OCCAM: OPTIMAL COMPUTATION OF COLLISION AVOIDANCE MANEUVERS

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ABSTRACT

OCCAM (Optimal Computation of Collision Avoidance Maneuvers) is a novel software tool aimed at computing minimum-fuel collision avoidance maneuvers in the short-term encounter scenario, which is generally applicable in LEO. Developed by the Space Dynamics Group of the Technical University of Madrid, it employs advanced modeling and optimization techniques, which make it an extremely fast and robust design tool. OCCAM features an extensive set of input parameters, different optimization strategies and output options to provide a high design flexibility for the user. Several methods of collision probability computation are also supported. Its user-friendly graphical interface and intuitive design logic make it really straightforward to master even for non-experts, and it can be employed either as a standalone tool or in conjunction with other satellite operation planning frameworks. A trial version of this tool called OCCAM lite is available on-line for the interested potential user at the web page of the Space Dynamics Group: `http://sdg.aero.upm.es/index.php/online-apps/occam-lite`.

***Index Terms*—** Collision Avoidance Maneuvers, Space Situational Awareness, Space Debris, Short-term encounter, Optimization

1. INTRODUCTION

As a collateral effect of the use of space by military, scientific and commercial spacecraft, tens of thousands of derelict objects have been left in orbit around our planet. They are usually referred to as space debris, and their number is growing year by year [1, 2, 3]. A conjunction or close approach between two objects is a potential risk not only to the involved objects as they can be damaged or destroyed in the case of a collision, causing mission termination if one of them is an active satellite, but also to other satellites because of the potential fragmentation cloud formed. Collision and posterior fragmentation has already happened in the past, see for instance the 2009 Iridium-Cosmos collision [4].

There are three main strategies to prevent the growth of space debris and the collisions between objects in orbit:

1. Post-Mission Disposal, which consists on a satellite moving out of its orbit after its mission has ended, either by reentering into the atmosphere or by moving to a cemetery orbit.

2. Active Debris Removal, when technologically possible, will allow to remove derelict objects that pose a menace. This would usually be performed for large objects, because of the amount of smaller debris they may generate and their larger cross-sectional area.

3. Collision Avoidance Maneuvers, routinely performed when the risk of two objects colliding is large enough. They consist in a small impulse (usually of the order of cm/s) to prevent the predicted collision, so one of the objects must still be active and have enough fuel to perform the maneuver.

One of the most crowded regions is the corresponding to Low Earth orbit (LEO), and the continuous growth of the population of objects there has lead to an increase of the conjunctions between active satellites and other bodies. Since a wide range of orbit inclinations are being employed in LEO, the relative velocity of two conjuncting objects is in general very large, of the order of the orbital velocity. It is mandatory to evaluate the risk these conjunctions pose and to design the corresponding collision avoidance maneuvers if necessary. For example, in November 2010 the U.S. military reported an average of 190 conjunctions and three collision avoidance maneuvers per week [2], and in 2011 Envisat reported 4 collision avoidance maneuvers [5].

The main source of conjunctions data is released by the American Joint Space Operation Center (JSPoC). The information is only disclosed to the involved satellite operators as a confidential document called Conjunction Summary Message (CSM) [6]. At the same time, the European Space Agency has developed their own system called CRASS (*Collision*

Risk Assessment Software) based on covariance estimates from TLE [7].

As the number of objects grows each year and the ground-based tracking systems improve, an increasing number of maneuvers are expected to be performed by every satellite in their lifetime. Each collision avoidance maneuver consumes part of the fuel carried onboard, so they should be carefully designed in order to consume as low fuel as possible. It's therefore paramount to devise high-fidelity and high-efficiency optimization strategies to be integrated into dedicated software applications.

Some software tools have been proposed in the past. In 2005 Alfano proposed a MATLAB tool linked to the software program STK for evaluating the effect of a collision avoidance maneuver using parametric studies [8].

Deimos Space developed the CORAM tool for the European Space Agency featuring parametric searches and also gradient-based optimization methods [9].

In 2014 the Space Dynamics Group of the Technical University of Madrid developed a novel tool called *OCCAM (Orbital Computation of Collision Avoidance Maneuvers)* aimed at computing minimum-fuel collision avoidance maneuvers in the short-term encounter scenario (usually applicable in LEO), presented in the present paper. The algorithms embedded in the core of this software allow for fast computation of the desired result, up to the point of being implemented as a web application in the trial version OCCAM lite. OCCAM is useful in the following scenario: *A maneuverable satellite S_1 and a non-maneuverable body S_2 will have a close encounter and may collide. Assuming that the relative velocity is large and that the positions of both objects are known with limited accuracy, what's the involved risk and the optimum collision avoidance maneuver?*.

