NEW ADAPTIVE SLIDING MODE CONTROL AND RENDEZ-VOUS USING DIFFERENTIAL DRAG

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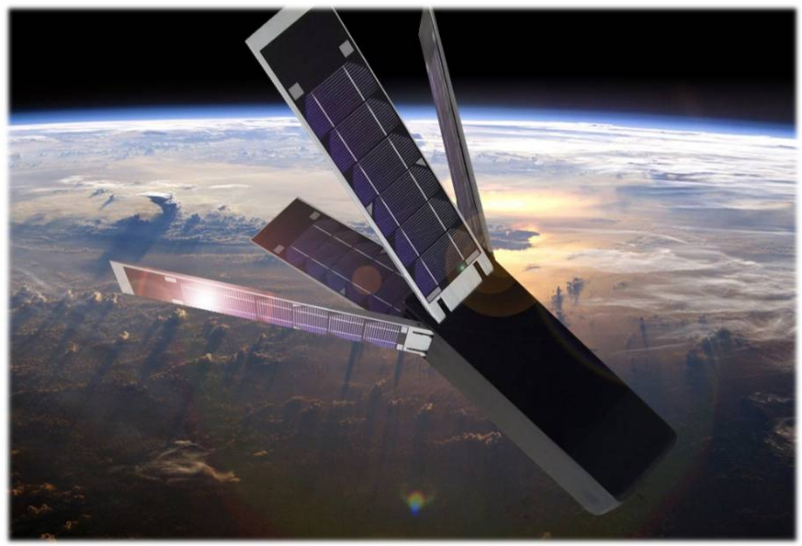

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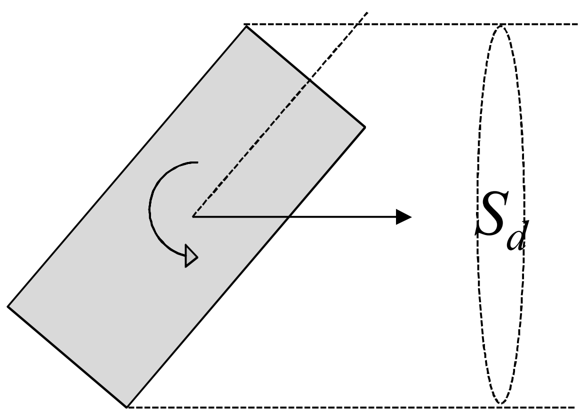
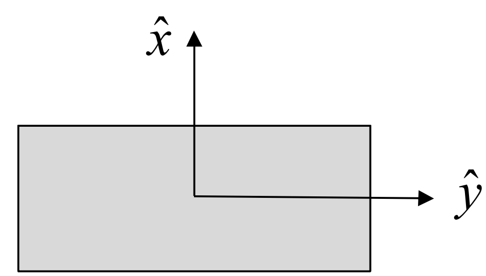
Rendez-vous Using Differential Drag

Assumptions

- Only in-plane control is considered.
- The target has a stable attitude. The rotation about the orbital plane of the chaser is controlled.
- The control output is S_d (cross-sectional area exposed to the atmosphere).
- S_d is bounded by $S_{min} \le S_d \le S_{max}$.
- The maneuver planner and the drag estimator are executed at the beginning of the maneuver.



