ORBIT DETERMINATION THROUGH GLOBAL POSITIONING SYSTEMS A LITERATURE SURVEY OF PAST AND PRESENT INVESTIGATIONS

Abdul Manarvi[1], Troy Henderson[2]

Student[1], Assistant Professor[2], Department of Aerospace Engineering, Embry-Riddle Aeronautical University

ABSTRACT

Positioning a spacecraft or a satellite in a predefined orbit was and is still considered as one of the most important process of any space mission design for military or commercial purposes. It was observed that findings from 1988 start off fruitfully, with sub-meter orbit accuracy. Data from orbit determination agrees with Very Long Baseline Inferometry (VLBI) solutions. Most investigations focused on increasing accuracy of prediction through minimization of errors. Various orbit estimation strategies process noise models for atmospheric fluctuations, while other methods combine processing of GPS phase and pseudo-range data. Orbit modeling is not restricted to one type of orbit rather it includes distinct categories of circular and elliptical orbits. This paper can be used for further investigations in Global Positioning Systems for orbit determination.

1. INTRODUCTION

The Global Positioning System (GPS) was developed by NASA to track remote sensing satellites at altitudes below 3,000 km with accuracies better than 10 cm [1]. Equipment required for GPS includes ground receivers operating in conjunction with a receiver onboard a satellite. This instruments estimate the satellite orbit and GPS satellite orbits simultaneously.

Satellite accuracies became stringent to a decimeter in the 1990’s. One example is TOPEX which includes a primary laser tracking system and a French Doppler instrument [2]. A GPS receiver aboard TOPEX and a network of ground receivers with precisely known positions. All receivers continuously track visible GPS satellites, measuring accumulated carrier phase (integrated Doppler) and one-way range (pseudorange, consisting of true range plus a bias due to the time offset between transmitter and receiver clocks) at approximately 1.2 and 1.6 GHz. Data from TOPEX and ground receivers will be combined to recover TOPEX and GPS satellite orbits in a reference frame defined by selected ground sites. The ground network for the TOPEX experiment will include NASA’s three deep-space tracking sites in California, Spain, and Australia, and at least three complementary sites operated by other government agencies.

2. TWO ORBIT SOLUTION STRATEGIES

GPS orbit solutions are dynamic, which means they are governed by the laws of motion. But, the user satellite orbit solutions can range from purely dynamic to purely kinematic. Where kinematic solutions are geometric solutions. Two techniques for tracking the 1992 TOPEX/Poseidon mission are discussed. The first technique uses an optimized combination of dynamics and kinematics, whereas the second technique uses gravity modeling to exploit data from repeated ground tracks [2].

To achieve decimeter tracking accuracy, a joint solution must be applied for the user and GPS satellite orbits. If GPS orbits are held fixed, user accuracy is typically limited at the meter level by GPS orbit error. Non-NASA ground-site locations and parameters such as atmospheric propagation delay and solar radiation pressure are also adjusted. Because the GPS satellites are in high 12-h orbits, a dynamic strategy exploiting the laws of motion is always preferable for the GPS orbit solutions. This provides ample accuracy and maximizes data strength for the more demanding user solution, which is our primary interest. Techniques presented here apply to any user orbiting below 3000 km, where it will remain within the main beams of the GPS satellites.

3. DYNAMIC ORBIT DETERMINATION

Until 1989 the classical dynamic solution technique was the only technique available for precise orbit determination [3]. This technique estimated satellite position and velocity at a single epoch with an extended arc of data. Observations at different times were related to the epoch state parameters by integrating the equations of motion, a process requiring accurate models of the forces acting on the satellites. Errors in the force models naturally introduce errors in the epoch state solution. In general, the further in time an observation is from the solution time, the greater the expected error from dynamic mismodeling. Consequently, the effect of force model errors tends to increase with increasing arc length.

TOPEX error studies illustrate the importance of accurate dynamic models. Figure 1 is taken from a covariance study of the altitude error for a single TOPEX orbit, at 1334 km, using GPS carrier data only [4]. A critical assumption is the gravity error model, which consists of the differences between more than 300 coefficients (20-by-20) from two gravity models, GEML10 and GEML2 [1] [4]. This reflects the approximate accuracy of the best gravity models in the early 1980's. In Figure 1, the estimated RMS altitude error is 26 cm. With a perfect gravity model this would fall to less than 6 cm. Clearly, a significant model improvement is needed to reach the 13-cm goal with a dynamic solution. Recent efforts at the Goddard Space Flight Center have substantially refined the gravity model, which is now expected to approach the required accuracy by 1992 [5]. Additional gravity model adjustment after launch using TOPEX tracking data may be needed.

