A fast and efficient algorithm for onboard LEO intermediary propagation*

Denis Hautesserres[‡] and Martin Lara[†]

(‡) *Centres de Competence Tech., CNES, Toulouse, France*

(†) *Space Dynamics Group, UPM, Madrid, Spain &*
GRUCACI – Univ. La Rioja, Logroño, Spain

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Background: Onboard propagation

- Common onboard orbit propagators
  - provide satellite navigation solution for short time spans
  - for instance: in case of GPS outage (minutes . . .)
- Accumulation of high-order effects is insignificant
  - main perturbation affecting operational LEO sats. is J_2
  - motion in this potential field known as *main problem*
- Hardware and software improvements:
  - numerical solution very fast and accurate
- Analytical approach: no need of integrating ODEs
  - "instant" but approximate solution
- Main problem is non-integrable $\longrightarrow$ intermediaries

- Intermediaries: **integrable** problems
  - comprise the bulk of the original main problem dynamics
  - solution limited in precision to 1st order effects of J_2
    - 7-8 meters in the ii.cc. of LEO
  - intermediary competitive vs. Runge-Kutta integration
    - Gurfil and Lara, Celest. Mech. Dyn. Astr. 2014
- Intermediary approach: no need of integration ODEs
  - less power consumption
    - important if power constraints, like Cubesats
  - more versatile: no need of step by step evaluation
  - less precise, but shares the same statistics as R-K
    - ii.cc. are known onboard within some uncertainty

Outline

- Perturbed Keplerian motion: Geopotential
- Main problem Hamiltonian
  - truncations in polar-nodal variables
  - *radial* and *zonal* intermediaries (common and natural)
- Deprit's radial, natural intermediary
  - 2nd order improvements
  - J_3 and J_4 effects
- Examples and comparisons
- Conclusions and future work

Perturbed Keplerian motion

- $V = -(\mu/r)\,[1 + \epsilon\, D(\boldsymbol{r}, \dot{\boldsymbol{r}}, t)]$, $\epsilon \ll 1$, $\mu = GM_\oplus$
  - the disturbing function D usually makes V non-integrable
  - expected solution: slightly distorted Keplerian orbit
- Geopotential: usual expansion in spherical harmonics
  - Earth case: $J_2 = -C_{2,0} = \mathcal{O}(10^{-3})$, $C_{n,m} = \mathcal{O}(J_2^2)$
- Main problem: $V = -(\mu/r)\left[1 - J_2\,(R_\oplus/r)^2\; P_{2,0}(\sin\varphi)\right]$
  - quite representative of the real dynamics, non-integrable
  - averaged dynamics: secular efects
    - precess. ellipse, regression/advance of the perigee
    - also, modification of the mean motion
  - non-integrable: solution by numerical integration
  - alternative: approximate analytical *intermediary* solutions

Truncations of the main problem

- Hamiltonian formalism ($\sim$ energy, canonical variables)
  - state scalar function from which we derive the e.o.m.
  - polar variables $(r, \theta, \nu, R = \dot{r}, \Theta = r^2\dot{\theta}, N = \Theta \cos i)$
- Expand the Legendre polynomial: $\sin\varphi = \sin i \sin\theta$

$\mathcal{H} = (1/2)(R^2 + \Theta^2/r^2) - \mu/r$	Kepler
$-(1/2)(\mu/r)\,(R_\oplus/r)^2\,J_2$	Main equatorial
$+(3/4)(\mu/r)\,(R_\oplus/r)^2\,J_2 \sin^2 I$	Cid intermediary
$-(3/4)(\mu/r)\,(R_\oplus/r)^2\,J_2 \sin^2 I \cos 2\theta$	Full problem

- Radial Hamiltonians: $(r, \theta, \nu, R, \Theta, N) \longrightarrow (r, -, -, R, \Theta, N)$
  - $\mathcal{H}(r, R; \Theta, N)$ 1-DOF $\Rightarrow$ integrable
- 3 different intermediaries of the main problem

- Good and bad radial intermediaries (classical definition)
  - Keplerian: $\mathcal{K} = \frac{1}{2}\left(R^2 + \frac{\Theta^2}{r^2}\right) - \frac{\mu}{r}$, $\quad \frac{1}{2\pi}\int_0^{2\pi}(\mathcal{H}-\mathcal{K})\mathrm{d}M \neq 0$
  - Equatorial main problem $\mathcal{E} = \mathcal{K} + \mathcal{P}_Q$