Chaser



Target

Rendez-vous Using Differential Drag

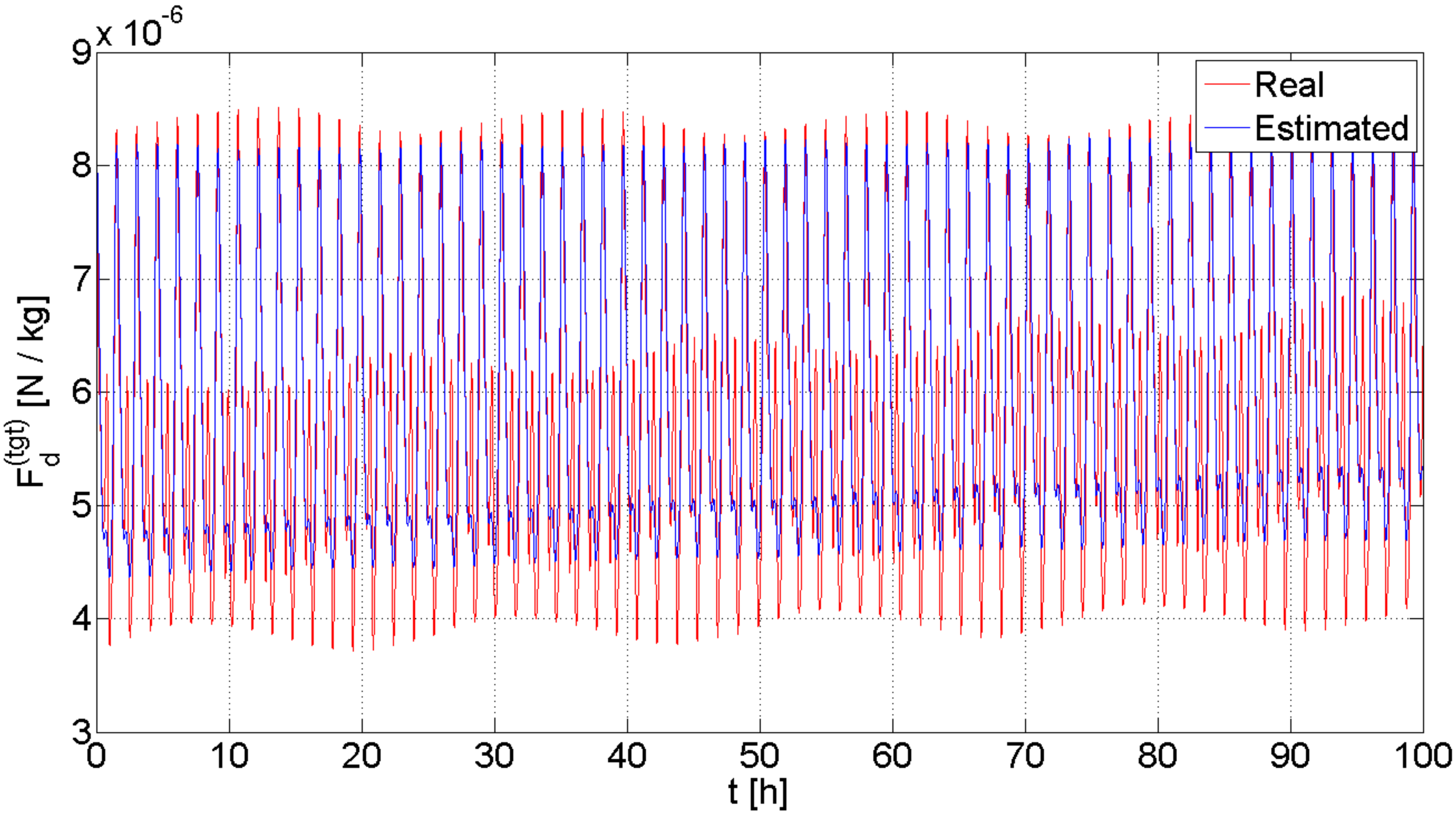
Maneuver Planner

- The maneuver planner computes the off-line optimal control strategy for the global rendezvous maneuver.
- It is obtained using an *hp*-adaptive Radau pseudospectral transcription using the software GPOPS.
- The dominant effects (secular J_2 effects and short period and altitude-dependent variations of drag) are modeled in the planner.
- For computational efficiency and accuracy, relative dynamics is expressed in terms of decomposed *curvilinear* variables and linear dynamics (Schweighart-Sedwick equations) is considered.

Drag Estimator

- To estimate the drag force on the two satellites, their accurate positions are monitored for at least two orbits.
- The estimation is performed by minimizing the mean square error between observed and simulated mean semi-major axis (generated through high-precision propagation based on the Jacchia 71 atmospheric model).

