

# The Pointing Error Engineering Tool (PEET): From Prototype to Release Version

*Marc Hirth<sup>(1)</sup>, Haifeng Su<sup>(1)</sup>, Thomas Ott<sup>(2)</sup>, Massimo Casasco<sup>(3)</sup>, Sohrab Salehi<sup>(4)</sup>*

(1) Astos Solutions GmbH

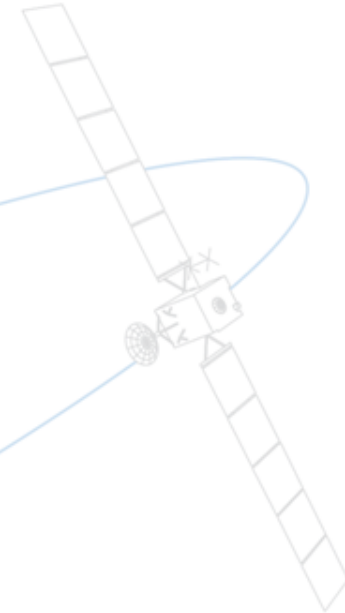
(2) Airbus DS – Space Systems

(3) ESA, ESTEC

(4) Rhea System for ESA

# Outline

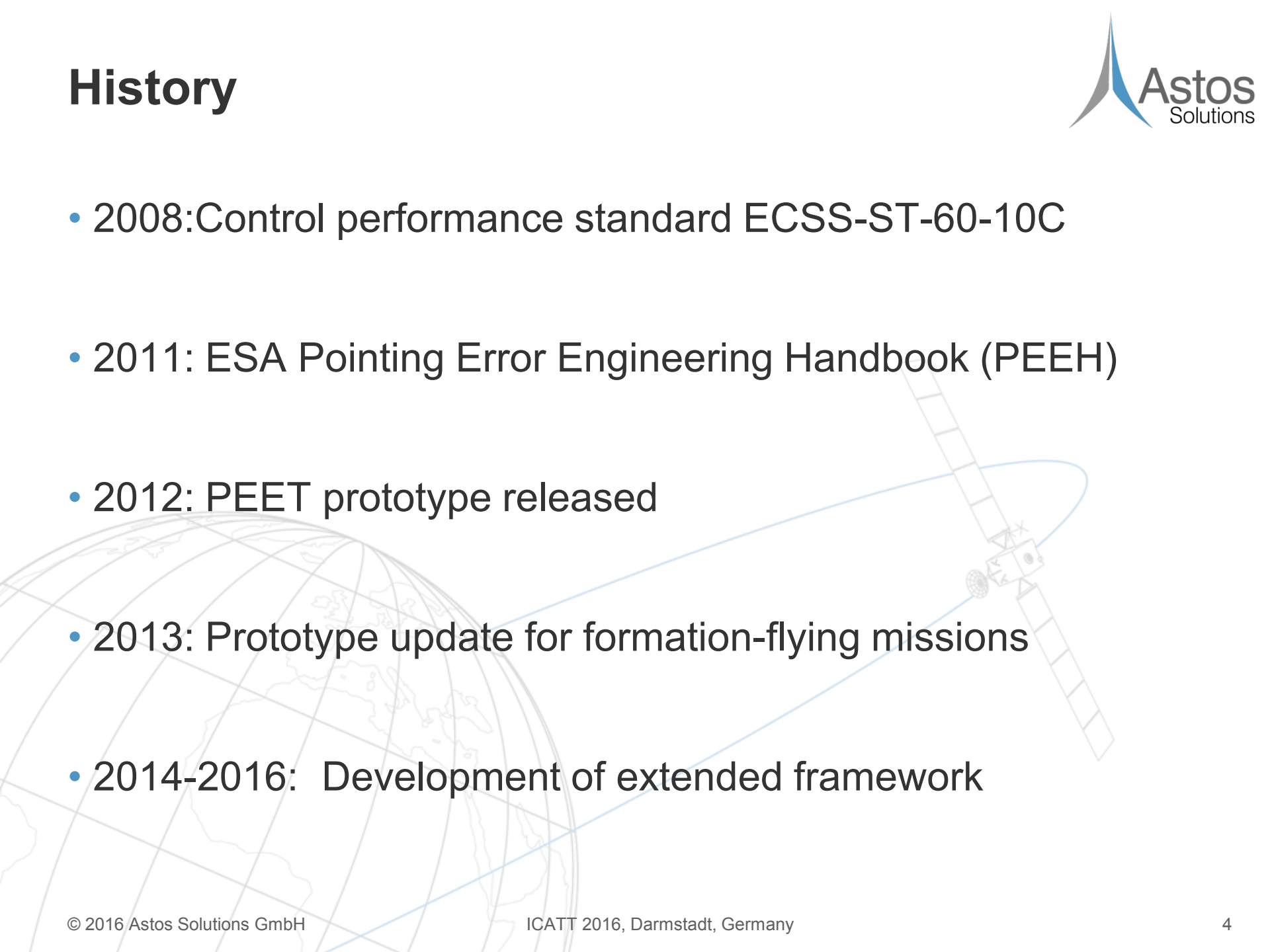
- Introduction
- PEEH Methodology in the Tool
- Software Overview
- “Advanced Statistical Method”
- Conclusion



# Introduction



# History

- 2008: Control performance standard ECSS-ST-60-10C
  - 2011: ESA Pointing Error Engineering Handbook (PEEH)
  - 2012: PEET prototype released
  - 2013: Prototype update for formation-flying missions
  - 2014-2016: Development of extended framework
- 

# Motivation

More and more stringent performance requirements in ESA projects (e.g. scientific or laser-communication missions)



Why PEEH?

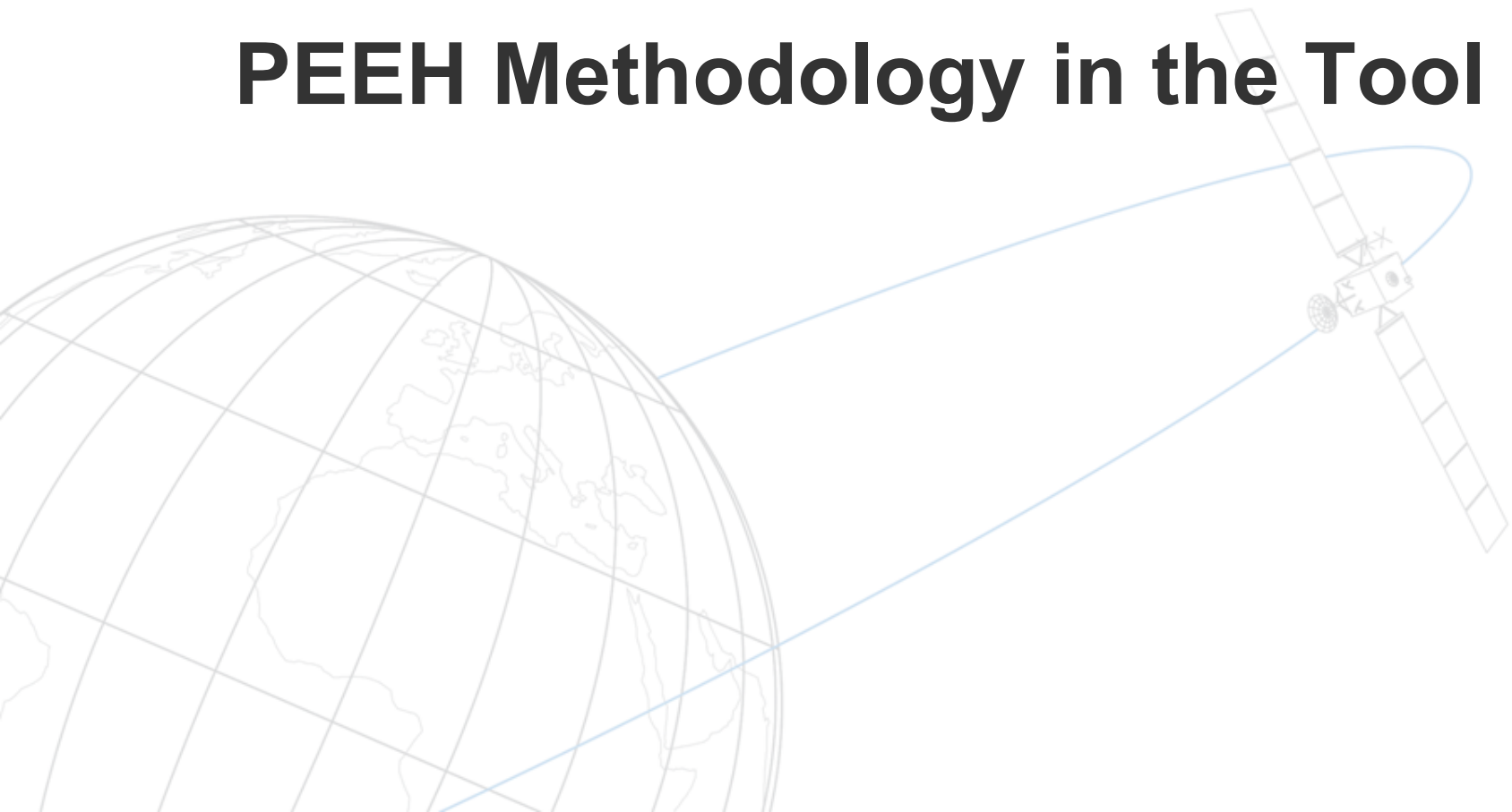
- Necessity for clear and accurate pointing error engineering methodology



