A TLE-BASED REPRESENTATION OF PRECISE ORBIT PREDICTION RESULTS

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ABSTRACT

The computing efficiency of the analytic SGP4 algorithm is particularly attractive in applications where orbit positions of thousands of objects are needed. This paper presents a two-step TLE-based algorithm to present the accurate numerically-predicted orbits. First, the numerically-predicted positions are used as observations to determine a set of TLE employing the SGP4 algorithm. Then, the differences in the along-track, cross-track and radial directions between the accurate positions and TLE-computed positions are modeled with three correction functions. In this way, numerically-predicted orbit positions are presented by a set of TLE and 3 correction functions. Users would first use the TLE/SGP4 to compute the position at any moment, and then correct the position using the correction functions.

1. INTRODUCTION

The NORAD catalogue uses two-line-elements (TLE) to present orbits of thousands of space objects, and users use the analytic SGP4 (SDP4 for high orbits) algorithm to compute the position and velocity of any object at the time of interest. Because of the open availability of TLEs and extreme computation efficiency, the TLE/SGP4 thus is widely used in many space applications. However, its advantage in the computation efficiency is shadowed by large errors in the predicted orbits. On the other hand, accurate orbit determination and prediction (OD and OP) can be achieved by taking complex perturbation forces in the OP, at a cost of more computation time.

With advances in the space tracking techniques and capacity, the number of tracked and catalogued space objects will rise from the present 17000 to hundreds of thousands in the future. One of requirements in developing such catalogue is to deliver orbit positions of space objects fast and accurately for applications such as the space conjunction analysis. The numerical OP methods are highly accurate, but computationally slow, thus may be not suitable for large numbers of space objects. Analytic OP methods are computationally fast, but the predicted orbits could have large errors. A possible solution to the balance of the orbit accuracy and computation efficiency is to use the semi analytic orbit propagation methods.

Generally, debris tracking data is sparse. When a differential orbit determination process is performed using a numerical orbit integration method, the orbit prediction is followed using the same integrator. The output of the orbit predictions is usually a file of positions and velocities at evenly-spaced epochs. Users obtain the position and velocity at the time of interest with an interpolation method. Depending on the prediction span and the time step length of the output, the file could have a size in MB. When a catalogue has hundreds of thousands of space objects, the total volume to keep the orbit prediction files could be in hundreds of GB. This would make the data transfer and storage inconvenient.

This paper presents a two-step method to present numerically-predicted accurate orbit. In the first step, TLE is generated from the numerically-predicted positions where the SGP4 algorithm is applied. Then, three correction functions are used to model the 3D positional differences between the TLE-predicted and numerical orbits. Therefore, users obtain the position at any moment from the sum of the TLE/SGP4-computed position and the corrections from the correction functions. Through this two-step procedure, users would be able to propagate the orbit in much better accuracy than the TLE/SGP4 algorithm.

The method is assessed with 4 geodetic SLR (satellite laser ranging) satellites at different altitudes: Larets (690 km), Starlette (815 km), Ajisai (1500km) and Lageos1 (5840 km). In what follows, the procedure to perform the orbit determination and orbit prediction for the four satellites are presented in Section 2, followed by the development of the two-step method in Section 3. Then, the results are presented in Section 4. Finally, some conclusions are given.

2. ORBIT DETERMINATION AND PREDICTION

POD (precise orbit determination) has become a routine practice for satellites tracked with advanced techniques such as GPS and SLR. This achievement is made possible through great accuracy improvements in Earth gravity modelling and highly accurate and dense tracking of the satellite. Some data processing techniques, for example introducing general accelerations and many air drag coefficients, are widely used to accommodate errors existing in less accurate models, such as the air drag.

Usually, a least-squares procedure is applied to perform the POD, where the satellite orbit elements, force model parameters, and bias parameters for observations are adjusted to fit the orbit trajectory into tracking data. When the accuracy of the observations like GPS carrier phases or SLR ranging data is in the order of millimeters, the resultant POD accuracy can be as good as a few centimeters.