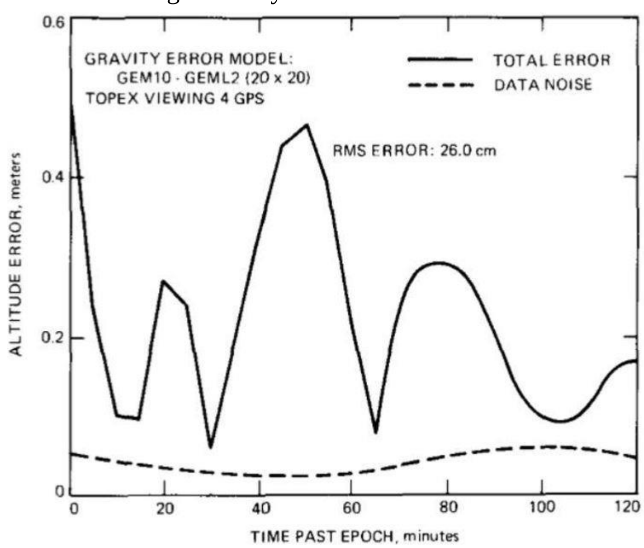

Figure 1. Predicted TOPEX altitude error with a dynamic solution using a gravity error model reflecting the estimated gravity model quality in the early to mid-1980's [2].

In Figure 1 TOPEX is assumed to track four GPS satellites at once. The TOPEX receiver will in fact have a six satellite capacity, and typically four to seven satellites will be within the nominal hemispherical field of view [4]. The actual TOPEX field of view may fall a few degrees short of a full hemisphere as a result of efforts to suppress reflected signals around the antenna. The effect of this, however, will be slight. In later studies we vary the receiver capacity from four to all satellites in view in order to evaluate performance tradeoffs.

When a dynamic solution is applied at lower altitudes, such as the 705 km planned for EOS or the 250-350 km typical of shuttle flights, errors from gravity and atmospheric drag soar. Platforms like the shuttle and space station, moreover, present serious additional complications from maneuvering and venting. In such cases, dynamic model error can easily climb to hundreds of meters. Therefore, at low altitudes or with dynamically unpredictable platforms we must turn to a geometric approach.

4. KINEMATIC ORBIT DETERMINATION

This approach to precise orbit determination developed in the late 80's was made possible by two features of GPS: 1) Pseudo-range measurements from many directions provide continuous geometric position determination, and 2) simultaneous carrier phase and pseudo-range data enabled long-term kinematic smoothing. Together, these offer a solution technique that is entirely geometric. The laws of motion are not used to infer position, hence no user-force models are needed, nor is knowledge of the satellite center of mass. Position is referred to the phase center of the GPS antenna, which can be defined with millimeter accuracy.

Briefly, the kinematic technique smooths a series of pseudo-range-based position solutions against the continuous record of position change obtained from carrier phase. Thus, carrier phase empirically supplies the state transition previously obtained from dynamic models. This is illustrated heuristically in Figure 2.

