TRAJECTORY DESIGN TOOLS FOR LIBRATION AND CISLUNAR ENVIRONMENTS

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ABSTRACT

Innovative trajectory design tools are required to support challenging multi-body regimes with complex dynamics, uncertain perturbations, and the integration of propulsion influences. Two distinctive tools, Adaptive Trajectory Design and the General Mission Analysis Tool have been developed and certified to provide the astrodynamics community with the ability to design multi-body trajectories. In this paper we discuss the multi-body design process and the capabilities of both tools. Demonstrable applications to confirmed missions, the Lunar IceCube CubeSat mission and the Wide-Field Infrared Survey Telescope Sun-Earth L_2 mission, are presented.

1. INTRODUCTION

The Sun-Earth libration and cislunar environments are challenging regimes for trajectory designers, with complex multi-body dynamics, perturbation modeling, and the addition of propulsive acceleration. Leveraging research on dynamical systems theory, several tools with applications to libration orbits and cislunar regions have been developed in cooperation between NASA's Goddard Space Flight Center and Purdue University [1,2,3]. One of these innovative tools, Adaptive Trajectory Design (ATD), is being used in conjunction with NASA-developed software, the General Mission Analysis Tool (GMAT), to design multi-body transfer trajectories for the upcoming Lunar IceCube CubeSat mission and the Wide-Field Infrared Survey Telescope (WFIRST) Sun-Earth L_2 mission [4,5]. As a payload deployed by the Exploration Mission-1 (EM-1) on the maiden flight of NASA's Space Launch System, Lunar IceCube will use a lunar-gravity assisted, multi-body transfer trajectory and an RF Ion engine to achieve lunar capture and delivery to the science orbit. WFIRST, however, is designed to employ a Sun-Earth L_2 quasi-halo orbit that will enable wide-field imaging and near-infrared sky surveys.

Trajectory design in support of lunar and libration point missions is becoming increasingly challenging as more complex mission designs are envisioned. To lessen these challenges, trajectory design software must incorporate an improved understanding of the Sun-Earth/Moon dynamical solution space as well as new numerical methods. As an example of the utility of enhancements to the trajectory design process, invariant manifolds, derived from dynamical systems theory, have been previously incorporated into the trajectory design of the Acceleration, Reconnection, Turbulence and Electrodynamics of the Moon's Interaction with the Sun (ARTEMIS) mission and the James Webb Space Telescope mission [6],[7]. The dynamical systems approach supplies insight into the natural dynamics associated with multi-body systems. In fact, such information enables a rapid and robust methodology for libration point orbit and transfer design when used in combination with numerical techniques such as targeting and optimization.

Strategies that offer interactive access to a variety of solutions enable a thorough and guided exploration of the trajectory design space. ATD is intended to provide access to well-known solutions that exist within the framework of the Circular Restricted Three-Body Problem (CR3BP) by leveraging both interactive and automated features to facilitate trajectory design in multi-body regimes [8]. In particular, well-known solutions from the CR3BP, such as body-centered, resonant and libration point periodic and quasi-periodic orbits, as well as any associated manifolds, are straightforwardly accessed and assembled using ATD. A screenshot of the CR3BP design environment within ATD appears in Figure 1, along with an interactively generated set of stable (green) and unstable (magenta) manifolds associated with a Sun-Earth L_1 Lyapunov orbit (blue). Furthermore, conic arc approximations may be constructed and imported. Each of these solutions may be incorporated via point-and-click arc selection, clipped to isolate trajectory segments, and connected using a corrections algorithm and impulsive maneuvers. Trajectories that traverse between multi-body systems may also be designed using a patched CR3BP approach. Once a trajectory has been assembled in the CR3BP within the ATD design modules, it can be corrected within an ephemeris model and then transferred to operational, high-fidelity software like GMAT.

Improved flexibility in trajectory design tools is essential in accommodating increased complexity in mission requirements, and has proven invaluable in the transfer trajectory design process for both the Lunar IceCube and WFIRST missions. In particular, ATD and dynamical systems research is employed to identify feasible transfer regions that connect an energetic initial deployment state to a desired lunar science orbit by exploiting solar gravity. In addition, ATD's powerful Poincaré mapping and orbit generation capabilities are employed to identify WFIRST science orbits that satisfy the mission constraints. On-demand manifold generation is also employed to construct transfer trajectories that leverage natural manifold trajectories to deliver the spacecraft from a low Earth orbit to the final science orbit. For both mission applications, these interactive trajectory design utilities permit the use of periodic and quasi-periodic orbits, as well as manifolds, to construct trajectories within multi-body regimes. These capabilities within ATD, along with its interface to GMAT, are demonstrated via application to the Lunar IceCube and WFIRST missions.

