SOME VALIDATION CHECKS OF "TriaXOrbitaL" TOOL : EARTH-MOON L2 ORBIT, SUN-MOON PERTURBATIONS

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ABSTRACT

The modelling becomes year after year a priority for each agency, each company in order to better foresee the features of space devices and spacecrafts. The paper presents a flight dynamic tool so-called "TriaXOrbital", freely distributed as promotional release, that has been used since 1989 with continuous improvements for constellation design, North-South station keeping manoeuvre, orbit transfers GTO or better Super-GTO to GEO, interplanetary flight and travel to Moon. The tool has been fully focused on electric propulsion long thrust arcs but can manage as well chemical propulsion shorter arcs of course. The tool is accessible and has been used by engineers or students. The main advantage of using such tool is for sure a simplification of the preliminary studies because the tool has been developed for being accessible for every engineer willing to improve his knowledge in the orbital manoeuvres field. But one of the drawbacks when using such tools is to be able to state about the validity and accuracy of the results provided by the tool. Since the beginning of its development, a great care has been taken with respect to the fundamental checks of the tool. Hence the paper exhibits some of the tests performed in order to reproduce the "everybody knows" specific behaviors of the flight dynamic: for example the evolution of the GEO due to real Sun, Moon and earth potential perturbations (J2) with inclination up to 15° for 54 years; the stability of the Earth-Moon Lagrangian point L2, and several other checks. The description of the tests performed may constitute a good starting point for using the tool and for the knowledge of its features.

Index Terms— Orbital evolution, tests GEO, test L2, Unstable manifold

1. INTRODUCTION

The modelling tool presented hereafter is based on a more than 26 years history of continuous improvements. As an introduction, the history review of the modelling tool is highlighted first.

A. CONSTELLATION IN 3 DIMENSIONS
The early beginning of such tools came with the need to be able to represent a constellation in three dimensions like the GPS one. The basis of the "TriaX" tools was born. Contrary to the other three dimensional software, the method used was relying on the intensive use of scalar product and vector product (for all the projections of the space on a plane). By prohibiting the classical use of trigonometric routines, the computation time (especially with the computers of those times) was largely improved. The computer language used was a basic language because it could be set up efficiently.

B. DELTA V LOSSES PER THRUST ARCS
The second step of the tools improvement came with the need of assessing the Delta V losses due to long thrust arcs. This was achieved with a Runge-Kutta 4th order (RK4) integration method. In order to perform faster computation, only two dimensions were used at that time with the polar co-ordinates $r(t), dr/dt, v(t), dv/dt$. The validation of the program was based on the quantification of the integration process errors.

C. NORTH-SOUTH STATION KEEPING
The third step came with the need to represent the North-South station keeping of a geostationary satellite. A three-dimension integration process based on the full Cartesian co-ordinates of $\vec{r}(t), \vec{v}(t)$ was then needed. In order to be able to represent clearly the changes during such manoeuvre, the visualisation 3D features were improved a lot. Inclination zoom with a cylindrical amplification factor (along Z axis) for small inclination views, Altitude zoom near the reference GEO orbit, Cubic views (with the three projections added simultaneously with the main plot)… are some of those special improved features. Integration process was improved using the 5th order (RK5) with automatic error / time step adjustment. The whole tool was transformed into the Visual Basic environment for a user-friendly package. The integration process was extended to the 9 planets and the sun: the 11 years cycle of the sun (due to Jupiter interaction was highlighted). The routines dealing with the 3D visualisation were then built into an independent general purpose so called "GlobalMultiUse DLL" (Dynamic Link Library), for further use in other software.

The tool has been intensively used for performing the modelling of GTO – GEO orbit transfer with high specific impulse and low thrust propulsion systems. Plots of that tool were used in the description of the author patents in order to check the patented claims and in many papers presented to international conferences.

Also, the landing of a spacecraft on the moon surface was modelled accurately in order to be able to evaluate the so called "deltaV losses" (used in the course of the Esa program Euromoon).

Interplanetary manoeuvres (Rosetta Wirtanen) have been computed and presented in papers for international conferences.

The SMART-1 trajectory coming first from in-house modellisation at Snecma and then coming directly from the data in flight has been presented and published in numerous of articles. Thanks to the accuracy of the computations, the differences of thrusting strategies to escape the Van Allen belts could be tested in flight: after suggesting that the better strategy was those with thrust perpendicular to the radius vector, the Flight dynamic service of ESOC took it into account successfully.

2. A MODELLING TOOL FOR ORBITAL EVOLUTIONS

The problem of the evolution from a particular starting orbit with the propulsion is not a simple problem for engineers. Tools are needed to give first assessments, and these tools shall be flexible enough in order to be usable for resolving as many problems as possible. The multi-purpose tool "TriaXOrbitaL" [1], is primarily a visualization tool because all outputs are shown in a powerful three dimensional dynamic space. The heart of the tool is based on the momentum equation (1) as follow:

$$\frac{d\,\vec{\omega}}{dt} = \vec{\Omega} \qquad (1)$$

with

$$\vec{\omega} = \begin{pmatrix} \vec{r} \\ \vec{v} \\ m \\ t \end{pmatrix} \qquad \vec{\Omega} = \begin{pmatrix} \vec{v} \\ -GM_{focus}\,\vec{r}/r^3 + \sum\left(\vec{T}_{thrust} + \vec{P}_{perturb}\right)\!/m \\ \sum\left\|\vec{T}_{thrust}\right\|/(g_0.Isp) \\ 1 \end{pmatrix}$$

where $\vec{r}$ is the radius vector from the focus body to the spacecraft in the inertial Galilean frame translated at the focus centre (m),

GM the gravitational constant time the Mass of the focus body (m^3/s^2),

$\vec{v}$ the velocity in that frame (m/s),

m spacecraft mass, changes as the thruster are on (kg),

t the local time (s), integrated for checks,

$\vec{T}_{thrust}$ the individual thrust vectors (N) where $\| \, . \, \|$ the norm,

$\vec{P}_{perturb}$ the perturbing force vectors coming from other bodies or from sun pressure or from non-spherical potential terms including J2 and equatorial ellipticity (N),

g_0Isp the specific impulse (m/s) of the thrusters used to produce the thrust with g_0=9.80665 (m/s^2).