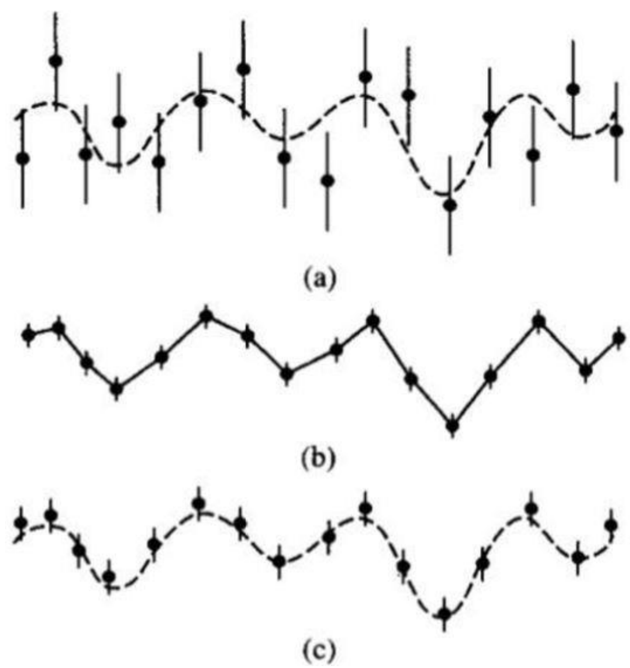

Figure 2. Conceptual illustration of GPS orbit determination via geometric point positioning and kinematic smoothing. (a)

Independent position solutions obtained from pseudo-range. (b) Precise track position change obtained from carrier phase. (c) Final position solutions after adjusting (b) against (a) [3].

A sequence of N independent position solutions obtained from pseudo-range is shown in Figure 2 (a). The dashed line represents the true satellite motion. Each position solution has an error up. The record of position change obtained from carrier phase over the same arc is shown in Figure 2 (b). This has a much smaller error $\sigma\Delta p$. The track of position change is now fit to the position points by adjusting a single (3-D) position bias to minimize the sum-of-squared residuals between the two sets of points. The mean of the position change curve is shifted to the mean of the point position solutions, and the adjusted curve closely tracks the true trajectory in Figure 2 (c). The smoothed position solutions, which are no longer independent, will have an approximate error as shown in Equation 1 [1]:

$$\hat{\sigma}_p = \sqrt{\frac{{\sigma_p}^2}{N} + {\sigma_{\Delta p}}^2} \qquad (1)$$

Taking values of 5 m for σp (1 second differential observations) and 5 cm for $\sigma\Delta p$ over a 2 hour arc (7200 observations), we have $\hat{\sigma}_p$ = 7.7 cm. To this we must add the effects of GPS ephemeris error, station location error, and troposphere. Although this example is illustrative, in reality the result is complicated by the solution for GPS satellite orbits, ground positions, and other parameters. Computer studies have therefore been carried out to evaluate the error in detail.

An example from TOPEX covariance studies [6] is given in Figure 3. Here TOPEX is allowed to track all visible GPS satellites, typically between 4 and 7. Although this offers no real advantage over the actual six-satellite TOPEX receiver capacity. Over a 4-h data arc the estimated RMS altitude error is 7.6 cm, with two peaks of about 12 cm. Because dynamic model errors are absent, this accuracy can be maintained down to the lowest satellite altitudes without concern for maneuvering or forces not modeled, so long as contact with GPS is maintained. And with no user models to compute, the kinematic solution is relatively simple and fast.

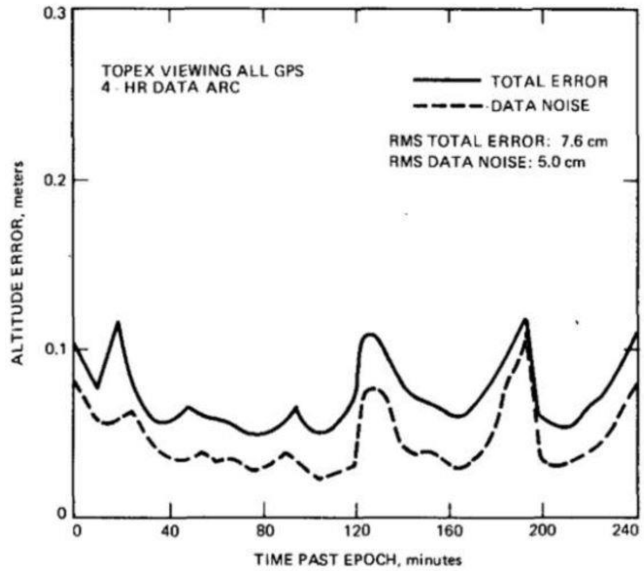

Figure 3. Predicted TOPEX altitude error with the kinematic strategy using pseudo-range and carrier phase over a 4 hour data arc [3].

