NEW ADAPTIVE SLIDING MODE CONTROL AND RENDEZ-VOUS USING DIFFERENTIAL DRAG

Hancheol Cho
Lamberto Dell’Elce
Gaëtan Kerschen

Space Structures and Systems Laboratory (S3L)
University of Liège, Belgium
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Rendez-vous Using Differential Drag

Assumptions

- Only in-plane control is considered.
- The target has a stable attitude. The rotation about the orbital plane of the chaser is controlled.
- The control output is $S_d$ (cross-sectional area exposed to the atmosphere).
- $S_d$ is bounded by $S_{\text{min}} \leq S_d \leq S_{\text{max}}$.
- The maneuver planner and the drag estimator are executed at the beginning of the maneuver.
Rendez-vous Using Differential Drag

Maneuver Planner

• The maneuver planner computes the off-line optimal control strategy for the global rendezvous maneuver.
• It is obtained using an $hp$-adaptive Radau pseudospectral transcription using the software GPOPS.
• The dominant effects (secular $J_2$ effects and short period and altitude-dependent variations of drag) are modeled in the planner.
• For computational efficiency and accuracy, relative dynamics is expressed in terms of decomposed curvilinear variables and linear dynamics (Schweighart-Sedwick equations) is considered.

Drag Estimator

• To estimate the drag force on the two satellites, their accurate positions are monitored for at least two orbits.
• The estimation is performed by minimizing the mean square error between observed and simulated mean semi-major axis (generated through high-precision propagation based on the Jacchia 71 atmospheric model).
Rendez-vous Using Differential Drag
Rendez-vous Using Differential Drag

Maneuver Planner

Controller

Plant

Drag Estimator

\( C_{b,t}^{est}, C_{d,c}^{est}, F_{d,t}^{est}, c_h \)

\( S_{d,r}, x_r, y_r, \dot{x}_r, \dot{y}_r \)

\( S_d \)
Controller Design

**Ideal (Reference) System**

\[
\begin{align*}
\ddot{x}_r &= 2\omega c\dot{y}_r + (5c - 2)\omega^2 x_r, \\
\ddot{y}_r &= -2\omega c\dot{x}_r + \Delta F_{d,r},
\end{align*}
\]

where \(\Delta F_{d,r} = F_{d,r}^{est} - F_{d,r}^{est} \left[ 1 - \frac{C_{d,c}^{est} S_{d,x}}{C_{b,t}^{est} m_c} (1 + c_h x_r) \right] = \alpha_r u_r + \beta_r, S_D \in [S_{min}, S_{max}], u_r \in [0,1].\)

**Real System** (including higher harmonics, SRP, third-body perturbations, NRLMSISE-00)

\[
\begin{align*}
\ddot{x} &= 2\omega c\dot{y} + (5c - 2)\omega^2 x + g_x, \\
\ddot{y} &= -2\omega c\dot{x} + \Delta F_d + g_y,
\end{align*}
\]

where \(\Delta F_d = (\alpha_r + \Delta \alpha) u + (\beta_r + \Delta \beta), u \in [0,1].\)
Controller Design

### Error Dynamics

\[
\begin{align*}
\ddot{e}_x &= 2\omega c \dot{e}_y + (5c - 2)\omega^2 e_x + g_x, \\
\ddot{e}_y &= -2\omega c \dot{e}_x + \alpha_r \Delta u + \left(\Delta \alpha \cdot u + \Delta \beta + g_y\right),
\end{align*}
\]

where \( \Delta u = u - u_r, \ e_x = x - x_r, \ e_y = y - y_r. \)
Controller Design

Sliding Surface

- Consider the sliding surface \( s = \begin{bmatrix} s_x & s_y \end{bmatrix}^T \) of the form:

\[
\begin{align*}
\dot{s} &= \dot{e} - Se, \\
S &= \begin{bmatrix}
-2\zeta \sqrt{2-c^2} \omega & \left(2-c^2\right)\omega \\
-2\omega c & 2c \\
-2\omega c & 0
\end{bmatrix}.
\end{align*}
\]

- Stability: \( \text{Re} \left[ \text{eig}(S) \right] < 0 \iff \zeta > 0 \)

\[
\begin{align*}
\lambda I - S &= \lambda^2 + 2\zeta \tilde{\omega} \lambda + \tilde{\omega}^2 = 0, \\
\tilde{\omega} &= \sqrt{2-c^2} \omega.
\end{align*}
\]
Controller Design

