A SEMI-ANALYTICAL ORBIT PROPAGATOR PROGRAM FOR HIGHLY ELLIPTICAL ORBITS

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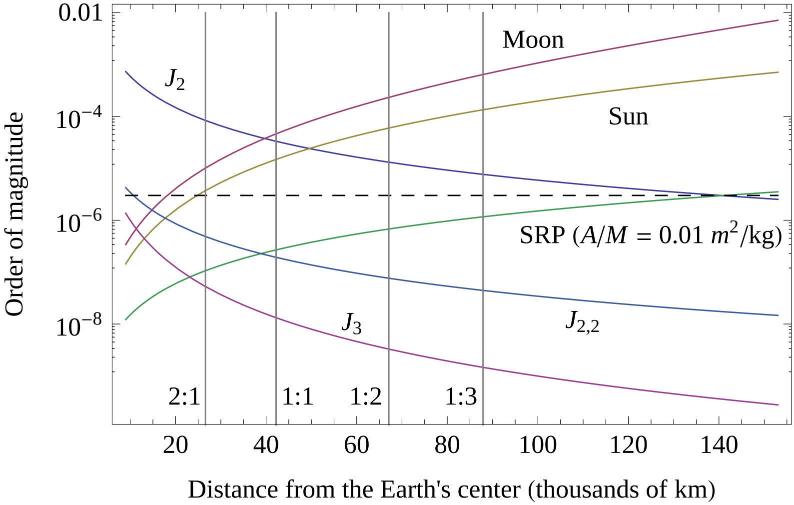
Outline

1. Motivation
2. Semi-analytical theory
3. Semi-analytical orbit propagator
4. Conclusions and future works

Motivation: Dynamics of a spacecraft in HEO



Semi-analytical theory: perturbation model

- Earth's gravitational potential:
  - zonal harmonics up to J_{10}, including J_2^2 effects
  - main tesseral harmonics affecting to the 2:1 resonance, which has an impact on Molniya orbits
- Lunisolar perturbations (mass-point approximation):
  - Legendre polynomial P_2 for the sun
  - Legendre polynomials P_2–P_6 for the moon
- Solar radiation pressure in the cannonball model approximation
- Atmospheric drag (Harris-Priester standard density model)

Semi-analytical theory: perturbation theory

The Lie–Deprit method (Deprit, 1969) looks for a generating function $\mathcal{W}$ of ϕ so that the terms $\mathcal{H}_n$, $\mathcal{K}_n$ $(= \mathcal{H}_{0,n})$ and $\mathcal{W}_n$ verify the partial differential equation, called Homological equation,

$$\boldsymbol{\mathcal{L}_{\mathcal{H}_0}(\mathcal{W}_n) + \mathcal{K}_n = \widetilde{\mathcal{H}}_{0,n}}$$

$\widetilde{\mathcal{H}}_{0,n}$, is computed from $\mathcal{H}_n$, $(\mathcal{W}_i)_{1\leq i\leq n-1}$ and $(\mathcal{H}_{p,q})_{p+q\leq n-1}$, where the latter are obtaineb by means of Deprit's equation

The solution of the Homological equation:

1. $\mathcal{K}_n \in \ker(\mathcal{L}_{\mathcal{H}_0})$
2. $\mathcal{W}_n + \mathcal{F}_n \in \mathrm{im}(\mathcal{L}_{\mathcal{H}_0})$ with $\mathcal{F}_n \in \ker(\mathcal{L}_{\mathcal{H}_0})$
   where $\mathcal{F}_n$ can be made at the same order (Kozai, 1962) or post-poned until next order (it is possible to consider $\mathcal{F}_n$ as the generating function of a new transformation, which can be used to remove other variables)

