A TLE-based Representation of Precise Orbit Prediction Results

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Contents

1. Introduction

Populations of space objects:

- **NASA:**
  quantity: ≥500,000;
  size : ≥1 cm;

- **NORAD catalogue:**
  quantity: 17,000 space objects, 6% are satellites;
  size : ≥10 cm;

1. Introduction

- **The number of space debris is still growing.**
- **Space collisions will occur more frequently.**
- **And, Kessler syndrome (chain collisions) could happen.**

What	Year	Pieces	Notes
Kosm			ium 33
STEP 2 R			
Irid			nos 2251
Kosm			ed
SPOT 1 R			
OV2-1 R			
Nimbus 4			
TES Rocket Body	2001	370	Explosion
CBERS 1 Rocket Body	2000	343	Explosion

1. Introduction

Situations:

- **Accurate and fast orbit prediction information is a necessary requirement for the space applicatons, such as conjunction analysis.**
- **Currently, most conjunction analysis are based on the NORAD TLE. However, its advantage in the computation efficiency is shadowed by the lack of high accuracy in the predicted orbits.** e.g. 7-day orbit prediction error may reach 8 km for LEO objects.
- **With deployments of more and more tracking facilities, the number of tracked and catalogued debris objects will rise from the present 17000 to hundreds of thousands in the near future.**

Requires: **Efficient and accurate orbit propagation methods.**

1. Introduction

The presented Method

Step 1: generate new TLE using the precise orbit predictions with differential corrections;

Step 2: bias corrections from a specific function is applied to improve the orbit prediction accuracy of the generated TLE (TLE^N).

Step 3: broadcast the TLE^N and the correction functions to users.

- Achieve highly accurate orbit predictions using TLE^N ;
- Reduce the storage of orbit prediction information.

2. Orbit determination and prediction

➢ Test objects:

- **Larets**
  altitude: 690 km

- **Starlette**
  altitude: 815 km

- **Ajisai**
  altitude: 1500 km

- **Lageos1**
  altitude: 5840 km

2. Orbit determination and prediction

➢Data source in the orbit determination

- **Simulated Debris Laser Ranging (DLR) Data**
  - ✓ **SLR Data: ILRS data center CDDIS**
  - ✓ **Corruption: Gaussian errors, σ~N (0,1.0 meter)**
- **NORAD TLE**
  - ✓ **https://www.space-track.org .**

➢Forces in the orbit determination and prediction

- **Earth gravity: JGM-3 gravity model (full)**
- **Third-body gravity: DE406**
- **Sun radiation pressure**
- **Atmospheric density model: MSIS-86**
- **Ocean tide model: CSR 3.0**
- **Solid tide model**

2. Orbit determination and prediction

- **Procedures**
  - The OD and OP are processed under a similar scenarios as that of space debris.
  - 100 computations for each satellite are conducted.
  - Initial orbit state vector is computed from the latest NORAD TLE before the OD fit.

OD: Two passes of DLR data, separated by 24 hours from a single station, are used to determine the orbits first.

OP: After the OD process, the orbits is propagated forward for 30 days using numerical integrator.

OD: orbit determination **OP: orbit propagation/prediction**

3. Algorithm

Generating TLE^N

1 The basic differential relation for the TLE elements

$$d\mathbf{r} = \frac{\partial \mathbf{r}}{\partial n} dn + \frac{\partial \mathbf{r}}{\partial e} de + \frac{\partial \mathbf{r}}{\partial i} di + \frac{\partial \mathbf{r}}{\partial \Omega} d\Omega + \frac{\partial \mathbf{r}}{\partial \omega} d\omega + \frac{\partial \mathbf{r}}{\partial M} dM + \frac{\partial \mathbf{r}}{\partial B^*} dB^*$$

n : mean motion
e : mean eccentricity
i : mean inclination
Ω : mean right ascension of the ascending node
ω : mean perigee argument
M : mean anomaly
B^* : TLE ballistic coefficient
$\mathbf{r}$: position vector

2 The derivative computations

$$\frac{\partial \mathbf{r}}{\partial \mathbf{X}} = \frac{\mathbf{r}(\mathbf{X}+\Delta\mathbf{X}) - \mathbf{r}(\mathbf{X})}{\Delta\mathbf{X}}$$

