ASTEROID PROXIMITY GNC ASSESSMENT THROUGH HIGH-FIDELITY ASTEROID DEFLECTION EVALUATION SOFTWARE (HADES)

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Why HADES?

Motivations
- Increasing number of missions to minor bodies (science, deflection)
- Relatively unknown environment (lack of knowledge prior arrival)
- Detailed preliminary analysis (operational orbit)
- System performance (e.g. thruster accuracy, sensors)

High-fidelity Asteroid Deflection Evaluation Software* capabilities
- Analysis of spacecraft motions at irregular objects
- Performance of different types of guidance schemes
- Relative navigation methods
- Performance of slow-push asteroid threat mitigation methods as Gravity Tractor (GT), Ion-beam Shepherd (IBS) and Laser Ablation (LA)

*Hades (ˈheɪdɪs; Ancient Greek: ᾍδης or Ἅδης, Haidēs) ancient Greek chthonic god of the underworld
HADES is a full Matlab software relying on a number of C routines
Propagation Module: Spacecraft Dynamics

- **Hill’s frame**
  \[
  \dot{r} + 2\hat{\omega}x\hat{r} + \hat{\omega}(\hat{\omega}x) = -\frac{\mu_s}{r^3}r + \mu_{\text{Sun}} \left( \frac{r_a}{r_a^3} - \frac{r_{sc}}{r_{sc}^3} \right) + \frac{\partial U}{\partial (\delta r)} + \text{SRP}(r_{sc}) + u
  \]

- **Asteroid’s frame**
  \[
  \dot{r} = -\frac{\mu_s}{\delta r^3}r + \mu_{\text{Sun}} \left( \frac{r_a}{r_a^3} - \frac{r_{sc}}{r_{sc}^3} \right) + \frac{\partial U}{\partial (\delta r)} + \text{SRP}(r_{sc}) + u
  \]

- **Body fixed frame**
  \[
  \dot{r} + 2\omega x \dot{r} + \omega(\omega x) = -\frac{\mu_s}{r^3}r + \mu_{\text{Sun}} \left( \frac{r_a}{r_a^3} - \frac{r_{sc}}{r_{sc}^3} \right) + \frac{\partial U}{\partial (\delta r)} + \text{SRP}(r_{sc}) + u
  \]

- **Perturbation**
  - SRP
    \[
    \text{SRP}(r_{sc}) = C_r s_{\text{SRP}} \left( \frac{r_{1AU}}{r_{sc}} \right)^2 \frac{r_{sc}}{r_{sc}} \frac{A}{m_{sc}}
    \]
  - Non uniform gravity field

Relative error between harmonics and shape model

Shape model gravity acceleration on (433) Eros
Control Module

- A continuous Lyapunov controller

- Discrete controllers based on the concept of control box

- Discrete control based on reflection method
  - invert the direction of the velocity when the spacecraft gets closer,
  - apply a manoeuvre along the radial direction such as to obtain the velocity reflected with respect to the tangential direction

- Discrete control based on dead band control

- Discrete LQR with integrative contribution

- Discrete control designed using stability criteria (Y. Liu et al, 2003)

Manoeuvre error in magnitude and direction: $u = R(\theta, \phi)u_{nom}(1 + r_{ex})$
Navigation Module

- **Problem definition**
  - Dynamics problem
  - Measurements
  - Estimate state variables

- **Performance model**

**H-infinity** bounds the maximum expected error with unknown statistics

\[
z_k = L_k x_k
\]

\[
I_h = \frac{1}{\rho} \|x_0 - \hat{x}_0\|_P^2 + \sum_{k=0}^{N-1} \left( \|z_k - \tilde{z}_k\|_S^2 - \frac{1}{\rho} \left( \|w_k\|_{Q_k}^2 + \|v_k\|_{R_k}^2 \right) \right) < 1
\]

\[
J_h = \min_{\hat{x}_k} \max_{w_k,v_k,x_0} I_h
\]