    $\mathcal{E} = \mathcal{K} - \frac{\mu}{2r}\frac{R_\oplus^2}{r^2}J_2 \quad \frac{1}{2\pi}\int_0^{2\pi}(\mathcal{H}-\mathcal{E})\mathrm{d}M \neq 0$
  - Cid-Lahulla intermediary $\mathcal{C} = \mathcal{K} + \mathcal{P}_Q + \mathcal{P}_R$,

    $$\mathcal{C} = \mathcal{K} - \frac{\mu}{2r}\frac{R_\oplus^2}{r^2}J_2\left(1 - \frac{3}{2}\sin^2 I\right) \qquad \frac{1}{2\pi}\int_0^{2\pi}(\mathcal{H}-\mathcal{C})\mathrm{d}M = 0$$
- Cid-Lahulla: paradigm of *common* intermediaries
  - $\langle\mathcal{C}\rangle \equiv \langle\mathcal{H}\rangle$ same secular rates as the main problem
  - actual orbit: short-period oscillations about Cid's orbit
    - ∗ except for effects of the 2nd order of J_2
  - analytical solution in elliptic integrals

Zonal intermediaries

- Add & subtract $\mathcal{A}$ to the main problem $\mathcal{H}$
  - reorganize: $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_1$ $\qquad (s \equiv \sin I,\ c \equiv \cos I)$

$$\mathcal{H}_0 = \frac{1}{2}\left(R^2 + \frac{\Theta^2}{r^2}\right) - \frac{\mu}{r} - \frac{\mu}{2p}\frac{R_\oplus^2}{r^2} J_2 \left(1 - \frac{3}{2}s^2 + \frac{3}{2}s^2 \cos 2\theta\right)$$

$$\mathcal{H}_1 = -\frac{\mu}{2p}\left(\frac{p}{r} - 1\right)\frac{R_\oplus^2}{r^2} J_2 \left(1 - \frac{3}{2}s^2 + \frac{3}{2}s^2 \cos 2\theta\right); \qquad \frac{p}{r} - 1 = e \cos f$$

  - $\mathcal{H}_0$ integrable (elliptic integrals); $\langle \mathcal{H}_1 \rangle = 0$
    - (Aksnes 1965, Astrophisica Norvegica)
  - low e: $e \sim \mathcal{O}(J_2) \Rightarrow \mathcal{H}_1 = \mathcal{O}(J_2^2)$
- Sterne 1957 AJ, Garfinkel 1958 AJ, . . . , Oberti 2005 A&A
- Vinti 1959 JR-NBS, Aksenov et al. 1961 P&SS:
  - accurate up to (some) 2nd order effects of J_2

Natural intermediaries

- Integrable after a contact transform. (Deprit 1981 CeMDA)
  - $(r, \theta, \nu, R, \Theta, N) \longrightarrow (r', \theta', \nu', R', \Theta', N')$
  - accurate up to $\mathcal{O}(J_2)$ secular and periodic effects
- Most of the *common* intermediaries can be naturalized
  - Cid-Lahulla, Aksnes, . . . : solution in elliptic integrals
- Deprit's radial intermediary:

$$\mathcal{H} = \frac{1}{2}\left(R'^2 + \frac{\tilde{\Theta}^2}{r'^2}\right) - \frac{\mu}{r'}, \qquad \tilde{\Theta} = \Theta' \sqrt{1 + J_2 \frac{R_\oplus^2}{p'^2}\left(\frac{1}{2} - \frac{3}{2}c'^2\right)}$$