$\mathbf{X}$: a vector representing the mean elements, $n, e, i, \Omega, \omega, M$, and B^*

$\Delta\mathbf{X}$: a vector representing the small increments of $n, e, i, \Omega, \omega, M$, and B^*

Generating TLE^N

3 Solutions using the least-square method

$$\underset{7\times1}{d\mathbf{X}} = (\underset{7\times N}{\mathbf{B}^{\mathrm{T}}}\ \underset{N\times7}{\mathbf{B}})^{-1}\ \underset{7\times N}{\mathbf{B}^{\mathrm{T}}}\ \underset{N\times1}{l}$$

where,

$$d\mathbf{X} = (dn, de, di, d\Omega, d\omega, dM, dB^*)^{\mathrm{T}}$$

$$l = (d\mathbf{r}_1, d\mathbf{r}_2, \cdots, d\mathbf{r}_N)^{\mathrm{T}}$$

$$\mathbf{B} = \begin{bmatrix} \frac{\partial \mathbf{r}_1}{\partial n} & \frac{\partial \mathbf{r}_1}{\partial e} & \frac{\partial \mathbf{r}_1}{\partial i} & \frac{\partial \mathbf{r}_1}{\partial \Omega} & \frac{\partial \mathbf{r}_1}{\partial \omega} & \frac{\partial \mathbf{r}_1}{\partial M} & \frac{\partial \mathbf{r}_1}{\partial B^*} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial \mathbf{r}_N}{\partial n} & \frac{\partial \mathbf{r}_N}{\partial e} & \frac{\partial \mathbf{r}_N}{\partial i} & \frac{\partial \mathbf{r}_N}{\partial \Omega} & \frac{\partial \mathbf{r}_N}{\partial \omega} & \frac{\partial \mathbf{r}_N}{\partial M} & \frac{\partial \mathbf{r}_N}{\partial B^*} \end{bmatrix}$$

4 Final form of the generated TLE

$$\mathbf{X}_{TLE^N} = \mathbf{X} + d\mathbf{X}$$

$\mathbf{X}_{TLE^N}$: a vector representing the TLE^N

$\mathbf{X}$: a vector representing the approximate TLE

$d\mathbf{X}$: a vector representing the corrections to the approximate TLE

3. Algorithm

Generating TLE^N

5 Orbit prediction accuracy assessment

- The numerically-propagated positions are highly accurate, they are used as the reference to compute the prediction errors of the TLE^N/SGP4-propagated.
- Two measures are used to assess the prediction errors:

Absolute Maximum Prediction Error

```
Max_Bias = max(fabs( bias[ i ] ) );
```

RMS of the prediction errors

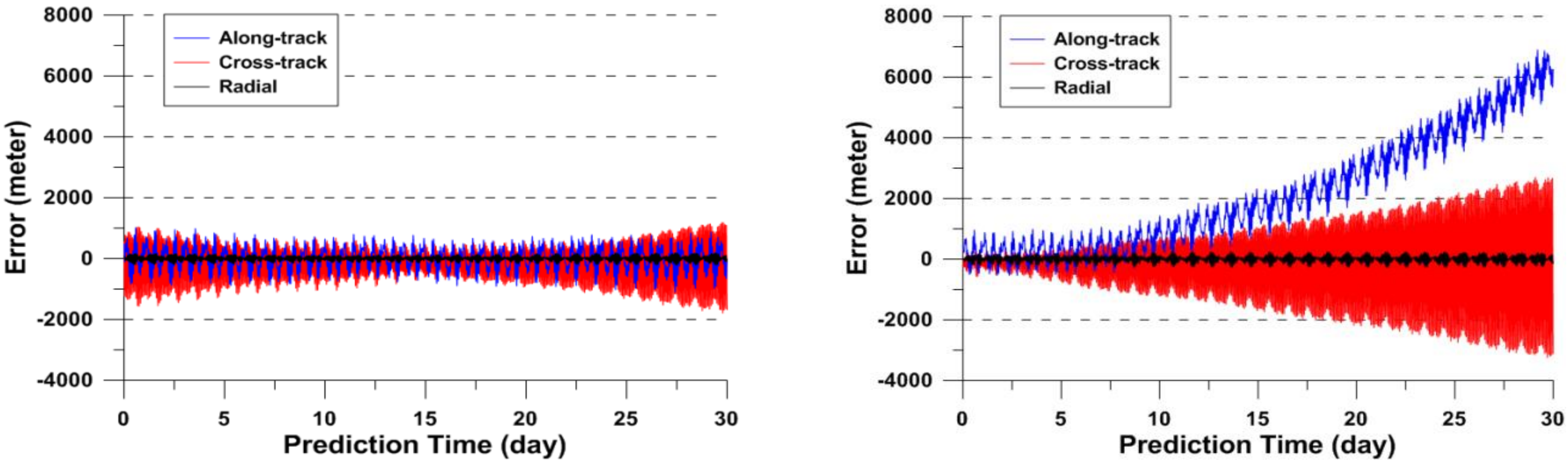
$$\mathrm{RMS} = \sqrt{\frac{\sum_{i=1}^{N}\left(\left|x_{\mathrm{OP}} - x_{\mathrm{reference}}\right|^2 + \left|y_{\mathrm{OP}} - y_{\mathrm{reference}}\right|^2 + \left|z_{\mathrm{OP}} - z_{\mathrm{reference}}\right|^2\right)}{3N}}$$