This general equation allows not only getting the evolution of the orbits with maneuvers around one main body, but also trajectories and maneuvers between planets because the writing of the perturbation forces equations allows to contain the gravity of every celestial body.

The data needed to start the integration of the equation (1) are numerous. This important aspect is briefly described in the following lines.

The starting point $\vec{\omega}_0$ at the integration time t=0 must be known. This point can be effectively the starting point of the manoeuvre, or as well the final point of the end of the manoeuvre when the integration step is negative.

The thrust strategy and thrust orientation $T_{thrust}(t)$ with respect to the time must be given.

The perturbing forces and their orientation $P_{perturb}(t)$ with respect to the time must be computed when requested for a given Julian date.

In order to simplify the fastidious work to be performed for specifying the input data and strategies, the tool implement several routines in dedicated windows and manage for friendly use a database for saving or recovering the starting points and strategies.

The organisation of the several inputs is thus relying in the current release on a DataBase that allows the selection of a recorded case in order to run it directly or for advanced user to modify it. In the case of advanced use, the following windows are in the current release, successively available:

Configuration: for defining the number and the orientation of the thrusters on the spacecraft, their feature (Isp, Thrust, thrust orientation strategy thanks to an expertise help included) and several other parameters (initial mass, allowed integration error on the position and on the velocity or time step).

Orbit: for defining the starting point of the integration, including the main focus planet selection, the initial orbit elliptical or hyperbolic, and some dedicated visualisation directives for the 3DD (three dimension dynamic) output. Those directives are particularly appreciated for making specific zoom on the manoeuvre evolution: Altitude Zoom for example GEO altitude +-20 km, and Inclination Zoom for the case of almost equatorial orbits by increasing the Z coordinate, etc. This window also include a Pork chop feature (based on Lambert's time theorem) for setting departure and arrival opportunity dates of travel –without thrust- from planet to planet, this Pork chop tool has been highlighted in the web by some users for its outstanding features.

Manoeuvre: dedicated to the number and location of thrust arcs along the orbit (including continuous thrust strategy), the criteria used to perform the manoeuvre and the setting of

the perturbation to be taken into account. Particular criteria included deals with the continuous transfer orbit between a GTO (or super-GTO) to GEO according to the patent [**2**] After this short description, the toll capabilities will be more obvious with the presentation of some examples of application in the next section.

3. EXAMPLES OF ORBITAL MANOEUVRES MODELLING WITH TRIAXORBITAL

A. CONTINUOUS EP ORBIT TRANSFER STRATEGY FROM GTO

The continuous thrust strategy is patented, the evolution from a starting orbit inclined 28° to GEO with eclipses is shown in Figure 1 in standard 3DD views with the red color being used when the thrust is on, the light green color for the case when the thrust is off, dark blue for the night when the sun eclipses occur. The cubic view (with plots of the projections on each plane) can help for the understanding of the evolution.
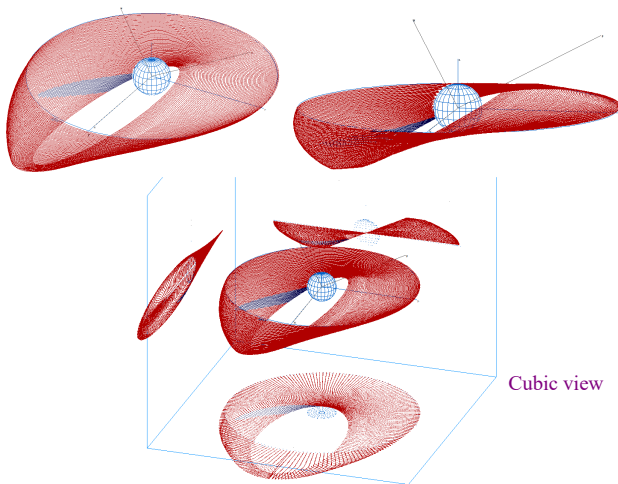
Figure 1 Continuous thrust transfer GTO-GEO

B. MOON FLYBY:

Figure 2 shows standard view and zoom view of a Moon flyby event followed by a free Earth return [**3**]
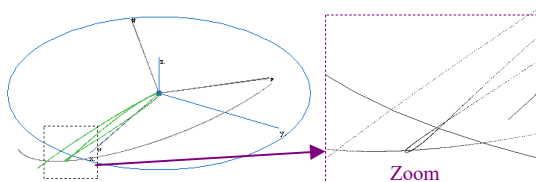