  - quasi-Keplerian system with variable angular momentum
  - solution in trigonometric functions
  - very simple periodic corrections

$$\xi - \xi' = -(1/2) J_2 (R_\oplus/p)^2 \Delta\xi, \qquad \xi \in (r, \theta, \nu, R, \Theta, N),$$

$$\Delta N = 0$$

$$\Delta r = (1/2) p \left(1 - 3s^2 - s^2 \cos 2\theta\right)$$

$$\Delta\theta = (pR/\Theta) \left[1 - 6c^2 + (1 - 2c^2) \cos 2\theta\right] + (1/4) \left[3 - 5c^2 - 4(1 - 3c^2)(p/r)\right] \sin 2\theta$$

$$\Delta\nu = (1/2)\, c\, [(pR/\Theta)(6 + 2\cos 2\theta) + (1 - 4p/r) \sin 2\theta]$$

$$\Delta R = (p\,\Theta/r^2)\, s^2 \sin 2\theta$$

$$\Delta\Theta = (1/2)\Theta\, s^2\, [(1 - 4p/r) \cos 2\theta - (p\,R/\Theta) \sin 2\theta]$$

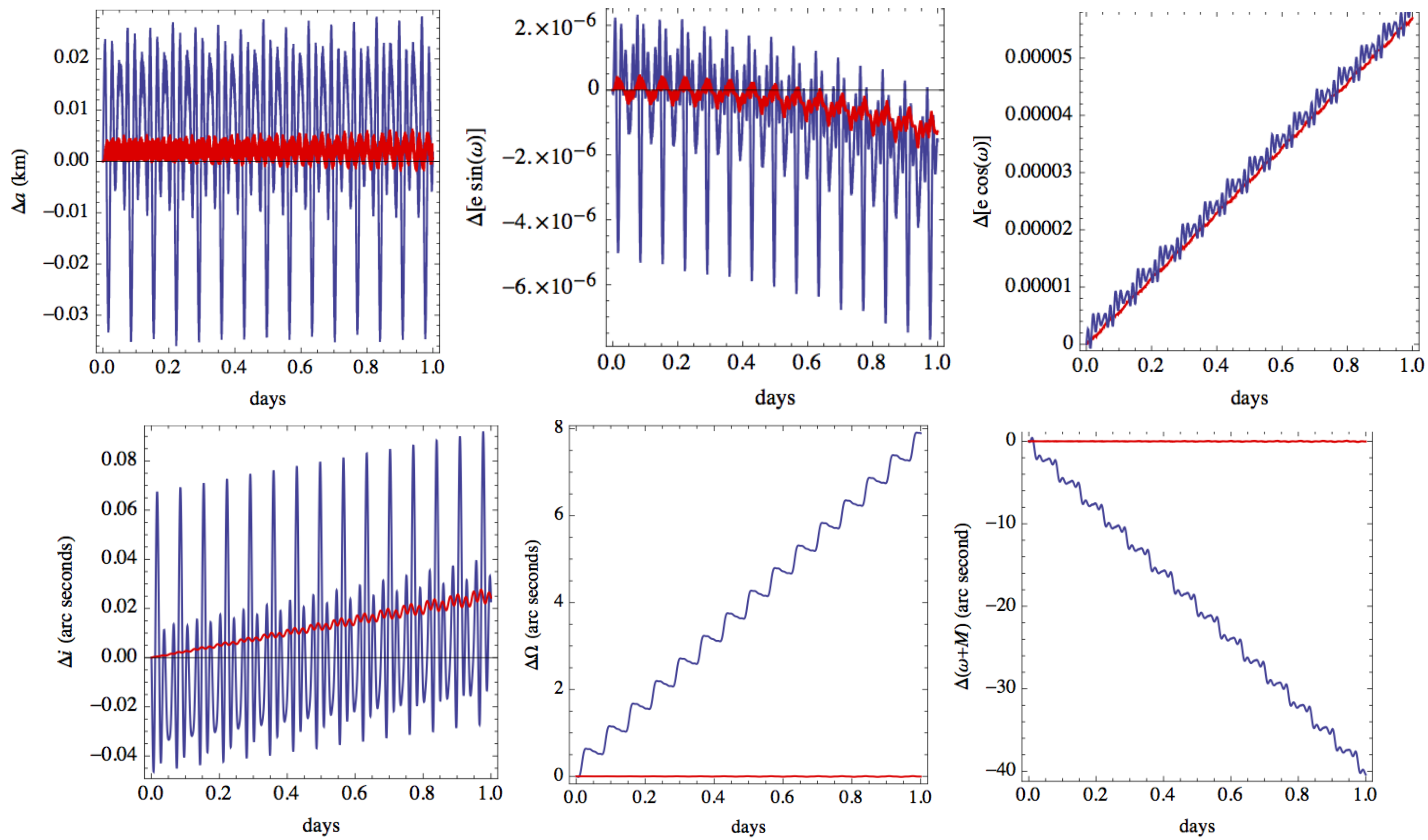
DRI: 2nd order improvements

- 2nd order transform. $(r, \theta, \nu, R, \Theta, N) \longrightarrow (r', \theta', \nu', R', \Theta', N')$
  - computed by "elimination of the parallax"
- New Hamiltonian term
  - $H_{0,2} = \frac{1}{4} J_2^2 \frac{\Theta^2}{r^2} \left(\Phi_1 + \frac{J_3}{J_2^2} \Phi_2 + \frac{J_4}{J_2^2} \Phi_3 \right)$
  - $\Phi_m = \Phi_m(p, e, i, \omega)$
  - $p = p(\Theta)$, $e \equiv e(r, -, R, \Theta)$, $i \equiv i(\Theta, N)$, $\omega \equiv \omega(r, \theta, R, \Theta)$
- No longer integrable, but $H_{0,2} = \Psi(r, -, R, \Theta) + \mathcal{O}(e J_2^2)$
- e small: $H_{0,2} \approx \Psi(r, -, R, \Theta)$
  - again radial (integrable) and quasi-Keplerian!!
  - limited to low e ... most common case in LEO
  - new periodic corrections more involved, yet manageable