The pseudo-range data noise assumed in Figure 3 corresponds to a single-channel precision of approximately 30 cm in 1 s. JPL's new Rogue GPS receiver, which will serve as the ground reference receiver, improves on this by about 40 percent [7]. The TOPEX receiver, built by Motorola, will have a pseudo-range precision of about 30 cm [3]. Doubling the data noise of Figure 3 increases the estimated altitude error only slightly to 8.8 cm. To achieve such a result in practice, sources of systematic error such as multipath and instrumental delay variations, must be controlled so that after several hours of smoothing their effect is small. Detailed simulations of the reflecting environment on TOPEX and at the ground sites indicate that after four hours, the net effect of multipath on orbit error will be less than 1 cm. Other instrumental effects are expected to be negligible.

5. REDUCED DYNAMIC STRATEGY

Kinematic tracking has one critical limitation: Performance is dependent upon the momentary observing geometry, as evidenced by the error fluctuations in Figure 3. Observing geometry depends upon the number and arrangement of ground receivers, the receiver viewing capacities, and the GPS satellite constellation. Loss of a ground site, GPS satellite, or user channel can cause the solution to degrade sharply or fail altogether. We can remedy this by reintroducing user dynamics, appropriately weighted according to model quality, while preserving the kinematic solution. User dynamics can then carry the solution through geometric weak spots while adding strength throughout.

5.1 Reduced Dynamic Formulation

To put this idea into practice we begin with a dynamic formulation and introduce a kinematic component by adding process noise to the user force models. The GPS orbit solution remains fully dynamic, while the user solution becomes partly dynamic and partly kinematic. Different solution characteristics, ranging from fully dynamic to kinematic, can be achieved by varying the parameters defining the process noise. The solution can be optimized for a particular dynamic model accuracy by carefully tuning those parameters. We present the reduced dynamic technique in a Kalman sequential filter formulation. This involves two steps: A time update, which uses a state transition model to propagate the state estimate and covariance from one time batch to the next, and a measurement update, which incorporates a new batch of measurements. These steps alternate until all batches are processed.

5.2 Reduced Dynamic Analysis

Both the TOPEX and ground receivers are assumed to observe all GPS satellites within their fields of view (typically 6 or 7), unless otherwise stated. Because the Kalman filter is a sequential estimator, the state covariances must be smoothed backward and then mapped to all time points at which data are taken.

An initial study confirmed the behavior of reduced dynamic tracking in its extreme forms: With τ set to 0 and σ0 and σ both large, the error estimate approached the result from a separate kinematic analysis; with τ set large and both σ0 and σ set to 0, the dynamic result was reproduced. Here we present results for intermediate values of τ, σ0, and σ. In general, when τ is long compared to the batch size, the results vary with the batch-to-batch uncertainty $\sigma bb \equiv (1m2)1/2\sigma$, rather than with the steady-state uncertainty σ and τ individually. Therefore, in the following a constant τ = 15 min and batch size of 5 min are used and only σ0 = σ is varied.

Figure 4 shows the estimated TOPEX altitude error as a function of the gravity error (which was varied by scaling GEM10-GEML2 from 0 to 100 percent) for different U, including the purely dynamic and kinematic forms. For any finite dynamic model error (in this case dominated by the gravity), a range of U exists within which the altitude error is lower than with either the dynamic or kinematic solutions. In other words, the reduced dynamic technique is superior, provided that the dynamic model is properly weighted.

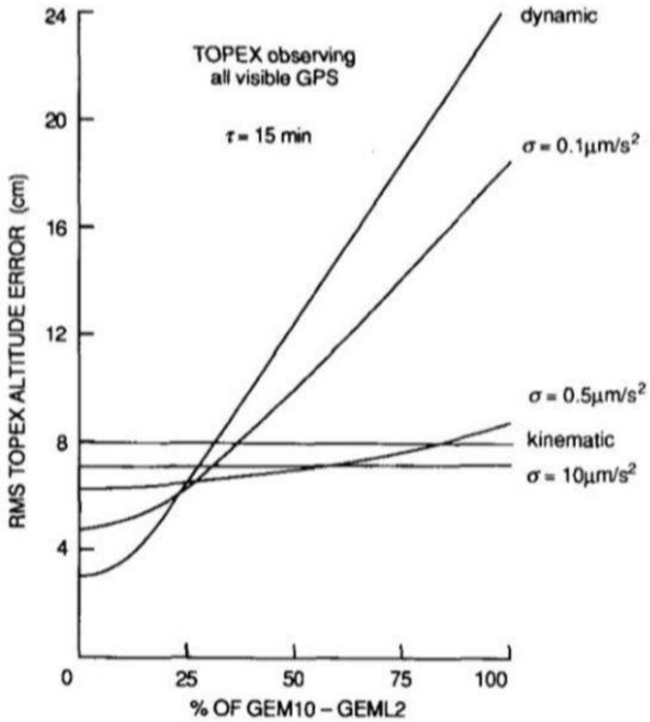

Figure 4. Predicted TOPEX altitude error using dynamic, kinematic and reduced dynamic strategies, shown as a function of the gravity model quality [8].