The accuracy, volume and temporal-spatial distribution of tracking data make the POD a very difficult task for general space objects. Early studies have shown that accurate debris orbit determination and prediction can be achieved using two passes of debris laser ranging (DLR) data from a single station, separated by 24h (Sang et al., 2013 a, b; Sang et al., 2014; Sang and Bennet, 2014; Bennet et al., 2015), if the accurate ballistic coefficient (BC) values of the space objects can be obtained beforehand. In this paper, the BC values for Larets and Starlette are 0.004681m^2/kg and 0.002292 m^2/kg, estimated from long-term historical TLEs (Sang and Bennet, 2014). For Ajisai and Lageos1, since their altitudes are relatively high, the method outlined in Sang et al. (2013a) to estimate the BC values is not suitable any more. According to the BC definition, $C_D \frac{A}{m}$, where C_D is the drag coefficient, A the cross-sectional area in the direction of object motion relative to the atmosphere, and m the object mass, the computed BC values for Ajisai and Lageos1 are 0.011552 m^2/kg (Lejba and Schillak, 2011), 0.001528 m^2/kg (Kucharski et al., 2013), respectively. All the BC values are fixed in all of the orbit determination and prediction computations.

The CRD-formatted SLR data for the four satellites in the period 1/7/2014 – 31/12/2014 is downloaded from the ILRS data center CDDIS (ftp://cddis.gsfc.nasa.gov/). The SLR data is corrupted with random errors of 1m standard deviation to reflect the DLR accuracy before input to the orbit determination software. The procedure to process the corrupted SLR data in the orbit determination is the same as that in Sang and Bennet (2014). It is noted that corrupted SLR data of only two passes separated 24h is used in the orbit determination. After the orbit determination process, the orbit prediction is followed.

In the orbit determination and prediction computations, the Earth gravity (the full 70×70 JGM-3 gravity model), the third-body gravities of the Sun, Moon and major planets, the solid earth tides and ocean tides, the air drag and solar radiation pressure are all considered. DE406 planetary ephemeris is used to compute the third body gravitational forces, MSIS86 atmospheric density model is used to compute the drag (Hedin, 1987). The drag coefficient, C_D, is fixed at 2.2, and the solar radiation pressure coefficient, C_R, is fixed at 1.1.

100 runs for orbit determination and prediction are conducted for each satellite. In each run, the orbit determination fit span is 2 days, and the orbit prediction span is 30 days, with the position and velocity output at 60s interval. Initial orbit state for the orbit determination is computed from the latest NORAD TLE before the orbit determination fit span.

The predicted positions are then used as observations for the TLE generation and estimation of the coefficients of the correction functions.

3. ALGORITHM

3.1. TLE generation

The numerically-predicted orbit positions after the orbit determination are regarded as pseudo-observations for generating TLE. As the determined orbit elements can be converted to any form of mean elements, given the definition of the demanding mean elements. In this paper, the osculating elements at the last observation epoch is computed first, and this epoch is defined as the TLE reference epoch. Then, the osculating elements are converted into a set of mean elements by removing the short-period effects of J_1, J_2, J_3 and J_{22} terms of the Earth gravitation (Hoots and France, 1987; Liu, 2000). This set of mean elements is used as the approximate TLE at the reference epoch in the TLE generation.

The basic differential relation between the position and the TLE elements is

$$\mathrm{d}\mathbf{r} = \frac{\partial \mathbf{r}}{\partial n}\mathrm{d}n + \frac{\partial \mathbf{r}}{\partial e}\mathrm{d}e + \frac{\partial \mathbf{r}}{\partial i}\mathrm{d}i + \frac{\partial \mathbf{r}}{\partial \Omega}\mathrm{d}\Omega + \frac{\partial \mathbf{r}}{\partial \omega}\mathrm{d}\omega + \frac{\partial \mathbf{r}}{\partial M}\mathrm{d}M + \frac{\partial \mathbf{r}}{\partial B^*}\mathrm{d}B^* \quad (1)$$

where $n, e, i, \Omega, \omega, M$ are the mean motion, eccentricity, inclination, right ascension of the ascending node, perigee argument and mean anomaly, B^* is the TLE ballistic coefficient. **r** is the position vector $(\mathrm{x}, \mathrm{y}, \mathrm{z})$ from the center of mass of the central body to the space object.