N : Number of the predicted orbit positions

3. Algorithm

Bias corrections – is it possible?

NORAD TLE:
1 07646U 75010A   14181.84362355 -.00000155 00000-0 -82272-5 0 9996
2 07646 049.8237 070.2576 0205718 029.4969 064.0347 13.82291354990142



(a) TLE^N (b) NORAD TLE

Figure 1: Prediction errors for 30 days for Starlette using NORAD TLE and TLE^N

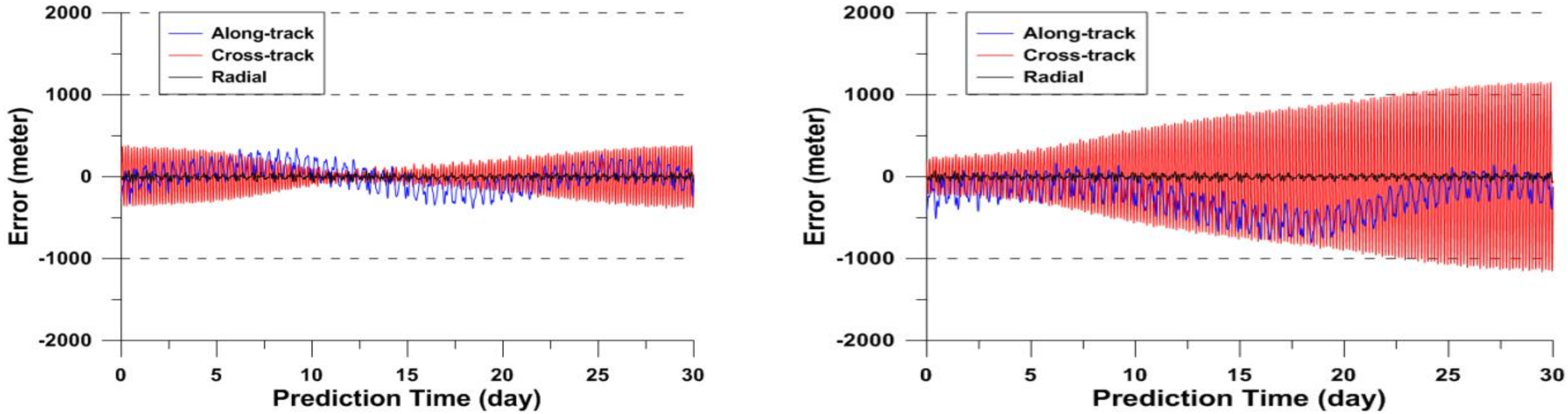
For TLE^N , max prediction error in the cross-track direction, which dominates, is **1.1 km**.

For NORAD TLE, max prediction error in the along-track direction, is **6.9 km**.

3. Algorithm

Bias corrections – is it possible?

NORAD TLE:
1 08820U 76039A   14182.40636373 .00000003 00000-0 00000+0 0 9993
2 08820 109.8323 122.8216 0044471 158.7282 000.7769 06.38664803634557



(a) TLE^N (b) NORAD TLE

Figure 2: Prediction errors for 30 days for Lageos1 using NORAD TLE and TLE^N

For TLE^N , max prediction error in the cross-track direction, which dominates, is **0.4 km**.

