Hybrid SGP4: tools and methods

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Outline

Motivation

How to improve the features of a Special perturbation, General perturbation or Semi-analytical method without loss of efficiency?

$$\ddot{\boldsymbol{r}} + \mu \frac{\boldsymbol{r}}{r^3} = \boxed{\boldsymbol{a}_d}$$

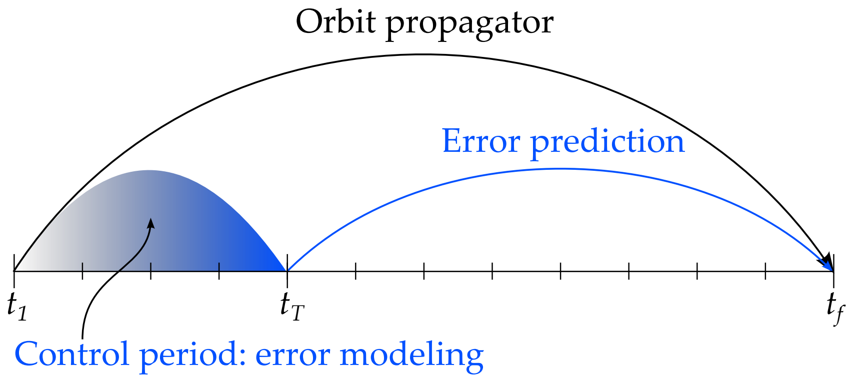
Statistical Time Series models or Computational Intelligence methods so as to model the perturbations and higher orders that are not considered in the Special perturbation, General perturbation or Semi-analytical methods.

Improve the features of SGP4 using hybrid methodology

The Simplified General Perturbations (SGP4) propagator is a standard AFSPACECOM propagator. It considers secular and periodic variations due to Earth oblateness, solar and lunar gravitational effects, gravitational resonance effects, and orbital decay using a drag model

Hybrid Orbit Propagator

1. A set of precise *observations*: $\{\boldsymbol{x}_t^{\mathcal{O}}\}_{t=1}^{T}$
2. A set of positions: $\{\boldsymbol{x}_t^{\mathcal{OP}}\}_{t=1}^{T}$

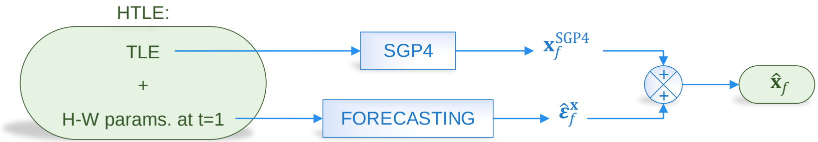


$\{\varepsilon_t\}_{t=1}^{T}$ with $\varepsilon_t = \boldsymbol{x}_t^{\mathcal{O}} - \boldsymbol{x}_t^{\mathcal{OP}}$

$\hat{\boldsymbol{x}}_f = \boldsymbol{x}_f^{\mathcal{OP}} + \hat{\varepsilon}_f$

Hybrid Orbit Propagator based on SGP4: HSGP4

1. **Statistical Time Series model**: Holt-Winters method
2. **Extend the TLE data**: Hybrid TLE (HTLE)



$$\Uparrow$$

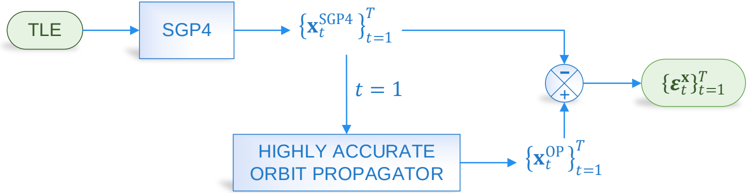
$$\hat{\varepsilon}^{\mathbf{x}}_{f|T} = A^{\mathbf{x}} + (f-1)B^{\mathbf{x}} + S^{\mathbf{x}}_{(f-1) \bmod s+1}$$

Hybrid Two-Line Element: HTLE

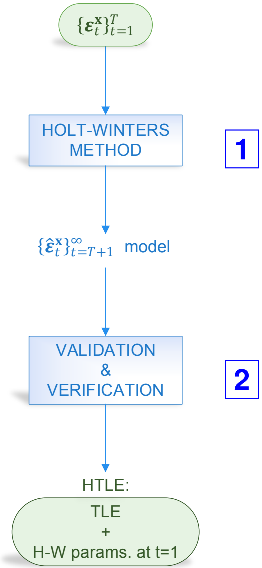
- **Initial conditions (TLE)**:

```
DEIMOS-1
1 35681U 09041A 11124.21233382 .00000325 00000-0 63164-4 0 9556
2 35681 098.0717 023.8270 0000845 081.0832 279.0474 14.69441166 94523
```