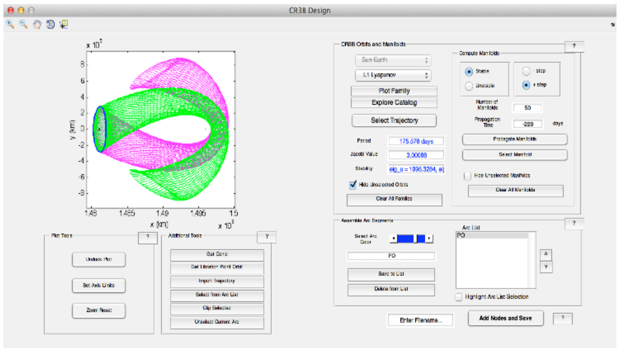

Fig. 1. ATD graphical interface with user-generated stable (green) and unstable (magenta) Sun-Earth L_1 Lyapunov manifolds.

2. TOOL AND TRAJECTORY DESIGN PREREQUISITES

Due to the chaotic nature of multi-body dynamical environments, trajectories must be modeled accurately. The software must integrate spacecraft trajectories precisely and model all accelerations including both impulsive and finite maneuvers. GMAT provides this capability by incorporating various high-order variable or fixed-step numerical integrators (e.g. Runge-Kutta, Bulirsch-Stoer, etc.). Precise force modeling includes an Earth and lunar gravity potential of up to 360 degree and order, solar radiation pressure, and multiple third-body perturbation effects. Trajectory targeting and optimization is accomplished by varying user-selected parameters to achieve the required goals. A differential corrector (DC) is routinely used to create continuous trajectories. These tools can incorporate B-plane and libration coordinate targets as well as intermediate targets such as Cartesian states, energy levels, and even stable and unstable mode directions. These software tools are also useful for prelaunch analysis and operations. In general, they include capabilities for maneuver and launch error analysis, launch window calculations, impulsive and finite maneuver modeling, and ephemeris generation.

2.1. Libration and Lunar Encounter Numerical Trajectory Design

In addition to a dynamical systems approach, the designer can also leverage numerical methods to compute and refine individual point solutions within an operational modeling environment [7]. In fact, trajectory design for lunar and libration orbit transfers and stationkeeping have been computed directly within ephemeris-based software such as GMAT and STK. These tools use a direct-shooting approach (forward or backward in time) as well as optimization techniques to compute trajectories that satisfy the mission constraints. These numerical methods employ partial derivatives, approximated via finite differencing, to iteratively refine an initial guess for a continuous trajectory. Additional constraints incorporated into the corrections procedure can include initial conditions, maneuver locations and/or direction, as well as orbital parameters such as period, position, velocity, and amplitude. A typical numerical targeting scenario for libration point orbit design within an operational modeling environment includes the following steps [7]:

- Target a trajectory energy that yields an escape trajectory towards a libration point with the Moon at the appropriate geometry
- Target the anti-Sun right ascension and declinations at the appropriate launch epoch
- Target the Solar-rotating coordinate system velocity of the Sun-Earth rotating coordinate *x-z* plane crossing condition to achieve a quasi-libration orbit, L_2 *x*-axis velocity ~ 0
- Target a second *x-z* plane crossing velocity which yields a subsequent *x-z* plane crossing, then target to a one-period revolution at L_2
- In all above conditions, vary the launch injection *C3* and parking orbital parameters (ω, Ω, parking orbit coast duration, and inclination)
- Incorporate conditions to achieve the correct orientation of the Lissajous pattern

Basic DC targeting procedures used to develop a baseline lunar gravity assist trajectory for a transfer trajectory to a Sun – Earth L_2 orbit include:

- Target the Moon at the appropriate encounter epoch to achieve an anti-Sun outgoing asymptote vector
- Target the lunar B-Plane condition to achieve gravity assist parameters and a perpendicular Sun-Earth rotating coordinate *x-z* plane crossing
- Target *x-z* plane crossing velocities which yield a second *x-z* plane crossing and target to a one-period revolution at L_2
- Re-target lunar B-plane conditions to achieve the correct orientation of the Lissajous pattern with respect to the ecliptic plane