With TOPEX observing all satellites, the geometry is consistently strong and kinematic tracking yields about an 8 cm accuracy, only 1 cm above the optimal. With TOPEX restricted to four satellites, geometry is often poor; kinematic performance collapses and the optimal solution is little better than the dynamic. If the gravity error is doubled, however, as in the case of a lower orbit, dynamic tracking error doubles to 24 cm, while reduced dynamic degrades only moderately, from 12 to 16 cm. Thus, even weak geometry can be valuable when dynamics are poorly modeled. With TOPEX observing five satellites, dynamic and kinematic tracking yield 12 and 16 cm, while the optimal solution improves to 9 cm.

Dynamic tracking yields higher error over regions where gravity is poorly known, such as ocean basins, while kinematic tracking is vulnerable to poor observing geometry. In the optimal combination, the two techniques complement one another and the solution is better balanced. Figure 5 compares TOPEX altitude accuracy over 2 h for all three techniques. Here, TOPEX observes five satellites and the gravity error is scaled at 50 percent. Both the dynamic and kinematic solutions show error peaks of 25 cm or higher, while reduced dynamic remains below 13 cm for the full arc. As observing geometry varies, the optimal estimator automatically adjusts the weight on dynamics to minimize overall error. Figure 6 gives the error breakdown of the three peaks of Figure 5. The near optimum use of state transition information in the reduced dynamic solution yields a more uniform distribution of errors.

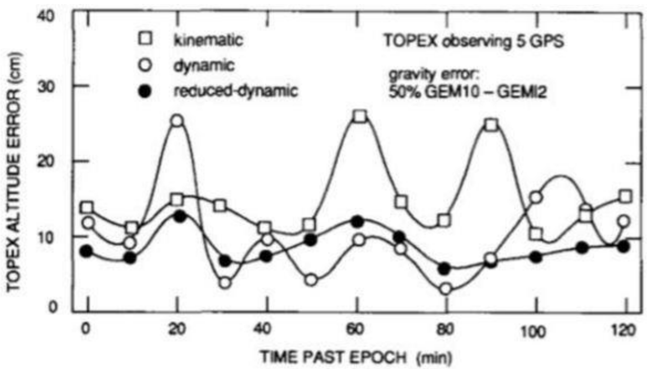

Figure 5. Detail of predicted TOPEX altitude error over a full 2 hour data arc for dynamic, kinematic, and reduced dynamic strategies **[3]**.

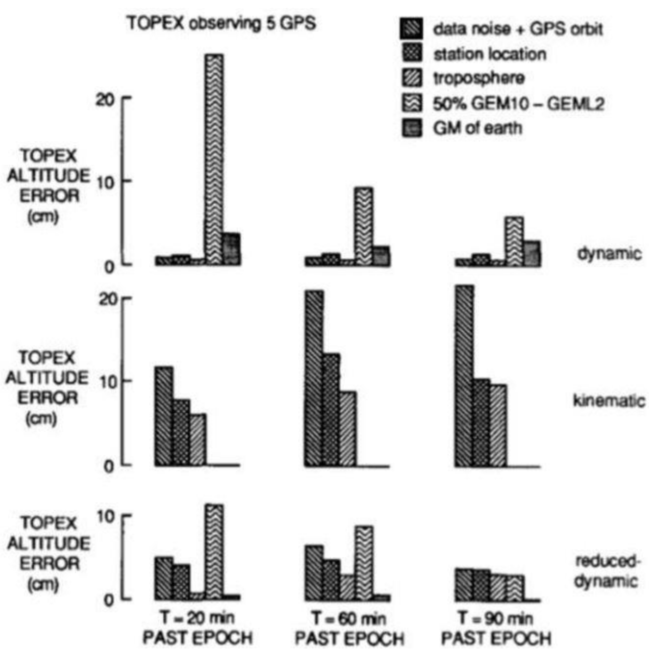

Figure 6. Breakdown of predicted TOPEX altitude error at three peak points of Figure 5 [3].