Physical Interpretation

- When \( s = 0 \) and \( \zeta \) is sufficiently small, the error on the mean component of the trajectory is 0 (but oscillating) because

\[
e_{x,m} = \frac{4c^2}{2 - c^2} e_x + \frac{2c}{(2 - c^2)\omega} \dot{e}_y = 0, \quad \dot{e}_x = \frac{2 - c^2}{2c} e_y - 2\zeta \sqrt{2 - c^2} \omega e_x, \quad \dot{e}_y = -2\omega c e_x.
\]

- Hence, the damping ratio \( \zeta > 0 \) is used to progressively reduce the size of the relative orbit of the error vector. Furthermore, when \( s = 0 \), the error dynamics is analogous to the dynamics of the oscillatory components:

\[
\dot{e}_{x,o} = \frac{(2 - c^2)\omega}{2c} e_{y,o} - \frac{2c}{(2 - c^2)\omega}\Delta F_d, \quad \dot{e}_x = \frac{2 - c^2}{2c} e_y - 2\zeta \sqrt{2 - c^2} \omega e_x, \quad \dot{e}_y = -2\omega c e_x.
\]
Controller Design

Lyapunov Function

- Let \( V = \frac{1}{2} s^T P s \), \( (P > 0) \), then

\[
\dot{V} = s^T P \dot{s} = \left( P_{11} s_x + P_{21} s_y \right) L^* + \left( P_{12} s_x + P_{22} s_y \right) (\alpha_r \Delta u + \Delta L)
\]

\[
= \left( P_{12} s_x + P_{22} s_y \right) (\eta L^* + \alpha_r \Delta u + \Delta L)
\]

\[
= \left( \frac{P_{12}}{P_{22}} s_x + s_y \right) P_{22} \left( \eta L^* + \alpha_r \Delta u + \Delta L \right), \quad (\text{Assume } P_{22} > 0)
\]

where \( L^* = \left( 5c^2 - 2 \right) \omega^2 e_x + 2\zeta \sqrt{2 - c^2} \omega \dot{e}_y + \frac{\left( 5c^2 - 2 \right) \omega}{2c} \dot{e}_y \): measurable

\[\Delta L = \Delta \alpha \cdot u + \Delta \beta + \eta g_x + g_y \]: uncertain, \( |\Delta L| \leq L_M \)

\[\eta = \frac{P_{11} s_x + P_{21} s_y}{P_{12} s_x + P_{22} s_y}\]
Controller Design

Control Input

- Let $\Delta u = -\frac{\Gamma}{\alpha_r} \left( \frac{P_{12}}{P_{22}} s_x + s_y \right) - \frac{\eta L^*}{\alpha_r}$, then

$$\dot{V} = \left( \frac{P_{12}}{P_{22}} s_x + s_y \right) P_{22} \left[ \Delta L - \Gamma \left( \frac{P_{12}}{P_{22}} s_x + s_y \right) \right]$$

$$\leq \left| \frac{P_{12}}{P_{22}} s_x + s_y \right| P_{22} L_M - P_{22} \Gamma \left( \frac{P_{12}}{P_{22}} s_x + s_y \right)^2.$$

Consider the region where $\left| \frac{P_{12}}{P_{22}} s_x + s_y \right| > \varepsilon$ holds, then

$$\dot{V} < \left| \frac{P_{12}}{P_{22}} s_x + s_y \right| P_{22} L_M - \varepsilon P_{22} \Gamma \left| \frac{P_{12}}{P_{22}} s_x + s_y \right|$$

$$= \left| \frac{P_{12}}{P_{22}} s_x + s_y \right| P_{22} \left( L_M - \varepsilon \Gamma \right) \leq 0.$$

$$\Gamma = \frac{L_M}{\varepsilon}$$
Adaptive Rules for the Gain $\Gamma$

<table>
<thead>
<tr>
<th>Observation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma$ $\rightarrow$ Controller $\rightarrow$ Plant $\rightarrow$ $\left</td>
</tr>
<tr>
<td>$2\Gamma$ $\rightarrow$ Controller $\rightarrow$ Plant $\rightarrow$ $\left</td>
</tr>
<tr>
<td>$\hat{\Gamma}$ $\rightarrow$ Controller $\rightarrow$ Plant $\rightarrow$ $\left</td>
</tr>
</tbody>
</table>

$\Gamma = \frac{L_M}{\varepsilon}$

$2\Gamma = \frac{2L_M}{\varepsilon} = \frac{L_M}{(\varepsilon / 2)}$

$\hat{\Gamma} = \frac{\hat{\Gamma} L_M}{\Gamma \varepsilon} = \frac{L_M}{(\varepsilon \Gamma / \hat{\Gamma})}$
### Adaptive Rules for the Gain $\Gamma$

