A TLE-based Representation of Precise Orbit Prediction Results

Jizhang Sang
School of Geodesy and Geomatics,
Wuhan University, China
1. Introduction

Populations of space objects:

- **NASA:**
  - quantity: ≥500,000;
  - size: ≥1 cm;

- **NORAD catalogue:**
  - quantity: 17,000 space objects, 6% are satellites;
  - size: ≥10 cm;
1. Introduction

- The number of space debris is still growing.
- Space collisions will occur more frequently.
- And, Kessler syndrome (chain collisions) could happen.

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<th>What</th>
<th>Year</th>
<th>Pieces</th>
<th>Notes</th>
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<td>CBERS 1 Rocket Body</td>
<td>2000</td>
<td>343</td>
<td>Explosion</td>
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1. Introduction

- Accurate and fast orbit prediction information is a necessary requirement for the space applications, such as conjunction analysis.
- Currently, most conjunction analysis are based on the NORAD TLE. However, its advantage in the computation efficiency is shadowed by the lack of high accuracy in the predicted orbits. e.g. 7-day orbit prediction error may reach 8 km for LEO objects.
- With deployments of more and more tracking facilities, the number of tracked and catalogued debris objects will rise from the present 17000 to hundreds of thousands in the near future.

Requires: Efficient and accurate orbit propagation methods.
1. Introduction

The presented Method

Step 1: generate new TLE using the precise orbit predictions with differential corrections;

Step 2: bias corrections from a specific function is applied to improve the orbit prediction accuracy of the generated TLE ($TLE^N$).

Step 3: broadcast the $TLE^N$ and the correction functions to users.

- Achieve highly accurate orbit predictions using $TLE^N$;
- Reduce the storage of orbit prediction information.
2. Orbit determination and prediction

- **Test objects:**
  - *Larets*
    - altitude: 690 km
  - *Starlette*
    - altitude: 815 km
  - *Ajisai*
    - altitude: 1500 km
  - *Lageos1*
    - altitude: 5840 km
2. Orbit determination and prediction

Data source in the orbit determination

- Simulated Debris Laser Ranging (DLR) Data
  - SLR Data: ILRS data center CDDIS
  - Corruption: Gaussian errors, $\sigma \sim N(0,1.0 \text{ meter})$
- NORAD TLE
  - [https://www.space-track.org](https://www.space-track.org)

Forces in the orbit determination and prediction

- Earth gravity: JGM-3 gravity model (full)
- Third-body gravity: DE406
- Sun radiation pressure
- Atmospheric density model: MSIS-86
- Ocean tide model: CSR 3.0
- Solid tide model
2. Orbit determination and prediction

- **Procedures**
  - The OD and OP are processed under a similar scenarios as that of space debris.
  - 100 computations for each satellite are conducted.
  - Initial orbit state vector is computed from the latest NORAD TLE before the OD fit.

OD: Two passes of DLR data, separated by 24 hours from a single station, are used to determine the orbits first.

OP: After the OD process, the orbits is propagated forward for 30 days using numerical integrator.

**OD:** orbit determination  **OP:** orbit propagation/prediction
3. Algorithm

Generating $TLE^N$

1. The basic differential relation for the TLE elements

$$dr = \frac{\partial r}{\partial n} dn + \frac{\partial r}{\partial e} de + \frac{\partial r}{\partial i} di + \frac{\partial r}{\partial \Omega} d\Omega + \frac{\partial r}{\partial \omega} d\omega + \frac{\partial r}{\partial M} dM + \frac{\partial r}{\partial B^*} dB^*$$

$n$: mean motion  
$e$: mean eccentricity  
$i$: mean inclination  
$\Omega$: mean right ascension of the ascending node  
$\omega$: mean perigee argument  
$M$: mean anomaly  
$B^*$: TLE ballistic coefficient  
r: position vector

2. The derivative computations

$$\frac{\partial r}{\partial X} = \frac{r(X + \Delta X) - r(X)}{\Delta X}$$

$X$: a vector representing the mean elements, $n,e,i,\Omega,\omega,M$, and $B^*$  
$\Delta X$: a vector representing the small increments of $n,e,i,\Omega,\omega,M$, and $B^*$
3. Algorithm