Semi-analytical theory: gravitational perturbations

- Deprit's perturbation algorithm (Deprit, 1969):
  - up to the second order of J_2, including Kozay-type terms in the mean elements Hamiltonian to get "centered" elements
  - closed-form of the eccentricity except for tesseral resonances
  - neglecting the coupling between J_2^2 and the moon's disturbing effects
- The theory is constructed in Delaunay variables (l, g, h, L, G, H):
  - $l = M$, $g = \omega$ and $h = \Omega$
  - Delaunay action $L = \sqrt{\mu a}$, the conjugate momentum to l
  - $G = L\sqrt{1 - e^2}$, the conjugate momentum to g
  - $H = G \cos i$, the conjugate momentum to h

Semi-analytical theory: gravitational perturbations

Hamiltonian ordering:

$$\mathcal{H} = \mathcal{H}_0 + \epsilon \mathcal{H}_1 + \frac{\epsilon^2}{2} \mathcal{H}_2$$

where

$$\begin{aligned}
\mathcal{H}_0 &= \mathcal{H}_{\mathcal{K}} + \mathcal{H}_{\omega_e} + \mathcal{H}_{☾} + \mathcal{H}_{\odot} \\
\mathcal{H}_1 &= \mathcal{H}_{\mathcal{Z}(J_2)} \\
\mathcal{H}_2 &= \mathcal{H}_{\mathcal{Z}(J_3 - J_{10})} + \mathcal{H}_{\mathcal{TR}_{2:1}} + \mathcal{H}_{☾} + \mathcal{H}_{\odot}
\end{aligned}$$

Semi-analytical theory: symbolic tools

- Variables:
  - $\mathcal{O} = (a, e, i, \omega, \Omega, M, f, E)$
  - $\mathcal{D} = (l = M, g = \omega, h = \Omega, L = \sqrt{\mu a}, G = \|\mathbf{x} \times \mathbf{X}\|, H = G \cos i)$
  - $\mathcal{W} = (r, \theta = \omega + f, \nu = \Omega, R = \dot{r}, \Theta = G, N = H)$
  - $\ldots$
- Relationship among the sets of variables:

$$\mathcal{R} = \{\eta = \sqrt{1 - e^2}, c = \cos i, s = \sin i, \phi = f - l, p = \frac{\Theta^2}{\mu}, C_s = e \cos g,$$

$$S_s = e \sin g, n = \sqrt{\frac{\mu}{a^3}}, \ldots\}$$

- Partial derivatives:

$$\left\{ \frac{\partial \mathcal{O}}{\partial \mathcal{D}}, \frac{\partial \mathcal{O}}{\partial \mathcal{W}}, \frac{\partial \mathcal{W}}{\partial \mathcal{D}}, \frac{\partial \mathcal{D}}{\partial \mathcal{W}}, \frac{\partial \mathcal{R}}{\partial \mathcal{D}}, \frac{\partial \mathcal{R}}{\partial \mathcal{W}}, \ldots \right\}$$

- Data Base of Integrals

Semi-analytical theory: perturbation theory

Elimination of the Parallax:

- Reduce the factors $\frac{\mu}{r}\left(\frac{p}{r}\right)^{\boldsymbol{n}} \to \frac{\mu}{r}\left(\frac{p}{r}\right)^{\mathbf{2}} \quad (n > 2)$
- Eliminate the explicit appearance of θ $(\theta = f + g)$

$$\mathcal{H}' = \mathcal{H}'_0 + \epsilon \mathcal{H}'_1 + \frac{\epsilon^2}{2}\mathcal{H}'_2 + \mathcal{O}(\epsilon^3)$$

where

$$\begin{aligned}
\mathcal{H}'_0 &= \mathcal{H}_{\mathcal{K}} + \mathcal{H}_{\omega_e} + \mathcal{H}_{☾} + \mathcal{H}_{\odot} \\
\mathcal{H}'_1 &= \mathcal{H}'_{\mathcal{Z}(J_2)} \\
\mathcal{H}'_2 &= \mathcal{H}'_{\mathcal{Z}(J_2^2)} + \mathcal{H}'_{\mathcal{Z}(J_3 - J_{10})} + \mathcal{H}_{\mathcal{TR}_{2:1}} + \mathcal{H}_{☾} + \mathcal{H}_{\odot}
\end{aligned}$$