Rendez-vous Using Differential Drag



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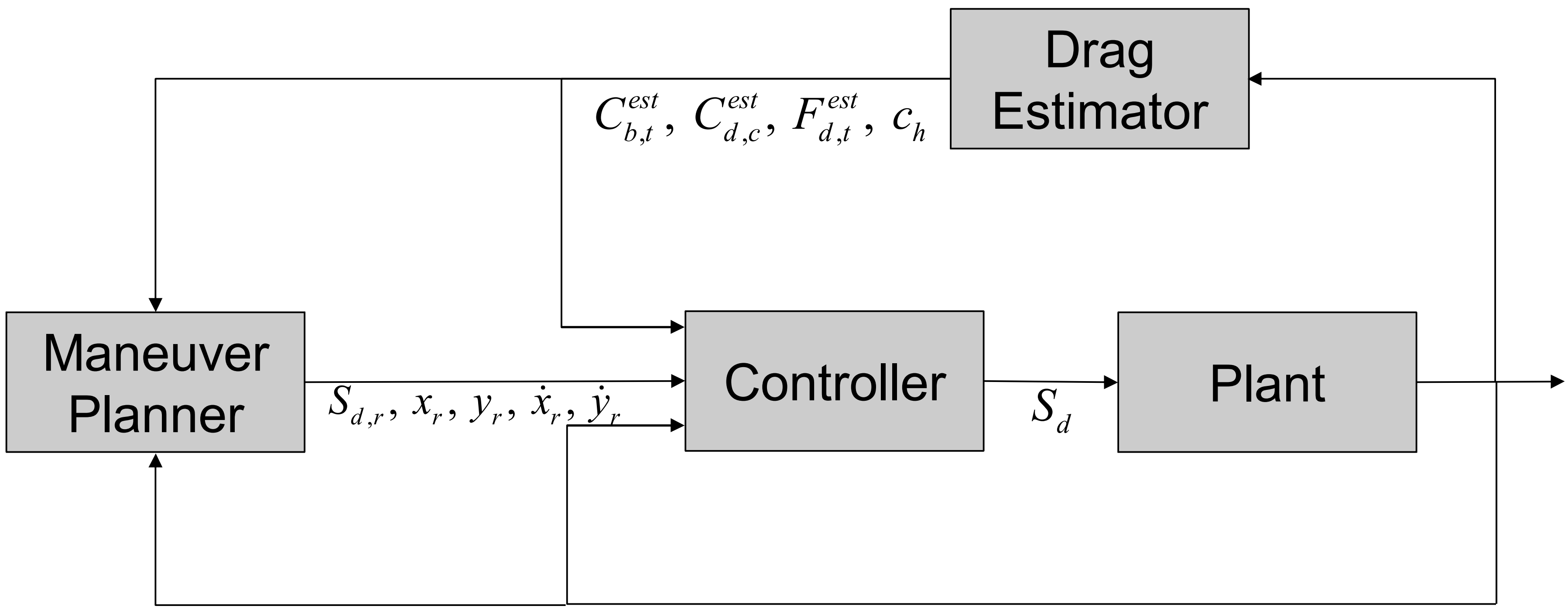
Rendez-vous Using Differential Drag



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Controller Design

Ideal (Reference) System

$$\ddot{x}_r = 2\omega c\dot{y}_r + (5c-2)\omega^2 x_r,$$

$$\ddot{y}_r = -2\omega c\dot{x}_r + \Delta F_{d,r},$$

where $\Delta F_{d,r} = F_{d,t}^{est}\left[1 - \frac{C_{d,c}^{est} S_{d,r}}{C_{b,t}^{est} m_c}(1 + c_h x_r)\right] = \alpha_r u_r + \beta_r,\ S_D \in [S_{\min}, S_{\max}],\ u_r \in [0,1].$

Real System (including higher harmonics, SRP, third-body perturbations, NRLMSISE-00)

$$\ddot{x} = 2\omega c\dot{y} + (5c-2)\omega^2 x + g_x,$$

$$\ddot{y} = -2\omega c\dot{x} + \Delta F_d + g_y,$$

where $\Delta F_d = (\alpha_r + \Delta\alpha)u + (\beta_r + \Delta\beta),\ u \in [0,1].$

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Controller Design

Error Dynamics

$$\ddot{e}_x = 2\omega c \dot{e}_y + (5c-2)\omega^2 e_x + g_x,$$

$$\ddot{e}_y = -2\omega c \dot{e}_x + \alpha_r \Delta u + \left(\Delta\alpha \cdot u + \Delta\beta + g_y\right),$$

where $\Delta u = u - u_r,\ e_x = x - x_r,\ e_y = y - y_r.$

Controller Design

Sliding Surface

- Consider the sliding surface $\boldsymbol{s} = \begin{bmatrix} s_x & s_y \end{bmatrix}^T$ of the form:

$$\boldsymbol{s} = \dot{\boldsymbol{e}} - \mathbf{S}\boldsymbol{e},$$

where $\boldsymbol{e} = \begin{bmatrix} e_x & e_y \end{bmatrix}^T$ and

$$\mathbf{S} = \begin{bmatrix} -2\zeta\sqrt{2-c^2}\,\omega & \dfrac{\left(2-c^2\right)\omega}{2c} \\ -2\omega c & 0 \end{bmatrix}.$$

- Stability: $\mathrm{Re}\left[\mathrm{eig}\left(\mathbf{S}\right)\right] < 0 \;\leftrightarrow\; \zeta > 0$

$$\because \; \left|\lambda\mathbf{I} - \mathbf{S}\right| = \lambda^2 + 2\zeta\tilde{\omega}\lambda + \tilde{\omega}^2 = 0,\; \tilde{\omega} = \sqrt{2-c^2}\,\omega.$$