Why PEET?

- Systematic and user-friendly application of methodology
- Automated performance management process
- Replacement of “manual” computations
- Support dissemination of the methodology

# PEEH Methodology in the Tool



# PEEH Methodology

PEEH provides explicit guideline for requirement definition and evaluation of error budgets using dedicated analysis steps:

- “AST-0” : Requirement specification
- AST-1: Error source characterization
- AST-2: Transfer analysis
- AST-3: Error index contribution analysis
- AST-4: Error evaluation

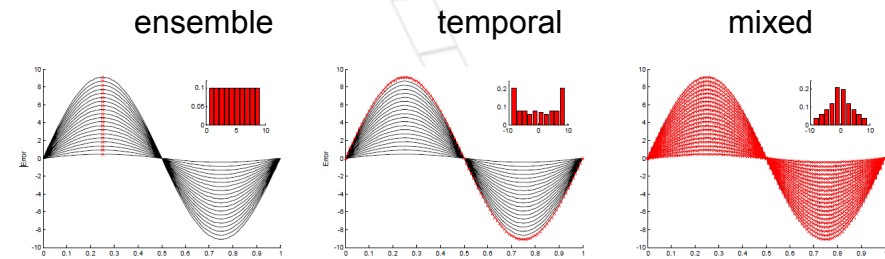
The PEET workflow is fully compatible with the methodology and provides all related parameters and setup options

# AST-0: Requirement specification

Unambiguous definition of requirements by parameters dependent on requirement type

- **Statistical requirements**

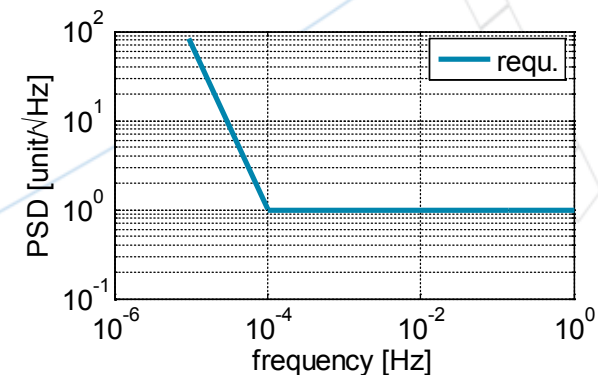
- Max. error value (per axis or half-cone)
- Related level of confidence
- Metric for time-windowed errors
- Statistical interpretation



Statistical interpretation (ECCS-ST-60-10C)

- **Spectral requirements**

- Spectral requirement function (PSD upper bound)
- (Metric for time-windowed errors)

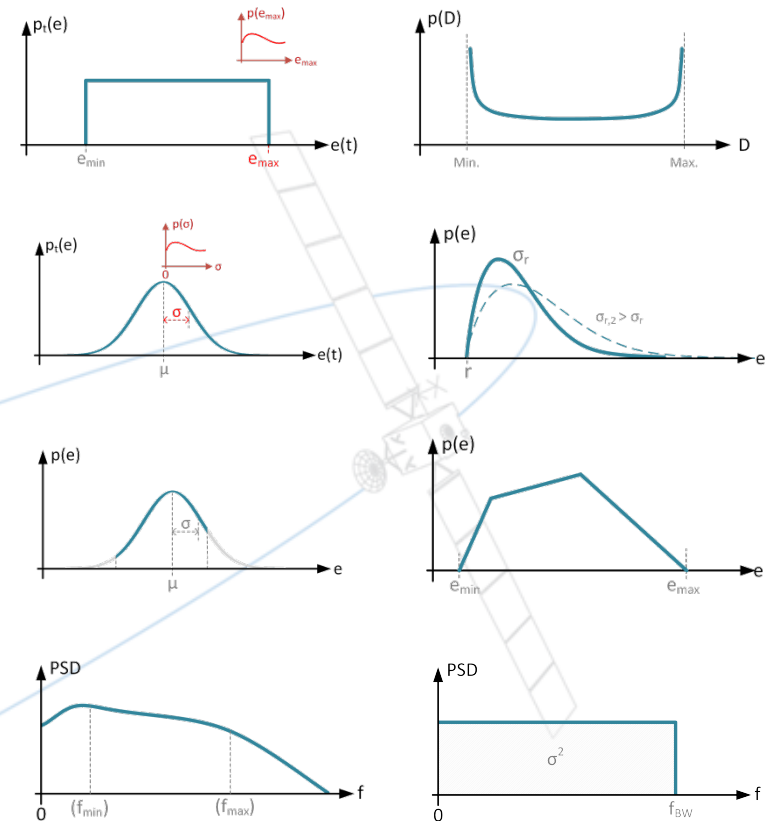




# AST-1: Error Source Characterization

Identification of potential error sources and classification based on characteristic properties:

- Random variables
  - Time-constant & Time-random
- Periodic errors
- Drift errors
- Random processes
  - BLWN or PSD

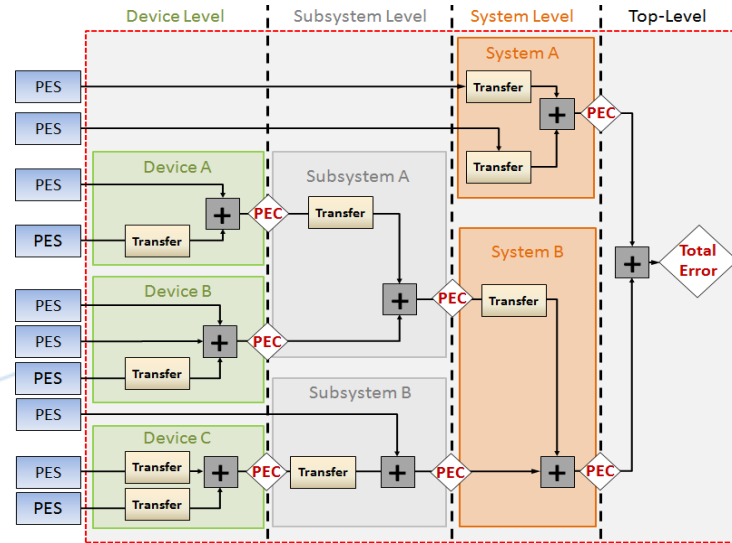


Statistical distributions describe temporal behavior and/or ensemble distribution of parameters