- weighted samples to propagate mean and covariance matrix through sigma-points

Sigma points \( \chi_i \)

\[
\tilde{x}_k = \sum_{l=0}^{2\tau} W_l^{(m)} \chi_{k|k-1}^l
\]

\[
\chi_{k|k-1}^l = f(\chi_{k|k-1}, u_k)
\]

\[
P_k^- = \sum_{l=0}^{2\tau} W_l^{(c)} (\chi_{k|k-1}^c - \tilde{x}_k)(\chi_{k|k-1}^c - \tilde{x}_k)^T.
\]

- update \( \tilde{x}_k = \tilde{x}_k^- + K(y_k - \tilde{y}_k^-) \)

- performance bound

\[
\theta_{k}^{-1} = \zeta \max \left( \text{eig} \left( P_k^- \right)^{-1} P_{xy,k} R_k^{-1} \right)^{-1}, \text{ Filter Gain } K = P_{xy,k} P_{y,k}^{-1}
\]
Navigation Module: Measurements Model

- Camera and LIDAR models
  - Simple based on ellipsoidal shape
    - Centroid identification \((x_c, y_c)\)
    - Local azimuth and elevation
    - LIDAR pointing towards the centroid
      \[ d = \left\| \delta r_{SC} - x_{surface}^c \right\| \]
  - Detailed based on actual shape models
    - Centre of brightness identified on the camera screen
    - LIDAR illuminates a spot close to the centre of brightness

- Modelisation of illumination and visibility

Example of image as seen on the screen of the camera and footprint of the LIDAR
Body Fixed with Asteroid Didymos

- Fixed hovering at 200m along a-axis
  - asteroid size $[c_1, c_2, c_3] = [1.05, 0.65, 0.45]$ km
  - 4th order gravity field

- 100 m position error
- 1 cm/s velocity error
- 2% (3σ) error in magnitude
- 1.5 deg (3σ) error in direction
- 20 m along track (1σ)
- 10 m cross track (1σ)
- 2 mm/s along track (1σ)
- 1 mm/s cross track (1σ)

### Continuous Thrust

<table>
<thead>
<tr>
<th>Method</th>
<th>Actuation interval</th>
<th>$\Delta v$ [m/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Continuous thrust</td>
<td>5</td>
<td>56.2</td>
</tr>
<tr>
<td>Control box (100m)</td>
<td>300</td>
<td>62.5</td>
</tr>
<tr>
<td>Dead-band control (100m)</td>
<td>300</td>
<td>64.3</td>
</tr>
<tr>
<td></td>
<td>600</td>
<td>61.7</td>
</tr>
<tr>
<td></td>
<td>900</td>
<td>54.4</td>
</tr>
<tr>
<td>Discrete LQR</td>
<td>300</td>
<td>55.8</td>
</tr>
<tr>
<td></td>
<td>600</td>
<td>54.9</td>
</tr>
<tr>
<td></td>
<td>900</td>
<td>53.4</td>
</tr>
<tr>
<td></td>
<td>1200</td>
<td>51.2</td>
</tr>
</tbody>
</table>

### Dead-band with 900 s sampling time

### 1200 s Discrete LQR

### LQR for hovering
1) 200 m above $a$, 2) 200 m above $b$, 3) 3D position 1300 m
Hill’s Hovering: Controlling the Illumination Angle

Objective: to maintain 5 degrees illumination angle

- Reference trajectory
  \[ x_{\text{ref}} = [-3.373 \text{km}, 0 \text{km}, 0 \text{km/s}, 0 \text{km/s}, 0 \text{km/s}] \]
- Initial trajectory
  \[ x_0 = [-3.423 \text{km}, 0.05 \text{km}, 0.05 \text{km/s}, 0.000031 \text{km/s}, -0.000051 \text{km/s}, 0.000020 \text{km/s}] \]
- 4th order gravity field