- **Precise observations,** $\{x_t^{\mathcal{O}}\}_{t=1}^{T}$: pseudo-observations simulated by using the numerical integration of a full-force model, including JGM-3 60×60 Earth gravitational potential, NRLMSISE-00 atmospheric drag, Sun and Moon 3rd-Body effect, solar radiation pressure including eclipses, Earth albedo, Earth IR, Earth solid tides, and relativistic effect (Elecnor Deimos)



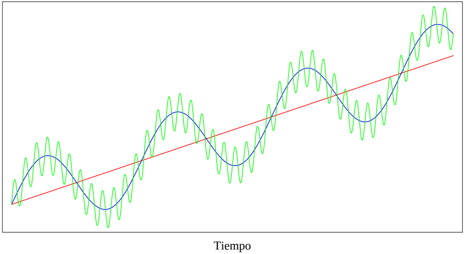
Hybrid Two-Line Element: HTLE



Classic **Holt-Winters method**:

$$\varepsilon_t = \mu_t + S_t + \nu_t$$

- μ_t is the trend
- S_t is the the seasonal component
- ν_t is the irregular component



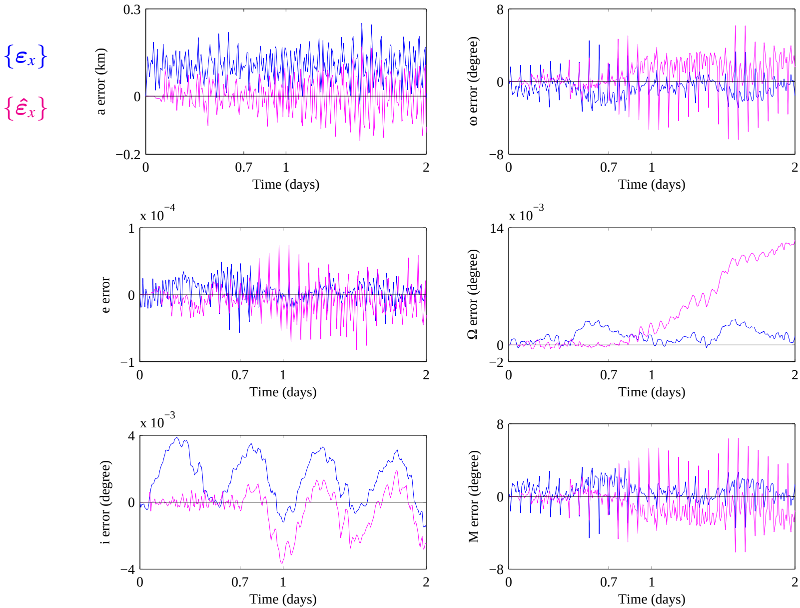
1.- Holt-Winters method

Require: s, c, h, and $\{\varepsilon_t\}_{t=1}^{T}$
Ensure: $\hat{\varepsilon}_{T+h|T}$

1: Estimate the values of $A_0, B_0, S_{-s+1}, \ldots, S_{-1}, S_0$
2: **for** $t = 1$; $t \leq T$; $t = t + 1$ **do**
3: $\quad A_t = \alpha(\varepsilon_t - S_{t-s}) + (1 - \alpha)(A_{t-1} + B_{t-1})$
4: $\quad B_t = \beta(A_t - A_{t-1}) + (1 - \beta)B_{t-1}$
5: $\quad S_t = \gamma(\varepsilon_t - A_t) + (1 - \gamma)S_{t-s}$
6: $\quad \hat{\varepsilon}_t = A_{t-1} + B_{t-1} + S_{t-s}$
7: **end for**
8: Select `error_measure` $\in$ {MSE, MAE, MAPE} and express it as a function of the smoothing parameters
9: Obtain the smoothing parameters that minimize `error_measure` using the L-BFGS-B method
10: Calculate $A_T, B_T, S_{T-s+1}, \ldots, S_{T-1}, S_T$ for the optimal smoothing parameters
11: $\hat{\varepsilon}_{T+h|T} = A_T + hB_T + S_{T-s+1+h \bmod s}$
12: **return** $\hat{\varepsilon}_{T+h|T}$

[San-Martín, M., *Métodos de propagación híbridos aplicados al problema del satélite artificial. Técnicas de suavizado exponencial [Ph.D. Thesis]*, University of La Rioja, 2014.]