Figure 2 Apollo 11 backup free return from the Moon

C. NORTH SOUTH STATION KEEPING FROM ANTI-EARTH SIDE

This example use the "Inclination Zoom" feature (Z coordinate 10000 times larger) in combination with a "Altitude Zoom" for setting a ring of +-20 km with respect to GEO circle. One can see directly on the plot Figure 3 the slight orbit shift after the thrust arc which is almost fully removed after the second thrust arc 12 hours later.
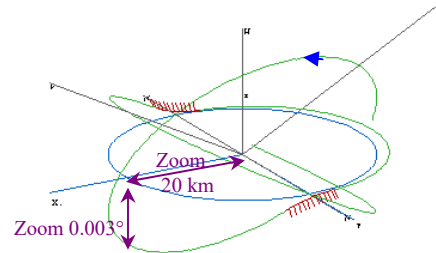

Figure 3 Zoom in altitude and inclination of a station keeping maneuver on a GEO spacecraft

D. MOON ORBIT INSTABILITY

The polar orbit around the Moon is shown in Figure 4 without any thrust, but with the Earth perturbation. The spacecraft will finally impact the Moon surface as the perilune is decreasing at each orbit (case of SMART-1).
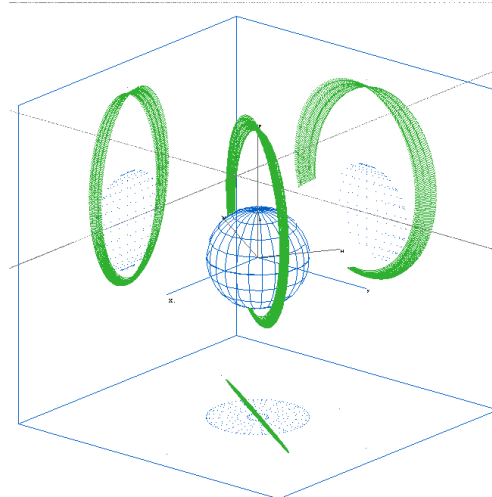

Figure 4 Natural Earth influence on a polar Moon orbit

E. CLASSICAL ORBIT TRANSFER:

With a thrust of 500 N in thee apogee centered thrust arcs, the classical GTO to GEO orbit transfer is simulated in Figure 5. The results include the losses due to the thrust arcs (sometimes called abusively gravity losses).
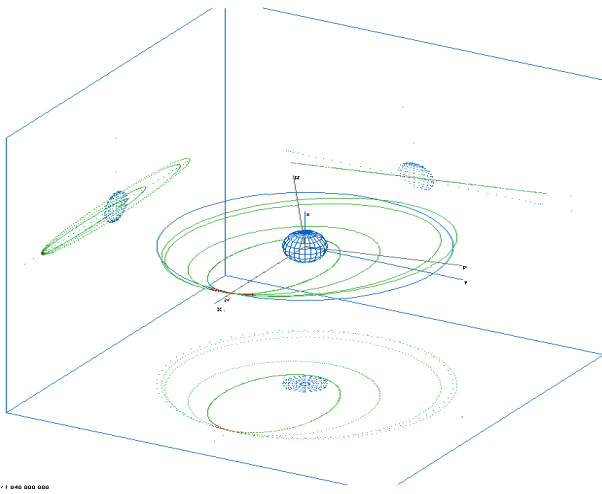
Figure 5 Chemical transfer GTO-GEO

F. SOFT LANDING ON THE MOON SOUTH POLE

The criterion of apo-lune altitude < 0 is used to terminate the computations. By adjustment of the initial altitude or adjustment of the thrust, one can simply reach the surface with a velocity of 0 m/s as in Figure 6
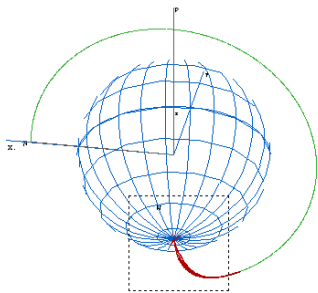

Figure 6 Automatic landing on the Moon South Pole

G. SUN PRESSURE PERTURBATION OF A GEOSYNCHRONOUS ORBIT

With the check box "sunpressure" checked in the tool and for a given front area of the spacecraft, the plot Figure 7 shows an Altitude Zoom around GEO orbit and the envelope of all the perturbed orbits (by the sole action of the sun pressure) for one year.
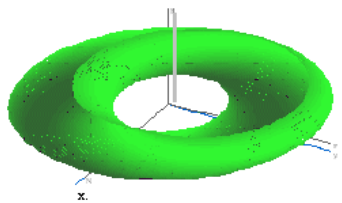

Figure 7 Zoom on the envelope of perturbed GEO orbits

H. EARTH-MARS TRAVEL IN 39 DAYS

With the use of a tangential thrust for performing a spiral and by using the pork chop feature (opportunity 16th July 2018), Figure 8 show a total delta-V (Earth-LEO to Mars-capture) = 49 km/s. In 2048, the local optimum case occurs on May 27th, but delta-V increases by +10km/s. It is to be highlighted that a better optimization could include in the last arm of the escape spiral an Earth fly-by to increase the Earth escape velocity, not presented here.
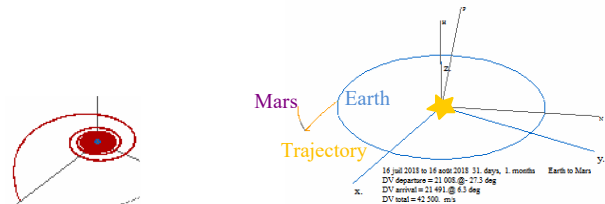

Figure 8 Earth LEO spiral and Go to Mars in 39 days

4. SYNTHESIS FOR ORBITAL MANOEUVRES MODELLING WITH TRIAXORBITAL

The list of the previous examples has shown that the tool is not specialized for one task but can be used for many very different tasks in the domain of flight dynamics: for this single tool of orbital manoeuvres modelling the range of application is non limited and thus this kind of tool "TriaXOrbitaL" is very well adapted to multipurpose studies.