For NORAD TLE, max prediction error in the cross-track direction, is **1.2 km**.

3. Algorithm

Bias correction

- Figures 1 & 2 show that TLE^N/SGP4-propagated orbit errors in the along-track, cross-track and radial directions are well-behaved.
- Use a function to fit each of the along-track, cross-track and radial biases, which are the differences between TLE^N/SGP4-propagated orbits and the numerically propagated orbits.

$$f(t) = \sum_{i=1}^{N} \left(a_i \sin\left(b_i t + c_i \right) \right)$$

t : time from the pre-set reference epoch, in hours;

N : number of the sine functions. Tests show that the fitting accuracy improves for larger values of N, but when N is larger than 8, the accuracy improvement slows. Therefore, N = 8 is chosen.

a_i, b_i, c_i : the unknown coefficients to be estimated;

3. Algorithm

Bias correction

Observations: Use the TLE^N-propagated orbit biases as pseudo-observations

Solution: least squares method is applied to estimate the unknown coefficients a_i, b_i, c_i

Result: Users could improve the accuracy of their TLE^N-propagated orbits by adding the bias corrections computed from the fitting function (M the transformation matrix).

$$\boldsymbol{X} = \boldsymbol{X}_{\mathrm{TLE}^N} + M \sum_{i=1}^{N} \boldsymbol{a_i} \sin(\boldsymbol{b_i} t + \boldsymbol{c_i})$$

4. Results

Position errors: TLE^N vs NORAD TLE

Table 1: Average max position errors for 100 computations for 30 days prediction using TLE^N and the corresponding NORAD TLE, in km.

TLE	Larets	Starlette	Ajisai	Lageos1
NORAD TLE	48.2	7.4	6.9	1.6
TLE^N	3.3	1.8	1.1	0.5

- The average maximum position errors for 30 days using TLE^N are 3.3km, 1.8km, 1.1km, 0.5km for Larets, Starlette, Ajisai and Lageos1, respectively, with the corresponding improvement in percentages about 93.2%, 75.7%, 84.1%, 68.8%.
- These results indicate that the TLE^N could achieve more accurate orbit predictions.

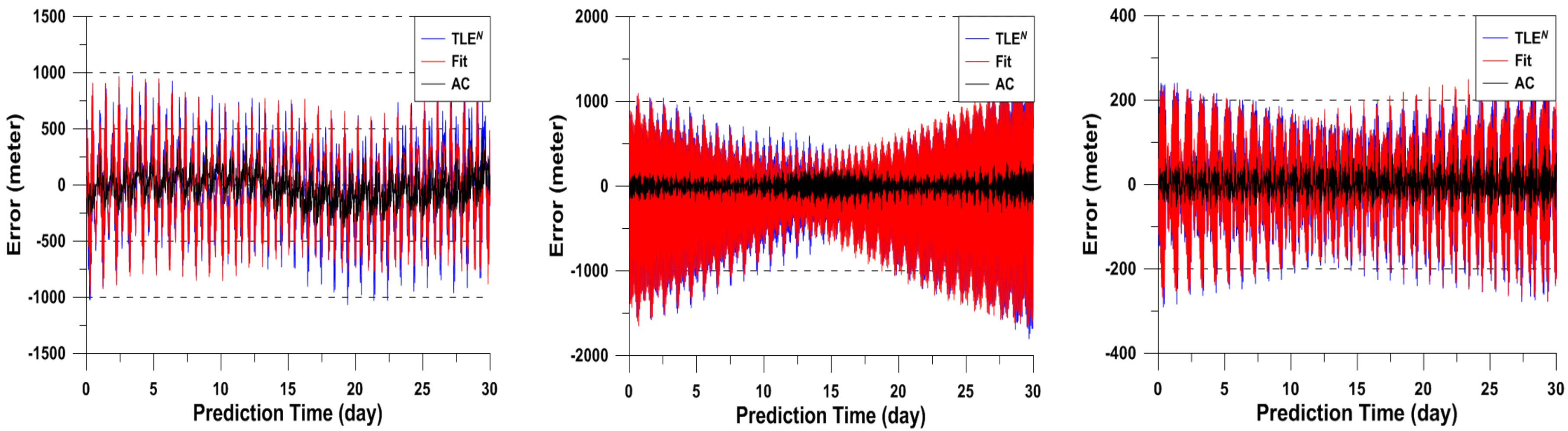
4. Results

Prediction errors - Starlette

TLE^N : TLE^N /SGP4-propagated orbit errors;

Fit: 30-day fitting results;

AC: TLE^N/SGP4-propagated orbit errors after corrections (AC).