The derivatives in the equations above can be computed using the numerical method as follows:

$$\frac{\partial \mathbf{r}}{\partial \mathbf{X}} = \frac{\mathbf{r}(\mathbf{X}+\Delta\mathbf{X}) - \mathbf{r}(\mathbf{X})}{\Delta\mathbf{X}} \quad (2)$$

where **X** is the vector of the mean elements $n, e, i, \Omega, \omega, M$ and B^*, $\Delta\mathbf{X}$ is the pre-set vector representing the very small increments of the **X**.

In Eq. (2), $\mathbf{r}(\mathbf{X})$ and $\mathbf{r}(\mathbf{X}+\Delta\mathbf{X})$ are computed using the SGP4 algorithm, given the approximate values of **X** and pre-set small increments $\Delta\mathbf{X}$. Thus, the partial derivatives, $\frac{\partial \mathbf{r}}{\partial n}, \frac{\partial \mathbf{r}}{\partial e}, \frac{\partial \mathbf{r}}{\partial i}, \frac{\partial \mathbf{r}}{\partial \Omega}, \frac{\partial \mathbf{r}}{\partial \omega}, \frac{\partial \mathbf{r}}{\partial M}$, and $\frac{\partial \mathbf{r}}{\partial B^*}$ are estimated.

With the numerically-predicted positions as observations, the corrections to the approximate TLE in Eq. (1), $\mathrm{d}n, \mathrm{d}e, \mathrm{d}i, \mathrm{d}\Omega, \mathrm{d}\omega, \mathrm{d}M$ and $\mathrm{d}B^*$, can be solved with the standard least squares method. The estimated corrections are

$$\mathrm{d}\mathbf{X} = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}\mathbf{l} \quad (3)$$

where

$$\mathbf{A} = \begin{bmatrix} \frac{\partial \mathbf{r}_1}{\partial n} & \cdots & \frac{\partial \mathbf{r}_1}{\partial B^*} \\ \vdots & \vdots & \vdots \\ \frac{\partial \mathbf{r}_N}{\partial n} & \cdots & \frac{\partial \mathbf{r}_N}{\partial B^*} \end{bmatrix}$$

$$\mathbf{l} = (\Delta\mathbf{r}_1 \quad \cdots \quad \Delta\mathbf{r}_N)^T$$

$$\mathrm{d}\mathbf{X} = (dn \quad \cdots \quad dB^*)^T$$

N is the number of the observations, $\mathbf{A}$ is the matrix of the error equations, $\mathbf{l}$ is the differences between the observations and its corresponding values computed from approximate TLE, $\mathrm{d}\mathbf{X}$ is the corrections to the approximate TLE.

The estimation is an iterative process. The final TLE after the convergence is denoted as TLE^N.

In the process, since the SGP4 algorithm is applied for computing the derivatives, $\frac{\partial \mathbf{r}}{\partial \mathbf{X}}$, therefore, the orbit prediction with TLE^N has to use the SGP4 algorithm.

3.2. Correction functions

Because the SGP4 uses simplified force models, the TLE^N/SGP4-predicted orbit still contains significant errors in the along-track, cross-track and radial directions, as shown in Fig. 1-2 below. Examination of these figures shows that the prediction errors in the along-track, cross-track and radial directions have clear periodic property. This suggests that the TLE^N/SGP4-predicted orbit errors could be modeled by 3 proper correction functions for the three directions. Different from the function forms given in Bennet et al. (2012), the correction function takes the form of the sum of several sine functions in this paper, which is similar to the Fourier series. The function shown to be able to model the TLE^N/SGP4-predicted orbit errors, as long as there are enough observations to estimate the unknown coefficients in the fitting functions. The correction function for any of the along-track, cross-track and radial errors is defined as

$$f(t) = \sum_{i=1}^{n}[a_i \sin(b_i t + c_i)] \qquad (4)$$

where n is the number of the sine functions; t is the time, in hours; a_i, b_i and c_i are the unknown coefficients.

4. METHOD EXPERIMENTS

The algorithm presented above is experimented with the 4 geodetic satellites (Larets, Starlette, Ajisai and Lageos1). The predicted positions discussed in Section 2 are used as the observations in the TLE generation and coefficient estimation of the correction functions.

5.1. Prediction errors using TLE^N

Fig. 1-2 shows the 30-day orbit prediction errors using TLE^N for Starlette and Lageos1. The start epoch to propagate the orbits for both satellites is at UTC midnight 1/7/2014.