This article is organized as follows. First, we introduce two useful reference systems in section 2 and [3]. Next, we present the linear dynamics formulation used in OCCAM in section 4, and the different methods implemented for the computation of the collision probability in section 5. We move then to the computationally-fast maneuver optimization method in section 6. Finally we present the tool OCCAM and the online trial version OCCAM lite in section 7 and 8 respectively.

2. ENCOUNTER PLANE

Also called *b-plane*, is a plane perpendicular to the relative velocity of the two conjuncting objects, and contains both of them at the moment of closest approach. To this end, we define the S_2-centered encounter plane reference system $< \xi, \eta, \zeta >$ as:

$$\boldsymbol{u}_\xi = \frac{\boldsymbol{v}_2 \times \boldsymbol{v}_1}{\|\boldsymbol{v}_2 \times \boldsymbol{v}_1\|},\ \boldsymbol{u}_\eta = \frac{\boldsymbol{v}_1 - \boldsymbol{v}_2}{\|\boldsymbol{v}_1 - \boldsymbol{v}_2\|},\ \boldsymbol{u}_\zeta = \boldsymbol{u}_\xi \times \boldsymbol{u}_\eta, \quad (1)$$

where $\boldsymbol{v_i}$ is the velocity vector of the object i.

Then, the encounter plane corresponds to the ξ-ζ plane, and the relative motion occurs on the η direction. Finally, S_1 will cross the encounter plane at the point $\boldsymbol{r_e} = (\xi_e, 0, \zeta_e)^\top$ expressed in the encounter plane reference frame.

3. UVW REFERENCE FRAME

In the CSM the relative position, velocity and covariance matrices for both bodies are expressed using the UVW reference frame, being *U* the radial, *V* the circumferential and *W* the out-of-plane component. This can be expressed mathematically as

$$\boldsymbol{u_U} = \frac{\boldsymbol{r}}{\|\boldsymbol{r}\|},\ \boldsymbol{u_W} = \frac{\boldsymbol{r} \times \boldsymbol{v}}{\|\boldsymbol{r} \times \boldsymbol{v}\|},\ \boldsymbol{u_V} = \boldsymbol{u_W} \times \boldsymbol{u_U}. \quad (2)$$

where $\boldsymbol{r}$ and $\boldsymbol{v}$ are the position and velocity vectors of the corresponding satellite.

In the UVW reference system, the components of an impulsive maneuver $\boldsymbol{\Delta v} = (\Delta v_U, \Delta v_V, \Delta v_W)^\top$ can be expressed as

$$\Delta v_U = \Delta v \cos\gamma \sin(\sigma + \alpha), \quad (3a)$$
$$\Delta v_V = \Delta v \cos\gamma \cos(\sigma + \alpha), \quad (3b)$$
$$\Delta v_W = \Delta v \sin\gamma \quad (3c)$$

where σ is an in-plane rotation of the velocity vector, opposite to the orbit angular momentum direction, and γ is a subsequent out-of-plane rotation towards the W axis. Finally, α is the flight-path angle, this is, the angle between between the V axis and the velocity vector.

The UVW reference system and the maneuver orientation angles are sketched in figure 1.

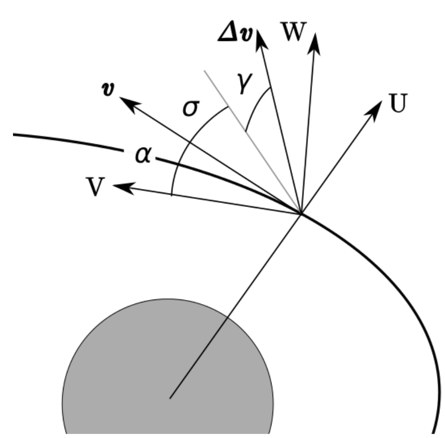

Fig. 1. UVW reference system and maneuver orientation angles.