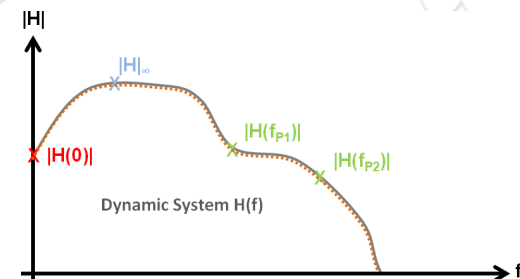
# AST-2: Transfer analysis

Determines how initial error sources affect the figure of merit, i.e. the “route” to the final error contribution

- Static systems
  - Generic models
  - Coordinate Transformations, etc.
- Dynamic (LTI) systems
  - Generic models
  - Flexible plant, gyro-stellar estimator
- Feedback systems



Transfer rules dependent on error signal class



# AST-3: Error Index Contribution Analysis

Impact of time-windowed errors for all metrics present in ECSS standard (e.g. APE, MPE):

- Frequency domain evaluation
  - Metric expressed as rational transfer functions
  - Applied to periodic and random process errors
- Evaluation according to ECSS rules
  - Random variables

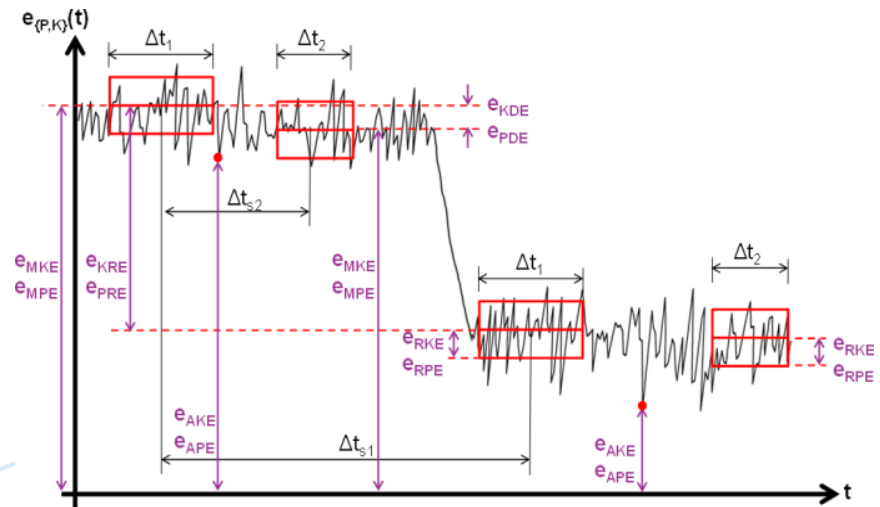
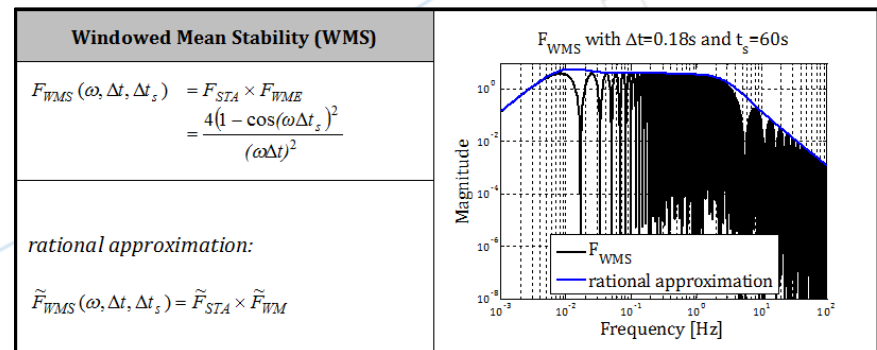


Table B-3: Budget contributions from zero mean Gaussian random errors

Index	S.I.	Distribution			Notes
		P(e)	$\mu(e)$	$\sigma(e)$	
APE	E	$P(3s) = \frac{1}{3}P(s)$	$3\mu$	$3\sigma$	See text. For P(s), $\mu$ and $\sigma_s$ see B.6
	T	$G(0, swc^2)$	0	swc	$G(m, V) =$ Gaussian with specified mean & variance, swc = worst case s
	M	$\int G(0, s^2)P(s)ds$	0	$\langle s \rangle$	For P(s), $\mu_s$ and $\langle s \rangle$ see B.6. For derivation see B.7



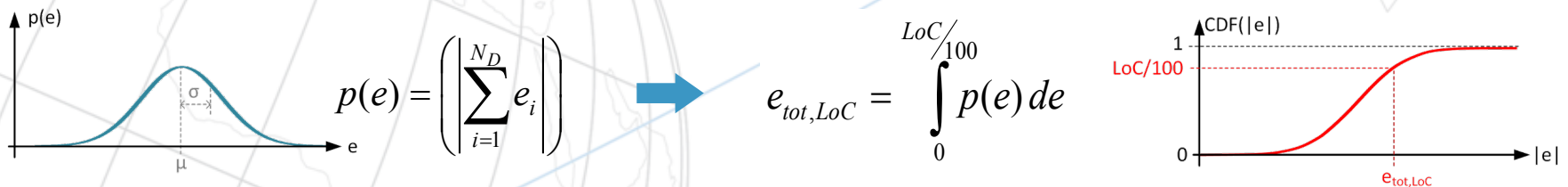
# AST-4: Error evaluation

Summation of final error contributions according to the selected statistical method:

- Simplified statistical method (used in prototype)
  - Evaluation based on stat. moments (central limit theorem) and a confidence factor
  - Simplified correlation (full/no correlation)

$$e(68\% / 95\% / 99.7\%) = \left| \sum \mu_i \right| + n_p \cdot \sum \sigma_i, \quad n_p = 1, 2, 3$$

- Advanced statistical method (used in release version)
  - Evaluation based on probability density functions & explicit level of confidence values
  - Specific correlation between contributions