Generating $TLE^N$

3  Solutions using the least-square method

$$dX = (B^T B)^{-1} B^T l$$

where,

$$dX = (dn, de, di, d\Omega, d\omega, dM, dB^*)^T$$

$$l = (dr_1, dr_2, \ldots, dr_N)^T$$

$$B = \begin{bmatrix}
\frac{\partial r_1}{\partial n} & \frac{\partial r_1}{\partial e} & \frac{\partial r_1}{\partial i} & \frac{\partial r_1}{\partial \Omega} & \frac{\partial r_1}{\partial \omega} & \frac{\partial r_1}{\partial M} & \frac{\partial r_1}{\partial B^*} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\frac{\partial r_N}{\partial n} & \frac{\partial r_N}{\partial e} & \frac{\partial r_N}{\partial i} & \frac{\partial r_N}{\partial \Omega} & \frac{\partial r_N}{\partial \omega} & \frac{\partial r_N}{\partial M} & \frac{\partial r_N}{\partial B^*}
\end{bmatrix}$$

4  Final form of the generated TLE

$$X_{TLE^N} = X + dX$$

$X_{TLE^N}$ : a vector representing the $TLE^N$

$X$ : a vector representing the approximate TLE

d$X$ : a vector representing the corrections to the approximate TLE
3. Algorithm

Generating $TLE^N$

5 Orbit prediction accuracy assessment

- The numerically-propagated positions are highly accurate, they are used as the reference to compute the prediction errors of the $TLE^N$/SGP4-propagated.
- Two measures are used to assess the prediction errors:

  **Absolute Maximum Prediction Error**
  \[\text{Max\_Bias} = \max(\text{fabs}(\text{bias}[i]));\]

  **RMS of the prediction errors**
  \[
  \text{RMS} = \sqrt{\frac{1}{3N} \sum_{i=1}^{N} (|x_{OP} - x_{\text{reference}}|^2 + |y_{OP} - y_{\text{reference}}|^2 + |z_{OP} - z_{\text{reference}}|^2)}
  \]

  $N$ : Number of the predicted orbit positions
3. Algorithm

Bias corrections – is it possible?

NORAD TLE:
1 07646U 75010A 14181.84362355 -00000155 00000-0 -82272-5 0 9996
2 07646 049.8237 070.2576 0205718 029.4969 064.0347 13.82291354990142

For \( TLE^N \), max prediction error in the cross-track direction, which dominates, is 1.1 km.

For NORAD TLE, max prediction error in the along-track direction, is 6.9 km.
3. Algorithm

Bias corrections – is it possible?

NORAD TLE:
1 08820U 76039A 14182.40636373 .00000003 00000-0 00000+0 0 9993
2 08820 109.8323 122.8216 0044471 158.7282 000.7769 06.38664803634557

Figure 2: Prediction errors for 30 days for Lageos1 using NORAD TLE and $TLE^N$

For $TLE^N$, max prediction error in the cross-track direction, which dominates, is **0.4 km**.

For NORAD TLE, max prediction error in the cross-track direction, is **1.2 km**.
3. Algorithm

Bias correction

- Figures 1 & 2 show that TLE$^N$/SGP4-propagated orbit errors in the along-track, cross-track and radial directions are well-behaved.
- Use a function to fit each of the along-track, cross-track and radial biases, which are the differences between TLE$^N$/SGP4-propagated orbits and the numerically propagated orbits.

\[
f(t) = \sum_{i=1}^{N} (a_i \sin (b_i t + c_i))
\]

\[t\] : time from the pre-set reference epoch, in hours;

\[N\] : number of the sine functions. Tests show that the fitting accuracy improves for larger values of \(N\), but when \(N\) is larger than 8, the accuracy improvement slows. Therefore, \(N = 8\) is chosen.

\[a_i, b_i, c_i\] : the unknown coefficients to be estimated;
3. Algorithm

**Bias correction**

**Observations:** Use the TLE\(^N\)-propagated orbit biases as pseudo-observations

**Solution:** least squares method is applied to estimate the unknown coefficients \(a_i, b_i, c_i\)

**Result:** Users could improve the accuracy of their TLE\(^N\)-propagated orbits by adding the bias corrections computed from the fitting function (\(M\) the transformation matrix).

\[ X = X_{\text{TLE}^N} + M \sum_{i=1}^{N} a_i \sin(b_it + c_i) \]
4. Results

Position errors: $TLE^N$ vs NORAD TLE

Table 1: Average max position errors for 100 computations for 30 days prediction using $TLE^N$ and the corresponding NORAD TLE, in km.