With any technique, as the data span increases the error from random data noise generally declines. In a dynamic solution the effects of force model errors tend to grow with arc length and eventually dominate, making it necessary to choose a span that balances data and dynamic errors. In the optimized solution, as the arc length and data strength increase, the filter automatically reduces dynamic weighting to maintain a balance between data and dynamic errors, and overall performance improves.

Because the weight on the dynamic model is reduced with longer data span, a reduced dynamic solution will tend to a kinematic solution as the span increases, assuming a fixed dynamic model. If force models are improved through tuning or other efforts, the optimum will shift back toward a dynamic solution. In the limit, as the force models approach perfection, the optimum solution becomes purely dynamic.

5.3 Reduced Dynamic Weighting

In a given application, the optimum initial weight for the reduced dynamic process noise can be estimated through a covariance analysis using a realistic dynamic error model. Misjudgment of the dynamic model error, which at some level is inevitable, will yield a suboptimal weight. Figure 7 shows estimated TOPEX altitude error as a function of gravity error for three reduced dynamic weights, along with the optimum performance curve. Note that performance is fairly insensitive to significant departures (a factor of two) from the optimum weight.

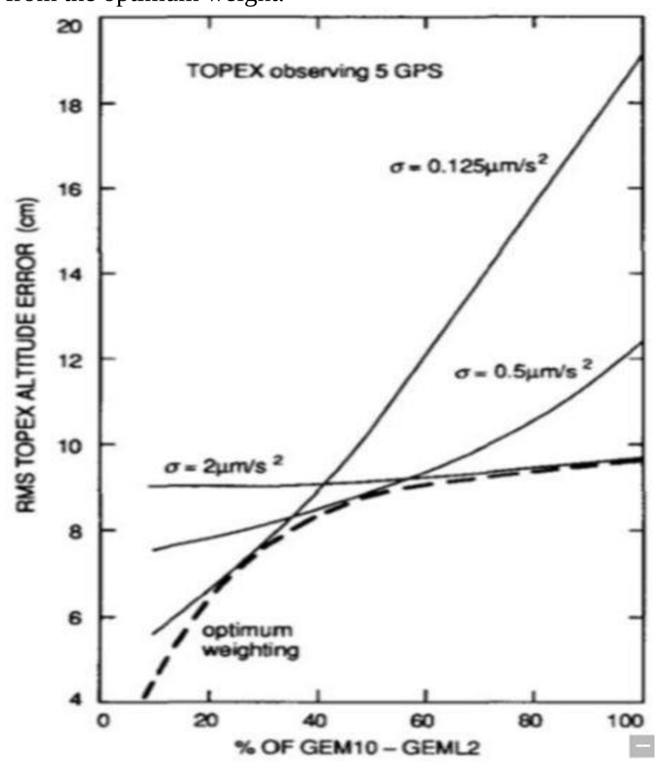

Figure 7. Predicted TOPEX altitude error for different reduced dynamic weightings, and optimal weighting, shown as a function of the gravity model quality [2].

6. LOW CIRCULAR ORBITERS

The first low user satellite to carry a high-precision GPS receiver was the US-French Ocean Topography Experiment

(TOPEX/Poseidon), launched in July 1992. TOPEX has a circular orbit at a nominal altitude of 1,336 km. Although a formal accuracy requirement of 13 cm has been set for the continuous determination of its altitude, TOPEX ocean science would benefit from an altitude accuracy comparable to the 2.5-cm precision of its radar altimeter. The experimental GPS receiver onboard TOPEX will have six dual-frequency channels allowing simultaneous observation of six GPS satellites.