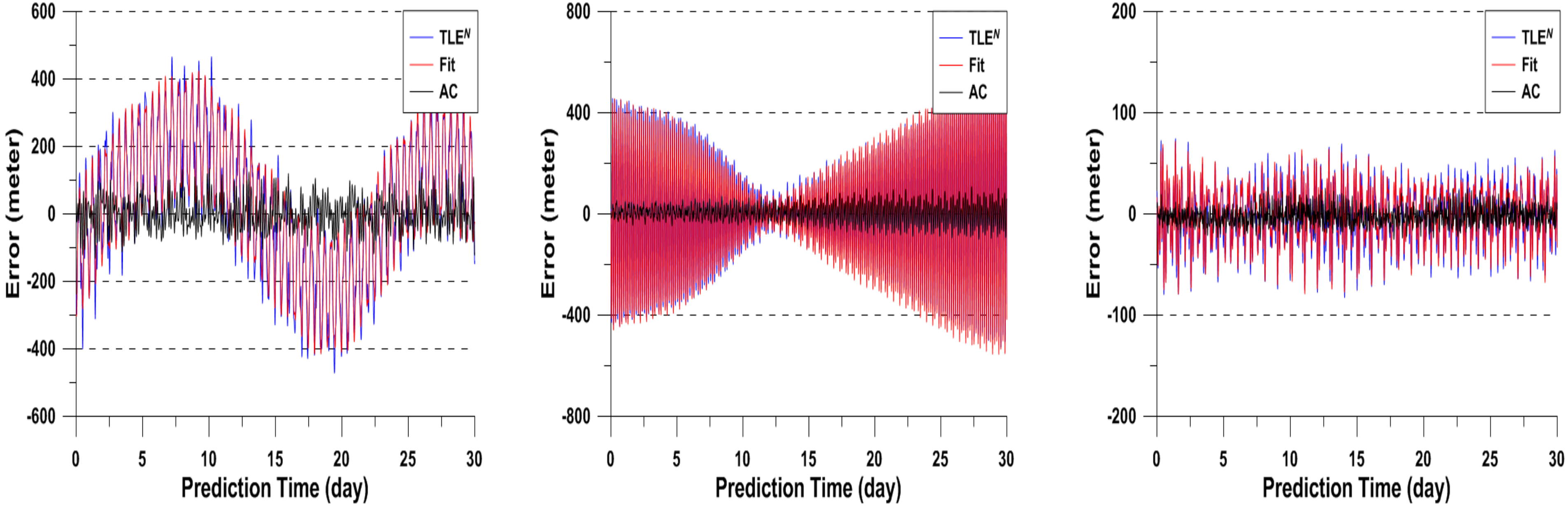


(a) Along-track (b) Cross-track (c) Radial

Figure 3-a: 30-day prediction errors using fitting functions for Starlette.