Many improvements can be added to the tool in order to solve more easily some particular problems like fly-by manoeuvres (already available for some cases thanks to the pork chop feature), solver and optimisers (already added some ZBrent solver for finding the right parameters for going to L2), adding more useful cases of input parameters, enable a full traceability of the result files including the feature of complete self sustainability traceability (for re-running any simulation cases directly from the output result files), etc…

The main advantage of using such tool is of course a simplification of the preliminary studies because the tool has been developed for being accessible for every engineer willing to improve his knowledge in the orbital manoeuvres field.

However the major risk using such tool could come from inaccurate problem or bugs or wrong behaviors and inaccuracies. The next chapter will show some of the tests performed to get confidence in the outputs of the tool. Only well known behaviors have been used to set-up the following checks.

5. CHECKS OF TRIAXORBITAL: GEO CASE

A very well known behaviour of the Geosynchronous orbits is the evolution of a free spacecraft (without any station

keeping) located first at GEO. The several perturbations occurring at GEO are coming from Moon and Sun, and from the Earth flatness. Those induce the inclination to change and to culminate at 15 ° in a cycle of 54 years [4]. Because a free GEO spacecraft lost quite fast its geosynchronism with Earth rotation, the perturbations due to ellipticity of the Earth equator are not really relevant in such evolution.

The test performed uses a spacecraft located at GEO with the perturbation flags set for the **Moon**, **Sun** perturbations and the Earth oblateness (**J2**) perturbations. The Moon orbit is set to be the right orbit for the date considered thanks to Bouiges formulaes [5].

The simulation covering more than the expected figure of 54 years is set for 21100 days (57 years). After one hour of computation, the plot of Figure 9 Figure 10 Details of the 54 years cycle for a free GEO satellite shows the expected results.

The details of the run are shown in Figure 10. the semi-major axis evolution (as given by a pure Keplerian osculating orbit at each period) is very periodic without long term trend after a so long run: that is other confirmation of the correctness of the tool's outputs. Inclination vector follows a single loop as expected, while the eccentricity vector is much more complex but bounded.
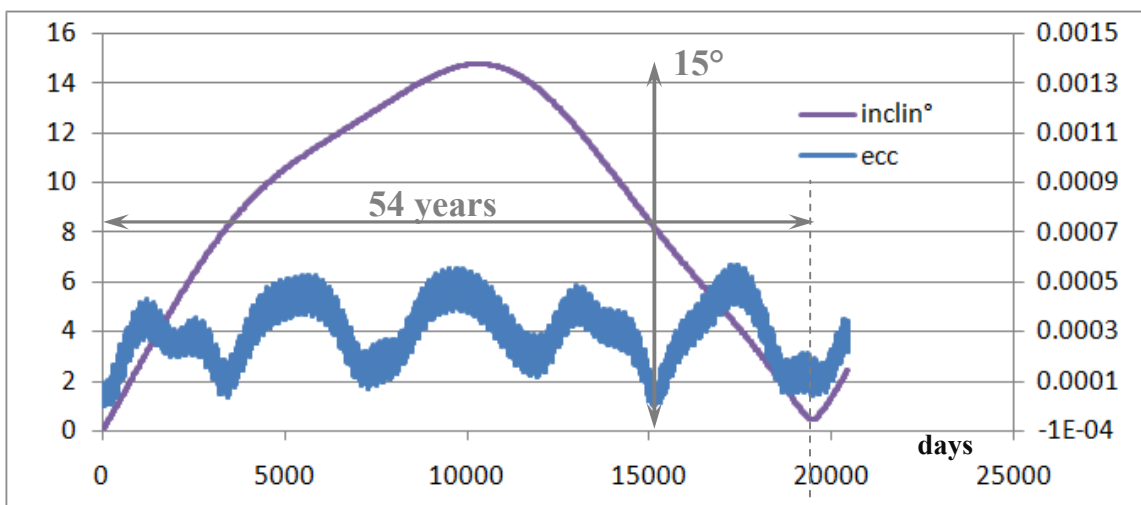

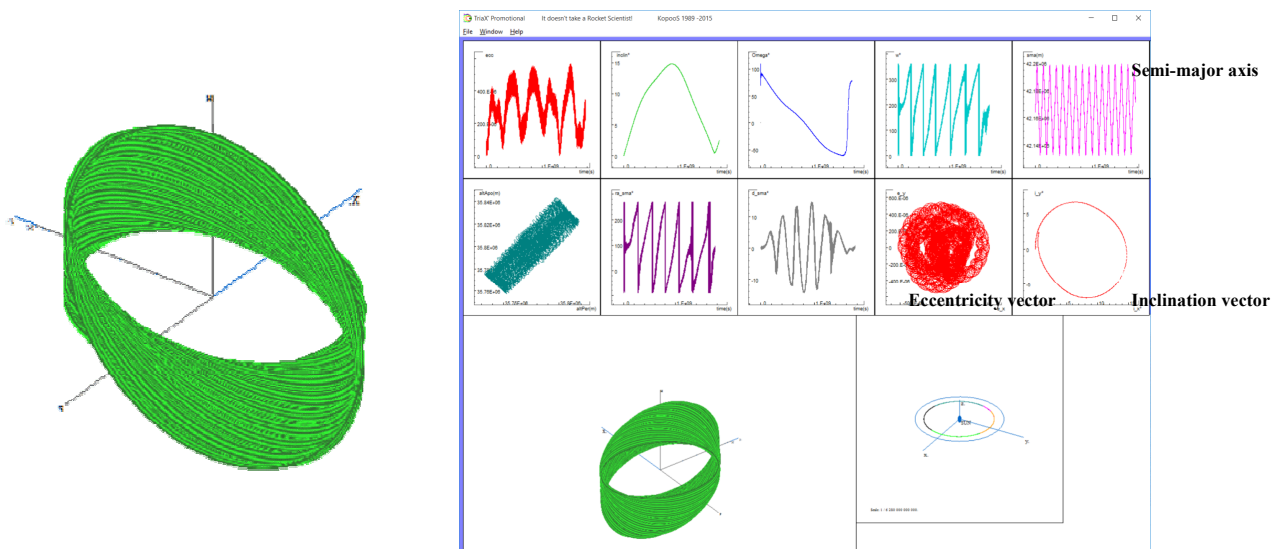**Figure 9 Check of the 54 years cycle for a free GEO satellite**