4. COLLISION AVOIDANCE DYNAMICS

Following references [10, 11] let us suppose that the time of closest approach is predicted when the maneuverable satellite S_1 has orbital true anomaly θ_c, semimajor axis a_0 and eccentricity e_0. Let the velocity of S_2 at that epoch be related to the velocity of S_1 by a in-plane rotation, opposite to the orbit angular momentum direction, of angle $-\pi < \phi < \pi$ around the S_1 orbital plane normal $\boldsymbol{u_{h1}}$

$$\phi = \text{atan2}\left[(\boldsymbol{v_1} \times \boldsymbol{v_2}) \cdot \boldsymbol{u_{h1}}, \boldsymbol{v_1} \cdot \boldsymbol{v_2}\right] \tag{4}$$

followed by an out-of-plane rotation $-\pi/2 < \psi < \pi/2$ in the direction approaching $\boldsymbol{u_{h1}}$:

$$\psi = \tan^{-1}\left[\frac{(\boldsymbol{v_2} \cdot \boldsymbol{u_{h1}}) \left\|\boldsymbol{v_2} \times \boldsymbol{u_{h1}}\right\|}{v_2^2 - (\boldsymbol{v_2} \cdot \boldsymbol{u_{h1}})^2}\right] \tag{5}$$

and by rescaling its magnitude v_1 by a factor $\chi = v_2/v_1$.

These transformations are illustrated in figure 2.

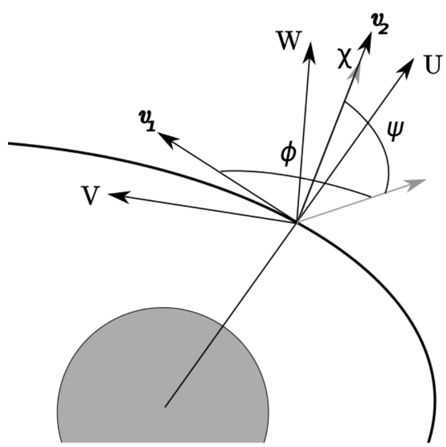

Fig. 2. Transformation from $\boldsymbol{v_1}$ to $\boldsymbol{v_2}$.

When S_1 performs an impulsive maneuver $\boldsymbol{\Delta v}$ expressed in the UVW reference system, the resulting relative position variation in the encounter plane reference system $\boldsymbol{r} = (\xi, \eta, \zeta)^\top$ obeys the linear relation

$$\boldsymbol{r} = \boldsymbol{r_e} + M\boldsymbol{\Delta v}, \tag{6}$$

where $\boldsymbol{r_e} = (\xi_e,\ 0, \zeta_e)^\top$ is the miss distance without maneuver. The $\boldsymbol{M}$ matrix is defined as in [10, 11] as a function of ϕ, ψ, χ, e_0, θ_c, θ_m and $\sqrt{\frac{a_0^3}{\mu}}$, where μ is the Earth's gravitational constant and θ_m is the true anomaly of S_1 at maneuver. The angular distance between the maneuver and the conjunction will be then $\Delta\theta = \theta_c - \theta_m$.

The M matrix can be expressed as the product of a rotation matrix R, a kinematics matrix K and a dynamics matrix D:

$$M = R\,K\,D. \tag{7}$$

The components of these matrices are carefully derived in [10, 11]. Among them, the D matrix is particularly important as it provides a linear relationship between $\boldsymbol{\Delta v}$ and the radial error and time delay at θ_c. In essence this matrix is an error state transition matrix, and compact expressions for its components were recently presented in [12].

5. COLLISION PROBABILITY CALCULATION

OCCAM can calculate the collision probability using many different possible methods in the short-term encounter scenario. This scenario is characterized by a very fast relative velocity and a small encounter characteristic time, which makes it possible to approximate the relative motion as rectilinear and consider the probability density functions (pdf) that describe the position of each body as frozen. Additionally, the bodies are replaced by their spherical envelopes and their center of masses are assumed to follow uncorrelated Gaussian distributions. This last point leads to consider the relative position covariance matrix as the sum of the position covariance matrices of each object.

With these hypothesis, the collision probability can be calculated as the three-dimensional integral of the relative position probability density distribution over the volume $\mathcal{V}$ swept by the sphere of radius s_A, with $s_A = s_1 + s_2$, the sum of the radius of each object.

$$P = \iiint_{\mathcal{V}} f_3\left(\boldsymbol{r}\right) \mathrm{d}\boldsymbol{r}. \tag{8}$$

Because of the fast rectilinear relative motion, this volume is a cylinder and it's possible to integrate analytically on the cylinder axis direction, reducing the problem to a two-dimensional integral over a circle $\mathcal{A}$ of radius s_A centered at $\boldsymbol{r_e}$ of the marginal pdf of two independent random variables in the encounter plane, both for the Gaussian and the non-Gaussian case [13].

$$P = \iint_{\mathcal{A}} f_2\left(\boldsymbol{r}\right) \mathrm{d}\boldsymbol{r}. \tag{9}$$

In the case of a Gaussian, the marginal pdf is also a Gaussian.