In both scenarios, target goals may include time (epoch, burn durations, and flight time), B-plane conditions (B.T B.R angle, B magnitude, outgoing asymptote vector and energy), libration Sun-Earth line crossing conditions (position, velocity, angle, energy, or mathematical computation of quantities such as eigenvectors), or other parameters at intermediate locations that are often used in the targeting process. Targets may be defined as a single event string, nested, or branched to allow repeatable targeting. Additionally, maneuvers can be inserted where appropriate. In fact, retargeting conditions via the addition of deterministic maneuvers can be used to achieve the correct orientation and Lissajous pattern size with respect to the ecliptic plane. This targeting procedure must be repeated for significant changes in the launch date or to include lunar phasing loop strategies.

Although point solutions can be computed within an operational modeling environment, this process is not well suited to rapid redesign to accommodate changing requirements. In particular, the numerical computation of individual end-to-end trajectories may not supply the intuitive understanding of the solution space necessary for the designer to make well-informed decisions throughout the trajectory design process. To facilitate the design of trajectories that satisfy mission constraints prior to differential corrections, the application of a dynamical system approach is incorporated into the overall trajectory design process.

3. ADAPTIVE TRAJECTORY DESIGN TOOL

Incorporating dynamical systems theory into a trajectory design environment supports rapid and efficient exploration of the complex solution space within chaotic multi-body systems [9,10]. During the last two decades, NASA GSFC and Purdue University have been proactive in exploiting dynamical systems techniques in the design of trajectories that enable missions to achieve complex scientific and technology demonstration objectives. ATD is an interactive design environment for constructing end-to-end trajectories within the Sun-Earth/Moon system using proven dynamical systems techniques. Implemented as a graphical user interface (GUI), ATD includes both interactive and automated modules for trajectory selection and corrections within the CR3BP and an ephemeris model. In fact, ATD provides the capability to select individual arcs, including periodic and quasi-periodic orbits, manifolds, and conics via on-demand trajectory generation and Poincaré mapping. This software was developed under the FY12 and FY13 NASA GSFC Innovative Research and Development programs. Currently, ATD is used by NASA GSFC to support Earth-Moon libration point orbit missions and other missions in cislunar space including the Transiting Exoplanet Survey Satellite mission, to investigate Earth-Moon habitat options, and to assist in the evaluation of orbit selection for the Asteroid Redirect Mission.

By leveraging dynamical systems theory, the trajectory design process can exploit a better understanding of the design space than a set of individual solutions. Existing commercial and NASA software trajectory design tools, such as STK/Astrogator and GMAT, are typically designed to deliver point solutions and mission support capabilities. In contrast, ATD allows trajectory segments to be generated and selected both in different frames (inertial, rotating, libration point) and models (conic, restricted three-body, ephemeris). Each of these individual arcs can then be connected to exploit the underlying natural dynamics within various regions and transitioned to a higher-fidelity ephemeris model via an interactive differential corrections process. The final trajectory can then be imported into GMAT for further analysis. The GSFC supported ARTEMIS mission successfully leveraged a similar trajectory design process for each segment of the trajectory, i.e., near Earth, Sun-Earth, and Earth-Moon. Each segment was individually selected and then connected to produce a continuous end-to-end trajectory. Knowledge of the underlying dynamics within multi-body regimes can also aid in the redesign of spacecraft trajectories as mission constraints and deployment conditions vary, potentially alleviating the excessive computational and time requirements associated with searching for a new point solution in a dynamically sensitive environment. Furthermore, the availability of a large assortment of orbits and trajectories within one mission design environment enables the user to efficiently construct and explore the design space of orbit options that satisfy a given set of mission requirements.

The models leveraged by trajectory design tools can impact the quality and accuracy of the design as well as the computational time associated with each simulation. At the beginning of the design process, a lower-fidelity model, such as the CR3BP, supplies an accurate and rapid assessment of the design space and facilitates the generation of an initial guess prior to corrections in a higher fidelity model. Within the ATD design environment, dedicated modules allow the user to compare and select periodic and quasi-periodic orbits that exist in the CR3BP, providing insight into the predicted transfer and station-keeping costs, stability, and geometrical properties of candidate orbits prior to analysis in an ephemeris environment.