# Software Overview



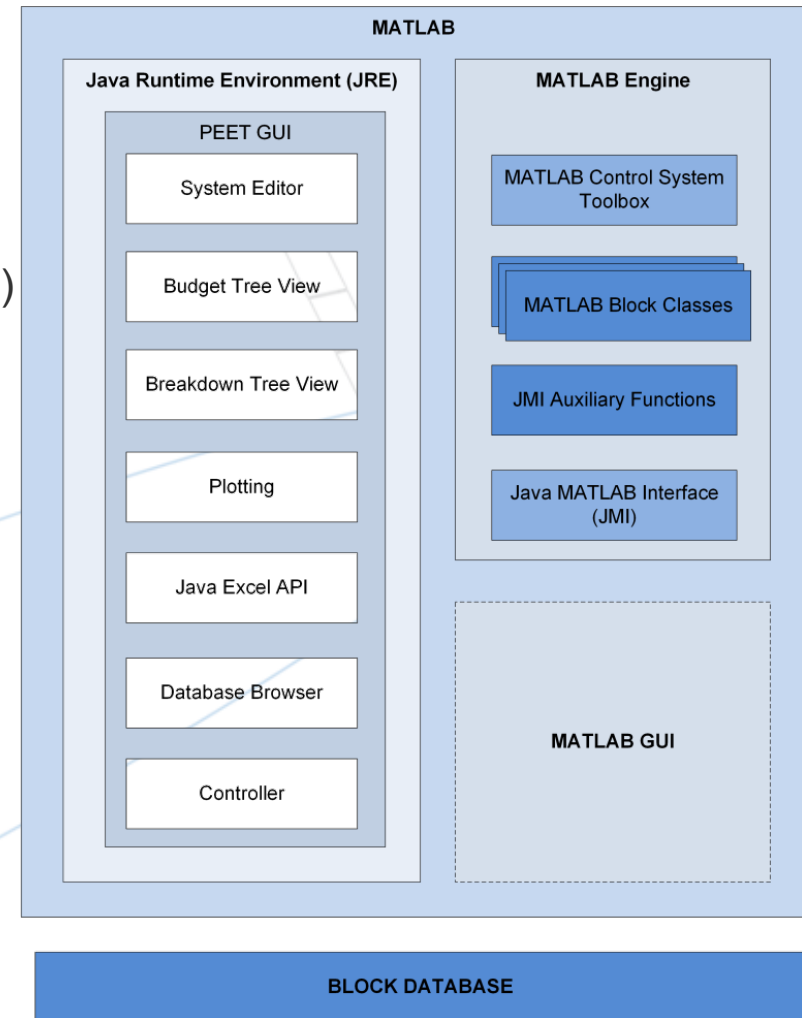
# Platforms and Requirements

- Designed as extension to Matlab with following requirements:
  - Matlab 2011b and later
  - Control System Toolbox
- Multiple platform support
  - Windows
  - Linux
  - Macintosh
- No “critical” hardware requirements
  - Runs on standard desktop PCs and laptops



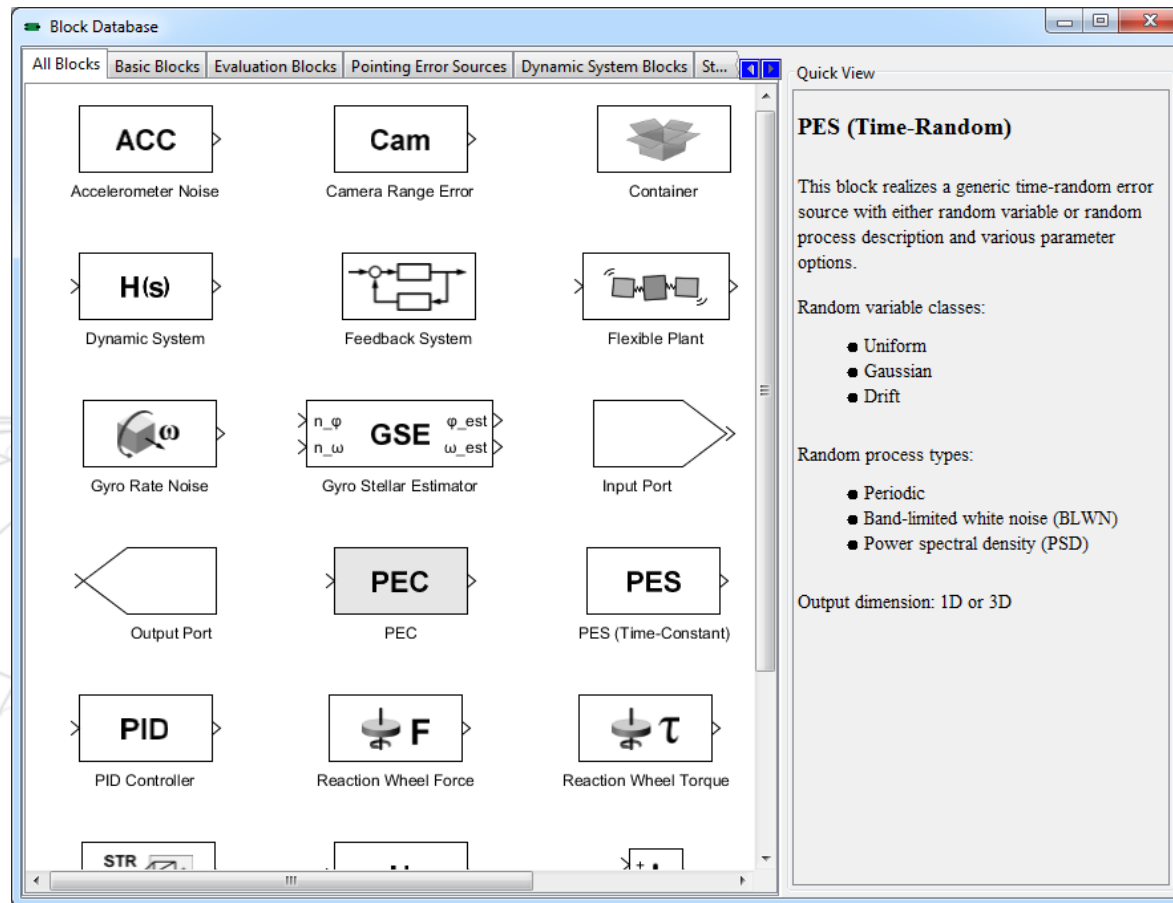
# Architecture and External Interfaces

- Main components:
  - Java GUI
  - Matlab core algorithms
  - Communication via Java-Matlab Interface (JMI) and XML scenario definition files
- Interfaces:
  - Import from MS Excel spreadsheets
  - Links to Matlab workspace variables
  - Configurable report to MS Excel
- Operation modes:
  - GUI mode
  - Script-base execution controlled by (user-defined) Matlab scripts



# Graphical User Interface

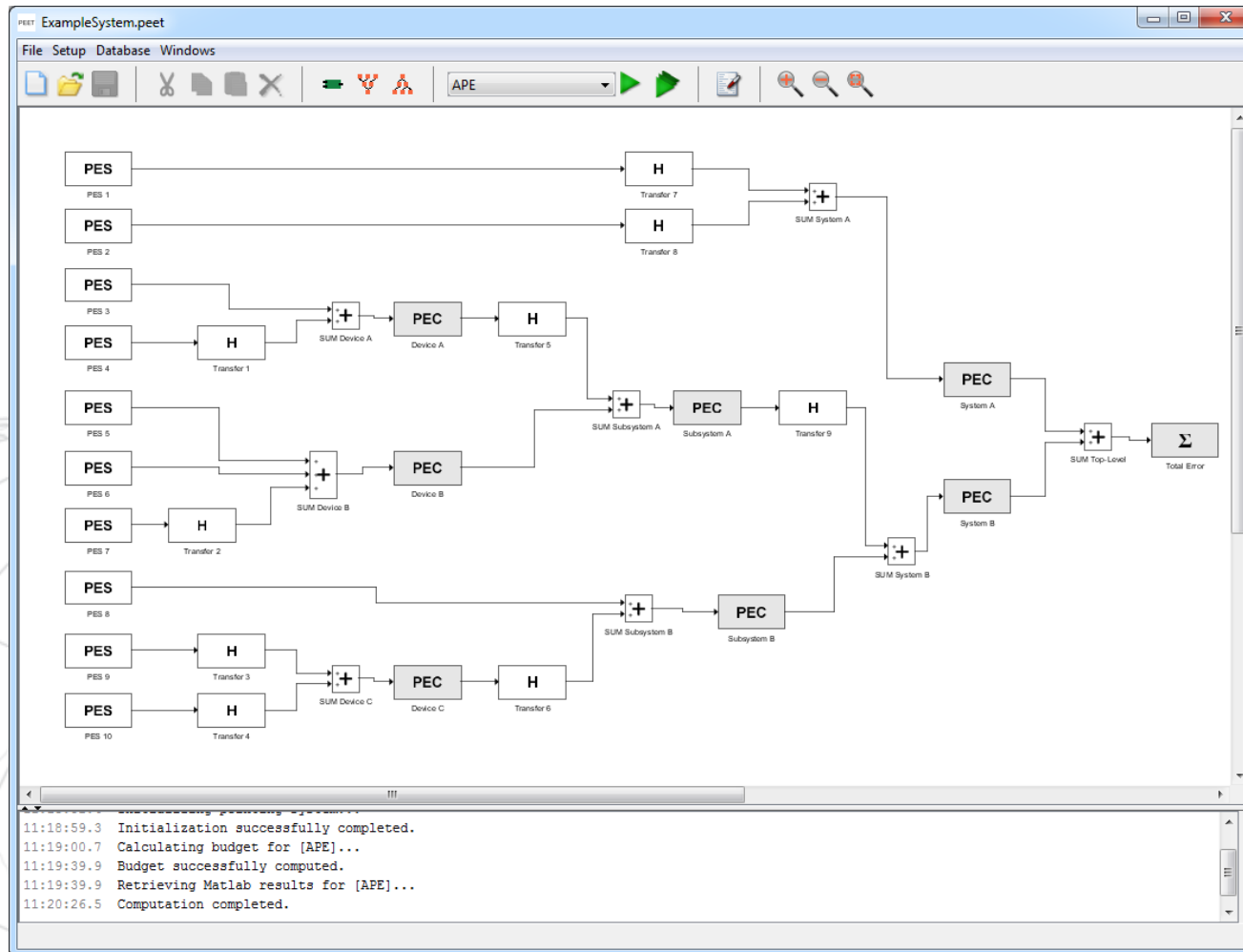
Block Database: Select identified error source and system models