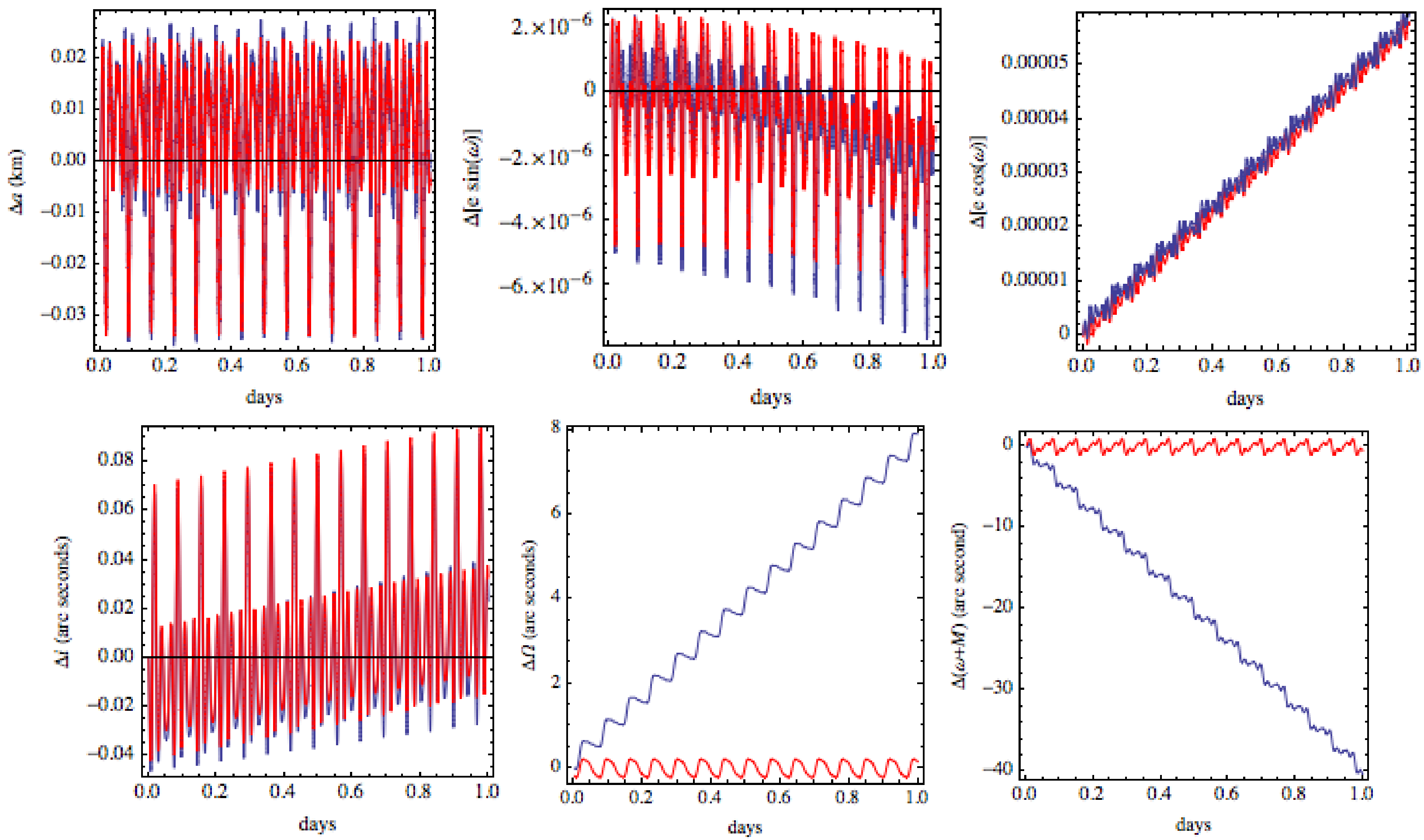
LEO Performance: examples

- Spot-type satellite:
  - $a = 7081.139$ km, $e = 0.0158$, $i = 98^\circ$,
  - $\Omega = 164.02^\circ$, $\omega = M = 0$
- Test cases for **one day**:
  - Numerical integration of the J_2–J_4 problem
  - Numerical integration of the J_2 (main) problem