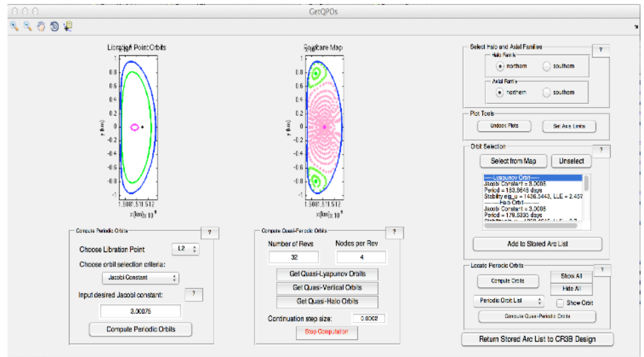

Fig. 2. Poincare mapping interface to identify periodic and quasi-periodic orbits available within ATD.

3.1. Poincaré Maps

ATD includes a module that employs Poincaré mapping, a technique from dynamical systems theory, to visualize and locate a wide variety of complex solutions near the libration points [11]. Poincaré mapping allows the designer to calculate large regions of trajectories and record their intersections with a hyperplane. These intersections can be represented on a lower-dimensional map to identify periodic and quasi-periodic orbits, visualize manifolds, and even design connections between trajectories. An example of the Poincaré mapping module within the ATD design environment appears in Figure 2, featuring a map that captures both periodic and quasi-periodic motion in the CR3BP near the Sun-Earth L_2 libration point. Such analysis via Poincaré mapping has been valuable to the design of transfers for both the Lunar IceCube mission and WFIRST.

3.2. Reference Catalog

To guide the orbit selection process, Purdue University and GSFC have incorporated an interactive reference catalog into ATD [12, 13]. This catalog provides the user with a guided approach to selecting libration, resonant and body-centered periodic and quasi-periodic orbits. Leveraging the autonomous CR3BP, the catalog contains a wide variety of orbits that have been generated and characterized. User-constructed trade spaces enable the identification of candidate orbits that satisfy mission constraints in the form of stability, transfer and station-keeping costs and geometry. Candidate orbits can then be exported to ATD and combined to produce a continuous trajectory or used to compute any associated manifolds. Screenshots of this interactive reference catalog interface and output are presented in Figures 3 and 4.

4. GENERAL MISSION ANALYSIS TOOL

GMAT was conceived and developed by an experienced team of aerospace engineers and software designers at NASA GSFC as an open-source high-fidelity space mission design tool [3]. This operational modeling environment supports trajectory design in cislunar and interplanetary space. Within the GMAT software package, users can leverage capabilities such as the interactive graphical interface, scripting, access to high-fidelity dynamical

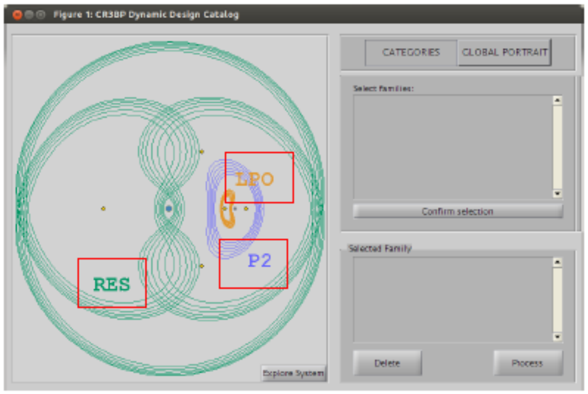

Fig. 3. Main panel of the interactive reference catalog within ATD.

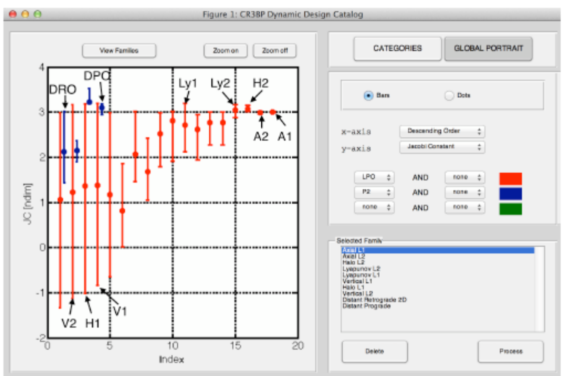

Fig. 4. Reference catalog interface to compare families of orbits within a user-defined trade space.

models, targeting and optimization algorithms, as well as plots and reports for analysis. The trajectory designer can perform complex design and analysis by first creating and customizing spacecraft, dynamical models, propagators, and targeters. Additional options exist to configure spacecraft dimensions, thrusters, tanks, impulsive and finite burns, additional celestial bodies, and coordinate systems. Furthermore, users can incorporate differential correctors, optimizers, and custom subroutines into the trajectory design process, while also exporting orbital parameters to data files. Figure 5 illustrates a recent application using GMAT to compute a trajectory solution that leverages a low-thrust propulsion system for a lunar CubeSat mission.