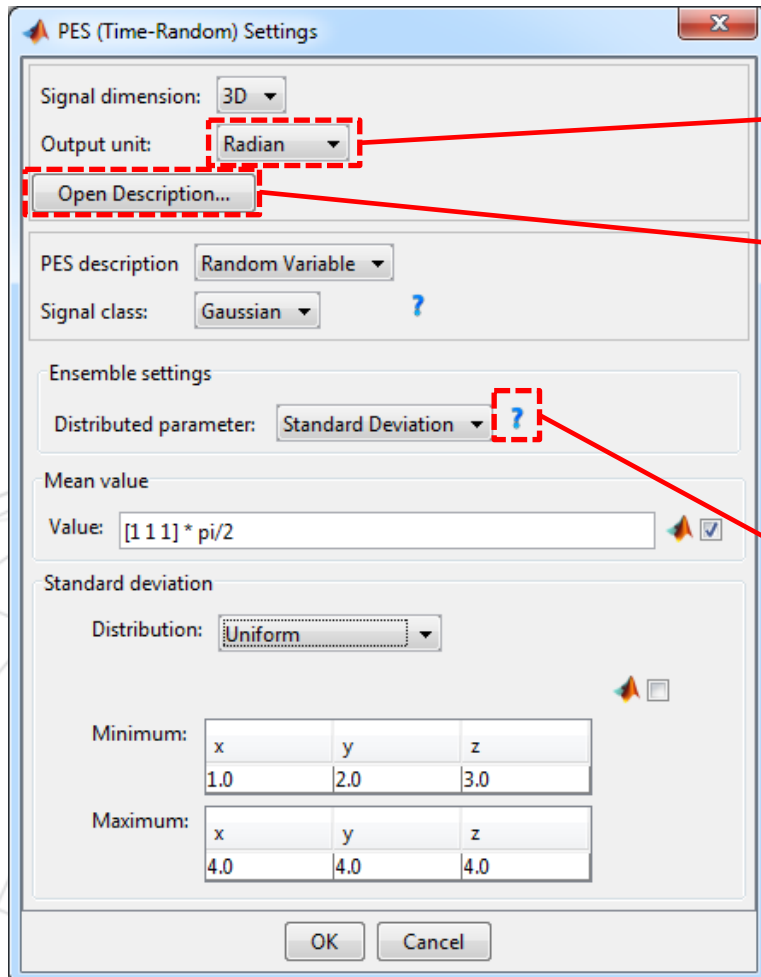
# Graphical User Interface (cont'd)

## System Editor: Define requirements & system interconnections



# Graphical User Interface (cont'd)

## Block Dialogs: Specify source/system characteristics



Signal dimension: 3D

Output unit: Radian

Open Description...