All the methods integrated in OCCAM solve Eq. (9) using different approaches. Ordered as in [13] from the fastest to the slowest method:

- García-Pelayo & Hernando-Ayuso method was recently presented as an analytical power series that solves exact and analytically Eq. (9) for any given pdf [13]. The method hinges on an expansion of the pdf as power series which lead to close expressions that involve Hermite Polynomials in the Gaussian case. Two terms are sufficient to achieve an acceptable accuracy for practical cases, making it $10\,\%$ faster than the next method.

- Serra's method is the other only completely analytical method for the computation of the collision probability. Serra et al derived an analytical power series premultiplied by an exponential function using Laplace transform and D-finite function properties [14]. The main advantage of this approach with respect of the García-Pelayo & Hernando-Ayuso method is that the pre-multiplier guarantees that all the terms in Serra's power series are positive, which means that upper and lower bounds for the collision probability may be easily obtained, and that are no oscillations between large positive and negative terms leading to loss of accuracy. However this oscillatory phenomena is not observed on practical cases.

- Chan's method is carefully presented in [15]. After deriving an exact power series for the isotropic case (this is, when the semiaxes of the covariance ellipse are equal), Chan approximated the general case replacing the circular integration region by an ellipse of equal area. This is referred to as Equivalent Area Approximation, and makes this method very accurate when the uncertainty is small compared to the sizes of the bodies, which is generally true in real missions. For large uncertainty or large anisotropy cases this method may be inaccurate, even if more terms in Chan's power series are considered. This method also implies that the collision probability is constant when the relative position lays on any point of same the error ellipse it belongs to, and decreases for larger error ellipses.

- Alfano's method integrates analytically once in the direction of one of the semiaxes of the uncertainty ellipse, then discretizes the integral in the other direction and writes it as a summation of terms [8]. In the process one of the terms had to be heuristically adjusted to correct an error tendency of the discretization method used, so it cannot be called an analytic method. However, it stands as a robust and accurate method slightly slower than the previous methods.

- Patera's method performs a first analytical radial integral, followed by a numerical radial integration [16]. It may have numerical problems when the nominal minimum relative distance is the same as s_A, since Eq. (9) becomes an improper integral that converges: a fraction in the integrand goes to $0/0$ but the integral exists and takes a finite value. As an advantage, Patera's method can be used to integrate not only over circles, but over generic shapes as well.

- Foster's method follows from a work of Foster and Estes [17] and consists of a numeric integration using polar coordinates with fixed step size. As the numerical integration is performed on a two dimensional domain, this is the slowest of the methods included in OCCAM. For most practical applications the step size is larger than required for a direct integration with a reasonable error tolerance (making it slower than an adaptive numeric integration method), but may lack accuracy for cases not usually found in real missions.

6. MANEUVER OPTIMIZATION

Different optimization strategies are possible in OCCAM: maximum miss distance or minimum collision probability for a fixed Δv magnitude, or minimum Δv to satisfy a given collision probability [11]. OCCAM uses a very fast optimization method based on reducing the optimization problem to a Quadratic Constraint Quadratic Program (QCQP), which can be solved in a fairly easy way compared to other optimization problems [18]. This is achieved by using two different relaxations.

On one hand, the linearization presented in section 4 leads to transforming quadratic forms of the relative position to quadratic forms of the applied impulse $\boldsymbol{\Delta v}$.

Then, an additional relaxation is introduced for the collision probability as suggested by Chan's method: the collision probability is (approximately) constant when the relative position lays on any point of same the error ellipse it belongs to and decreases for larger error ellipses. This is a good approximation when the uncertainty is small compared to the size of the bodies, this is, $\frac{s_A^2}{\det(C_{\xi\zeta})} \ll 1$. Note that the determinant of the covariance matrix in the encounter plane is simply the product of the two eigenvalues of that matrix. The condition for the miss distance to lay on an error ellipse can be expressed mathematically as a constant value of squared distance using the inverse of the covariance matrix as metric tensor, and corresponds to the v parameter used by Chan in his method [15] or the Depth of intrusion as defined by Lázaro et al [19] as *"Scale factor to be applied to the debris covariance ellipsoid in order to have the spacecraft trajectory tangent to it"*.