5. MISSION DESIGN APPLICATIONS

To demonstrate the use of ATD and GMAT as well as their graphical interfaces, these software tools are used to design trajectories for both the Lunar IceCube and WFIRST missions. Furthermore, the roles of well-known orbits in facilitating transport within multi-body systems are shown, emphasizing the value in design tools that enable rapid and well-informed trajectory design.

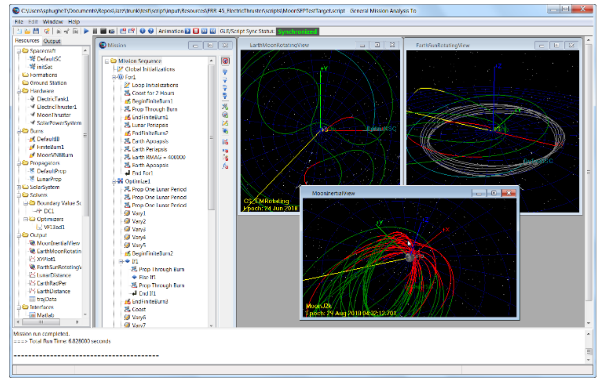

Fig. 5. GMAT graphical interface.

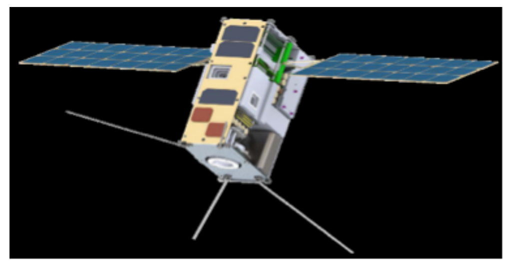

Fig. 6. Lunar IceCube spacecraft design.

5.1. Lunar IceCube Application

Lunar IceCube, a 6U CubeSat depicted in Figure 6, has been selected for participation in the Next Space Technologies for Exploration Partnerships, which leverages partnerships between public and private entities to develop the deep space exploration capabilities necessary for the next steps in human spaceflight. The Lunar IceCube mission is led by the Space Science Center at Morehead State University and supported by scientists and engineers from the NASA GSFC, Busek, and Catholic University of America. GSFC is providing the trajectory design, maneuver and navigation support, as well as tracking support.

Lunar IceCube will ride onboard the Orion EM-1 vehicle, currently scheduled for launch in 2018. Secondary payloads are deployed after the Interim Cryogenic Propulsion Stage (ICPS) disposal maneuver, placing Lunar IceCube on a high-energy trajectory. Due to uncertainties in the ejection mechanism, Lunar IceCube's exact deployment state is not known in advance. However, with no additional maneuvers, the highly energetic nominal deployment state would result in Lunar IceCube quickly departing the Earth-Moon system. To decrease the spacecraft energy and achieve a transfer that approaches a low-altitude lunar orbit, the Lunar IceCube spacecraft is equipped with a low-thrust propulsion system. This iodine-fueled engine is a Busek Ion Thruster 3-cm (BIT-3) system, which is currently designed to deliver a maximum 1.2 mN of thrust with an I_{sp} of 2500s and a fuel mass of approximately 1.5kg. For the Lunar IceCube mission, the BIT-3 system enables finite-duration low-thrust arcs to be leveraged along a transfer trajectory that connects the initial high-energy deployment state to the final lunar science orbit.

5.1.1. Designing the Lunar IceCube Trajectory

Although feasible end-to-end transfers may be obtained within an operational modeling environment, a combined dynamical systems and numerical approach offers significant insight into the available transfer geometries and the corresponding regions of existence, which can be incorporated into the design process [4]. Individual point solutions may be highly sensitive to uncertainties in the deployment state and epoch, as well as any additional on-orbit perturbations. In fact, for relatively large perturbations in the deployment or flyby conditions, Lunar IceCube may not possess sufficient propulsive capability to achieve the desired reference trajectory. Alternatively, another transfer geometry may provide an operationally feasible solution. To facilitate the identification and computation of these solutions, a trajectory design framework is constructed and demonstrated using ATD. First, the complete transfer trajectory is split into three segments: the post-deployment lunar encounter, the Sun-Earth-Moon transfer, and the lunar science orbit approach. Concepts from dynamical systems theory are applied over each segment to models of varying levels of fidelity, from the CR3BP to an ephemeris model. In addition, mapping techniques are employed to identify connections between each trajectory segment [4]. Using the resulting analysis, a reasonable initial guess is obtained for corrections in an ephemeris model to obtain a high-fidelity, low-thrust-enabled, end-to-end transfer in GMAT.