Figure 10 Details of the 54 years cycle for a free GEO satellite

6. POWERFUL CHECKS WITHIN THE EARTH-MOON SYSTEM

TriaXOrbital is fully featured with the possibility to make the Moon orbit real (using Bouiges formulae [5]) or circular. This last case enables simulations in the so called CR3BP (circular restricted 3 body problem)[7].

Contrary to most other tools dealing with Earth-Moon (E-M) libration points trajectories, TriaXOrbitaL is not using any special frame in barycentric rotating coordinates for the integration of the dynamic equations, neither any normalization is needed, neither backward time integration (even if the tool is fully able to integrate backward in time): For Earth-Moon trajectories, TriaXOrbitaL is integrating forward time the dynamic equations (1) with respect the main focus (Earth) in Centered Inertial frame (ECI) with of course the full perturbations from any other body selected. Hence it is important to show that the results provided by TriaXOrbitaL do not provide any deviation with respect to some known results.

A. FIRST RELEVANT CHECK: STABILITY AT L2

It is well known in the Earth-Moon system, that once the spacecraft is placed at L2 with the same velocity of L2 wrt Earth, it must stay there indefinitely. Such test has been performed: First a two impulses Transfer Trajectory is performed according to Farquhar [6], [7]. Then a delta V at L2 is automatically added to the spacecraft for staying at L2.

But because L2 is unstable, any small perturbation in position or velocity has large consequences for the further trajectory evolution: the third body (the spacecraft) is to be ejected soon from L2 in any cases.

The plot below is performed (automatically) in the ECI (Earth inertial frame) which is the integration frame, and for information views in Moon centered inertial frame or views in the synodic frame (rotating with respect to the Earth-Moon --those bodies are set along the x axis with y axis being in plane --)
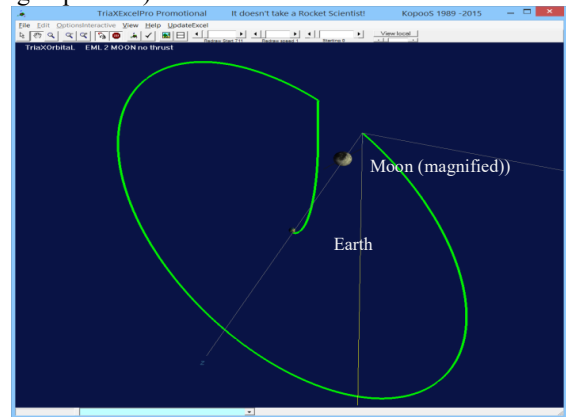

Figure 11: 3D view (under TriaXExcelPro, tool freely available) of the two impulses Transfer Trajectory followed by a stay at L2 for one Moon period, in ECI
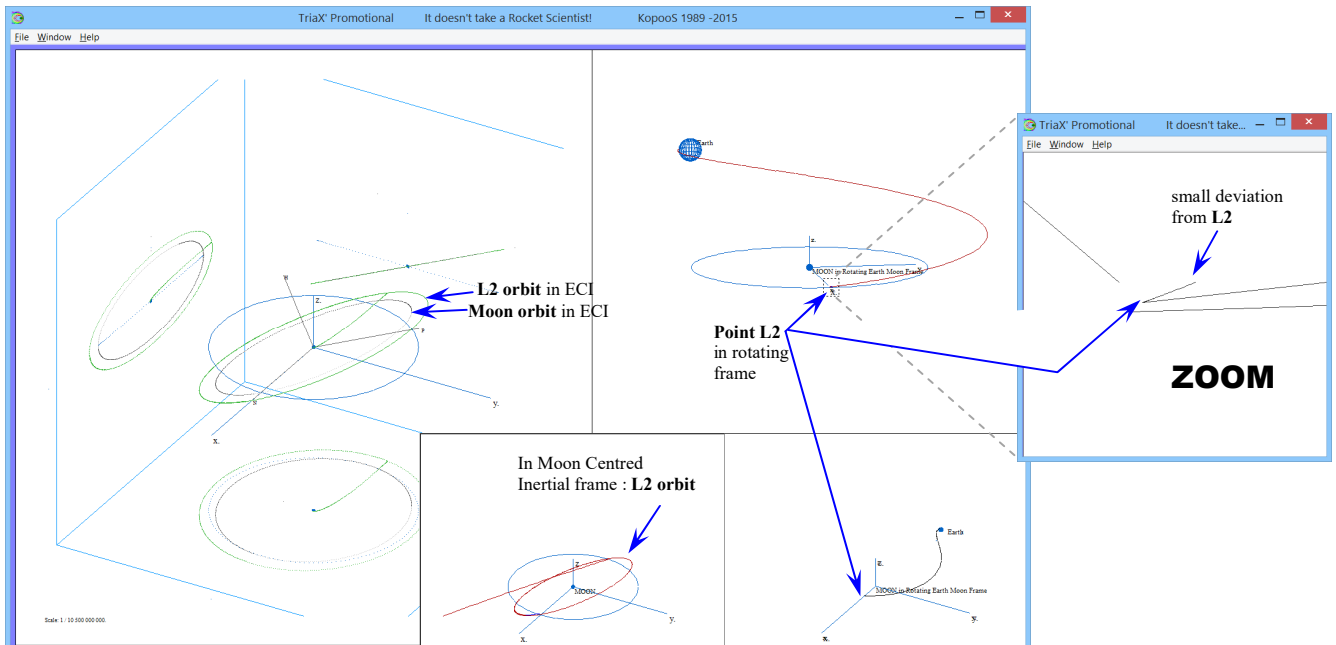