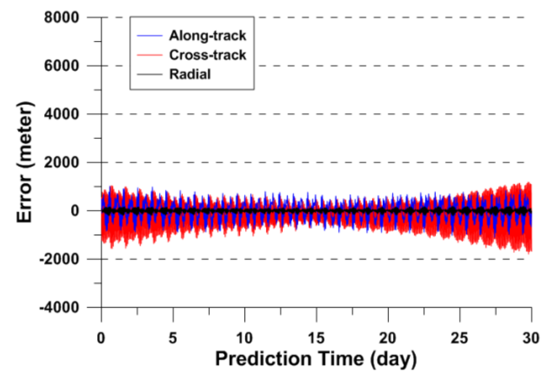


Fig. 1. 30-day prediction errors for **Starlette** using TLE^N.

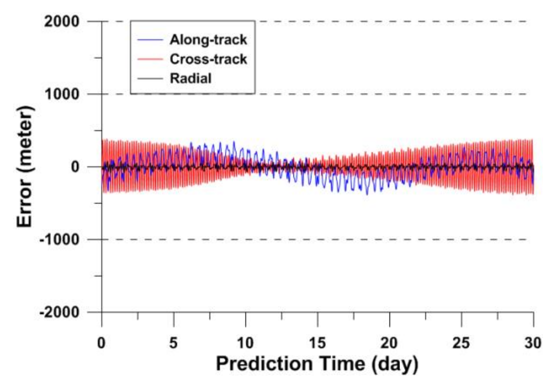


Fig. 2. 30-day prediction errors for **Lageos1** using TLE^N.

From the figures we can see that the prediction errors in each of the three directions have clear periodic property. For Starlette, the absolute maximum error over 30 days in the along-track direction is 1.1 km using TLE^N. For the higher Lageos1, the absolute maximum error over 30 days is in the cross-track direction with a value of 0.4 km. Table 1 gives the average of maximum three-dimensional 30-day prediction errors obtained from 100 computations for each of the 4 test satellites.

Table1. Averages of maximum 30-day prediction errors from 100 computations, in km.

Larets	Starlette	Ajisai	Lageos1
3.3	1.8	1.1	0.5

4.2 Prediction errors after corrections

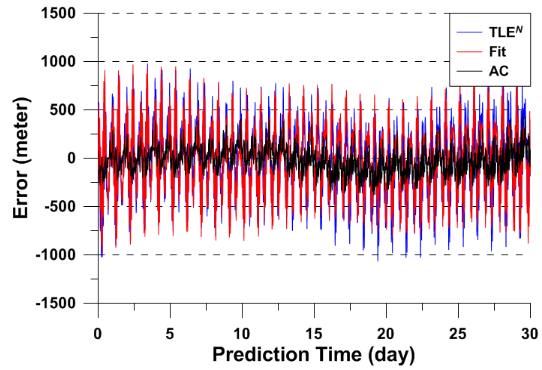
Because the SGP4 algorithms uses simplified force models, the TLE^N/SGP4-predicted orbit still contains significant errors in the along-track, cross-track and radial directions. These errors are modeled 3 correction functions for the along-track, cross-track and radial directions, respectively. Tests have shown that the fitting accuracy improves when the number of the sine functions in the correction function, n, is increased. For the prediction span of 30 days, $n = 8$ is found to be appropriate for the error fitting. Using the prediction errors at each epoch as observations, the nonlinear least-square method is applied to estimate the unknown coefficients a_i, b_i and c_i $(i = 1, \cdots, 8)$, . After determining the coefficients in the correction functions, the prediction errors at any epoch are computed using Eq. (4), and then used to correct the TLE^N/SGP4-predicted orbits to obtain more accurate orbit.

Fig. 3-4 shows the fitting results for Starlette and Lageos1 over the 30-day orbit prediction span. TLE^N (blue line) means the prediction errors using the TLE^N/SGP4; **Fit** (red line) means the corrections computed from the correction functions; **AC** (black line) means the prediction errors after applying corrections to the TLE^N/SGP4-predicted orbits.