Controller Design

Physical Interpretation

- When $\boldsymbol{s}$ = 0 and ς is sufficiently small, the error on the mean component of the trajectory is 0 (but oscillating) because

$$e_{x,m} = \frac{4c^2}{2-c^2} e_x + \frac{2c}{\left(2-c^2\right)\omega} \dot{e}_y = 0,$$
$$e_{y,m} = e_y - \frac{2c}{\left(2-c^2\right)\omega} \dot{e}_x = \frac{4\zeta c}{\sqrt{2-c^2}} e_x.$$

$\Leftarrow$

$$\dot{e}_x = \frac{\left(2-c^2\right)\omega}{2c} e_y - 2\zeta\sqrt{2-c^2}\,\omega e_x,$$
$$\dot{e}_y = -2\omega c e_x.$$

- Hence, the damping ratio $\varsigma > 0$ is used to progressively reduce the size of the relative orbit of the error vector. Furthermore, when $\boldsymbol{s}$ = 0, the error dynamics is analogous to the dynamics of the oscillatory components:

$$\dot{e}_{x,o} = \frac{\left(2-c^2\right)\omega}{2c} e_{y,o} - \frac{2c}{\left(2-c^2\right)\omega} \Delta F_d,$$
$$\dot{e}_{y,o} = -2\omega c e_{x,o}.$$

$\Leftarrow$

$$\dot{e}_x = \frac{\left(2-c^2\right)\omega}{2c} e_y - 2\zeta\sqrt{2-c^2}\,\omega e_x,$$
$$\dot{e}_y = -2\omega c e_x.$$

Controller Design

Lyapunov Function

- Let $V = \frac{1}{2}\mathbf{s}^T\mathbf{P}\mathbf{s},\ (\mathbf{P} > \mathbf{0})$, then

$$\mathbf{P} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \rightarrow P_{11} > 0,\ P_{11}P_{22} > P_{12}P_{21}$$

$$\begin{aligned} \dot{V} &= \mathbf{s}^T\mathbf{P}\dot{\mathbf{s}} \\ &= \left(P_{11}s_x + P_{21}s_y\right)L^* + \left(P_{12}s_x + P_{22}s_y\right)\left(\alpha_r\Delta u + \Delta L\right) \\ &= \left(P_{12}s_x + P_{22}s_y\right)\left(\eta L^* + \alpha_r\Delta u + \Delta L\right) \\ &= \left(\frac{P_{12}}{P_{22}}s_x + s_y\right)P_{22}\left(\eta L^* + \alpha_r\Delta u + \Delta L\right),\ \left(\text{Assume } P_{22} > 0\right) \end{aligned}$$

where $L^* = \left(5c^2 - 2\right)\omega^2 e_x + 2\zeta\sqrt{2 - c^2}\,\omega\dot{e}_y + \frac{\left(5c^2 - 2\right)\omega}{2c}\dot{e}_y$: measurable

$\Delta L = \Delta\alpha \cdot u + \Delta\beta + \eta g_x + g_y$: uncertain, $\left|\Delta L\right| \le L_M$

$$\eta = \frac{P_{11}s_x + P_{21}s_y}{P_{12}s_x + P_{22}s_y}$$

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Controller Design

Control Input

- Let $\Delta u = -\dfrac{\Gamma}{\alpha_r}\left(\dfrac{P_{12}}{P_{22}}s_x + s_y\right) - \dfrac{\eta L^*}{\alpha_r}$, then

$$\dot{V} = \left(\frac{P_{12}}{P_{22}}s_x + s_y\right)P_{22}\left[\Delta L - \Gamma\left(\frac{P_{12}}{P_{22}}s_x + s_y\right)\right]$$

$$\leq \left|\frac{P_{12}}{P_{22}}s_x + s_y\right| P_{22}L_M - P_{22}\Gamma\left(\frac{P_{12}}{P_{22}}s_x + s_y\right)^2.$$

Consider the region where $\left|\dfrac{P_{12}}{P_{22}}s_x + s_y\right| > \varepsilon$ holds, then

$$\dot{V} < \left|\frac{P_{12}}{P_{22}}s_x + s_y\right| P_{22}L_M - \varepsilon P_{22}\Gamma\left|\frac{P_{12}}{P_{22}}s_x + s_y\right|$$

$$= \left|\frac{P_{12}}{P_{22}}s_x + s_y\right| P_{22}\left(L_M - \varepsilon\Gamma\right) \leq 0.$$