4. Results

Prediction errors – Lageos1

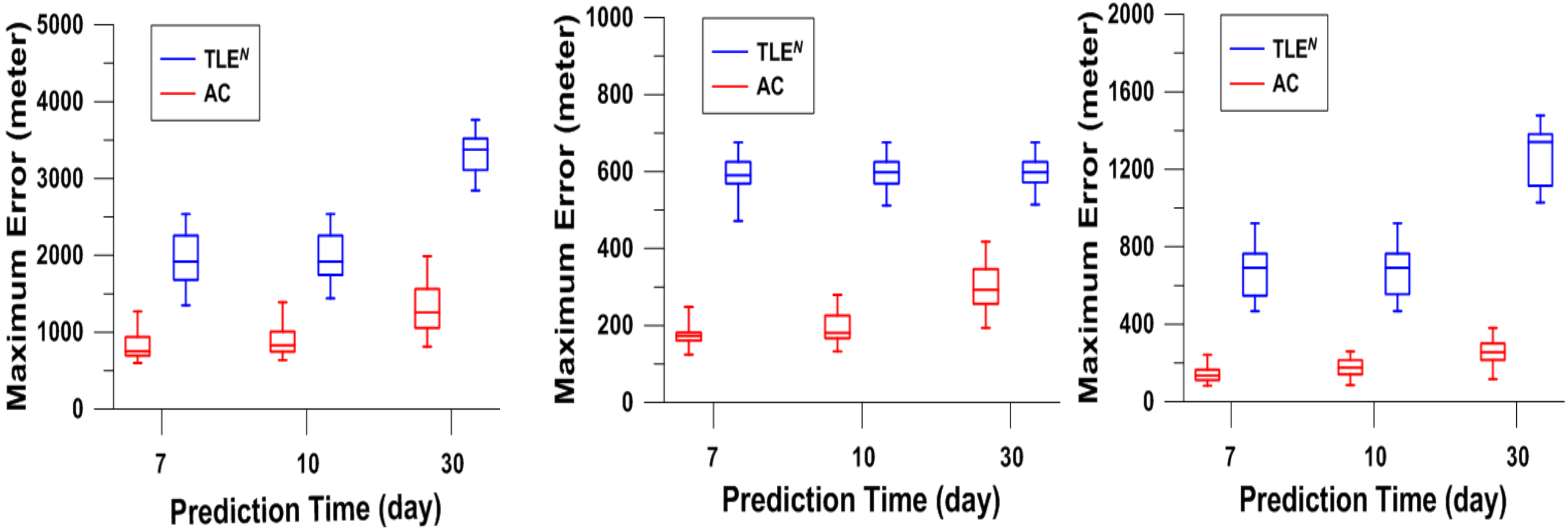


(a) Along-track (b) Cross-track (c) Radial

Figure 4-a: 30-day prediction errors using fitting functions for Lageos1.