Figure 12: Orbits of L2 in ECI, in Moon Centered Inertial and view of point L2 in the rotating synodic E-M frame
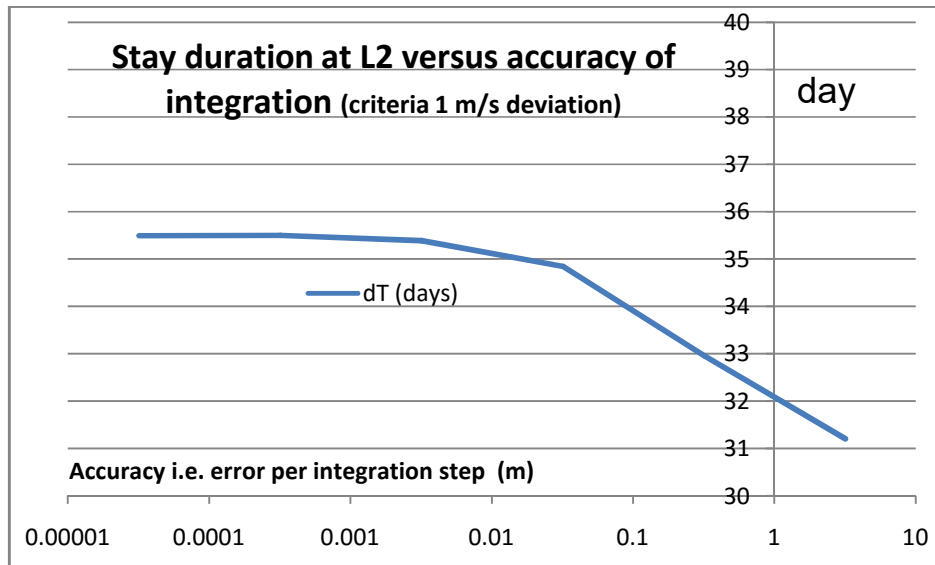
Figure 13: Evolution of the duration at L2 before deviation occurs versus integration errors levels

The duration at the point L2 before having any visible divergence in the trajectory of the spacecraft depends slightly on the integration errors. This can be used for a measure of the fidelity of the integration.

For rough accuracy of Runge-Kutta (error of the order of 1 m per integration step), the stay is 30 days before shoving a discrepancy in the velocity of 1 m/s (that is the deltaV that would be needed for coming back to L2), see above the zoom of the rotating frame near the end of the x axis where the point L2 is located: a small deviation from L2 becomes visible.

The effect of much more accurate levels is plotted in Figure 13: it appears that 30 days is already the sign of very high fidelity because for much more accurate integrations (up to integrations errors of 10µm at each step), the stay at L2 culminates at 35.5 days.

This concludes the check successfully.

B. SECOND RELEVANT CHECK: UNSTABLE MANIFOLD FROM L2

It is interesting to see in the references (for example [**8**]) what happens after a perturbation is intentionally provided to the third body (the spacecraft) that was staying at L2.

After the previous check of the stay at L2, the natural perturbations due to integration errors provides indeed systematically the same trajectory on the so called very well known "unstable manifold".

Moreover, a continuation of the integration of the equations allows to further continue the trajectory along a converging manifold i.e. stable invariant manifold (where for other tools

a common practice is to integrate backward in time from L2) that ends with TriaXOrbital near L2 but not "exactly".

One shall highlight that with TriaXOrbital no switch is needed for selecting the stable or unstable invariant manifold (specialists of the CR3BP would say that like an homoclinic connection appears automatically with the tool), but the L2 point is not reached exactly, actually quite near to it, see the following plot zoom. This concludes the checks successfully.
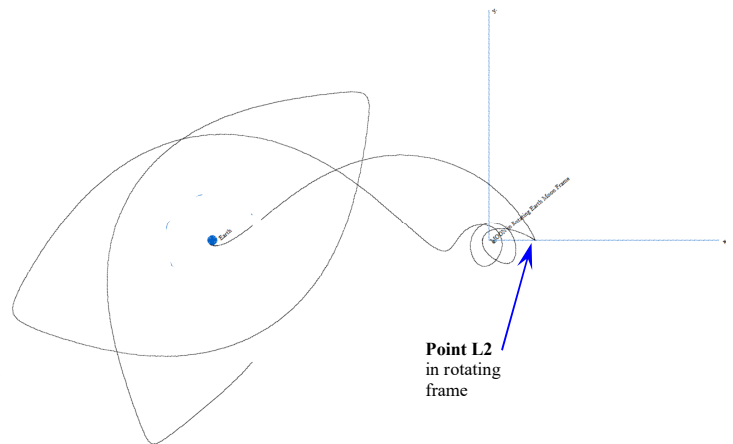