<table>
<thead>
<tr>
<th>Method</th>
<th>Actuation interval (min-max-mean) [min]</th>
<th>( \Delta v ) [m/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Continuous thrust</td>
<td>8</td>
<td>16.9</td>
</tr>
<tr>
<td>Control box</td>
<td>421/533/485</td>
<td>24.2</td>
</tr>
<tr>
<td>Reflection method</td>
<td>325/1792/890</td>
<td>11.2</td>
</tr>
<tr>
<td>Dead-band control</td>
<td>8/440/140</td>
<td>25.9</td>
</tr>
<tr>
<td>Discrete LQR (fixed)</td>
<td>360</td>
<td>24.1</td>
</tr>
<tr>
<td>Discrete LQR (fixed)</td>
<td>240</td>
<td>24.0</td>
</tr>
<tr>
<td>Discrete LQR (fixed)</td>
<td>180</td>
<td>23.1</td>
</tr>
<tr>
<td>Stable PD (fixed)</td>
<td>360</td>
<td>18.2</td>
</tr>
<tr>
<td>Stable PD (fixed)</td>
<td>240</td>
<td>18.2</td>
</tr>
<tr>
<td>Stable PD (fixed)</td>
<td>180</td>
<td>18.2</td>
</tr>
</tbody>
</table>

\( \Delta v \) budget for 60 days

- 20 m along track (1\( \sigma \))
- 10 m cross track (1\( \sigma \))
- 2 mm/s along track (1\( \sigma \))
- 1 mm/s cross track (1\( \sigma \))

\( \text{Reference trajectory} \)
\( \text{Initial trajectory} \)
\( \text{4th order gravity field} \)
Hill’s hovering: Navigation Errors

Fixed hovering
- Initial and reference trajectory
  \[ x_0 = 5 \cdot [-1\text{km}, 1\text{km}, \sqrt{2}\text{km}, 0\text{km/s}, 0\text{km/s}, 0\text{km/s}] \]
- 100 m initial position estimate error
- 1 cm/s initial velocity error

Measurements assembly characteristics

<table>
<thead>
<tr>
<th></th>
<th>Lidar mounting error (1σ)</th>
<th>Lidar range error (1σ)</th>
<th>Lidar range bias (1σ)</th>
<th>Number of pixels per side</th>
<th>Camera FoV</th>
<th>Camera side</th>
<th>Attitude error (1σ)</th>
<th>Attitude bias (1σ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1:</td>
<td>0.001 deg</td>
<td>10 m</td>
<td>1 m</td>
<td>2048</td>
<td>20 deg</td>
<td>10 cm</td>
<td>0.0057 deg</td>
<td>0.0006 deg</td>
</tr>
<tr>
<td>Case 2:</td>
<td>0.001 deg</td>
<td>10 m</td>
<td>1 m</td>
<td>2048</td>
<td>20 deg</td>
<td>10 cm</td>
<td>0.0057 deg</td>
<td>0.0006 deg</td>
</tr>
<tr>
<td>Case 3:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case 4:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\( \Delta v \) budget for 3 day

<table>
<thead>
<tr>
<th>Case</th>
<th>( \Delta v ) [cm/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>9.18</td>
</tr>
<tr>
<td>Case 2</td>
<td>9.14</td>
</tr>
<tr>
<td>Case 3</td>
<td>9.25</td>
</tr>
<tr>
<td>Case 4</td>
<td>9.18</td>
</tr>
</tbody>
</table>

10% difference between different models

Case 1:
- True world -> ellipsoid shape
- Filter 4\textsuperscript{th} order gravity field
- Measurements model simple

Case 2:
- True world -> ellipsoid shape
- Filter 4\textsuperscript{th} order gravity field
- Measurements model detailed

Case 3:
- True world -> 1708 facets*
- Filter -> 1708 facets*
- Measurements model detailed

Case 4:
- True world -> 7790 facets*
- Filter -> 1708 facets*, 1% error on \( R_{\text{mean}} \)
- Measurements model detailed

*shape from asteroid (433) Eros
Station Keeping at 10 km Station for AIM

- Initial trajectory
  \[ x_0 = 5 \cdot [-1\text{km}, 1\text{km}, \sqrt{2}\text{km}, 0\text{km/s}, 0\text{km/s}, 0\text{km/s}] \]