For the maximum miss distance case, the squared miss distance is used as objective function since it's a quadratic function of the encounter plane position:

$$J_r = \xi^2 + \zeta^2 = \boldsymbol{r}^\top Q \boldsymbol{r} \tag{10}$$

with

$$\mathbf{Q} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{11}$$

The probability minimization case after relaxation has the following objective function

$$J_p = \boldsymbol{r}^\top Q^* \boldsymbol{r} \tag{12}$$

with $Q^* = QC_{\xi\eta\zeta}^{-1}Q$. Comparing Eq. (10) and Eq. (12), we find that both have the same mathematical structure, a semidefinite positive quadratic form. Consequently if a method can optimize one of the objective functions, it will be able to optimize the other, so we will present from here onwards the maximum miss distance case.

Introducing the linear dynamics relation from Eq. (6) into the objective function to be maximized, Eq. (10), it reads

$$\begin{aligned} J_r &= (\boldsymbol{r_e} + M\boldsymbol{\Delta v})^\top Q\,(\boldsymbol{r_e} + M\boldsymbol{\Delta v}) \\ &= \boldsymbol{r_e}^\top Q\boldsymbol{r_e} + \boldsymbol{\Delta v}^\top A\boldsymbol{\Delta v} + 2\boldsymbol{r_e}^\top QM\boldsymbol{\Delta v} \end{aligned} \tag{13}$$

where $A = M^\top QM$.

As a constraint, the magnitude of the applied impulse must verify $\|\boldsymbol{\Delta v}\| \leq \Delta v_0$. It's convenient to express this constraint as a quadratic function:

$$f(\boldsymbol{\Delta v}) = \boldsymbol{\Delta v}^\top\boldsymbol{\Delta v} - \Delta v_0^2 \leq 0. \tag{14}$$

After dropping the constant term $\boldsymbol{r_e}^\top Q\boldsymbol{r_e}$ from Eq. (13) and multiplying it by Δv_0^2, the optimization problem can be be conveniently written in a standard form:

$$\begin{aligned} &\text{maximize} \quad \tilde{J}_r(\boldsymbol{u}) = \boldsymbol{u}^\top A\boldsymbol{u} + 2\boldsymbol{b}^\top\boldsymbol{u} \\ &\text{subject to} \quad f(\boldsymbol{u}) = \boldsymbol{u}^\top\boldsymbol{u} - 1 \leq 0 \end{aligned} \tag{15}$$

where we set

$$\boldsymbol{u} = \boldsymbol{\Delta v}/\Delta v_0, \quad \boldsymbol{b}^\top = \boldsymbol{r_e}^\top QM/\Delta v_0. \tag{16}$$

This is a non-convex quadratic optimization problem, which can be reduced to the following convex problem [18, p. 229]

$$\begin{aligned} &\text{minimize} \quad \frac{\left(\boldsymbol{s_1}^\top\boldsymbol{b}\right)^2}{\lambda - \lambda_1} + \frac{\left(\boldsymbol{s_2}^\top\boldsymbol{b}\right)^2}{\lambda - \lambda_2} + \lambda \\ &\text{subject to} \quad \lambda \geq \lambda_1 \end{aligned} \tag{17}$$

where λ_1, λ_2 are the two nonzero eigenvalues of A in descending order, and $\boldsymbol{s_1}$, $\boldsymbol{s_2}$ are the corresponding eigenvectors.

Equation (17) leads to the condition

$$\begin{cases} \left(\dfrac{\boldsymbol{s_1}^\top\boldsymbol{b}}{\lambda - \lambda_1}\right)^2 + \left(\dfrac{\boldsymbol{s_2}^\top\boldsymbol{b}}{\lambda - \lambda_2}\right)^2 - 1 = 0 \\ \lambda \geq \lambda_1 \end{cases}, \tag{18}$$

which can be easily solved with Newton's method for example, providing λ_{opt}.

Once λ_{opt} has been determined, the corresponding $\boldsymbol{\Delta v}$ can be obtained as ([18, p. 229])

$$\boldsymbol{\Delta v_{opt}} = -\Delta v_0\,(A - \lambda_{opt}I)^\dagger\,\boldsymbol{b} \tag{19}$$

where the dagger sign represents the pseudoinversion matrix operation. Finally, the miss distance coordinates are obtained by substituting Eq. (19) into Eq. (6).