5.1.2. Sample Lunar IceCube Transfers

To validate the overall design process for trajectories that meet the spacecraft constraints of mass, area, propulsion capability and thrust levels, several point designs have been numerically generated using an operational-level modeling environment. One sample transfer, depicted in Figure 7, features a long predominantly natural segment, indicated by blue arcs, that resembles the Sun-Earth L_1 Lyapunov manifold structures. By including finite duration burns, colored red, this natural motion can be adjusted to ensure capture into a lunar science orbit. This design uses an EM-1 launch epoch of December 15, 2017 and the post-ICPS deployment state made available at the time of the Lunar IceCube proposal. The post ICPS deployment information will be updated once the EM-1 design has been finalized, thus requiring a redesign of the trajectory and an understanding of the transfer trajectory trade space. Redesign may also be required following a significant perturbation to the outbound lunar flyby conditions. Rather than targeting back to the original reference trajectory, displayed in Figure 7, an alternative transfer geometry can be incorporated into the design process, providing a feasible flight time and propellant mass budget. Using ATD, alternative manifold structures from the CR3BP can be identified and incorporated into the design of a trajectory that connects the lunar encounter to the final science orbit.

5.1.3. Manifolds of Periodic Orbits

Motion within the CR3BP is guided by an underlying dynamical structure that includes families of periodic orbits and their associated manifolds. In the Sun-Earth system, well-known periodic orbits in the Earth vicinity include the planar Lyapunov

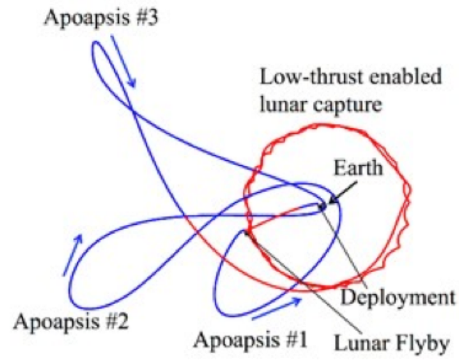

Fig. 7. Sample Lunar IceCube trajectory design in the Sun-Earth rotating frame within an ephemeris model including natural (blue) and low-thrust (red) arcs.

and three-dimensional halo orbits near the L_1 and L_2 equilibrium points. Both of these families include periodic orbits that possess stable and unstable manifolds, causing nearby trajectories to naturally flow towards or away from the periodic orbit, respectively. Within these manifolds, trajectories can pass through the L_1 and L_2 gateways, departing the Earth vicinity. For planar motion, the manifold structures associated with the L_1 and L_2 Lyapunov orbits serve as separatrices, identifying the boundary between two types of motion that are qualitatively different. To demonstrate this concept, consider Figure 8 which displays a sample (a) stable manifold in blue and (b) unstable manifold in green associated with a Sun-Earth L_1 Lyapunov orbit, as generated in ATD. Using Figure 8(a) as a reference, trajectories on the blue surface lie directly on the stable manifold, which has been integrated backwards in time in the Sun-Earth CR3BP for approximately 210 days. Accordingly, these trajectories asymptotically approach the reference L_1 Lyapunov orbit. Motion that possesses both position and velocity states that lie within the boundaries of the blue manifold surface pass through the L_1 gateway and depart the Earth's vicinity. When designing CubeSat trajectories that are close to planar, the stable manifolds of the L_1 Lyapunov orbit can supply approximate bounds on motion, i.e., regions within the stable manifold must be avoided to ensure that a trajectory does not depart the Earth vicinity. Furthermore, this structure may influence motion near the Earth after deployment. On the contrary, motion on the green surface in Figure 8(b) lies on the unstable manifold associated with the L_1 Lyapunov orbit, which is integrated forward in time for 210 days. Trajectories interior to the boundaries of this manifold structure originate from the vicinity of the Sun. However, the unstable manifold may still guide motion that flows towards the Earth. In fact, arcs from both of these manifold structures may be combined to identify nearby trajectories that temporarily depart the Earth vicinity and achieve the necessary energy and phasing parameters to reach the desired lunar science orbit. Although these structures exist in the simplified and autonomous CR3BP, they are approximately retained in the true ephemeris model of the Sun, Earth and Moon, providing rapid and valuable insight into the existence and the associated boundaries for predominantly natural transfer geometries for the Lunar IceCube mission.