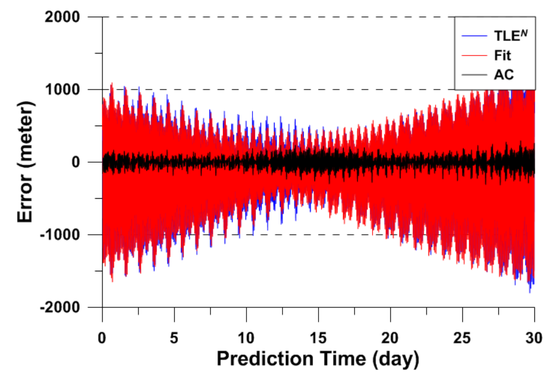
From Fig. 3-4, it can be seen that the correction functions with $n = 8$ could model the TLE^N/SGP4 prediction errors very well. When the corrections from the correction function are applied, the prediction errors are reduced dramatically, both in magnitude and variance. This is also seen in the distributions of the absolute maximum prediction errors before and after the application of the corrections, see the box-and-whisker plots in Fig. 5-8. In these figures, three separate prediction time span are considered: 7, 10 and 30 days. The analysis of the maximum errors is to show the worst-case scenario.

Table. 2 gives the representative RMS values of the 30-day prediction errors for the 100 runs. It shows that the correction function method results in a much smaller position errors when the TLE^N/SGP4-predicted orbit is corrected. The average RMSs of corrected prediction errors are reduced by nearly 75% for Larets, Starlette, and Ajisai, and 80% for Lageos1.

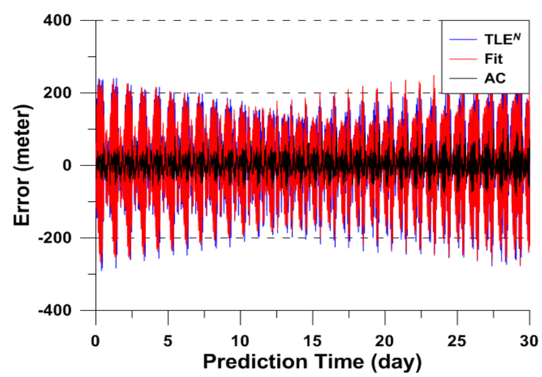
The reduction in the RMS of the prediction errors shows the numerically-predicted orbits can be properly presented with the proposed algorithm. A comparison with the semi-analytically-predicted orbit indicates that the performance of the presented method is similar for high orbits, and better than that for lower orbits.



(a) Along-track

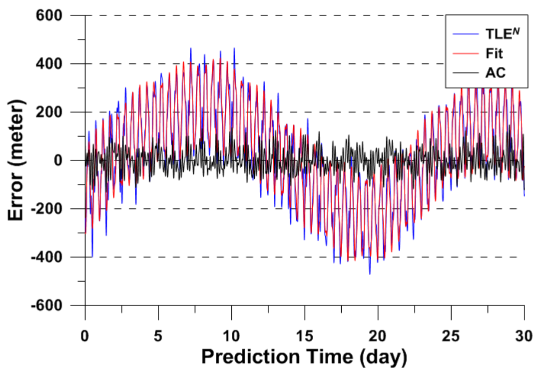


(b) Cross-track

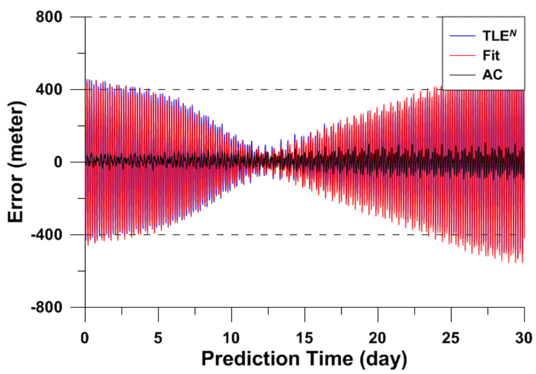


(c) Radial

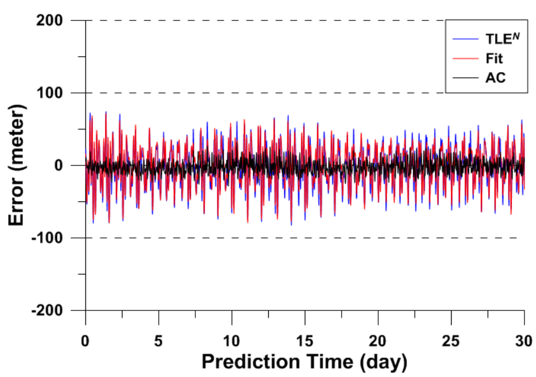
Fig. 3. Correct the 30-day TLE^N/SGP4 prediction errors for **Starlette**.