7. OCCAM

All the previous methods and algorithms were integrated into the software tool OCCAM (Optimal Computation of Collision Avoidance Maneuvers). This novel software tool aimed at computing minimum-fuel collision avoidance maneuvers in the short-term encounter scenario was developed by the Space Dynamics Group of the Technical University of Madrid (UPM). It employs the advanced modeling and optimization techniques presented in this paper, which make it an extremely fast and robust design tool. OCCAM features an extensive set of input parameters, different optimization strategies and output options to provide a high design flexibility for the user. The different methods of collision probability computation explained in section 5 are supported. Its user-friendly graphical interface and intuitive design logic make it really straightforward to master even for non-experts, and it can be employed either as a standalone tool or in conjunction with other satellite operation planning frameworks. It's possible to run OCCAM on computers Windows or Linux.

OCCAM was submitted to the Copyright Registry Office of Madrid soon after the completion of its first version in 2014 (record entry 16/2014/8395). Later that year it was presented to Innovatech, a technology commercialization seminar organized by the UPM. It was positively evaluated by a panel of experts from the industry and the University, and a commercial sheet was included in the UPM portfolio of technologies [20]. Despite the great performance of the tool, the characteristics of the field limit the number of potential users and suggests a personalized approach. A customized collaboration could be proposed on a per-partner basis, adapting the tool to their existing frameworks.

7.1. Input

OCCAM user interface (see figure 3) shows at once all the input options available to the user. In the first place, the user can choose between three different optimization strategies; Maximum miss distance for fixed Δv, minimum collision probability for fixed Δv, and minimum Δv for a required collision probability.

Additionally, the user can choose if OCCAM should prefer prograde or retrograde maneuvers in case of multiple solutions, and the number of orbits ahead the conjunction where the solution will be calculated.

Next, the user can choose one method for calculating the collision probability among the ones presented in this paper.

The conjunction parameters relate the velocity of S_1 and S_2 as in section 4, and the coordinates in the encounter plane before maneuver are also to be given.

After that the user should input the orbit information for S_1.

Finally, the user must set the covariance matrix in the UVW reference frame and the spherical envelope radius for

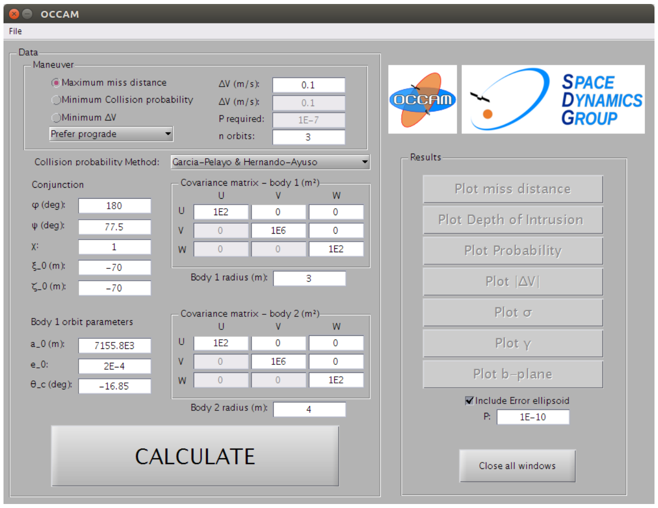

Fig. 3. OCCAM user interface.

each body, and press the *Calculate* button.

The user can save the scenario he has input as a text file and load it at a later time. This feature also allows inputing the data automatically from an external program.

OCCAM workflow is shown on the diagram in figure 4.

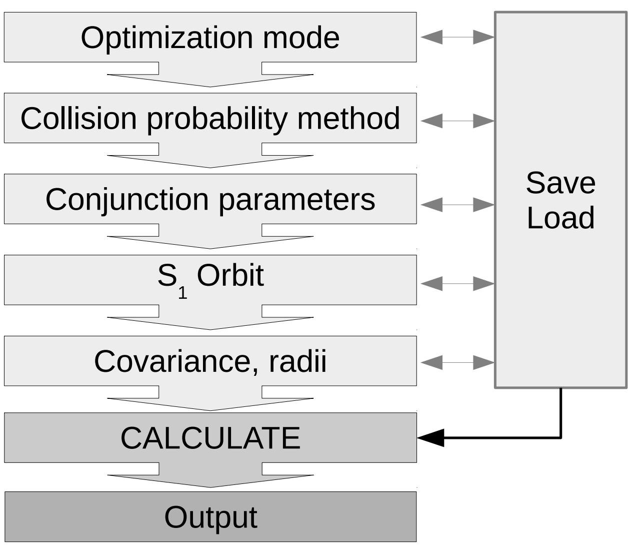

Fig. 4. OCCAM input workflow.