Semi-analytical theory: perturbation theory

Delaunay Normalization:

$$\mathcal{L}_{\mathcal{H}_0}(\mathcal{W}_n) + \mathcal{K}_n = \widetilde{\mathcal{H}}_{0,n}$$

- $\mathcal{L}_{\mathcal{H}_0} = n\frac{\partial}{\partial l} - \omega_e \frac{\partial}{\partial h} + n_{☾} \frac{\partial}{\partial l_{☾}} + n_{\odot} \frac{\partial}{\partial l_{\odot}}$
- Solution:

$$\mathcal{K}_n = \frac{1}{2\pi}\int_0^{2\pi} \widetilde{\mathcal{H}}_{0\,n} \mathrm{d}l$$

$$\mathcal{W}_n = \frac{1}{n}\int (\widetilde{\mathcal{H}}_{0\,n} - \mathcal{K}_n)\mathrm{d}l + \mathcal{F}_n = \mathcal{W}_n^* + \mathcal{F}_n$$

$\mathcal{F}_n$ is used to remove the long-period terms from $\mathcal{W}_n$

$$\mathcal{F}_n = -\frac{1}{2\pi}\int_0^{2\pi} \mathcal{W}_n^* dl$$

Semi-analytical theory: perturbation theory

- Zonal terms:

$$\int \frac{\sin mf}{r^n} dl \quad \text{and} \quad \int \frac{\cos mf}{r^n} dl \Longrightarrow dl = \frac{r^2}{a^2\eta} df$$

- Third body:

$$\int r^n dl \Longrightarrow dl = \frac{r}{a} dE$$

$$\mathcal{H}'' = \mathcal{H}''_0 + \epsilon\mathcal{H}''_1 + \frac{\epsilon^2}{2}\mathcal{H}''_2 + \mathcal{O}(\epsilon^3)$$

where

$$\begin{aligned}
\mathcal{H}''_0 &= \mathcal{H}_{\mathcal{K}} + \mathcal{H}_{\omega_e} + \mathcal{H}_{☾} + \mathcal{H}_{\odot} \\
\mathcal{H}''_1 &= \mathcal{H}''_{\mathcal{Z}(J_2)} \\
\mathcal{H}''_2 &= \mathcal{H}''_{\mathcal{Z}(J_2^2)} + \mathcal{H}''_{\mathcal{Z}(J_3-J_{10})} + \mathcal{H}_{\mathcal{TR}_{2:1}} + \mathcal{H}'_{☾} + \mathcal{H}'_{\odot}
\end{aligned}$$

Semi-analytical theory: equations of motion

- Long-period terms:

$$\frac{d(l,g,h)}{dt} = \frac{\partial \mathcal{H}''}{\partial(L,G,H)}$$

$$\frac{d(L,G,H)}{dt} = -\frac{\partial \mathcal{H}''}{\partial(l,g,h)}$$

Delaunay variables are not singular for high eccentricity orbits

Semi-analytical theory: non-gravitational forces

- Solar radiation pressure in the cannonball model approximation
- Atmospheric drag (Harris-Priester standard density model)

$$
\begin{aligned}
\frac{d(l,g,h)}{dt} &= \frac{\partial \mathcal{H}''}{\partial (L,G,H)} + \mathcal{P}_{(l,g,h)} \\
\frac{d(L,G,H)}{dt} &= -\frac{\partial \mathcal{H}''}{\partial (l,g,h)} - \mathcal{Q}_{(l,g,h)}
\end{aligned}
$$

Semi-analytical orbit propagator

First version of the semi-analytical orbit propagator (not include the resonant effects)

- 3rd body ephemeris: Meeus (1998) approximate formulas
- Atmospheric density model: Harris-Priester (1962)
- Higher-order, variable step size, Runge-Kutta numerical integration routine

Conclusions

- Earth's gravitational potential:
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Future works

- Double-averaged third body models
- Third-averaged Sun model (fully analytical)
- Short-period terms (direct and inverse transformations)
- Non singular variables
- Other perturbations
- Third order semi-analytical theory