PES description: Random Variable

Signal class: Gaussian

Ensemble settings

Distributed parameter: Standard Deviation

Mean value

Value: [1 1 1] \* pi/2

Standard deviation

Distribution: Uniform

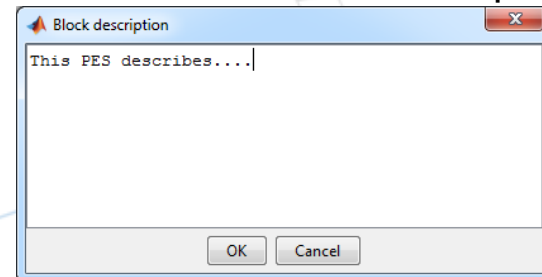
Minimum:	x	y	z
	1.0	2.0	3.0

Maximum:	x	y	z
	4.0	4.0	4.0

OK Cancel

Custom or default units

User-defined block description

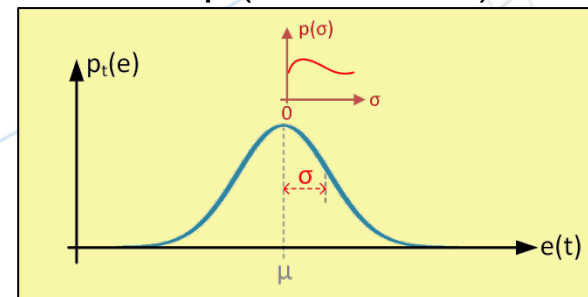


Block description

This PES describes...|

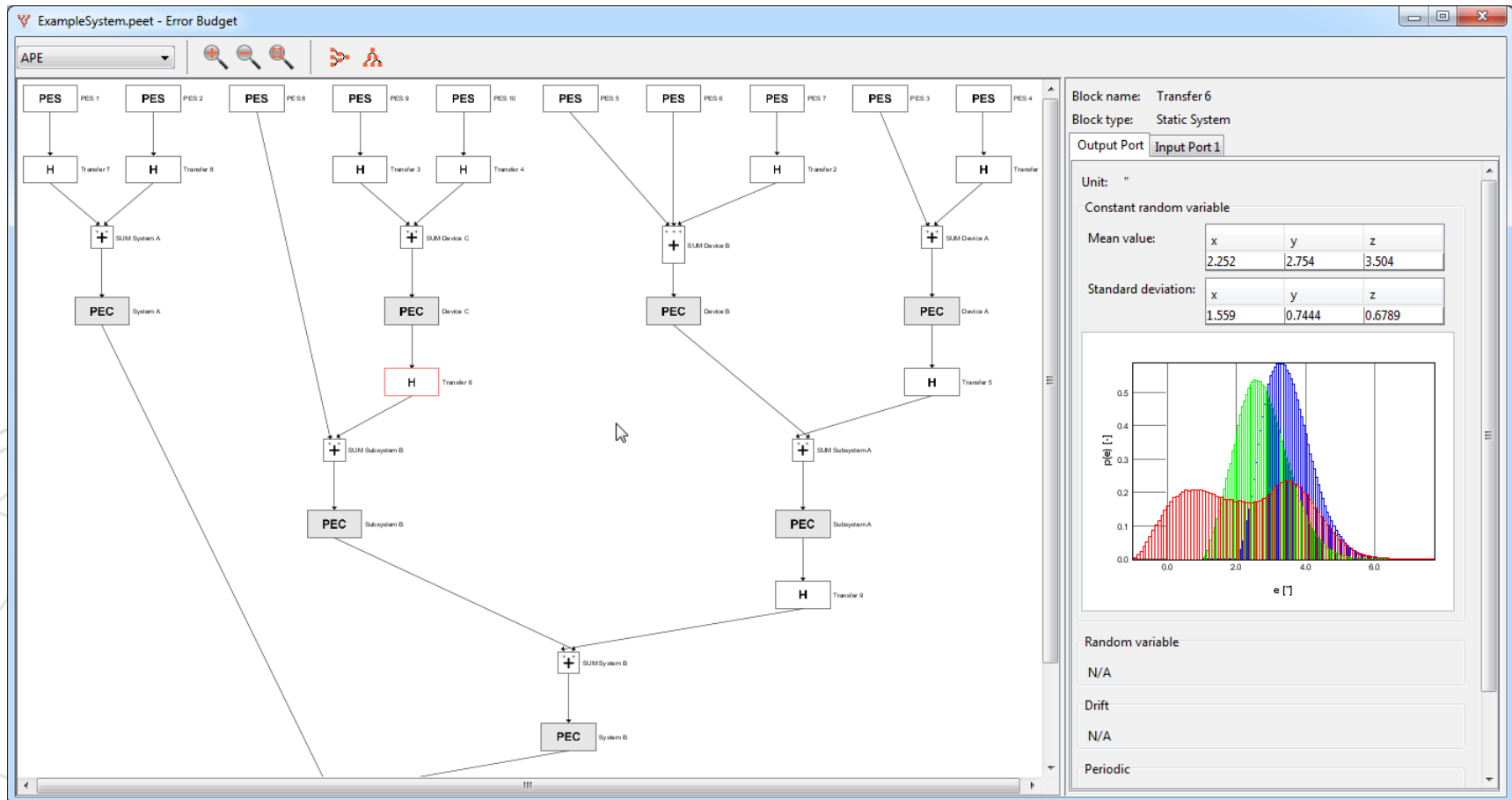
OK Cancel

Quick-help (mouse-over)



# Graphical User Interface (cont'd)

## Budget Tree View: Analyze statistics of error signal components



The screenshot displays the 'ExampleSystem.peet - Error Budget' window. The main area shows a hierarchical tree of error components. At the top level, there are 10 PES (Power Spectral Density) blocks labeled PES 1 through PES 10. These are connected to various transfer functions (H) and summed (SUM) blocks. The tree structure is as follows:

- PES 1 and PES 2 connect to Transfer 7 and Transfer 8, which then connect to SUM System A.
- PES 3 and PES 4 connect to Transfer 3 and Transfer 4, which then connect to SUM Device C.
- PES 5 and PES 6 connect to Transfer 5 and Transfer 6, which then connect to SUM Device B.
- PES 7 and PES 8 connect to Transfer 2 and Transfer 1, which then connect to SUM Device A.
- PES 9 and PES 10 connect to Transfer 9 and Transfer 10, which then connect to SUM Subsystem B.
- Transfer 6 (highlighted in red) connects to SUM Subsystem A.
- Transfer 10 connects to Transfer 9.
- Transfer 9 connects to Transfer 8.
- Transfer 8 connects to Transfer 7.
- Transfer 7 connects to Transfer 6.
- Transfer 6 connects to Transfer 5.
- Transfer 5 connects to Transfer 4.
- Transfer 4 connects to Transfer 3.
- Transfer 3 connects to Transfer 2.
- Transfer 2 connects to Transfer 1.
- Transfer 1 connects to Transfer 0.
- Transfer 0 connects to Transfer -1.
- Transfer -1 connects to Transfer -2.
- Transfer -2 connects to Transfer -3.
- Transfer -3 connects to Transfer -4.
- Transfer -4 connects to Transfer -5.
- Transfer -5 connects to Transfer -6.
- Transfer -6 connects to Transfer -7.
- Transfer -7 connects to Transfer -8.
- Transfer -8 connects to Transfer -9.
- Transfer -9 connects to Transfer -10.
- Transfer -10 connects to Transfer -11.
- Transfer -11 connects to Transfer -12.
- Transfer -12 connects to Transfer -13.
- Transfer -13 connects to Transfer -14.
- Transfer -14 connects to Transfer -15.
- Transfer -15 connects to Transfer -16.
- Transfer -16 connects to Transfer -17.
- Transfer -17 connects to Transfer -18.
- Transfer -18 connects to Transfer -19.
- Transfer -19 connects to Transfer -20.
- Transfer -20 connects to Transfer -21.
- Transfer -21 connects to Transfer -22.
- Transfer -22 connects to Transfer -23.
- Transfer -23 connects to Transfer -24.
- Transfer -24 connects to Transfer -25.
- Transfer -25 connects to Transfer -26.
- Transfer -26 connects to Transfer -27.
- Transfer -27 connects to Transfer -28.
- Transfer -28 connects to Transfer -29.
- Transfer -29 connects to Transfer -30.
- Transfer -30 connects to Transfer -31.
- Transfer -31 connects to Transfer -32.
- Transfer -32 connects to Transfer -33.
- Transfer -33 connects to Transfer -34.
- Transfer -34 connects to Transfer -35.
- Transfer -35 connects to Transfer -36.
- Transfer -36 connects to Transfer -37.
- Transfer -37 connects to Transfer -38.
- Transfer -38 connects to Transfer -39.
- Transfer -39 connects to Transfer -40.
- Transfer -40 connects to Transfer -41.
- Transfer -41 connects to Transfer -42.
- Transfer -42 connects to Transfer -43.
- Transfer -43 connects to Transfer -44.
- Transfer -44 connects to Transfer -45.
- Transfer -45 connects to Transfer -46.
- Transfer -46 connects to Transfer -47.
- Transfer -47 connects to Transfer -48.
- Transfer -48 connects to Transfer -49.
- Transfer -49 connects to Transfer -50.
- Transfer -50 connects to Transfer -51.
- Transfer -51 connects to Transfer -52.
- Transfer -52 connects to Transfer -53.
- Transfer -53 connects to Transfer -54.
- Transfer -54 connects to Transfer -55.
- Transfer -55 connects to Transfer -56.
- Transfer -56 connects to Transfer -57.
- Transfer -57 connects to Transfer -58.
- Transfer -58 connects to Transfer -59.
- Transfer -59 connects to Transfer -60.
- Transfer -60 connects to Transfer -61.
- Transfer -61 connects to Transfer -62.
- Transfer -62 connects to Transfer -63.
- Transfer -63 connects to Transfer -64.
- Transfer -64 connects to Transfer -65.
- Transfer -65 connects to Transfer -66.
- Transfer -66 connects to Transfer -67.
- Transfer -67 connects to Transfer -68.
- Transfer -68 connects to Transfer -69.
- Transfer -69 connects to Transfer -70.
- Transfer -70 connects to Transfer -71.
- Transfer -71 connects to Transfer -72.
- Transfer -72 connects to Transfer -73.
- Transfer -73 connects to Transfer -74.
- Transfer -74 connects to Transfer -75.
- Transfer -75 connects to Transfer -76.
- Transfer -76 connects to Transfer -77.
- Transfer -77 connects to Transfer -78.
- Transfer -78 connects to Transfer -79.
- Transfer -79 connects to Transfer -80.
- Transfer -80 connects to Transfer -81.
- Transfer -81 connects to Transfer -82.
- Transfer -82 connects to Transfer -83.
- Transfer -83 connects to Transfer -84.
- Transfer -84 connects to Transfer -85.
- Transfer -85 connects to Transfer -86.
- Transfer -86 connects to Transfer -87.
- Transfer -87 connects to Transfer -88.
- Transfer -88 connects to Transfer -89.
- Transfer -89 connects to Transfer -90.
- Transfer -90 connects to Transfer -91.
- Transfer -91 connects to Transfer -92.
- Transfer -92 connects to Transfer -93.
- Transfer -93 connects to Transfer -94.
- Transfer -94 connects to Transfer -95.
- Transfer -95 connects to Transfer -96.
- Transfer -96 connects to Transfer -97.
- Transfer -97 connects to Transfer -98.
- Transfer -98 connects to Transfer -99.
- Transfer -99 connects to Transfer -100.