7.2. Output

The output in form of plots can be easily obtained with the buttons on the right part of the interface (see figure 3).

Some of them are related to the quantity to be optimized (like the miss distance, Depth of Intrusion or Probability, or the Δv), while other give the user important information like

(a) (b) (c)

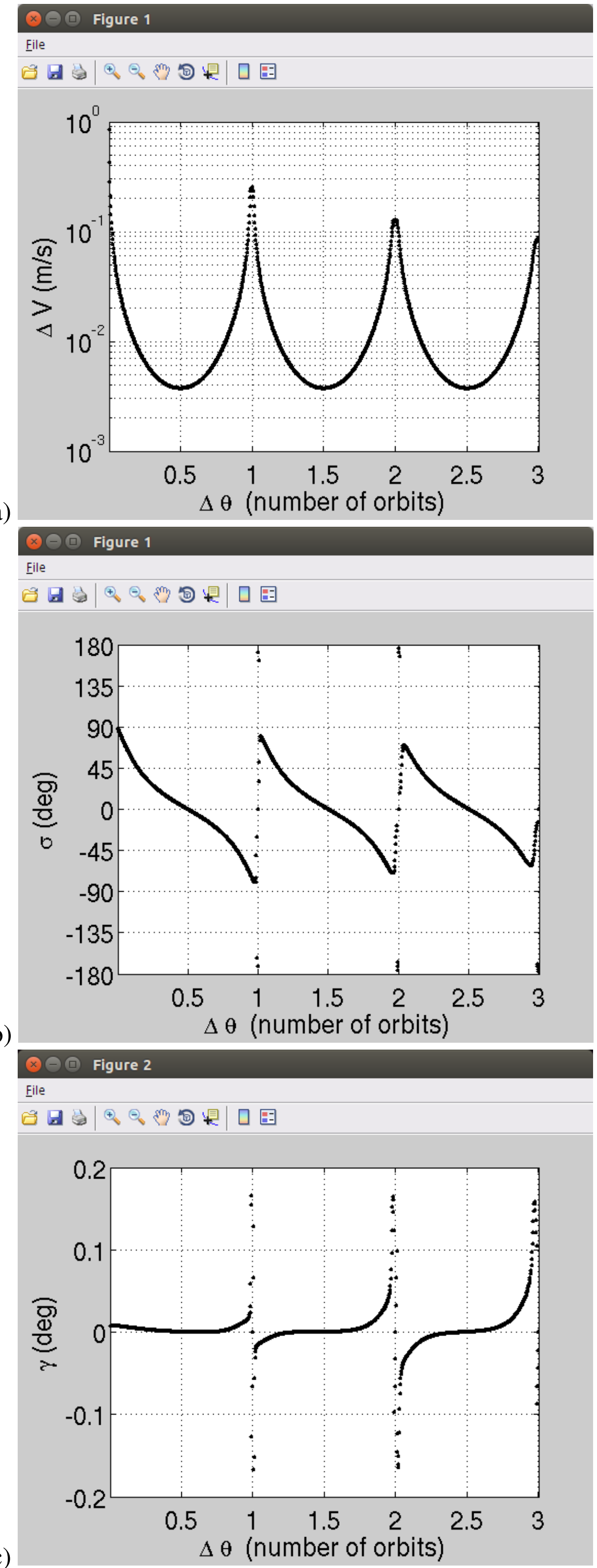

Fig. 5. OCCAM output example. (a) is the required Δv, (b) and (c) are the in-plane and out-of-plane maneuver angles.

the maneuver orientation angles (in-plane σ and out-of-plane γ) or how the intersection of the relative motion and the encounter plane shifts after applying the maneuver for different anticipation times $\Delta\theta$.

An example of example outputs can be seen on figure 5, where the default scenario in OCCAM was used and the optimal maneuvers for minimum Δv with $p_{required} = 10^{-10}$ were calculated.

8. OCCAM LITE

Some of the algorithms used in OCCAM have been also implemented in an open web application called OCCAM lite. It's available online in the Space Dynamics Group webpage `http://sdg.aero.upm.es/index.php/online-apps/occam-lite`.