  - Quasi-Keplerian intermediary in mean elements + . . .
    - full 2nd order inverse & direct transformation eqs.
    - simplified, 2nd order inverse + 1st order direct eqs.

- J_2–J_4 model vs.: J_2-numerical and intermediary (full)



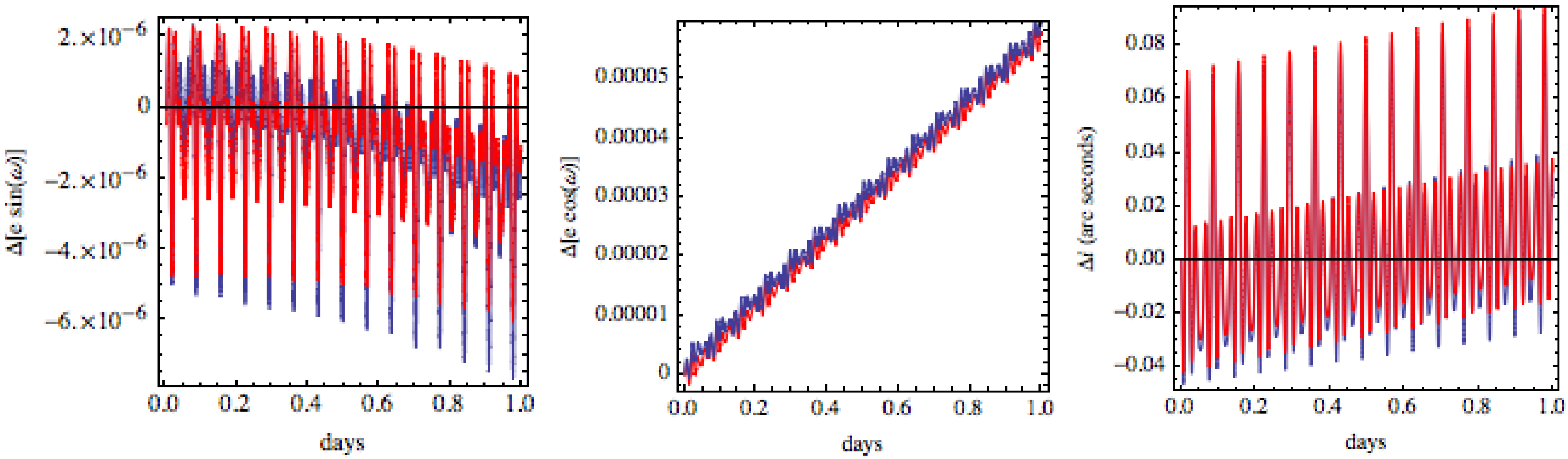
- J_2–J_4 vs. J_2 numeric & intermediary (simp.): 3 times faster