- Control box 1.5 km side
  - 2\% (3\sigma) error in magnitude
  - 1.5 deg (3\sigma) error in direction

- Performance model
  - 20 m along track (1\sigma)
  - 10 m cross (1\sigma)
  - 2 mm/s along track (1\sigma)
  - 1 mm/s along track (1\sigma)
Station Keeping at 10 km Station for AIM

- Synthetic results displayed: single case

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total control budget</td>
<td>1.9154 m/s</td>
</tr>
<tr>
<td>Mean Time between actuations</td>
<td>4547.3684 min</td>
</tr>
<tr>
<td>Min Time between actuations</td>
<td>4392.0903 min</td>
</tr>
<tr>
<td>Max Time between actuations</td>
<td>4858.802 min</td>
</tr>
</tbody>
</table>

- MC simulation
  - 20 m along track (1σ)
  - 10 m cross (1σ)
  - 2 mm/s along track (1σ)
  - 1 mm/s along track (1σ)
  - 50 m per component position dispersion (1σ)
  - 1 cm/s per component velocity dispersion (1σ)
  - 2% (3σ) error in magnitude
  - 1.5 deg (3σ) error in direction

- MC statistics (30 days)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total control budget</td>
<td>0.93109 m/s (dispersion 1-σ: 0.048834 m/s)</td>
</tr>
<tr>
<td>Mean Time between actuations</td>
<td>3.1824 days (dispersion 1-σ: 0.03609 days)</td>
</tr>
<tr>
<td>Min mean Time between actuations</td>
<td>3.0032 days (dispersion 1-σ: 0.087042 days)</td>
</tr>
<tr>
<td>Max mean Time between actuations</td>
<td>3.3528 days (dispersion 1-σ: 0.068887 days)</td>
</tr>
<tr>
<td>Min mean dy actuations</td>
<td>0.042674 m/s (dispersion 1-σ: 0.023174 m/s)</td>
</tr>
<tr>
<td>Max mean dy actuations</td>
<td>0.067255 m/s (dispersion 1-σ: 0.0010453 m/s)</td>
</tr>
</tbody>
</table>
Deflection Module

- **Laser Ablation**
  - 3D Interaction laser-matter (Thiry et al. 2015)
  - Thrust aligned to local surface normal
  - Affected by rotational velocity
  - No contamination included
  - 2 laser pointing strategies
    - Pointing to obtain desired thrust direction
    - Fixed laser pointing

- **Ion Beam shepherd**
  - Ion plume Gaussian expansion (Goebel et al. 2008)
  - Ion plume constant axial velocity
  - 2 thrusters required
  - No contamination included

- **Gravity Tractor**
  - Avoid asteroid impingement by expansion plume

- **Orbital and rotational dynamics computational intensive:**
  - Mean directions of thrust and efficiency based on the actual geometry of the asteroid over a control period of 7 days
  - Variational approach implemented (Gauss equations)
  - No actual spacecraft control integrated
Deflecting a 100 m Asteroid

- (433) Eros scaled down to 100 m
- mass 9.263 \cdot 10^8 \text{ kg}
- 20 years operations
- 20 kW maximum power for laser beam (\( \mu = 50\% \)) or thrusters (\( \mu = 60\% \))
- Systems placed at 200 m along track

Results

- GT Maximum deflection
  - spacecraft mass circa 16500 kg
- LA with variable pointing 20% more efficient than the fixed pointing
  - operationally more complex
- IBS appears less efficient but
  - more lightweight and less affected by contamination

Laser ablation deflection: 1) controlling laser pointing, 2) maintaining laser pointing fixed.