$\Rightarrow$ $$\Gamma = \frac{L_M}{\varepsilon}$$

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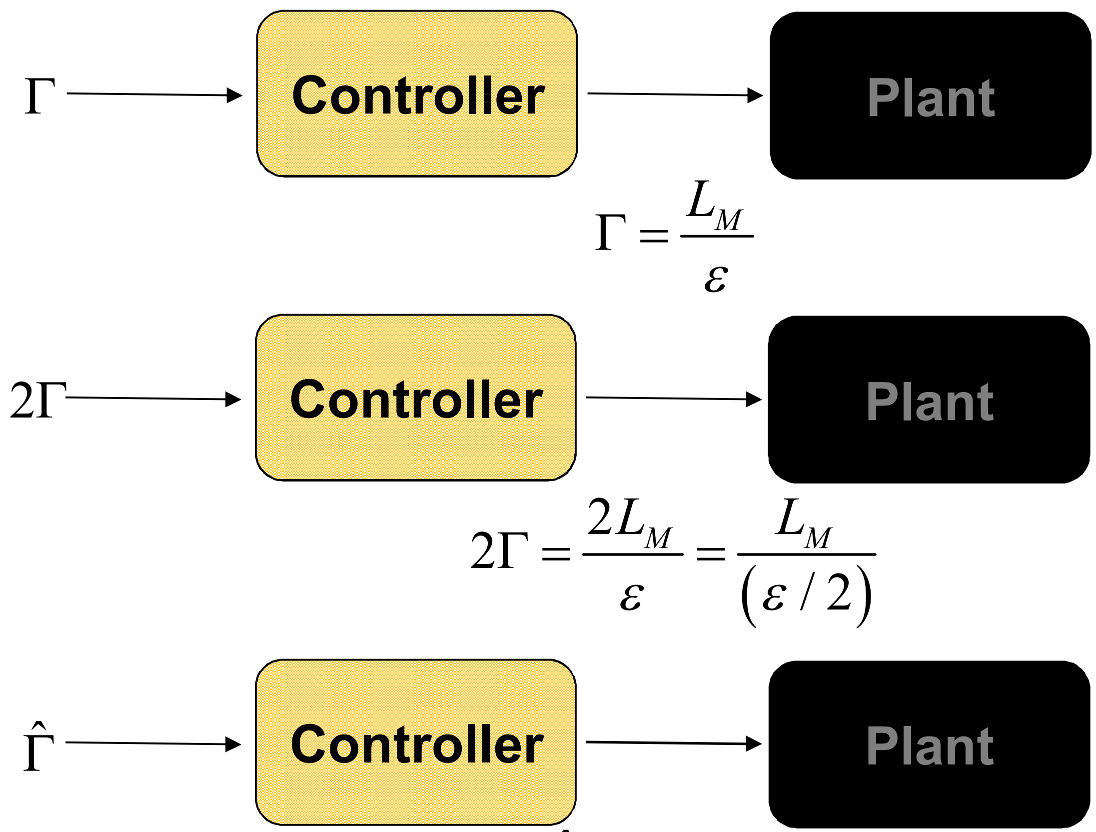
Adaptive Rules for the Gain Γ

Observation



$$\left|\frac{P_{12}}{P_{22}} s_x + s_y\right| \le \varepsilon$$

$$\Gamma = \frac{L_M}{\varepsilon}$$

$$\left|\frac{P_{12}}{P_{22}} s_x + s_y\right| \le \frac{\varepsilon}{2}$$

$$2\Gamma = \frac{2L_M}{\varepsilon} = \frac{L_M}{(\varepsilon/2)}$$

$$\left|\frac{P_{12}}{P_{22}} s_x + s_y\right| \le \varepsilon \frac{\Gamma}{\hat{\Gamma}}$$

$$\hat{\Gamma} = \frac{\hat{\Gamma}}{\Gamma}\frac{L_M}{\varepsilon} = \frac{L_M}{\left(\varepsilon\Gamma/\hat{\Gamma}\right)}$$

Adaptive Rules for the Gain Γ

Estimation of Γ

Assume that a rough estimate $\hat{\Gamma}$ is applied, and the resultant sliding variable is bounded by $\hat{\varepsilon}$ so that $\left|\left(P_{12}/P_{22}\right)s_x + s_y\right| \le \hat{\varepsilon}$. Then, the real (unknown) gain Γ can be estimated by

$$\Gamma = \frac{\hat{\varepsilon}}{\varepsilon}\hat{\Gamma}. \quad (\varepsilon : \text{ desired})$$

Adaptive Rule

Let Γ_0 be the initial estimate. At each instant of time, $\left|\left(P_{12}/P_{22}\right)s_x + s_y\right|$ is compared with ε, and the real-time adaptive law for Γ is given by the following rules:

$$\Gamma^{(k+1)} = \Gamma^{(k)}, \qquad \text{if } \left|\frac{P_{12}}{P_{22}}s_x^{(k)} + s_y^{(k)}\right| \le \varepsilon,$$

$$\Gamma^{(k+1)} = \Gamma^{(k)} \cdot \min\left(\delta, \left|\frac{P_{12}}{P_{22}}s_x^{(k)} + s_y^{(k)}\right| / \varepsilon\right), \quad \text{if } \left|\frac{P_{12}}{P_{22}}s_x^{(k)} + s_y^{(k)}\right| > \varepsilon.$$