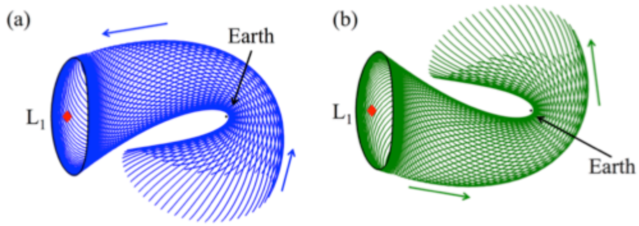

Fig. 8. (a) Stable and (b) unstable manifolds generated in ATD for Lunar IceCube transfer trajectory design.

5.1.4. Feasible Transfer Regions

Techniques from dynamical systems theory are applied to the construction of Earth apoapsis maps, which facilitate the identification of feasible transfer regions and their associated geometries for the Sun-Earth-Moon segment of the Lunar IceCube transfer trajectory [4]. To demonstrate this process, consider an apoapsis map constructed using prograde initial conditions, i.e., counter-clockwise motion about the Earth, at a Jacobi constant of C = 3.00088 for trajectories that complete two revolutions around the Earth, as depicted in Figure 9. The gray-shaded portions of the figure indicate forbidden regions, while red diamonds locate the equilibrium points, the light blue circle at the center indicates the location of the Earth and the purple curve depicts the lunar orbit, approximated as circular. On this apoapsis map, apoapses for each feasible transfer region are colored by the geometry of the subsequent transfer path, determined using the velocity direction at each apoapsis, i.e. prograde or retrograde. For instance, red regions in Figure 8 indicate transfers that possess two subsequent apoapses that are retrograde, such as the transfer displayed in the bottom right inset. This feasible transfer region lies close to the zero velocity curves of the CR3BP and the transfers resemble the sample end-to-end trajectory in Figure 7, constructed as a point solution using an operational modeling environment. When supported by concepts from dynamical systems theory, apoapsis maps also supply insight into some preliminary bounds on the feasible regions of motion near the Earth. For instance, the white region in the lower left quadrant of Figure 9 is contained within the curve corresponding to the first apoapses along the Sun-Earth L_1 Lyapunov stable manifold. Specifically, each apoapsis within this white region quickly departs the Earth vicinity through the L_1 gateway. As the model fidelity is improved, these preliminary bounds and feasible transfer regions may be shifted and distorted within the phase space. However, knowledge of these regions corresponding to dynamical structures in the CR3BP may supply preliminary insight into the sensitivity of any nearby trajectories and facilitate exploration of the available transfer geometries.

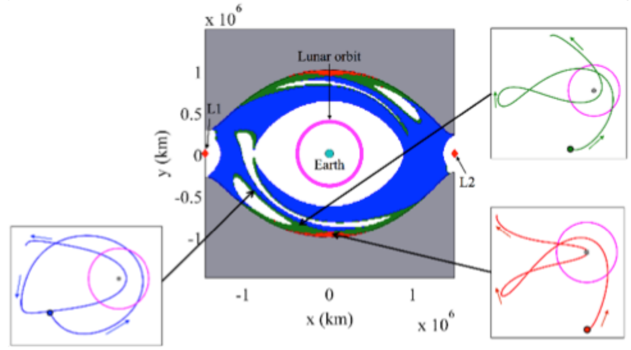

Fig. 9. Sample apoapsis map in the CR3BP for prograde initial conditions, with colored regions differentiating transfer geometries as illustrated via the inset images.

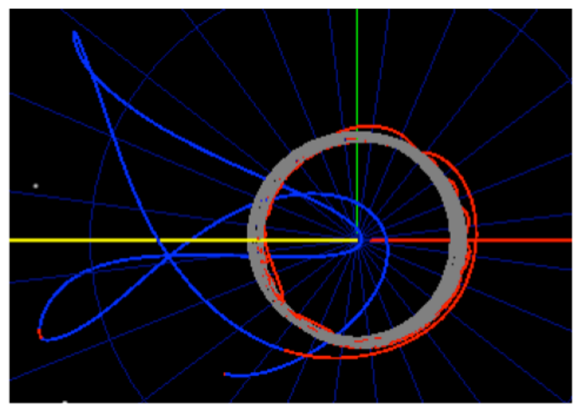

Fig. 10. Corrected sample transfer in ephemeris incorporating natural (blue) and low-thrust (red) arcs.