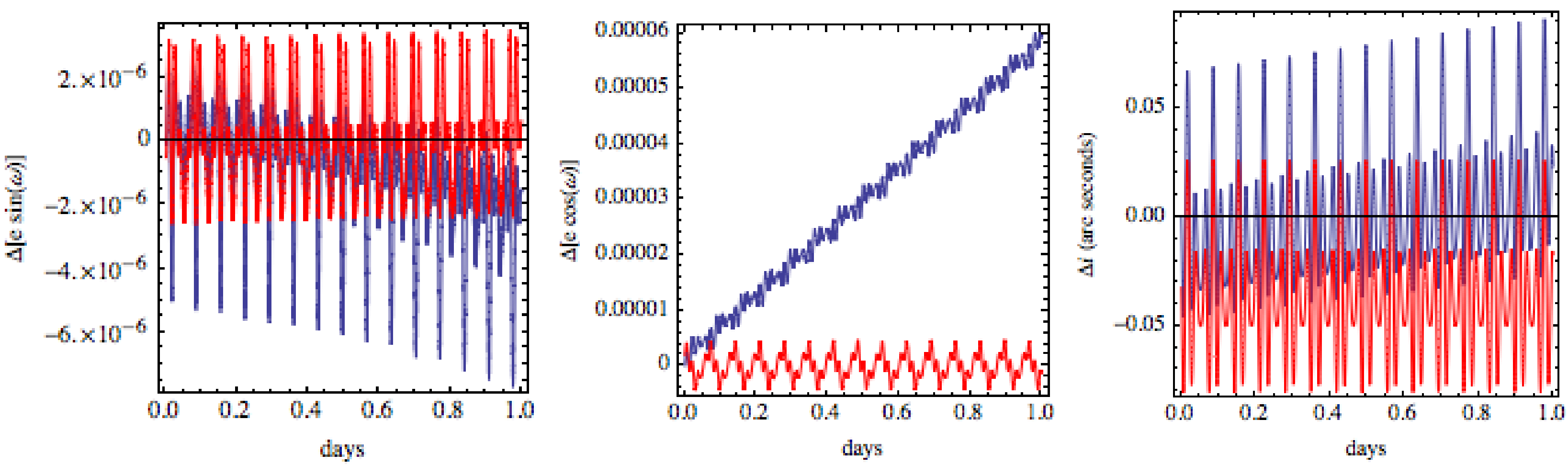
J_3 long-period effects

- Clearly noted since the beginning of the propagation
  - perigee dynamics, e, i dynamics
  - intermediary only deals with J_3 short-period effects
- New canonical transformation contrary to truncation
  - Alfriend & Coffey's *elimination of the perigee* (1984)
  - extremely simple formulas for the case of LEO
- Sequence:
  - short-period inverse corrections (J_2, J_2^2, J_3, J_4)
  - long-period inverse corrections (J_3)
  - quasi-Keplerian intermediary evaluation (J_2, J_2^2 and J_4)
  - long-period direct corrections (J_3)
  - short-period inverse corrections (J_2)