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Numerical Simulation

Control Parameters

$$L_M = 10^{-5},\ \varepsilon = 0.005,\ \zeta = 10^{-4},\ \delta = 1.01,\ P_{11} = 3000,\ P_{12} = 1000,\ P_{21} = 0,\ P_{22} = 10^6.$$

Mean elements of the target	Semi-major axis	$6728 \cdot 10^3$ m
	Eccentricity	0
	Inclination	98 deg
	RAAN	90 deg
	Argument of perigee	0 deg
	True anomaly	0 deg
Initial gap of the chaser	Along-track	$50 \cdot 10^3$ m
	Radial	100 m
Target properties	Ballistic coefficient	0.014 m^2kg^{-1}
Chaser properties	Mass	4 kg
	Dimensions	0.3×0.1×0.1 m^3
	Drag coefficient	2.8

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Numerical Simulation

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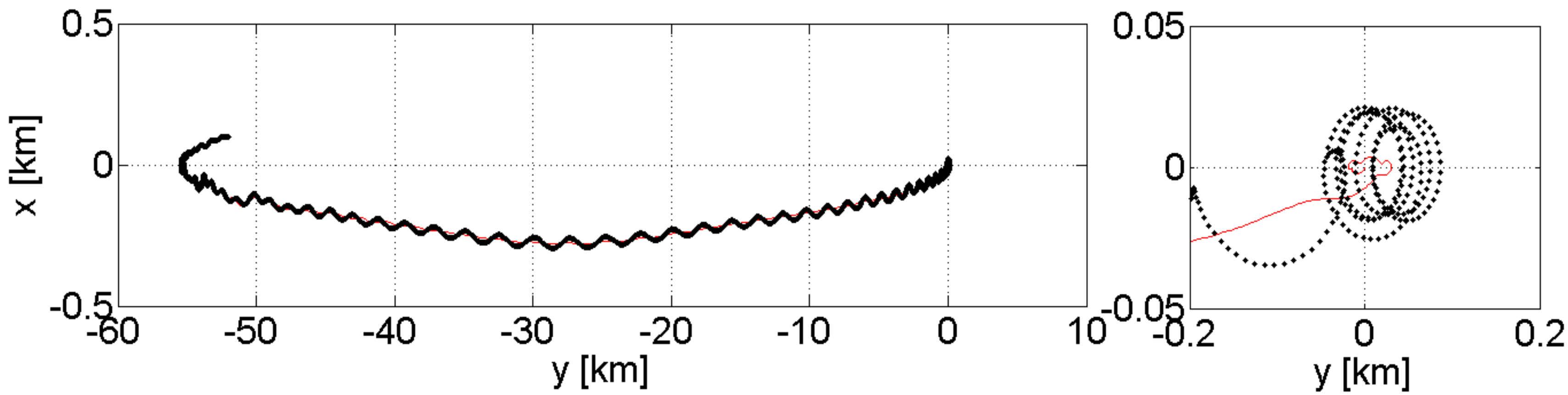

Global trajectories in *y*-*x* plane (left) and the final phase (right)