4. Results

Maximums of prediction errors: TLE^N vs AC – Larets

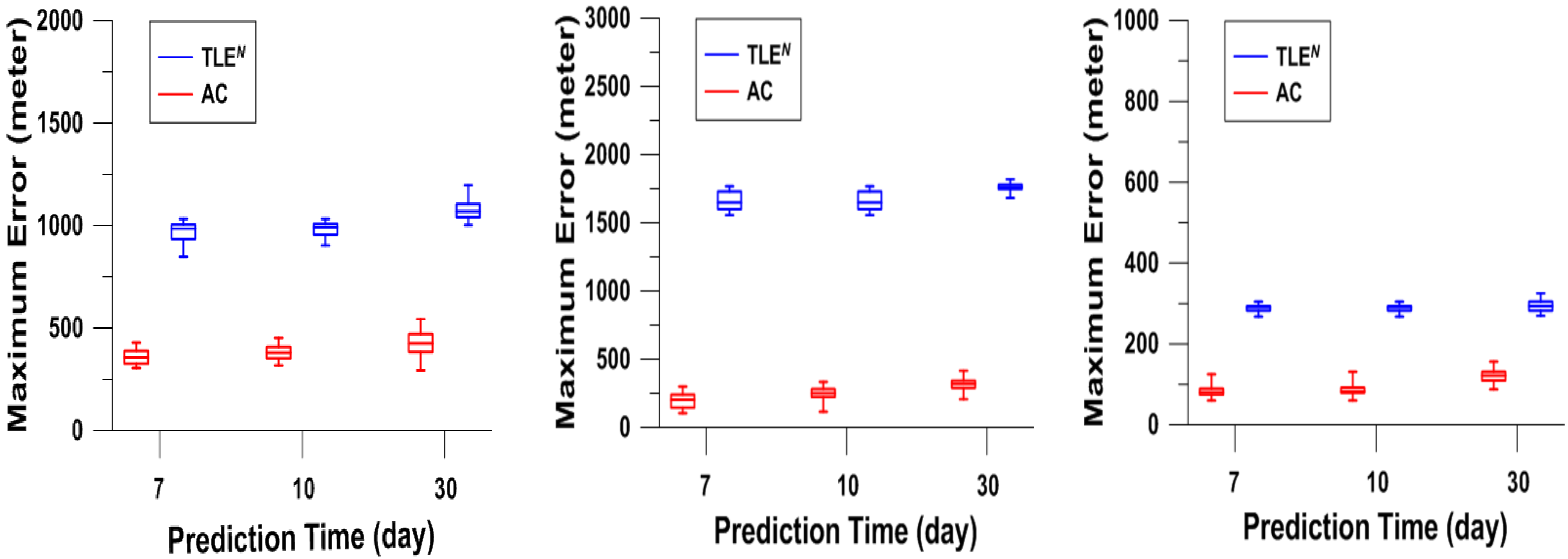


(a) Along-track (b) Cross-track (c) Radial

Figure 5: Comparison of absolute maximum prediction errors using TLE^N before and after bias corrections for **Larets** for 7, 10 and 30 day predictions from the 100 computations.

4. Results

Maximums of prediction errors: TLE^N vs AC – Starlette

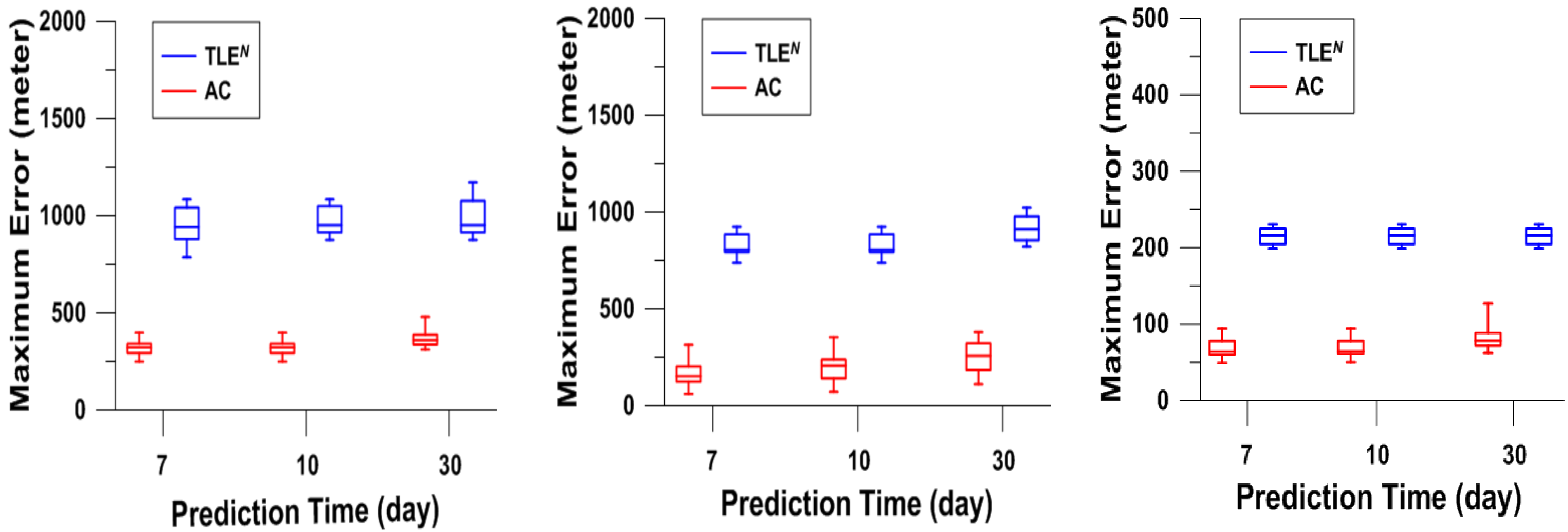


(a) Along-track (b) Cross-track (c) Radial

Figure 6: Comparison of absolute maximum prediction errors using TLE^N before and after bias corrections for **Starlette** for 7, 10 and 30 day predictions from the 100 computations.