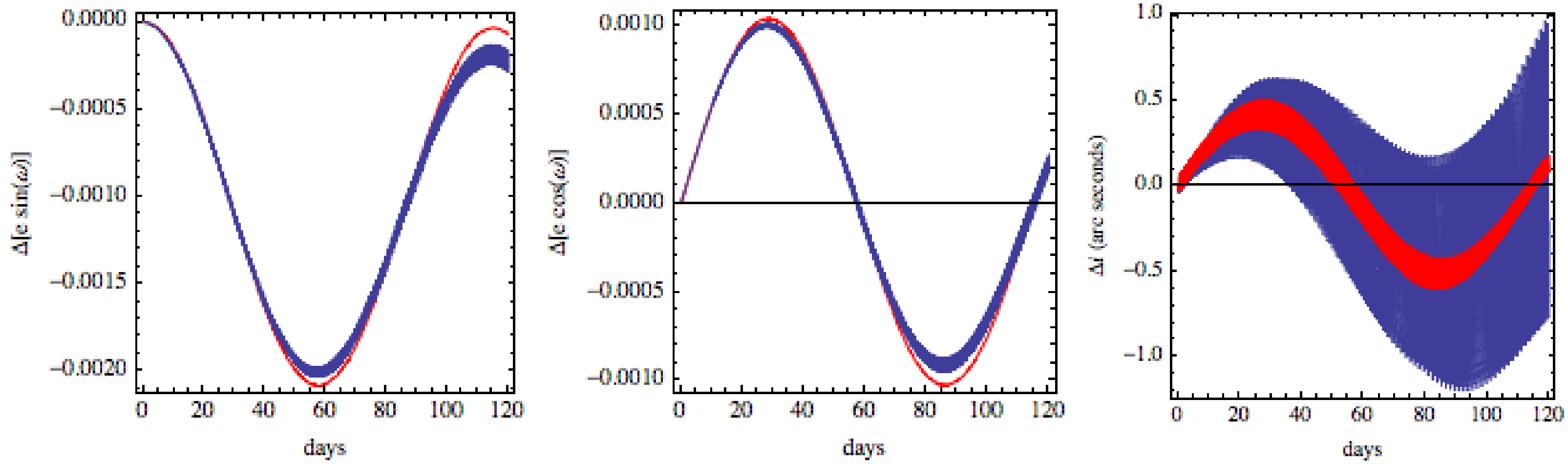
- without long-period corrections



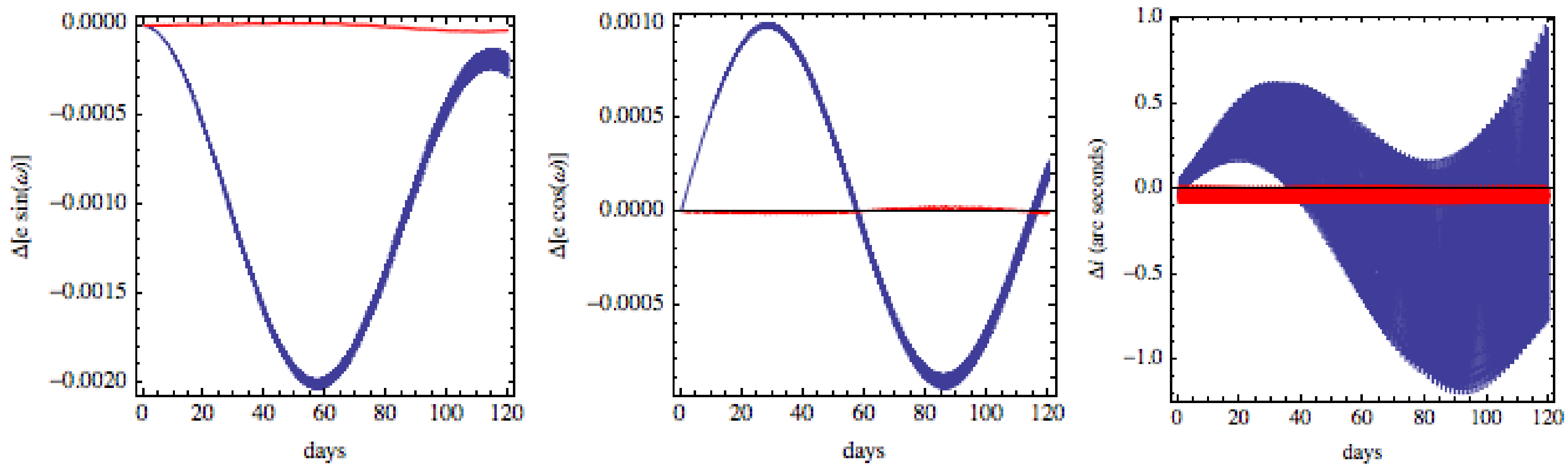
- with J_3 long-period corrections: observable improvements



- 4 months without long-period corrections



- 4 months with J_3 long-period corrections



Conclusions

- Higher order geopotential: improves propagation of LEOs
  - penalizes Cowell integration in terms of computing time
- Increase in computational burden $\longrightarrow$ power consumption
  - can be radically alleviated for the lower eccentricity orbits
  - *intermediary* solution, within a reasonable accuracy.
    - neglect terms $\mathcal{O}(e^2 J_2^2)$ of the perigee dynamics
- Our intermediary: higher order secular and periodic effects
  - compact form of straightforward evaluation (polar vars.)
  - useful for onboard orbit propagation: restricted power
- Future improvements and work
  - include atmospheric drag effects
  - try other intermediaries: Aksnes, Vinti, 2 fixed centers