Numerical Simulation

Control Parameters

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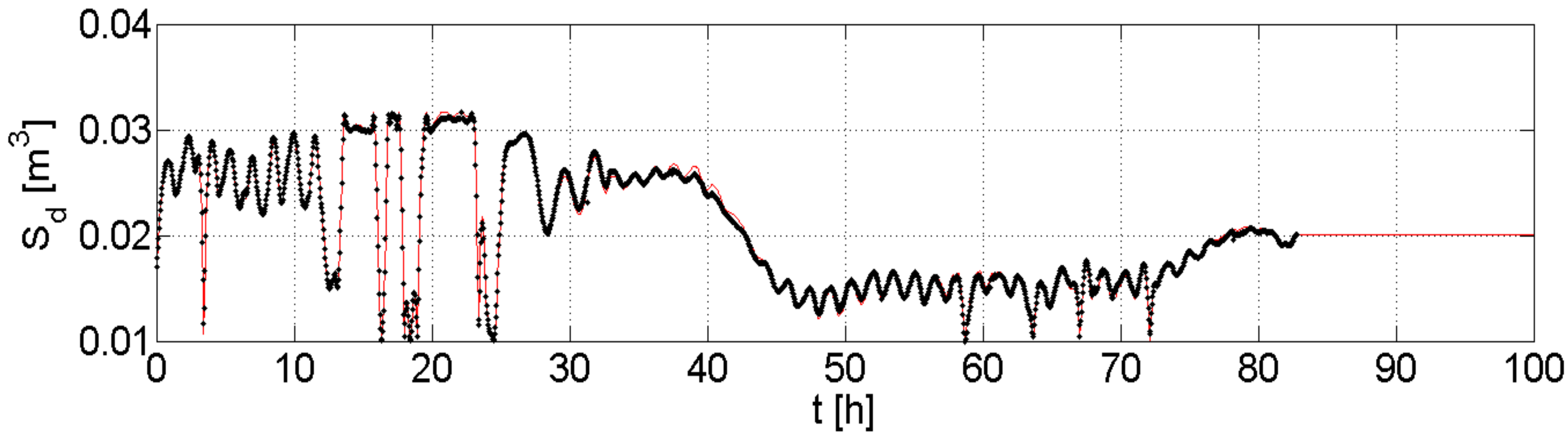
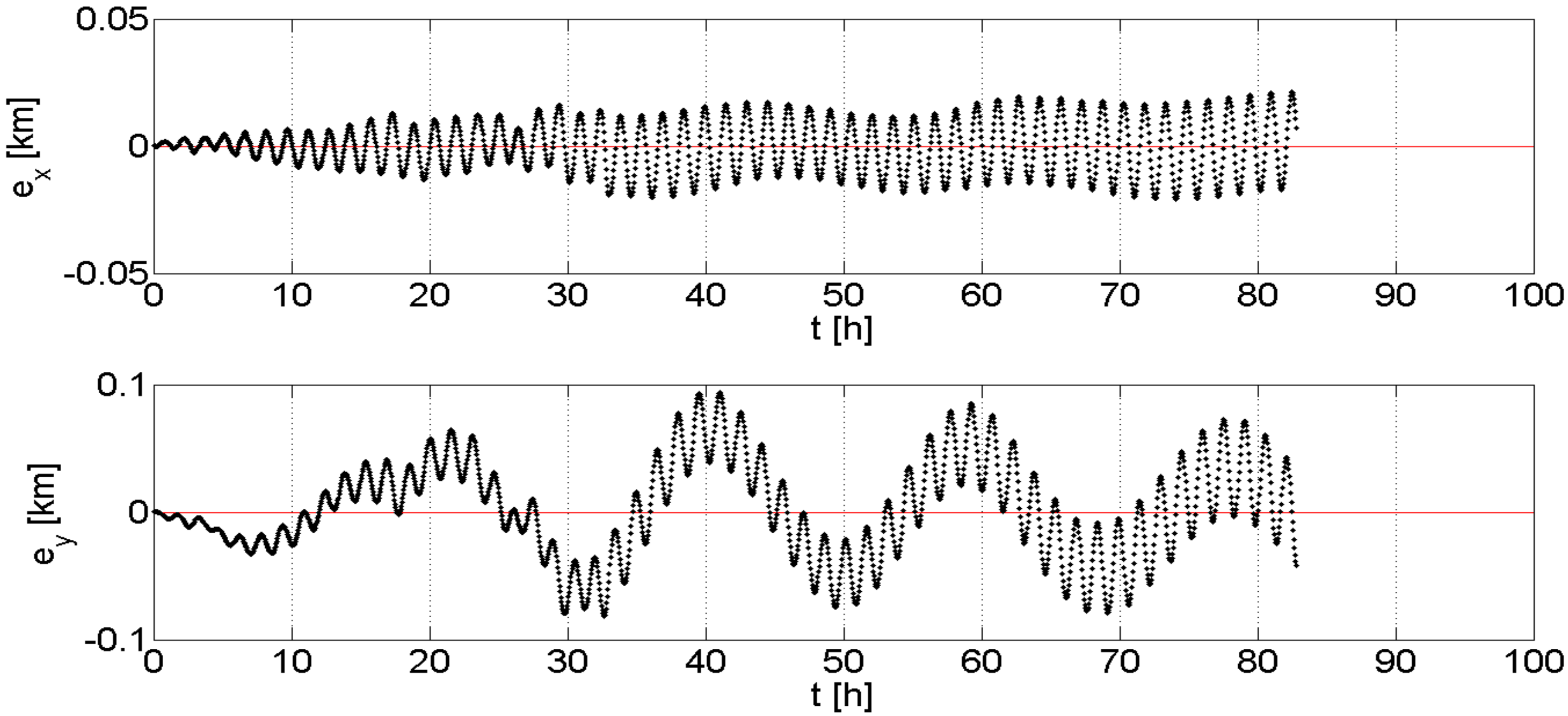