4. Results

Maximums of prediction errors: TLE^N vs AC – Ajisai

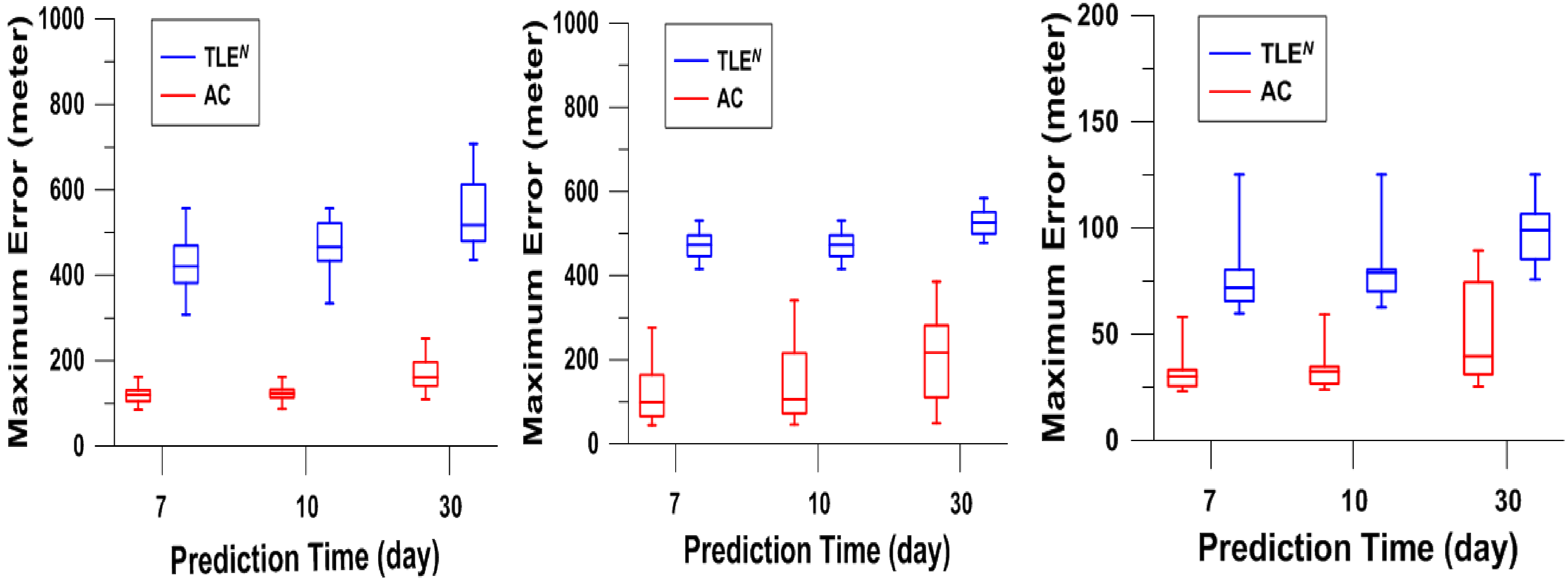


(a) Along-track (b) Cross-track (c) Radial

Figure 7: Comparison of absolute maximum prediction errors using TLE^N before and after bias corrections for **Ajisai** for 7, 10 and 30 day predictions from the 100 computations.

4. Results

Maximums of prediction errors: TLE^N vs AC – Lageos1



(a) Along-track (b) Cross-track (c) Radial

Figure 8: Comparison of absolute maximum prediction errors using TLE^N before and after bias corrections for **Lageos1** for 7, 10 and 30 day predictions from the 100 computations.

RMS of the prediction errors: TLE^N vs AC

Table 2: RMS for 30-day orbit prediction errors of each satellite for 100 computations, in meters.

Satellite	TLE^N			AC			improved by
	max	min	average	max	min	average	
Larets	705.3	548.8	641.3	218.9	147.7	163.8	74.5%
Starlette	317.7	285.6	303.7	85.4	73.4	76.0	74.9%
Ajisai	271.8	255.8	264.0	76.3	60.9	68.0	74.2%
Lageos1	147.8	115.7	125.2	34.0	23.3	25.9	79.3%

5. Conclusions

- The ever-increasing space debris and deployments of more and more tracking facilities challenge efficient and accurate OD/OP for debris.
- TLE generated (TLE^N) from precise orbit predictions could represent orbit in reasonable accuracy.
- Applying bias corrections to TLE^N-propagated orbits can further improve the OP accuracy.
- The file size for TLE^N and correction functions is only 4KB.
- The proposed method will be expanded to provide covariance information for the propagated orbits.
- It may be possible to generate TLE^N directly from original observation data.

Thank you !