Ion Beam Shepherd deflection

Gravity Tractor deflection.
Debris Analysis around a 100 m Asteroid

- Possible impact on a 100 m asteroid
- Time to clear the proximity of the asteroid

Hill radius: circa 15 km  
Asteroid 2013XK22: Shape Geographos  
\([a, e, i, \Omega_a, \omega_a] = [1.045\text{AU}, 0.2034, 0.1221\text{rad}, 3.187\text{rad}, 4.6374\text{rad}]\)

- 30,000 uniformly distributed samples on the surface \(E \leq 0\)
- different value of area to mass ratios (A2M)
- fractions \(k_\omega\) of the asteroid’s nominal angular velocity

The initial conditions play important role in the number of surviving particles; if all the SRP is the same, particles with lower initial tangential velocity will have more probability to survive for longer period

The SRP will affect the survivability – A2M = 0.001 kg/m² produces more surviving samples because particles are more affected by the asteroid’s gravity and less by the SRP

At the beginning the particles with higher energy (close to 0) will experience SRP and third body effects leading to escape/impact
Conclusions & Future Works

- Main features and architecture of HADES
  - Dynamics in 3 different frames
  - Several control techniques available
  - Different measurement models
  - Use of asteroid actual shapes
  - GNC
  - Deflection analysis

- Possibility of performing broad range of simulations
  - Assessment different control laws
  - Assessment impact of environment knowledge
  - MC simulations
  - Debris analysis
  - LA, IBS and GT deflections

- Improvements
  - Integration of deflection systems
    - Propagation Module
    - Control Module
    - Navigation Module
      - Estimation of the deflective action
  - Effect of contamination

LA laser pointing control with contamination
Thank you

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www.stardust2013.eu
twitter.com/stardust2013eu

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Control Module

\[ \dot{x} = \left[ -\frac{\mu_a}{\delta r^3} \begin{bmatrix} I_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & I_{3 \times 3} \end{bmatrix} \right] \delta x + \begin{bmatrix} 0_{3 \times 3} \\ \beta \end{bmatrix} \delta x + \begin{bmatrix} 0_{3 \times 3} \\ I_{3 \times 3} \end{bmatrix} u = \begin{bmatrix} 0_{3 \times 3} \\ \beta \end{bmatrix} \delta x + \begin{bmatrix} 0_{3 \times 3} \\ I_{3 \times 3} \end{bmatrix} u \]

\[ \delta x_{k+1} = \left[ I + \frac{\Delta t^2}{\beta \Delta t} \begin{bmatrix} I_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & I_{3 \times 3} \end{bmatrix} \right] \delta x_k + \begin{bmatrix} [\Delta t I_{3 \times 3}] \\ I_{3 \times 3} \end{bmatrix} u_k = A_k \delta x_k + B_k u_k \]

- **Discrete LQR with integrative contribution**

\[ J(u) = \sum_{k=1}^{\infty} x_k^T Q x_k + u_k^T R u_k \]

\[ K = (B_k^T S B_k + R)^{-1} B_k^T S A_k \]

\[ A_k^T S A_k - S - A_k^T S A_k (B_k^T S B_k)^{-1} B_k^T S A_k + Q = 0 \]

- **Discrete control designed using stability criteria (Y. Liu et al, 2003)**

\[ x = f(t, x) \quad t \neq t_k \]

\[ z = h(x) \quad t \neq t_k \]

\[ \Delta x = u_k(x) \quad t = t_k \]

\[ x(t_0) = x_0 \quad t = t_0 \]

1- There has to exist a constant \( l_k \)

\[ D^+ V(t, x) \leq \frac{l_k}{\Delta t_k} c_k(V(t, x)) \quad \gamma = \frac{l_k}{\Delta t_k} \]

\( D^+ \) is the right-hand generalized derivative

\( \Delta t_k \) control time

\( c_k(\ldots) \) suitable scalar function

2- There has to exist a constant \( l_k \)

\[ V(t_k^+, x + u_k) \leq V(t_k, x) + v_k d_k(V(t, x)) \]

\[ l_k + v_k \leq 0 \]

for suitable neighbourhood of \( s(q)/\|x\| \leq q \)

\[ c_k(s) \leq d_k(s) \quad \text{if} \quad v_k < 0 \]

and

\[ d_k(s) \leq c_k(s) \quad \text{if} \quad l_k < 0 \]

\[ s + v_k d_k(s) \leq 0 \quad d_k(s) = c_k(s) = s \]