(a) Along-track

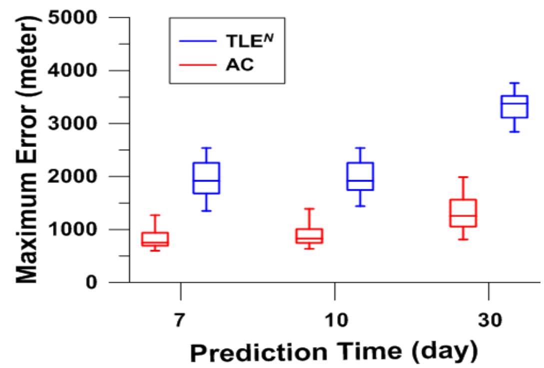


(b) Cross-track

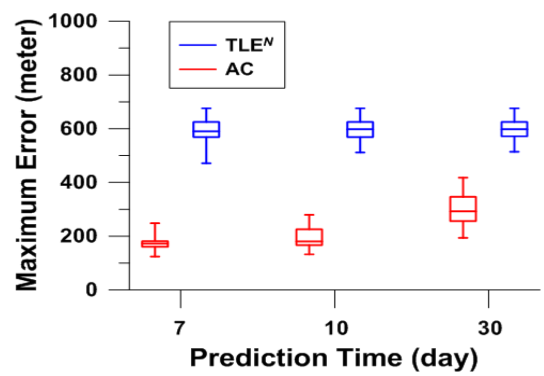


(c) Radial

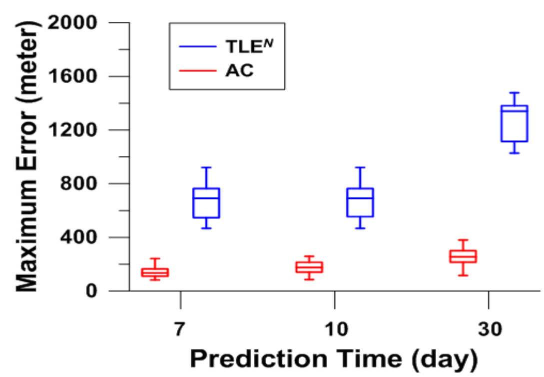
Fig. 4. Correct the 30-day TLE^N/SGP4 prediction errors for **Lageos1**.



(a) Along-track



(b) Cross-track



(c) Radial

Fig. 5. The absolute maximum prediction errors before and after correction for **Larets**.