<table>
<thead>
<tr>
<th>TLE</th>
<th>Larets</th>
<th>Starlette</th>
<th>Ajisai</th>
<th>Lageos1</th>
</tr>
</thead>
<tbody>
<tr>
<td>NORAD TLE</td>
<td>48.2</td>
<td>7.4</td>
<td>6.9</td>
<td>1.6</td>
</tr>
<tr>
<td>$TLE^N$</td>
<td>3.3</td>
<td>1.8</td>
<td>1.1</td>
<td>0.5</td>
</tr>
</tbody>
</table>

- The average maximum position errors for 30 days using $TLE^N$ are 3.3km, 1.8km, 1.1km, 0.5km for Larets, Starlette, Ajisai and Lageos1, respectively, with the corresponding improvement in percentages about 93.2%, 75.7%, 84.1%, 68.8%.
- These results indicate that the $TLE^N$ could achieve more accurate orbit predictions.
4. Results

Prediction errors - Starlette

\( TLE^N \): \( TLE^N \)/SGP4-propagated orbit errors;
**Fit:** 30-day fitting results;
**AC:** \( TLE^N \)/SGP4-propagated orbit errors after corrections (AC).

(a) Along-track  
(b) Cross-track  
(c) Radial

Figure 3-a: 30-day prediction errors using fitting functions for Starlette.
4. Results

Prediction errors – Lageos1

(a) Along-track  (b) Cross-track  (c) Radial

Figure 4-a: 30-day prediction errors using fitting functions for Lageos1.
4. Results

Maximums of prediction errors: $TLE^N$ vs AC – Larets

(a) Along-track

(b) Cross-track

(c) Radial

Figure 5: Comparison of absolute maximum prediction errors using $TLE^N$ before and after bias corrections for Larets for 7, 10 and 30 day predictions from the 100 computations.
4. Results

Maxima of prediction errors: $TLE^N$ vs AC – Starlette

Figure 6: Comparison of absolute maximum prediction errors using $TLE^N$ before and after bias corrections for Starlette for 7, 10 and 30 day predictions from the 100 computations.

(a) Along-track  (b) Cross-track  (c) Radial
4. Results

Maximums of prediction errors: $TLE^N$ vs AC – Ajisai

Figure 7: Comparison of absolute maximum prediction errors using $TLE^N$ before and after bias corrections for **Ajisai** for 7, 10 and 30 day predictions from the 100 computations.
4. Results

**Maximums of prediction errors:** $TLE^N$ vs AC – Lageos1

![Graphs showing maximum prediction errors for TLE^N and AC for Lageos1 over along-track, cross-track, and radial predictions.](image)

(a) Along-track  
(b) Cross-track  
(c) Radial

Figure 8: Comparison of absolute maximum prediction errors using $TLE^N$ before and after bias corrections for Lageos1 for 7, 10 and 30 day predictions from the 100 computations.
## RMS of the prediction errors: $TLE^N$ vs AC

Table 2: RMS for 30-day orbit prediction errors of each satellite for 100 computations, in meters.

<table>
<thead>
<tr>
<th>Satellite</th>
<th>$TLE^N$</th>
<th>AC</th>
<th>improved by</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>max</td>
<td>min</td>
<td>average</td>
</tr>
<tr>
<td>Larets</td>
<td>705.3</td>
<td>548.8</td>
<td>641.3</td>
</tr>
<tr>
<td>Starlette</td>
<td>317.7</td>
<td>285.6</td>
<td>303.7</td>
</tr>
<tr>
<td>Ajisai</td>
<td>271.8</td>
<td>255.8</td>
<td>264.0</td>
</tr>
<tr>
<td>Lageos1</td>
<td>147.8</td>
<td>115.7</td>
<td>125.2</td>
</tr>
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5. Conclusions

- The ever-increasing space debris and deployments of more and more tracking facilities challenge efficient and accurate OD/OP for debris.
- TLE generated ($TLE^N$) from precise orbit predictions could represent orbit in reasonable accuracy.
- Applying bias corrections to $TLE^N$-propagated orbits can further improve the OP accuracy.
- The file size for $TLE^N$ and correction functions is only 4KB.
- The proposed method will be expanded to provide covariance information for the propagated orbits.
- It may be possible to generate $TLE^N$ directly from original observation data.
Thank you!