The right-hand panel provides detailed statistics for 'Transfer 6':

- Block name: Transfer 6
- Block type: Static System
- Output Port: Input Port 1
- Unit: "
- Constant random variable
- Mean value: 

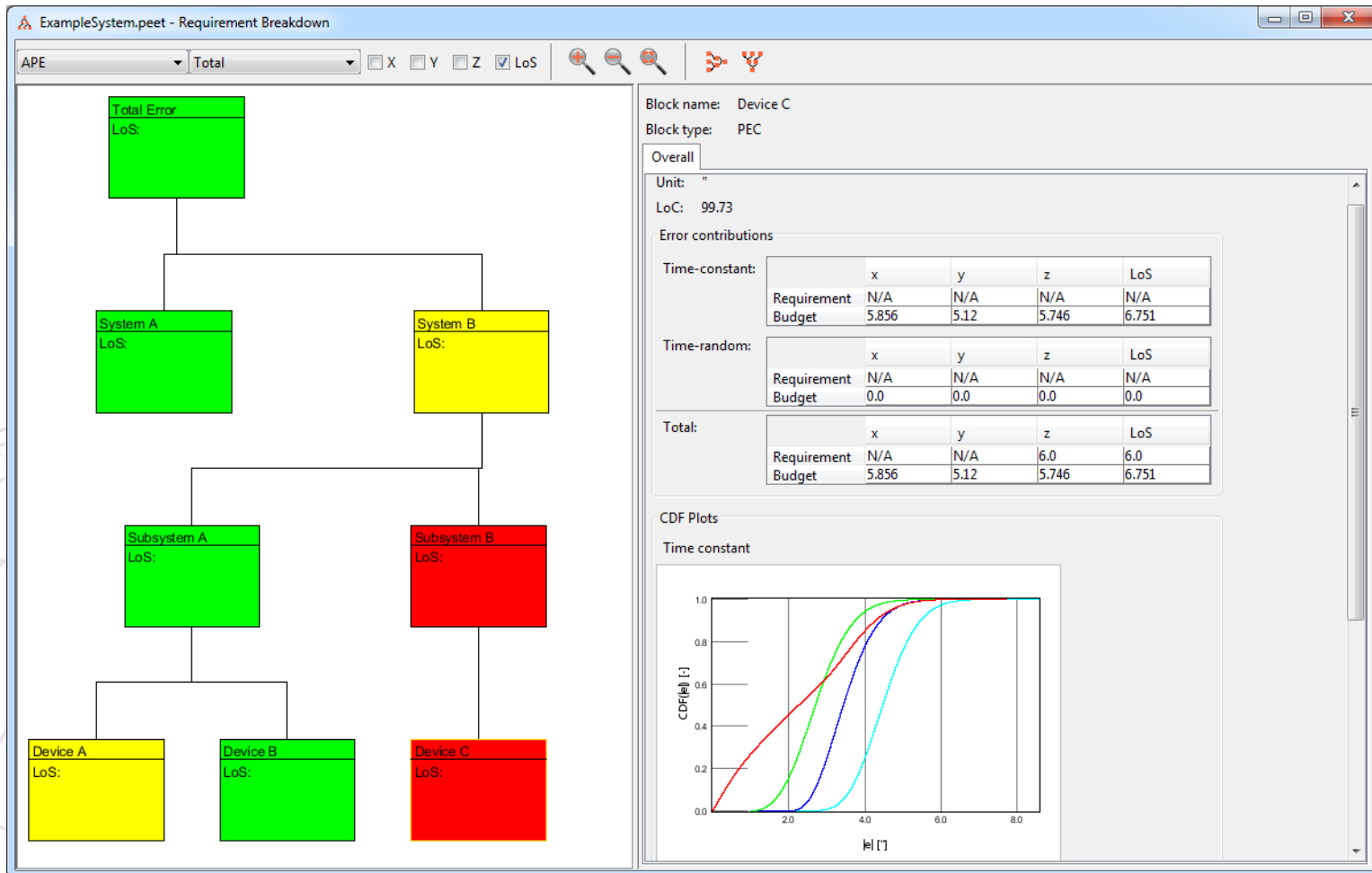
x	y	z
2.252	2.754	3.504
- Standard deviation: 

x	y	z
1.559	0.7444	0.6789
- Random variable: N/A
- Drift: N/A
- Periodic: No

Below the statistics is a histogram showing the probability density function (PDF) of the error signal component. The x-axis is labeled 'e [']' and ranges from 0.0 to 6.0. The y-axis is labeled 'P(e ['])' and ranges from 0.0 to 0.5. The histogram shows a distribution with a peak around 3.5, with a red shaded area under the curve.

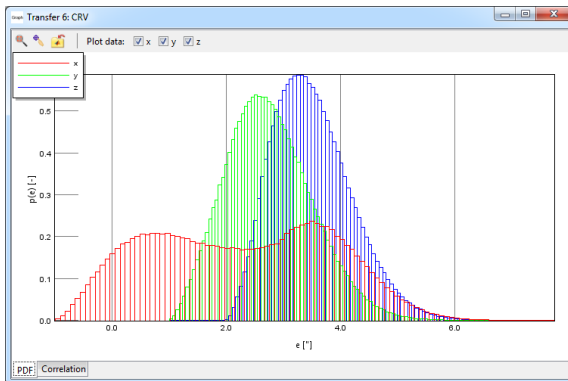
# Graphical User Interface (cont'd)

## Breakdown Tree View: Analyze compliance with requirements

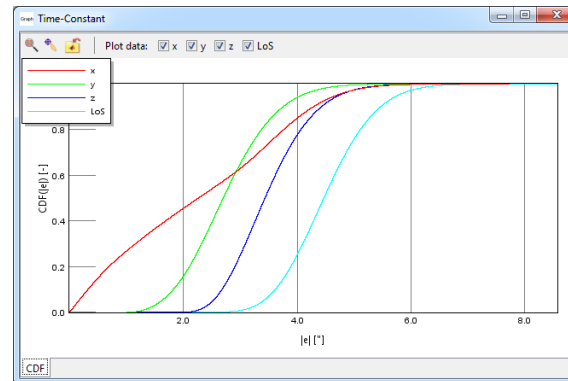


# Graphical User Interface (cont'd)

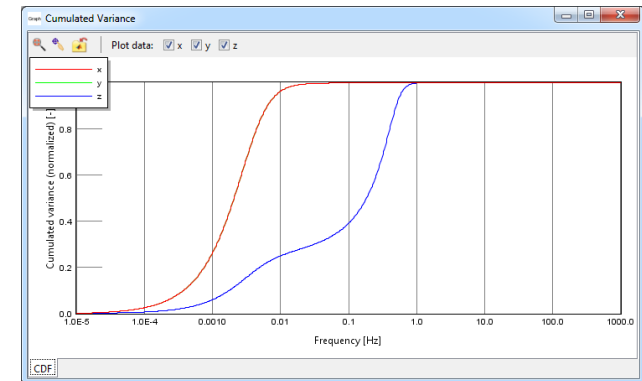
## Plot Viewer: Detailed result inspection with various plot types