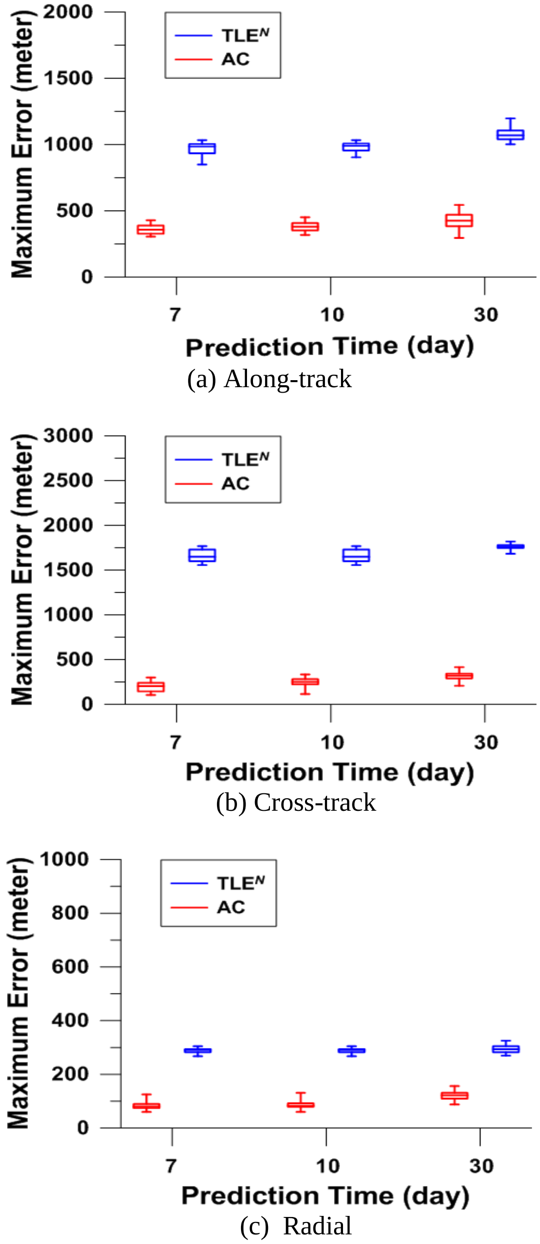


Fig. 6. The absolute maximum prediction errors before and after correction for **Starlette**.

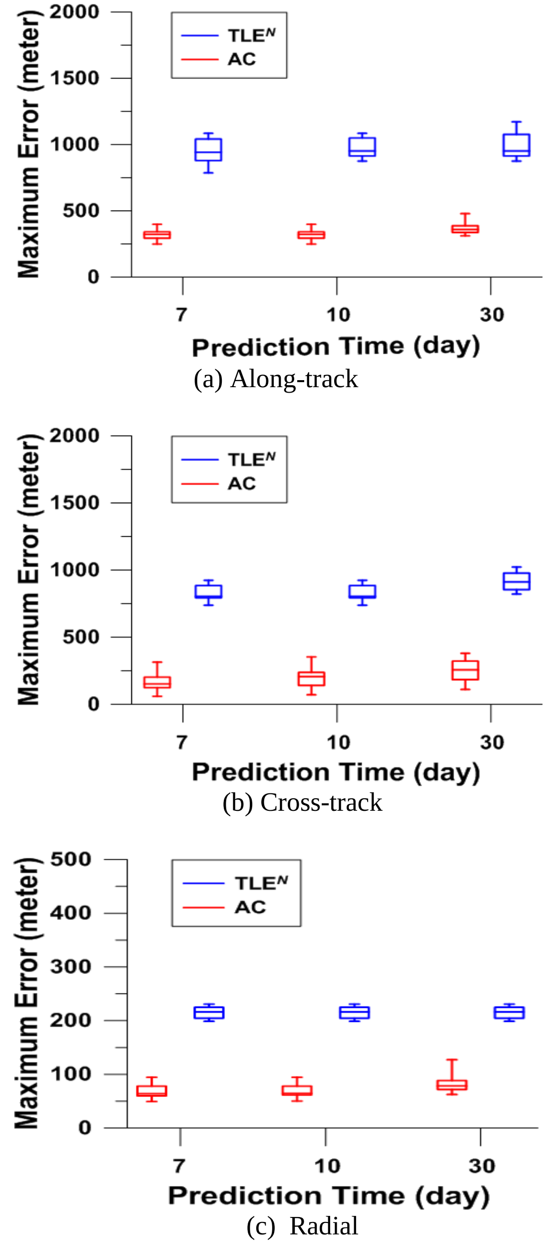
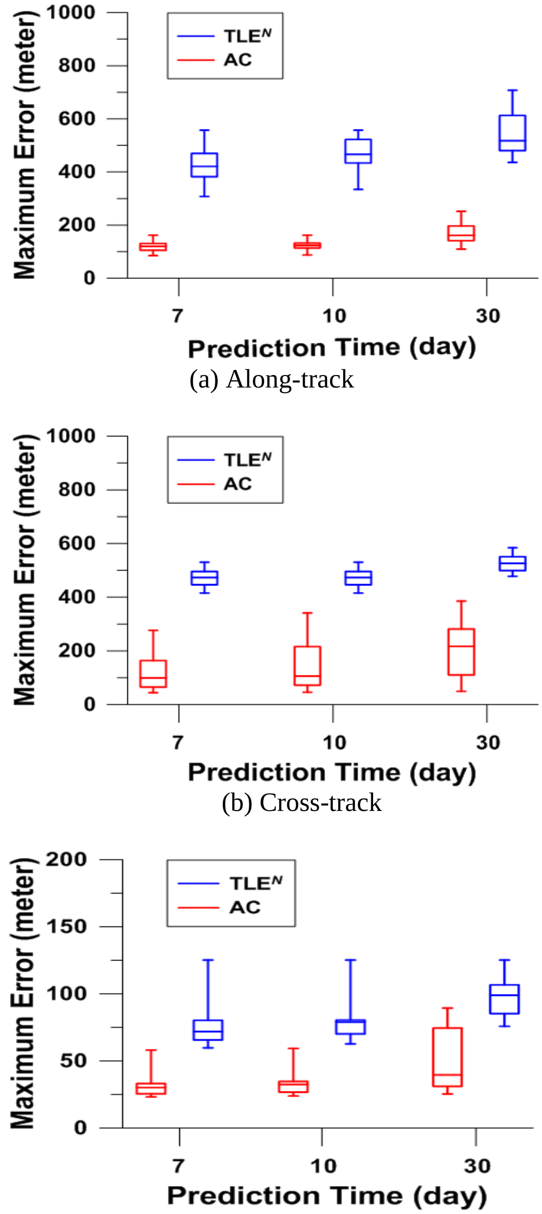