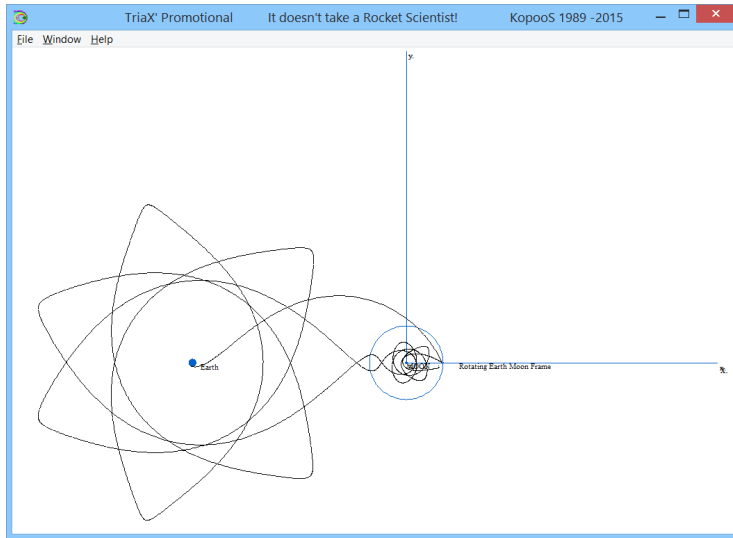**Figure 14: TriaXOrbital view Transfer Trajectory followed by a stay at L2 for 30 days and then diverging**

Figure 15: TriaXOrbital free evolution after stay at L2 for 30 days: diverging and converging again back to near L2
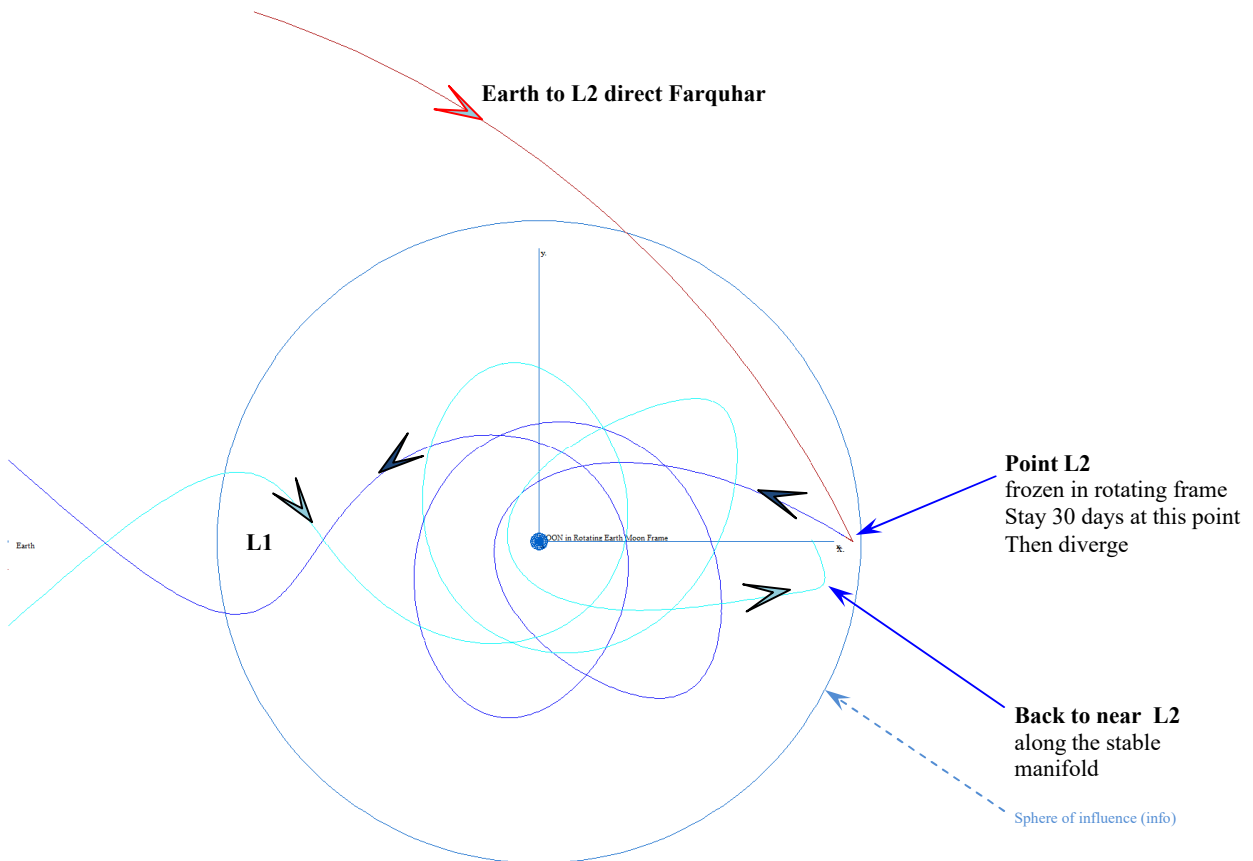