\[ V(t, x) \leq a(\|x\|) \]

\[ V(t, x) = \frac{1}{2} (x^2 + y^2 + z^2 + T^2 v_x^2 + T^2 v_y^2 + T^2 v_z^2) \]

\[ u_k(\delta x) = [0.0.0,-ax,-ay-bv_y,-az-bv_z]^T \]

\[ D^+ V(t, x) = xx_x + yy_y + zz_z + \beta_1 T^2 xx_x + \beta_2 T^2 yy_y + \beta_3 T^2 zz_z \leq \gamma \frac{1}{2} (x^2 + y^2 + z^2 + T^2 v_x^2 + T^2 v_y^2 + T^2 v_z^2) \]

\[ V(t_k^+, x + u_k) = \frac{1}{2} (x^2 + y^2 + z^2 + T^2 (v_x + ax + bv_z)^2 + T^2 (v_y + ay + bv_y)^2 \]

\[ + T^2 (v_z + az + bv_z)^2) \leq \gamma \frac{1}{2} (x^2 + y^2 + z^2 + T^2 v_x^2 + T^2 v_y^2 + T^2 v_z^2) \]

\[ x^T A x \geq 0 \]
Infos

- HADES runs completely in **matlab**
- **Routines** both in Matlab® and **mex-fied in C for speed purpose**
  - dynamics equations (mainly for gravity field purposes)
  - visibility functions (shapes, shadows)
- **Integration of ODE**
  1. matlab ode **Runge-Kutta 45** (for events to be located)
  2. integrator **ODEMEXv12** package
    - **✓** 10-50 times faster
    - **✗** No events handling
    - **✗** asteroid shape dynamics to be included
- **Hints**
  - Coupled Control-Navigation intensive
    - **✓** performance methods for long simulations
    - **✓** simulations to draw necessary statistics and move to performance model
  - Continuous, discrete LQR and control box tends to be more precise and robust
Gravity Based on Polyhedral Model

Potential gradient defined using polyhedron described by triangular facets

\[ \nabla U = -G\rho \sum_{e=edges} L_e \hat{E}_e \cdot \hat{r}_e + G\rho \sum_{f=facets} \omega_f \hat{E}_f \cdot \hat{r}_f \]

Geometrical characterisation of the surface (normals, edges etc)

\[ L_e = \frac{r_i + r_j + e_{ij}}{r_i + r_j - e_{ij}} \]
\[ \omega_f = 2 \arctan \left( \frac{\hat{r}_i \cdot \hat{r}_j \times \hat{r}_k}{r_i r_j r_k + r_i (\hat{r}_j \cdot \hat{r}_k) + r_j (\hat{r}_i \cdot \hat{r}_k) + r_k (\hat{r}_i \cdot \hat{r}_j)} \right) \]

\[ \hat{E}_e = \vec{n}_A \vec{n}_{21}^A + \vec{n}_B \vec{n}_{12}^B \]
\[ \hat{E}_f = \vec{n}_f \vec{n}_f \]

\( r_f \) spacecraft distance from facet’s centre
\( r_e \) spacecraft distance from edge centre

Asteroid (433) Eros - acceleration on the surface: comparison with paper Winkler

Winkler

 Implemented model

Solution overlapping – difference due to number of facets and density (assumed 2500 kg/m\(^3\))
Spherical Harmonics and Reflection

Ellipsoid of arbitrary size: harmonics coefficients (Boyce 1997)

\[ C_{2n,2m} = \frac{3}{2\pi R^2} \frac{n!(2n-2m)!}{2^{2m}(2n+3)(2n+1)!} \left( 2 - \delta_{0m} \right) \sum_{i=0}^{\text{int}\left(\frac{n-m}{2}\right)} \frac{(a^2 - b^2)^{m+2i} + 1/2(a^2 + b^2)^{n-m-2i}}{16i(n-m-2i)! (m+i)!} \]

Gravity attraction error on the circumscribing sphere at 3 km

Mesh 14400 facets

Reflection: unable to maintain 5 degrees illumination angle