#### Estimation of $\Gamma$
Assume that a rough estimate $\hat{\Gamma}$ is applied, and the resultant sliding variable is bounded by $\hat{\epsilon}$ so that $\left| \frac{P_{12}}{P_{22}} s_x + s_y \right| \leq \hat{\epsilon}$. Then, the real (unknown) gain $\Gamma$ can be estimated by

$$\Gamma = \frac{\hat{\epsilon}}{\epsilon} \hat{\Gamma}. \quad (\epsilon: \text{desired})$$

#### Adaptive Rule
Let $\Gamma_0$ be the initial estimate. At each instant of time, $\left| \frac{P_{12}}{P_{22}} s_x + s_y \right|$ is compared with $\epsilon$, and the real-time adaptive law for $\Gamma$ is given by the following rules:

$$\Gamma^{(k+1)} = \Gamma^{(k)}, \quad \text{if} \quad \left| \frac{P_{12}}{P_{22}} s_x^{(k)} + s_y^{(k)} \right| \leq \epsilon,$$

$$\Gamma^{(k+1)} = \Gamma^{(k)} \cdot \min \left( \delta, \left| \frac{P_{12}}{P_{22}} s_x^{(k)} + s_y^{(k)} \right| / \epsilon \right), \quad \text{if} \quad \left| \frac{P_{12}}{P_{22}} s_x^{(k)} + s_y^{(k)} \right| > \epsilon.$$
### Numerical Simulation

**Control Parameters**

\[
L_M = 10^{-5}, \quad \varepsilon = 0.005, \quad \zeta = 10^{-4}, \quad \delta = 1.01, \quad P_{11} = 3000, \quad P_{12} = 1000, \quad P_{21} = 0, \quad P_{22} = 10^6.
\]

<table>
<thead>
<tr>
<th>Mean elements of the target</th>
<th>Semi-major axis</th>
<th>6728 \cdot 10^3 m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eccentricity</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Inclination</td>
<td></td>
<td>98 deg</td>
</tr>
<tr>
<td>RAAN</td>
<td></td>
<td>90 deg</td>
</tr>
<tr>
<td>Argument of perigee</td>
<td></td>
<td>0 deg</td>
</tr>
<tr>
<td>True anomaly</td>
<td></td>
<td>0 deg</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Initial gap of the chaser</th>
<th>Along-track</th>
<th>50 \cdot 10^3 m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radial</td>
<td></td>
<td>100 m</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Target properties</th>
<th>Ballistic coefficient</th>
<th>0.014 m^2kg^{-1}</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Chaser properties</th>
<th>Mass</th>
<th>4 kg</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dimensions</td>
<td>0.3\times0.1\times0.1 m^3</td>
<td></td>
</tr>
<tr>
<td>Drag coefficient</td>
<td>2.8</td>
<td></td>
</tr>
</tbody>
</table>
Numerical Simulation

Control Parameters

\[ L_M = 10^{-5}, \quad \varepsilon = 0.005, \quad \zeta = 10^{-4}, \quad \delta = 1.01, \quad P_{11} = 3000, \quad P_{12} = 1000, \quad P_{21} = 0, \quad P_{22} = 10^6. \]

Global trajectories in $y$-$x$ plane (left) and the final phase (right)
Numerical Simulation

Control Parameters

\[ L_M = 10^{-5}, \ \varepsilon = 0.005, \ \zeta = 10^{-4}, \ \delta = 1.01, \ P_{11} = 3000, \ P_{12} = 1000, \ P_{21} = 0, \ P_{22} = 10^6. \]

Cross-sectional area of the chaser \( S_d \)
Numerical Simulation

Control Parameters

\[ L_M = 10^{-5}, \, \varepsilon = 0.005, \, \zeta = 10^{-4}, \, \delta = 1.01, \, P_{11} = 3000, \, P_{12} = 1000, \, P_{21} = 0, \, P_{22} = 10^6. \]

Errors in the \( x \)-axis (upper) and \( y \)-axis (lower)
Numerical Simulation

Control Parameters

\[ L_M = 10^{-5}, \ \varepsilon = 0.005, \ \zeta = 10^{-4}, \ \delta = 1.01, \ P_{11} = 3000, \ P_{12} = 1000, \ P_{21} = 0, \ P_{22} = 10^6. \]

\( P_{12} / P_{22} \) \( s_x + s_y \) (upper) and the gain \( \Gamma \) (lower) updated by the adaptive law.
Numerical Simulation

Summary

• The small $\epsilon$ is, the smaller error is.
• If the initial $\Gamma_0$ is too large, the control output $S_d$ is easily saturated and the errors start to oscillate with large amplitudes from the beginning of the maneuver.
• The response to the variation of $u$ is slow, so the gain seems to be ended up with a larger value than the real one.
• How to determine $\delta$ and $P_{ij}$?