Hybrid SGP4: tools and methods

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6th International Conference on Astrodynamics Tools and Techniques (ICATT)
Darmstadt, March 14-17, 2016
Motivation

Hybrid Orbit Propagator

HSGP4 Orbit Propagator

Hybrid Two-Line Element (HTLE)

Conclusions
Motivation

How to improve the features of a Special perturbation, General perturbation or Semi-analytical method without loss of efficiency?

\[ \ddot{r} + \mu \frac{r}{r^3} = a_d \]

Statistical Time Series models or Computational Intelligence methods so as to model the perturbations and higher orders that are not considered in the Special perturbation, General perturbation or Semi-analytical methods.
**Improve** the features of SGP4 using hybrid methodology

The Simplified General Perturbations (SGP4) propagator is a standard AFSPACECOM propagator. It considers secular and periodic variations due to Earth oblateness, solar and lunar gravitational effects, gravitational resonance effects, and orbital decay using a drag model.
1. A set of precise observations: \( \{ x_t^O \}_{t=1}^T \)

2. A set of positions: \( \{ x_t^{OP} \}_{t=1}^T \)

Orbit propagator

Error prediction

Control period: error modeling

\( \{ \epsilon_t \}_{t=1}^T \) with \( \epsilon_t = x_t^O - x_t^{OP} \)

\( \hat{x}_f = x_f^{OP} + \hat{\epsilon}_f \)
1. **Statistical Time Series model**: Holt-Winters method

2. **Extend the TLE data**: Hybrid TLE (HTLE)

\[ \hat{\epsilon}_f^x | T = A^x + (f - 1)B^x + S^{x}_{(f-1) \mod s+1} \]
Initial conditions (TLE):

DEIMOS-1
1 35681U 09041A 11124.21233382 .00000325 00000-0 63164-4 0 9556
2 35681 098.0717 023.8270 0000845 081.0832 279.0474 14.69441166 94523

Precise observations, \( \{ \mathbf{x}_t^O \}_{t=1}^T \): pseudo-observations simulated by using the numerical integration of a full-force model, including JGM-3 60 \( \times \) 60 Earth gravitational potential, NRLMSISE-00 atmospheric drag, Sun and Moon 3rd-Body effect, solar radiation pressure including eclipses, Earth albedo, Earth IR, Earth solid tides, and relativistic effect (Elecnor Deimos)
Hybrid Two-Line Element: HTLE

\[ \{ \varepsilon^x_t \}_{t=1}^T \]

HOLT-WINTERS METHOD

\[ \{ \hat{\varepsilon}^x_t \}_{t=T+1}^\infty \] model

VALIDATION & VERIFICATION

HTLE:

TLE + H-W params. at t=1
1.- Holt-Winters method

Classic **Holt-Winters method**: 

\[ \varepsilon_t = \mu_t + S_t + \nu_t \]

- \( \mu_t \) is the trend
- \( S_t \) is the seasonal component
- \( \nu_t \) is the irregular component
1.- Holt-Winters method

Require: \( s, c, h, \) and \( \{\varepsilon_t\}_{t=1}^{T} \)

Ensure: \( \hat{\varepsilon}_{T+h T} \)

1: Estimate the values of \( A_0, B_0, S_{-s+1}, \ldots, S_{-1}, S_0 \)
2: \textbf{for} \( t = 1; \ t \leq T; \ t = t + 1 \) \textbf{do}
3: \[ A_t = \alpha(\varepsilon_t - S_{t-s}) + (1-\alpha)(A_{t-1} + B_{t-1}) \]
4: \[ B_t = \beta(A_t - A_{t-1}) + (1-\beta)B_{t-1} \]
5: \[ S_t = \gamma(\varepsilon_t - A_t) + (1-\gamma)S_{t-s} \]
6: \[ \hat{\varepsilon}_t = A_{t-1} + B_{t-1} + S_{t-s} \]
7: \textbf{end for}
8: Select \texttt{error.measure} \( \in \{\text{MSE, MAE, MAPE}\} \) and express it as a function of the smoothing parameters
9: Obtain the smoothing parameters that minimize \texttt{error.measure} using the L-BFGS-B method
10: Calculate \( A_T, B_T, S_{T-s+1}, \ldots, S_{T-1}, S_T \) for the optimal smoothing parameters
11: \[ \hat{\varepsilon}_{T+h T} = A_T + hB_T + S_{T-s+1+h \mod s} \]
12: return \( \hat{\varepsilon}_{T+h T} \)

[San-Martín, M., \textit{Métodos de propagación híbridos aplicados al problema del satélite artificial. Técnicas de suavizado exponencial [Ph.D. Thesis]}, University of La Rioja, 2014.]
2- Validation and verification: 2-day propagation

\{e_x\}, \{\tilde{e}_x\}

Time (days)

\(a\) error (km)

\(w\) error (degree)

\(e\) error

\(\Omega\) error (degree)

\(i\) error (degree)

\(M\) error (degree)
2.- Validation and verification: 30-day propagation

\{\varepsilon_x\}
\{\hat{\varepsilon}_x\}

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2.- Select the HTLE: minimize the distance error

**Table:** Distance error (km) of SGP4 versus HSGP4 modeling different sets of Delaunay variables

<table>
<thead>
<tr>
<th>Prop. span</th>
<th>SGP4</th>
<th>HSGP4(_{(l,g)})</th>
<th>HSGP4(_{(l,g,L,G)})</th>
<th>HSGP4(_{(l,g,h,L,G,H)})</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7 days</td>
<td>10.551</td>
<td>1.088</td>
<td>0.994</td>
<td>1.019</td>
</tr>
<tr>
<td>1 day</td>
<td>14.235</td>
<td>3.206</td>
<td>3.195</td>
<td>3.198</td>
</tr>
<tr>
<td>2 days</td>
<td>28.650</td>
<td>5.883</td>
<td>5.893</td>
<td>5.708</td>
</tr>
<tr>
<td>7 days</td>
<td>101.164</td>
<td>20.477</td>
<td>20.500</td>
<td>19.483</td>
</tr>
<tr>
<td>30 days</td>
<td>486.738</td>
<td>41.100</td>
<td>41.099</td>
<td>42.969</td>
</tr>
</tbody>
</table>
2.- Select the HTLE: minimize the distance error

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+ 

Holt-Winters parameters for $\hat{\epsilon}_t^l$ and $\hat{\epsilon}_t^g$
HSGP4 \((l, g)\)+ HTLE 30-day propagation

SGP4 vs numerical integration and HSGP4 vs numerical integration errors

Distance error

Along-track error

Cross-track error

Radial error
Conclusions

- Apply the hybrid methodology to SGP4 in order to develop an HSPG4

\[ \text{HTLE} = \text{TLE} + \{A_1, B_1, S_{-s+2}, \ldots, S_0, S_1\}_x \text{ with } x \in \{l, g, h, L, G, H\} \]

- Apply HSGP4 to DEIMOS-1

The distance error of HSGP4 after 30 days, 41.1 km, is equivalent to SGP4 error after only 2.9 days