Extensive analyses have been carried out for TOPEX orbit determination with differential GPS [3] [4] [5]. The analyses indicate that the gravity model error accounts for major error source with dynamic tracking of TOPEX. For this reason, reduced-dynamic tracking was recommended. The assumptions used in this analysis include: 5 cm and 0.5 cm in pseudo-range and carrier phase data noises with 5-minute intervals, 2 m in GPS ephemerides which were adjusted together with TOPEX orbit, 5 cm in coordinates for the three DSN sites and 20 cm for the other three sites, 1 cm in zenith tropospheric delay, a gravity error which is equal to half the difference between GEM10 and GEML2, a 10% solar pressure, and 10^{-8} in GM of earth. The peak error at 20 minutes past epoch for the kinematic solution is due to poor momentary geometry; the two peaks at 60 and 90 minutes past epoch for the dynamic solution are a result of poor gravity modeling. Reduced dynamic removes these peaks and the error over the entire pass are below 13 cm [3].

In the above analysis, we have assumed a GPS tracking system with somewhat limited capability. On the other hand, the multipath on pseudo-range measurements, which could potentially become a major error source, has not been accounted for. The effects of multipath are very difficult to assess. Means to reduce the multipath effects have been investigated. One approach which has proved effective in reducing multipath is to place the GPS antenna in a conducting back-plate made up of concentric choke rings [9]. TOPEX will employ such a back-plate and elevate the antenna back-plate on a boom to keep it well above large reflecting surfaces such as the solar panel, the high-gain data relaying antenna and the TOPEX main body. Simulation analyses indicate that the multipath effects are reduced by 20 dB with a boom 4.3 m long [3].

TOPEX nicely illustrates the benefits of reduced-dynamic tracking since its ~1300-km altitude, its six-satellite receiver capacity, and its relatively compact dimensions permit both good observing geometry and reasonably well-modeled dynamics. A far greater modeling challenge is presented by several other current or planned NASA space platforms: the large (14 m) platforms of the polar orbiting Earth Observing System (Eos), which will carry heavy slewing instruments and fly at 700 km; the actively maneuvering space shuttle at altitudes as low as 300 km; and the sprawling (155 m) Space Station Freedom at about 400 km. All will eventually carry experiments seeking tracking accuracies of better than 10 cm. Indeed, a recent international workshop on Space Geodesy set a goal of “no more than 1 cm RMS error, single pass, without orbit discontinuities” [10] for tracking future orbiting ocean altimeters, such as the one that will fly on Eos.

Since we cannot expect to approach centimeter or even decimeter accuracy in modeling the dynamics of such ungainly platforms, the optimal orbit solution strategy will be almost purely kinematic, to maximize performance under kinematic tracking, we must maximize geometric observing strength. With that in mind, we have taken the examples of Eos (98° inclination, 705 km altitude) and Space Station Freedom (28°, 400 km) and carried out covariance studies under a more robust set of assumptions: the GPS constellation is increased to 24 satellites, as is expected to occur by 1995; the flight antennas will enable both upward and downward viewing from Eos to track all GPS satellites in view down to the earth limb (typically a dozen or more); the ground network is expanded to 10 sites with the three fixed ground sites are assumed known to 2 cm in each component, which is expected to be achieved or surpassed by very long baseline interferometry within the next few years [10].

7. ELLIPTICAL ORBITS

A number of elliptically orbiting missions in the coming decade are expected to carry sophisticated sensors which will require precise calibration for position and velocity. Here, the focus is on missions which will carry radio telescopes into high-earth elliptical orbits. These elliptical orbits pose unique orbit determination challenges, since the viewing geometries and dynamics can vary considerably with the altitude of the satellite. The orbit requirements for the Japanese VLBI Space Observatory Program (VSOP) MUSES-B spacecraft are examined and planned for launch in 1995, and for the International VLBI Satellite (IVS), for which a late-1990s launch is envisioned.

The highly elliptical orbit of VSOP will have its apogee at -20,000 km and perigee at -1,000 km in altitude, as shown alongside a circular GPS orbit in Figure 8. Although the position requirements of 130 m can easily be met with ground-based Doppler tracking, the velocity requirement of 0.4 cm/sec is not, especially at the perigee where the error can be as large as 1-2 cm/sec [11].

