OCCAM:
Optimal Computation of Collision Avoidance Maneuvers

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Increasing number of close encounters between active satellites and other bodies

- 2010: *Every week an average 190 conjunctions and 3 collision avoidance maneuvers* were reported
- 2011: *Envisat alone* reported *4 collision avoidance maneuvers*.

Software tools are needed to plan the execution of these maneuvers in an optimum way and with:

- Versatility
- Efficiency
- Capability to explore a wide parameter space
- Modularity
What OCCAM can do

- **Accurately** predict the outcome of a collision avoidance maneuver (CAM)

- **Globally** optimize the direction of the maneuver $\Delta v$ for minimum collision probability or maximum miss distance

- Finding the **global** minimum $\Delta v$ to reduce the collision probability to a given threshold
Scenario

A maneuverable satellite $S_1$ and a non-maneuverable body $S_2$ experience a close encounter and may collide.

What do we know? (Input)
- $S_1$ and $S_2$ nominal positions and velocities at close encounter
- $S_1$ and $S_2$ covariance matrices at close encounter
- $S_1$ and $S_2$ spherical envelopes

What do we want to know (Output)
- Collision probability
- Best way to reduce it (if critical)
Calculating Collision Probability

- Short-term encounter scenario
- Encounter plane \((\xi - \zeta)\) projection
- 2D integral over combined cross-section area

\[
P_c = \iint_{A} f(r) \, dr
\]
OCCAM supports six collision probability calculation methods (from the fastest to the slowest method):

1. García-Pelayo & Hernando-Ayuso method (JGCD, in press)
2. Serra et al. method (JGCD, 2016)
3. Chan’s method
4. Alfano’s method
5. Patera’s method
6. Foster’s method
Avoiding the collision

- Suppose the conjunction is critical (e.g. $P_c > 10^{-4}$)
Avoiding the collision

- Suppose the conjunction is critical (e.g. $P_c > 10^{-4}$)
- What is the best way to reduce $P_c$ with limited $\Delta v$?
- When should we maneuver?
Avoiding the collision

- Suppose the conjunction is critical (e.g. $P_c > 10^{-4}$)
- What is the best way to reduce $P_c$ with limited $\Delta v$?
- When should we maneuver?

- Displace the conjunction point away from the high $P_c$ area
- Find the optimum orbital maneuver orientation and timing
Impulsive maneuver in UVW
(Local Horizontal Local Vertical)

\[ \Delta \mathbf{v} = (\Delta v_U, \Delta v_V, \Delta v_W) \]

\[ \Delta v_U = \Delta v \cos \gamma \sin (\sigma + \alpha), \]
\[ \Delta v_V = \Delta v \cos \gamma \cos (\sigma + \alpha), \]
\[ \Delta v_W = \Delta v \sin \gamma. \]
Impulsive maneuver in UVW
(Local Horizontal Local Vertical)
\[ \Delta v = (\Delta v_U, \Delta v_V, \Delta v_W)^\top \]

\[ \Delta v_U = \Delta v \cos \gamma \sin (\sigma + \alpha), \]
\[ \Delta v_V = \Delta v \cos \gamma \cos (\sigma + \alpha), \]
\[ \Delta v_W = \Delta v \sin \gamma. \]

\( v_2 \) is related to \( v_1 \) by:

- a \( S_1 \) in-plane rotation \( \phi \)
- a \( S_1 \) out-of-plane rotation \( \psi \)
- a scaling \( \chi = v_2 / v_1 \)
Impulsive collision avoidance dynamics

**Linear** impulsive maneuver approximation:

\[ r \simeq r_e + M \Delta v, \quad M = R K D \]

- **R**: Rotation matrix
- **K**: Kinematics matrix
- **D**: Dynamics matrix (error state transition matrix)

M is a function of \( \phi, \psi, \chi, e_0, \theta_c, \theta_m \) and \( \sqrt{\frac{a_0^3}{\mu}} \):

- \( e_0 \): eccentricity of unperturbed orbit of \( S_1 \)
- \( a_0 \): semimajor axis of unperturbed orbit of \( S_1 \)
- \( \theta_c, \theta_m \): true anomaly of \( S_1 \) at conjunction and maneuver

Maneuver anticipation angle: \( \Delta \theta = \theta_c - \theta_m \)
- In general: non-linear, non-convex optimization problem
- Computationally heavy
- To solve it, two **relaxations** are introduced:
  1. Linear dynamics $r = r_e + M \Delta v$
  2. Constant $P_c$ when $r$ lays on a constant pdf ellipse. Good approximation if the bodies are large with respect to the uncertainty
- The problem is transformed into a Quadratically Constrained Quadratic Program equivalent to a convex problem
- **Finally reduced to an eigenvalue and eigenvector problem** + 1D non-linear equation
- Extremely fast
OCCAM (Input)

OCCAM user interface

OCCAM input diagram flow
OCCAM (Ouput)

- **required $\Delta v$**
- **In-plane maneuver orientation angle**
- **Out-of-plane maneuver orientation angle**
OCCAM lite (Try it online!)

Reduced number of features
Online web-app

http://sdg.aero.upm.es/index.php/online-apps/occam-lite

(or Google: “OCCAM lite”)

<table>
<thead>
<tr>
<th>Maneuver Parameters</th>
<th>Body Sizes</th>
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</thead>
<tbody>
<tr>
<td>$\Delta V = \frac{0.1 \text{ m s}^{-1}}{3}$</td>
<td>$r_1 = 3 \text{ m}$</td>
</tr>
<tr>
<td>$n_{orbits} = 3$</td>
<td>$r_2 = 4 \text{ m}$</td>
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</tbody>
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Body 1 orbit parameters:
- $a_0 = 7155.831 \text{ m}$
- $e_0 = 2E-4$
- $\theta_c = -16.85 \text{ deg}$

Body 1 covariance matrix:
- $\sigma_{1T} = 1000 \text{ m}$
- $\sigma_{1N} = 100 \text{ m}$
- $\sigma_{1H} = 100 \text{ m}$

Body 2 covariance matrix:
- $\sigma_{2T} = 1000 \text{ m}$
- $\sigma_{2N} = 100 \text{ m}$
- $\sigma_{2H} = 100 \text{ m}$

Collision geometry:
- $\phi = 180 \text{ deg}$
- $\psi = 77.5 \text{ deg}$
- $\chi = 1$

Collision Probability calculation method:
- Garcia-Pelayo & Hernando-Ayuso
- Serna
- Chen
- Alfano
- Patera
- Foster (VERY SLOW, limited time resolution)
OCCAM was developed at the Space Dynamics Group (UPM)
Submitted to the Copyright Registry Office of Madrid and presented at UPM Innovatech 2014 (technology commercialization seminar)
OCCAM allows satellite operators to rapidly analyze a wide range of possible strategies
Its graphical user interface makes it easy to learn and use, while retaining a high design flexibility
It can be used as a standalone tool, or in conjunction with other satellite operation planning frameworks
A customized integral service could be offered to each partner
Thank you

Questions?