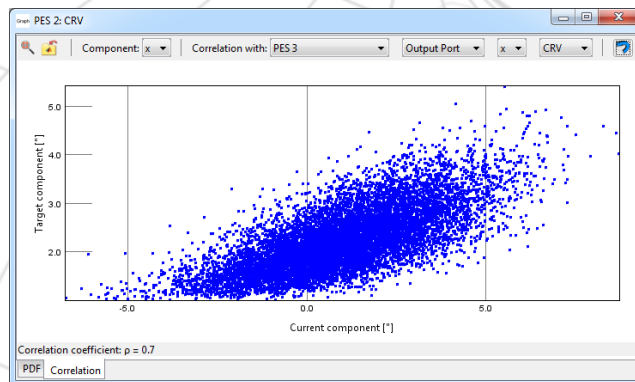
PDF plots



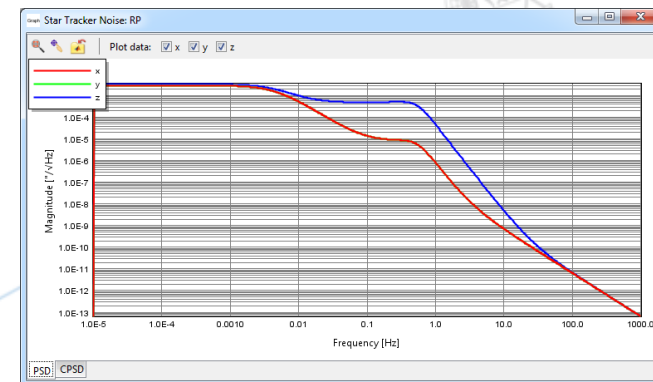
CDF plots



Cumulated variance  
(of random process)



Correlation „scatter“ plots



PSD plots (auto- and cross-spectra)


# **“Advanced Statistical Method”**




# Background

Simplified statistical method used in software prototype:

- Based on statistical moments ( $\mu$ ,  $\sigma$ ) and confidence factors only
- Relies on applicability of central limit theorem



For a valid level of confidence evaluation, total error needs to follow (at least nearly) a Gaussian distribution

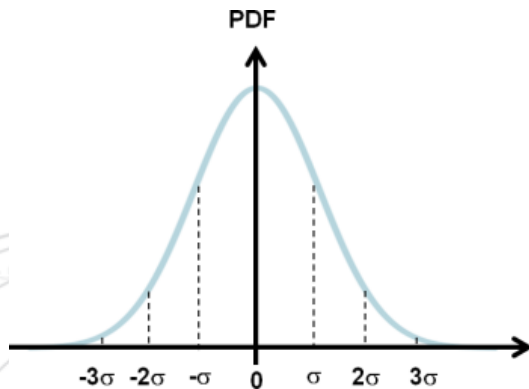


If dominant non-Gaussian errors are present, this leads to significant systematic errors

# Limitations of the Simplified Method (1)

Example:

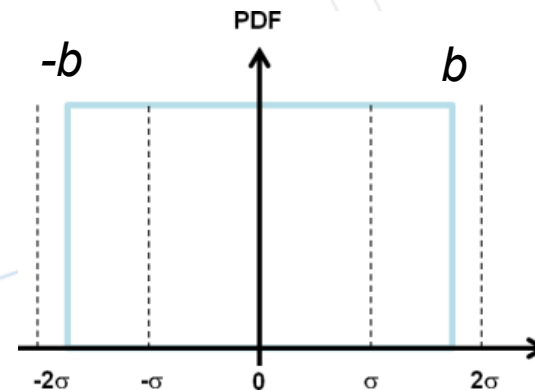
„The error shall be smaller than X with 99.73% ( $3\sigma$ ) probability“  
(applied to a Gaussian and a uniform distribution)



$$e = 3\sigma = 3\sigma_G$$



correct “by definition”



$$e = 3\sigma = 3\sigma_U = \sqrt{3} b$$



Correct value:  $0.9973 b$



$\approx 80\%$  systematic error



Even exceeds given bounds



# Limitations of the Simplified Method (2)

Calculation of line-of-sight (LoS) errors:

- PEEH provides derived expression for instantaneous (or deterministic) LoS errors
- ECSS provides approximate solution for „statistical“ errors (valid for zero-mean Gaussian with closely equal  $\sigma$ )
- Exact description via PDF

$$e_{LOS} = \sqrt{e_x^2 + e_y^2}$$

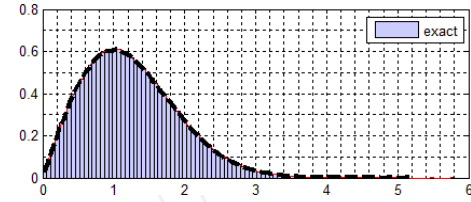
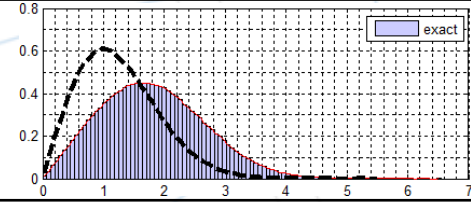
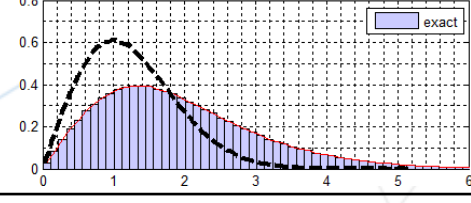
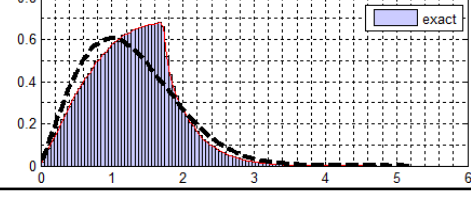
$$e_{LOS} = \max(\sigma_x, \sigma_y) \sqrt{-2 \log(1 - p_c)}$$

$$e_{LOS} = \int_0^{LoC} p \left( \sqrt{e_x^2 + e_y^2} \right) de$$

➔ “Careless” application of the first expressions to non-matching conditions again leads to systematic errors

# Limitations of the Simplified Method (3)

Calculation of line-of-sight (LoS) errors: 68,3% LoC

Case	$e_{LoS}$			PDF
	"Deterministic"	ECSS approx.	"Exact" (num.)	
X: G(0,1) Y: G(0,1)	-7%	0%	1.5158	
X: G(1,1) Y: G(1,1)	30%	-30%	2.190	
X: G(0,1) Y: G(0,2)	-22%	+30%	2.305	
X: U(-√3, √3) Y: G(0,1)	-10%	-3%	1.57	

# Implementation (1)

Analytical treatment (PDF convolution and CDF determination) in conflict with several constraints:

- SW requirement: No further MATLAB toolboxes shall be used (e.g. the Symbolic Toolbox)
- Even with symbolic computation, closed-form solutions could not be guaranteed (for arbitrarily complex systems)
- With a numerical description of the PDF convolution, the joint PDF of all error sources is required
  - Usually not known by the user
  - In best case, knowledge of (marginal) PDFs and correlation in terms of correlations coefficients expected

# Implementation (2)

Therefore, the chosen approach is entirely numerical:

- Sample-based (around  $1e6$  samples per source, 10000 PDF bins)
- Dedicated inverse transform sampling method allows both „imprinting“ correlation and PDF information

➔ [see paper for more information](#)

- Intrinsic drawback: loss of accuracy with respect to the analytical computation
  - Error expected to  $< 1\%$  in entire computation chain
  - Thus safely negligible compared to potentially large systematic errors of simplified method

# Conclusion



# Conclusion

- PEET is a tool to accurately compute statistical and spectral error budgets (release: mid 2016).
- PEET is not restricted to pointing applications, but can also be used in other engineering fields.
- PEET is well-suited for integration in analysis tool chains with the available Excel & Matlab interfaces.
- PEET files provide much better transparency (model assumptions) and flexibility (model adaption) than purely “tabular” budgets.
- Limitation: PEET cannot explicitly account for non-linearities and transient system behavior, i.e. it shall not be understood as a replacement for E2E simulators.