Fig. 7. The absolute maximum prediction errors before and after correction for **Ajisai**.



(a) Along-track

(b) Cross-track

(c) Radial

Fig. 8. The absolute maximum prediction errors before and after correction for **Lageos1**.

4.3 Storage for the TLE^N and coefficients of correction functions

Numerically-predicted orbits are usually output at a fixed time interval, which depends on the orbit altitude. The size of the output file could be in MB if the prediction span is long. For example, if the output interval is 60s, and the prediction span is 30 days, the file size is about 5MB. However, using the presented method, a file of only 4 KB is needed, where 1 KB for the TLE and 3 KB for the coefficients of the correction functions. When hundreds of thousands of objects are considered, the save in computer hard space and file transmission is huge.

Table. 2. RMS for 30-day orbit prediction errors of each satellite, in meters.

TLE				
Satellite	max	min	average	
Larets	705.3	345.8	641.3	
Starlette	317.7	285.6	303.7	
Ajisai	271.8	255.8	264.0	
Lageos1				
AC				
Satellite	max	min	average	average improved by
Larets	218.9	147.7	163.8	75.5%
Starlette	85.4	73.4	76.0	74.9%
Ajisai	76.3	60.9	68.0	74.2%
Lageos1	34.0	23.3	25.9	79.3%

5. CONCLUSIONS

The TLE/SGP4 method provides a fast way to propagate orbits for space objects but lacks the required high-accuracy in the predicted orbits. More applications require more accurate orbit prediction results, this calls for the use of more rigorous numerical orbit determination and prediction methods considering more complex force models. The output of numerically-predicted orbits is usually a file of position and velocity vectors at a fixed time interval, and the size could be in MB depending on the prediction span and time interval. Considering hundreds of thousands of space objects for the space situational awareness application, the delivery of numerically-predicted orbits to users could be a challenge because of the huge size of the orbit files.

In this paper, a two-step method to present the numerically-predicted orbits is proposed. First, the numerically-predicted positions are used as observations to determine a set of TLE employing the SGP4 algorithm. Then, the differences in the along-track, cross-track and radial directions between the numerically-predicted positions and TLE-computed positions are modeled with three correction functions. In this way, numerically-predicted orbit positions are presented by TLE^N and 3 correction functions. Users would first use the TLE/SGP4 to compute the position at any moment, and then correct the position using the correction functions.

The method is experimented with four geodetic satellites at different altitudes. The orbits are determined using laser ranging data of two passes separated 24 hours, and then predicted forward for 30 days. The RMS of the 30-day prediction errors using the TLE^N/SGP4 is several hundred meters or less. After the corrections to the TLE^N-predicted orbits, the final prediction accuracy is improved significantly. The RMS of the prediction errors is, reduced by more than 74% compared with that of the TLE^N-predicted orbit errors. With the proposed algorithm, users would only need very little effort to compute the corrections using the correction functions to obtain improved orbit prediction accuracy.

In addition to the accuracy and computing efficiency, the proposed algorithm also has advantage over numerical orbit prediction methods on the file storage and transmission, if hundreds of thousands of orbit are considered. Another possible advantage is that the proposed algorithm would be able to generate covariance information for the predicted orbits, which is essential for the space conjunction analysis.

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