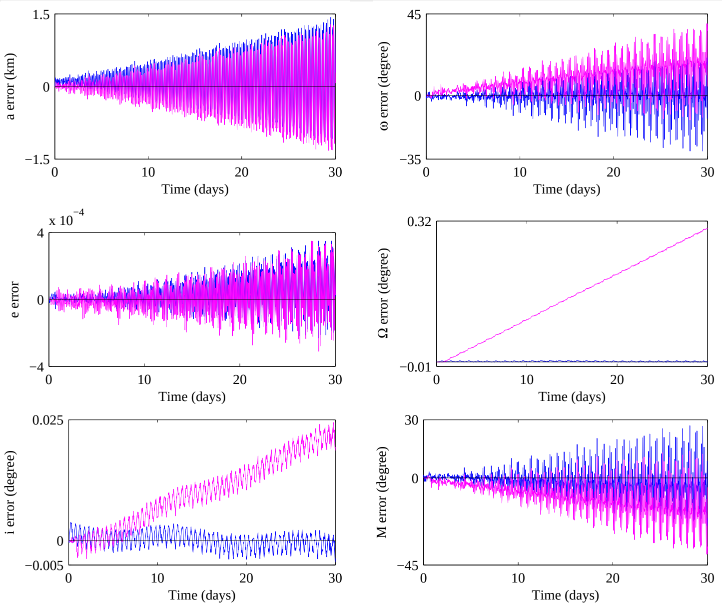
2- Validation and verification: 2-day propagation



2.- Validation and verification: 30-day propagation

$\{\varepsilon_x\}$

$\{\hat{\varepsilon}_x\}$



2.- Select the HTLE: minimize the distance error

Table: Distance error (km) of SGP4 versus HSGP4 modeling different sets of Delaunay variables

Prop. span	SGP4	HSGP4$_{(l,g)}$	HSGP4$_{(l,g,L,G)}$	HSGP4$_{(l,g,h,L,G,H)}$
0.7 days	10.551	1.088	0.994	1.019
1 day	14.235	3.206	3.195	3.198
2 days	28.650	5.883	5.893	5.708
7 days	101.164	20.477	20.500	19.483
30 days	486.738	41.100	41.099	42.969

2.- Select the HTLE: minimize the distance error

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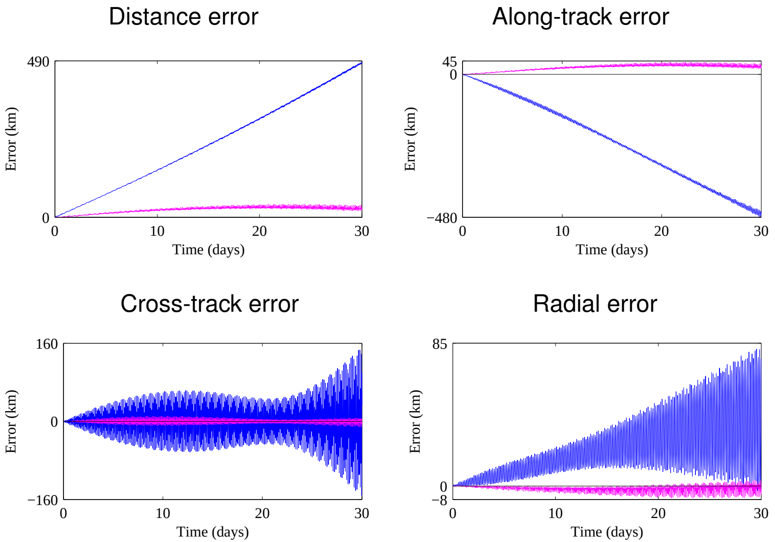
+

Holt-Winters parameters for $\hat{\varepsilon}_t^l$ and $\hat{\varepsilon}_t^g$

HSGP4 (l, g)+ HTLE 30-day propagation

SGP4 vs numerical integration and HSGP4 vs numerical integration errors



Conclusions

- Apply the hybrid methodology to SGP4 in order to develop an HSPG4
- **HTLE** = TLE + $\{A_1, B_1, S_{-s+2}, \ldots, S_0, S_1\}_x$ with $x \in \{l, g, h, L, G, H\}$
- Apply HSGP4 to DEIMOS-1

  The distance error of HSGP4 after **30 days, 41.1 km**, is equivalent to SGP4 error after only **2.9 days**