Figure 16: ZOOM of TriaXOrbital view at L2 in the rotating frame: diverging L2 and converging back again near L2
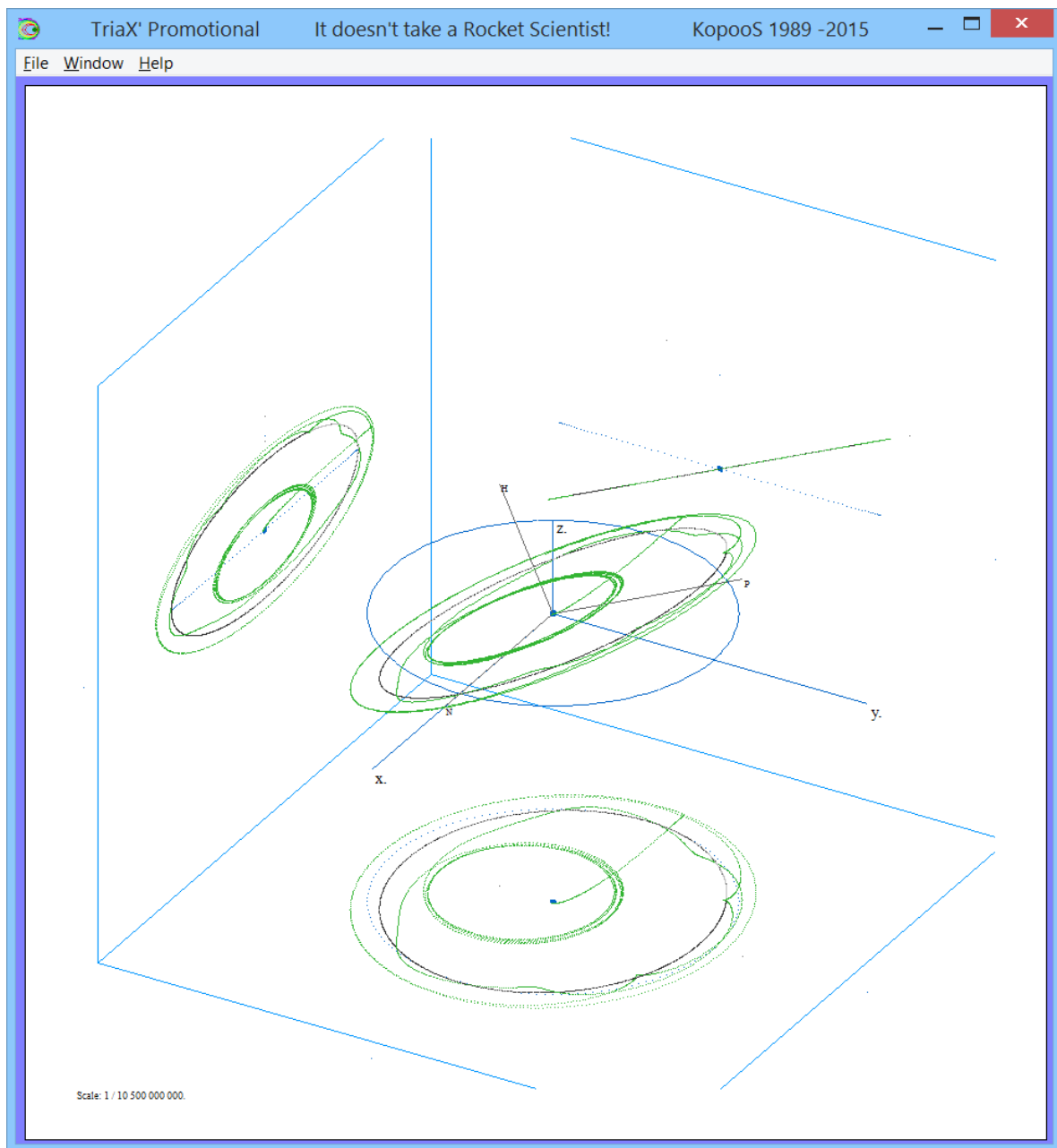
Figure 17: TriaXOrbital free evolution after stay at L2 for 30 days: diverging and converging again back to near L2

7. APPLICATION EARTH TO MOO L2

This chapter presents a final note regarding the application performed after the successful checks of the simulation results from the tool: while the rotating E-M synodic frame exhibits some specific looping and star shapes, those kind of shapes do not help for understanding the real meaning, the capability of the tool to show a conventional plot of the third body (the spacecraft) into ECI is almost mandatory. This is

performed in the next plot: clearly, the star with 6 orbits followed by the spacecraft after L1 neck and before going back again to near L2 through proximity with L1 neck are quite "normal ellipses" with respect to Earth.

The 6 "elliptic orbits" around the Earth have the same inclination as the Moon, and it is to be highlighted that those have very similar orbital parameters with roughly perigee altitude of 125 000 km and apogee altitude of 290 000 km. Hence, those orbits are very well suited for a departure to

the libration point L2 without any propellant cost (except very slight corrections) and thus especially no high impulses are needed at injection Moon or L2. This is particularly interesting for Earth to Moon transfers when using low thrust electric propulsion for going safely to the Earth to Moon orbits like L2 or Halo orbits.

8. CONCLUSIONS

The paper has presented in simple words the tool TriaXOrbitaL. The major checks performed with the tool can give a good confidence in the results of the simulations. Several interesting applications of the tool have been presented: those make the single tool a multipurpose tool for orbit evolution problems with or without thrust.

The last application presented shows that this simple tool allows to simulate quite complex problems sometimes better known in the rotating synodic frame but with the capability for the user to understand what happens in more conventional inertial frame. The output of a free orbit transfer from elliptical orbit around the Earth to the point L2 seems to be valuable for the Electric propulsion.

The tool TriaXOrbitaL "as is" is freely distributed.

9. REFERENCES

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