As OCCAM lite is an on-line trial version for the interested potential user, only some of the capabilities of OCCAM are available. The user cannot select a non-zero miss distance before maneuver, the covariance matrices are assumed to be diagonal, and only miss distance maximization is offered to the user.

OCCAM lite can run not only on typical workstations, but also on mobile devices such as tablets and smartphones, thanks to the fast algorithms presented in this paper. This is a proof of the outstanding velocity of OCCAM.

The user interface is presented in figure 6, and an example of some of the available results are plotted in figure 7.

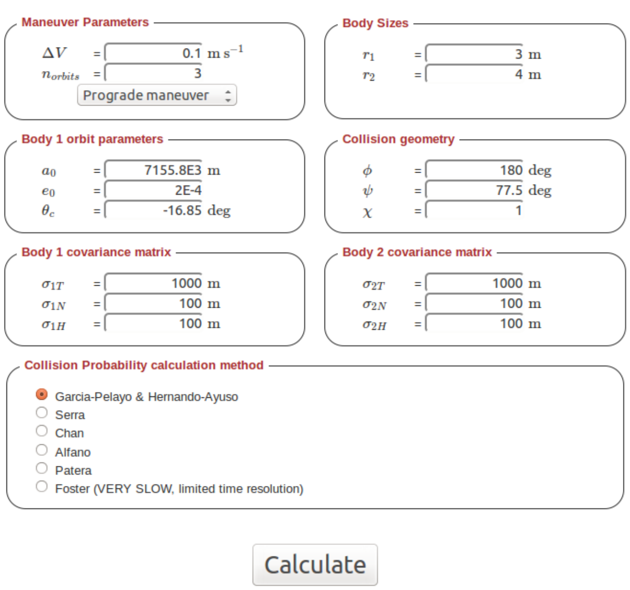

Fig. 6. OCCAM lite user interface.

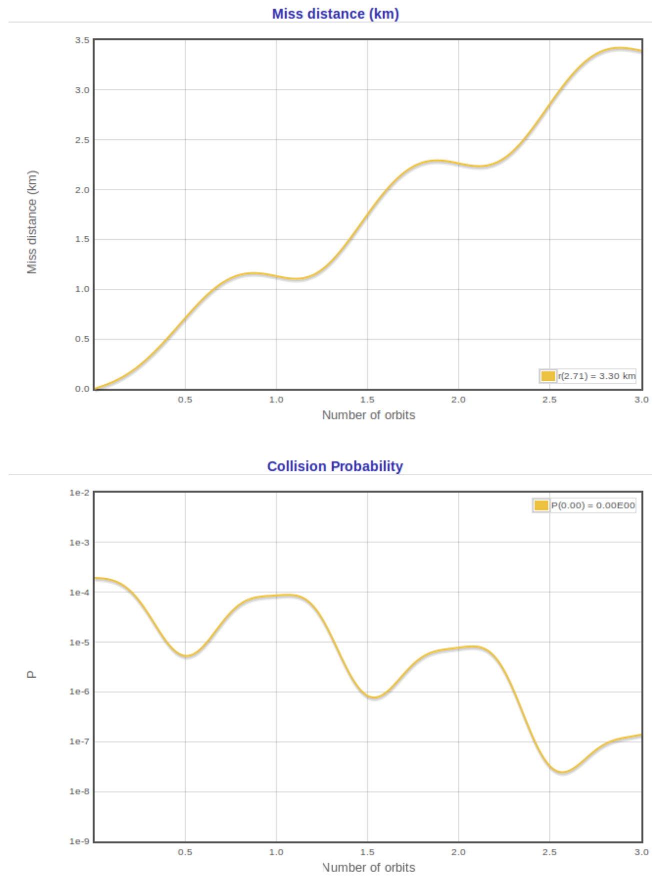

Fig. 7. OCCAM lite results example (default scenario).

9. CONCLUSIONS

OCCAM is novel software tool aimed at computing minimum-fuel collision avoidance maneuvers in the short-term encounter scenario was developed by the Space Dynamics Group of the Technical University of Madrid. It employs the advanced modeling and optimization techniques presented in this paper, which make it an extremely fast and robust design tool.

In an increasingly complex operational scenario, OCCAM does what other collision avoidance planning tools do but in a fraction of their computation time, making it a fast and reliable design and planning tool for the space operators seeking to minimize the cost of their collision avoidance maneuvers.

Some of the algorithms used in OCCAM have been also implemented in an open web application called OCCAM lite.

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