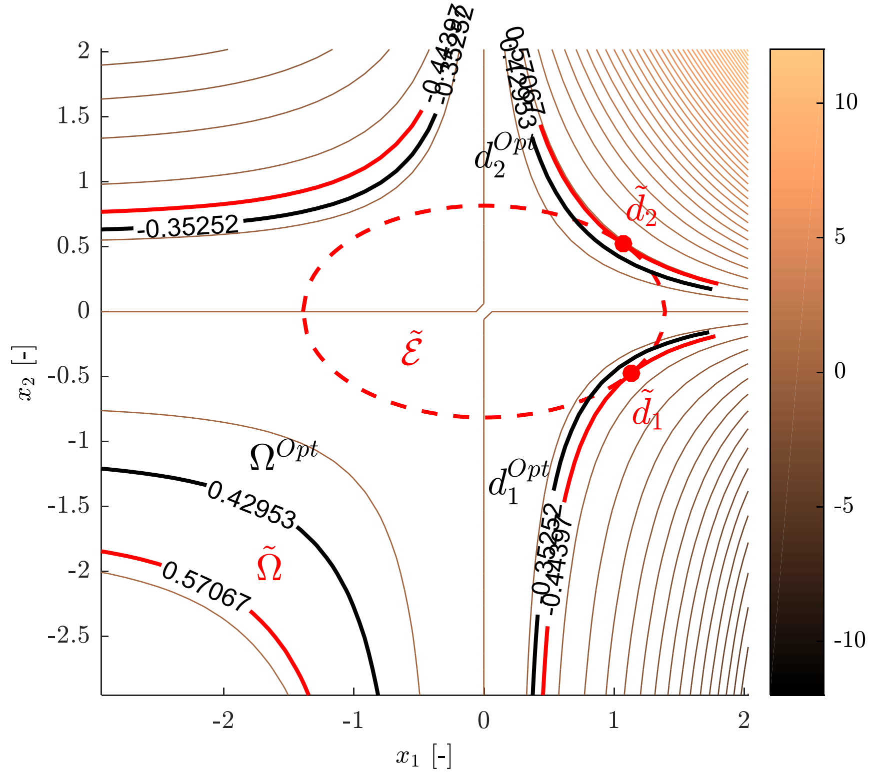
Then

$$\tilde{d}_1 \le d_1^{Opt} \quad \text{and} \quad \tilde{d}_2 \ge d_2^{Opt}$$

i.e. the result in terms of safety boxes dimensions is always conservative with respect to the optimal solution

$$f(x_1, x_2) = (e^{x_1} - 1)(e^{x_2/2} - 1)$$

Contour lines illustration



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Characteristics of the method
Direct computation of the probability

The Inputs' statistics method:

1. **implements directly the international requirement**

   → The probability of the fall-back zone is the probability of the inputs and not a probability derived from an estimation of statistical distribution of the fragments (as for MC simulations). Working on the input domain, the probability can be directly and exactly computed.

2. **has an explicit physical meaning**

   → It works directly with the statistic distribution of the inputs and so with the causes of the rare event.

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Characteristics of the method
Computational speed

The Inputs' statistics method:

3. **decreases the computation time of the safety boxes by more than one order of magnitude (typically from hours/days to minutes).**
   → The time Δt_{integr} of a single integration of atmospheric re-entry dynamics requires about 1s, then:

State of the Art computational time:

$$T_{SoA} \cong T_{Montecarlo} = \\ = N_{samples} \cdot \Delta t_{integr} \\ = 10^6 \cdot O(1s) \\ = O(10^6 s) \\ = O(days)$$

Inputs' statistics computational time:

$$T_{InpStat} \cong T_{OptIter} \cdot N_{integr} \cdot \Delta t_{integr} = \\ = O(10) \cdot O(10) \cdot O(1s) \\ = O(10^2 s) \\ = O(minutes)$$

4. **requires a computational effort that doesn't depend on the computed probability**

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Characteristics of the method
Computational speed

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$$T_{SoA} \cong T_{Montecarlo} = \\ = \underbrace{N_{samples}} \cdot \Delta t_{integr} \\ = 10^6 \cdot O(1s) \\ = O(10^6 s) \\ = O(days)$$

Strongly dependent on the computed probability

Inputs' statistics computational time:

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All independent on the computed probability!

4. **requires a computational effort that doesn't depend on the computed probability**

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Characteristics of the method
Error estimation

The Inputs' statistics method:

5. **gives results whose conservatism is difficult to be estimated**
   → No control on the distance from the optimal solution: it does not provide the smallest safety box, but a larger one.
   → The minimum safety box is approached from a conservative direction.
   → Accurate results are not guaranteed for whichever transfer function.

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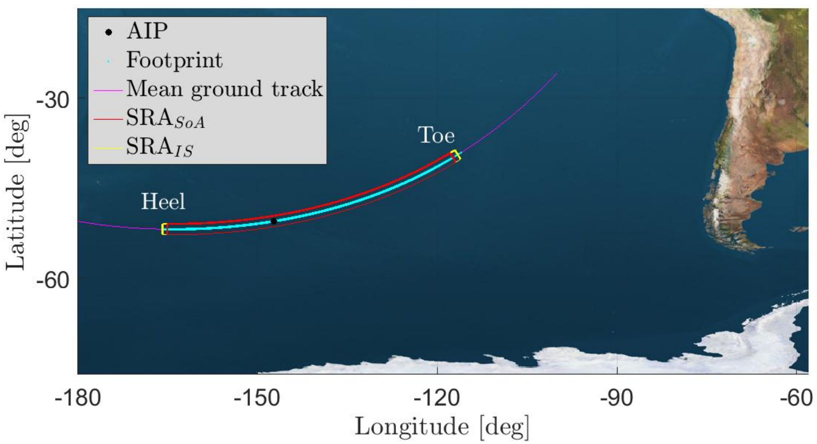
Application to the ATV-GL Shallow re-entry

Input variables:
4 normally distributed random variables

- magnitude of the ΔV of the second de-orbitation manoeuvre (DEO2) ΔV_{man};
- explosion altitude h_{Expl},
- pitch angle of thrust orientation δ,
- vehicle overall mass m_0.

Probability requirement:
Safety Re-entry Area $\alpha = 10^{-5}$



Results given by Inputs' Statistics method:
- Number of iterations of the programming algorithm: 33
- Computational time: 850 seconds
- SRA dimension: 4310 km

Results given by Monte Carlo Simulation + Peaks over threshold method:
- Number of samples: 40000 (half for short frag. and half for the long frag.)
- Computational time: about 22 hours
- SRA dimension: 3900 km

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Conclusions

- Presentation of a new approach for the estimation of rare events applied to the computation of the SRA
  - It is always conservative and it approximates the optimal solution
  - Computational time is few order of magnitude smaller than state of art methods
  - Not sensitive to the dimensions of the input parameters
  - Computed solution is close to the optimal one

- Good performances in comparison with state of art methods
  → SRA of ATV-GL shallow re-entry

- Several potential applications for engineered problems
  - Destructive controlled re-entry of large structures (e.g. ISS and visiting vehicles at its EoL)
  - Destructive re-entry of uncooperative satellites orbiting LEO and MEO (Active Debris Removal)
  - Destructive controlled re-entry of last stages launchers

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