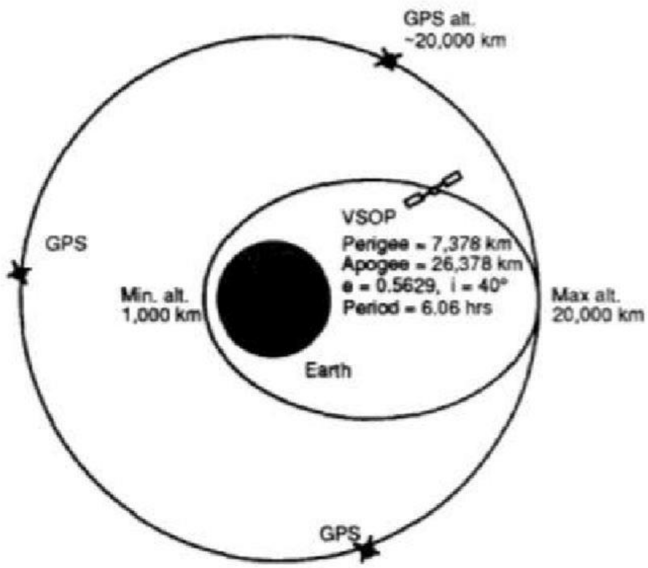

Figure 8. VSOP orbit geometry [1].

While ground-based GPS measurements can be used to calibrate the Doppler data for improved tracking, the inclusion of differential GPS measurements between VSOP and a global network of ground sites should be even better. Due to its wide range of altitudes an ingenious combination of different tracking techniques should be considered for optimal accuracy. For altitudes below 2,000 km, reduced-dynamic tracking (with a steady-state sigma of 0.5 μm/sec2 and a 15-minute time constant, which were the same as used in Figure 9 for TOPEX) was utilized. At higher altitudes, the acceleration parameters were still adjusted, but treated as constant instead of process-noise parameters. A ground network of 10 globally distributed sites was assumed.

8. CONCLUSION

A brief literature review of the application of Global Positioning Systems to orbit determination has been presented. Although orbital accuracy in 1988 was promising, research has shown an improvement in tracking accuracy using GPS for orbits of various eccentricities. Although a limited tracking network existed in the past, centimeter accuracy over distances of several thousand kilometers was possible. Current techniques of applying numerical methods and future advances in computation time will allow for more precise ephemerides for any orbit.

9. REFERENCES

[1] T. P. Y. S. M. L. B. J. H. a. R. P. M. S. C. Wu, "GPS-Based Precise Tracking of Earth Satellites from Very Low to Geosynchronous Orbits," 1992.

[2] J.-T. W. C. L. T. T. P. Y. S-C Wu, "Precise Tracking of Remote Sensing Satellites with the Global Positioning System," in *IEEE Transactiona on Geoscience and Remote Sensing*, 1989.

[3] T. P. Y. G. A. H. S-C Wu, "Toward Decimeter Topex Orbit Determination Using GPS," in *AAS/AIAA Astrodynamics Specialist Conference*, Stowe, 1978.

[4] S.-C. W. Thomas P. Yunk, "Reduced dynamic technique for precise orbit determination of low earth satellites," in *AAS/AIAA Astrodynamics Specialist Conf.*, Kalispell, 1987.

[5] G. Rosborough, "Orbit error due to gravity model error," in *AAS/AIAA Astrodynamics Specialist Conf.*, Kalispell, 1987.

[6] T. P. Y. a. S. C. Wu, "Non-dynamic decimeter tracking of earth satellites using the global positioning system," in *AIAA 24th Aerospace Sciences Meeting*, Reno, 1986.

[7] T. M. e. al., "Rogue: A new high accuracy digital GPS receiver," in *IUGG XIX General Assembly*, Vancouver, 1987.

[8] S.-C. W. J.-T. W. C. L. T. Thomas P. Yunck, "Precise Tracking of Remote Sensing Satellites With the Global Positioning System," in *IEEE Transactions on Geoscience and Remote Sensing*, 1989.

[9] e. a. T. K. Meehan, "Baseline Results of the ROGUE Digital GPS Receiver," in *American . Geophysical Union Spring Meeting*, Baltimore, 1988.

[10] I. I. M. a. S. Z. (eds.), "The Interdisciplinary Role of Space Geodesy," *Springer-Verlag*, p. 116, 1989.

[11] S. M. L. J. A. Estefan, "High-Precision Orbit Determination for High-Earth Elliptical Orbiters Using the Global Positioning System," in *AIAA/AAS Astrodynamics Conf.*, Portland, 1990.