Cross-sectional area of the chaser S_d

Numerical Simulation

Control Parameters

$$L_M = 10^{-5},\ \varepsilon = 0.005,\ \zeta = 10^{-4},\ \delta = 1.01,\ P_{11} = 3000,\ P_{12} = 1000,\ P_{21} = 0,\ P_{22} = 10^6.$$



Errors in the *x*-axis (upper) and *y*-axis (lower)

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Numerical Simulation

Control Parameters

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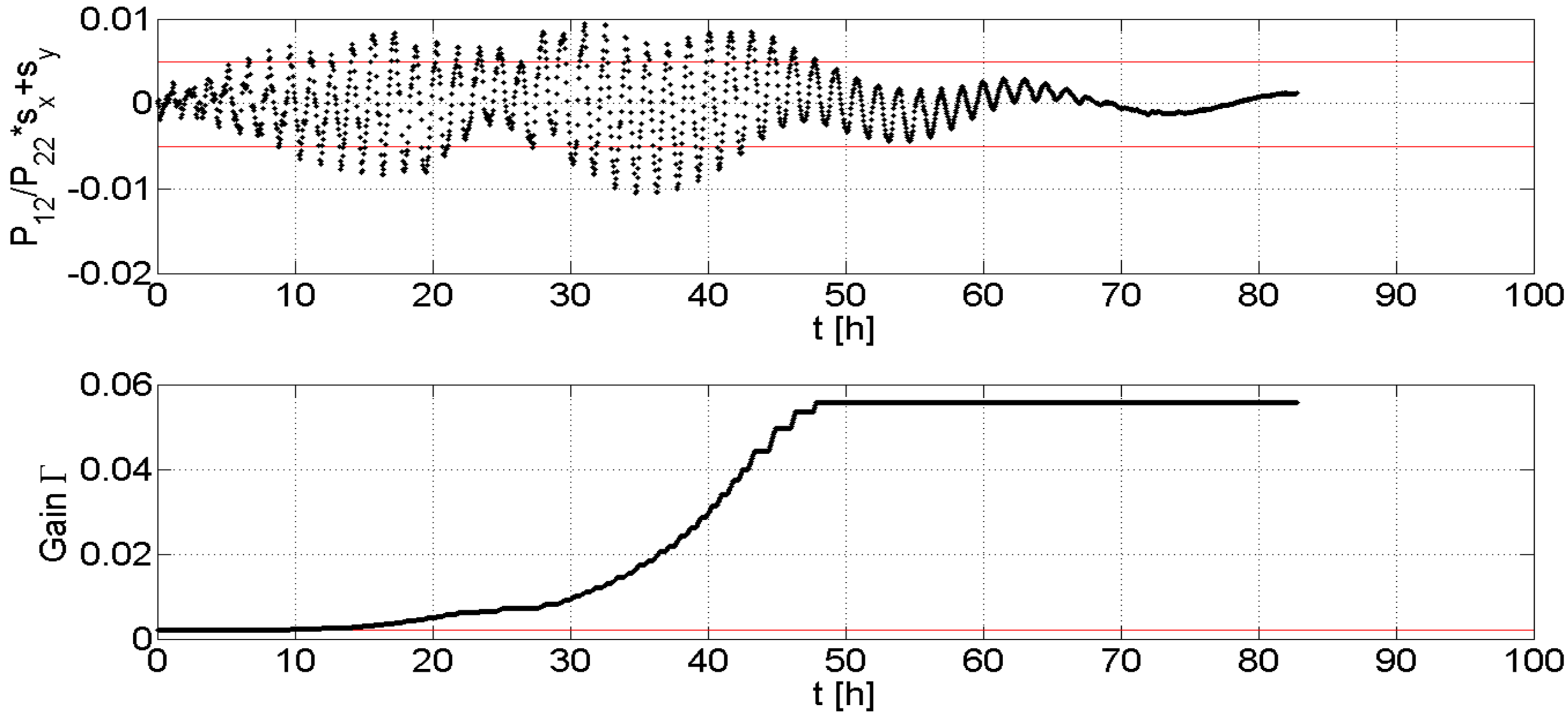

$\left|(P_{12}/P_{22})s_x + s_y\right|$ (upper) and the gain Γ (lower) updated by the adaptive law

Numerical Simulation

Summary

- The small ε is, the smaller error is.
- If the initial Γ_0 is too large, the control output S_d is easily saturated and the errors start to oscillate with large amplitudes from the beginning of the maneuver.
- The response to the variation of u is slow, so the gain seems to be ended up with a larger value than the real one.
- How to determine δ and P_{ij}?

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