5.1.5. Connections between Transfer Segments

By leveraging ATD and the outlined mapping strategies, a sample trajectory can be constructed by directly selecting and assembling each of the three trajectory segments prior to corrections in an ephemeris model [4]. Consider the point solution identified using operational-level software, as depicted in Figure 7. This sample solution can be approximately reconstructed by employing apoapsis maps, which guide the trajectory design process. For instance, the apoapsis map in Figure 9 can be used to locate the desired geometry for the Sun-Earth segment of the trajectory, corresponding to the red feasible transfer regions. These maps can also be transitioned to a higher fidelity model such as the bicircular restricted four-body problem to incorporate lunar gravity [4]. These apoapsis maps are also constructed for the remaining trajectory segments and overlaid to enable the assembly of a reasonable initial guess. This initial guess can then be corrected in an ephemeris model and exported to GMAT to produce a continuous transfer, as displayed in Figure 10, incorporating both natural (green) and low-thrust (red) arcs. Using techniques from dynamical systems theory, feasible transfers exhibiting various geometries can be constructed rapidly and efficiently for constrained secondary payloads.

5.2 WFIRST Application

WFIRST is a NASA-led observatory mission currently in the preliminary design development stage. The WFIRST mission concept is designed as a six-year mission, intended for launch in 2024, to perform observations that will study dark energy, the origin and evolution of the universe and exoplanets that may harbor life, while also supporting a guest observer program. To achieve these scientific objectives, WFIRST is designed to leverage an existing 2.4 meter telescope, as well as a wide field instrument that possesses a field of view 100 times wider than the Hubble Space Telescope, and a coronagraphic instrument. This mission involves a partnership between NASA GSFC and NASA Jet Propulsion Laboratory, with NASA GSFC's Navigation and Mission Design Branch providing trajectory design, maneuver, navigation and tracking support.

5.2.1. Orbit Selection

To achieve the primary objectives of the WFIRST mission concept, a baseline scientific orbit in the Sun-Earth L_2 region is currently of interest. While meeting the thermal and dynamical requirements of the science instruments, the orbit and transfer trajectory must satisfy additional geometrical constraints. In particular, the selected libration point orbit must continually avoid Earth shadow and maintain a Sun-Earth L_2 to vehicle (SEL2V) angle that is less than 36°. An SEL2V angle that is greater than 36° would cause the spacecraft to lie too far below or above the horizon during the summer and winter seasons, potentially impeding clear communications with ground stations. These two requirements can each be translated into constraints on the geometry of the selected libration point orbit. For instance, the communications-driven requirement that the maximum SEL2V angle remain below 36° can be translated into an approximate constraint on the maximum *y*- and *z*- amplitudes of the orbit. As depicted in Figure 11, a 36°-angle cone beginning at the Earth and centered on the *x*-axis possesses a radius of 1.09 x 10^6 km at L_2. Thus, the maximum *y*- and *z*- amplitudes of a candidate L_2 orbit in a rotating libration point (RLP) frame can be approximated as 1.09 x 10^6 km. Next, to avoid Earth shadow, the spacecraft must remain outside of the umbral and penumbral shadows. These shadow regions are depicted in Figure 12 with the Sun, Earth and L_2 located along the *x*-axis, the umbral shadow shaded grey and the penumbral shadow colored blue. By trigonometry, the tip of the umbral shadow cone intersects the *x*-axis to the left of L_2. Accordingly, the penumbral shadow cone dominates characterization of the boundaries of the Earth shadow region near L_2. As portrayed in Figure 12, trigonometry can be used to determine the radius of the penumbral shadow cone at L_2 to equal approximately 13,000 km. This Earth shadow avoidance constraint can be incorporated into the orbit selection process by ensuring that any candidate orbits in the CR3BP do not pierce a 0.51° cone centered along the Earth to L_2 line as depicted in Figure 11. These constraints are applied to candidate orbits in the Sun-Earth CR3BP within ATD to enable thorough and guided exploration of the orbit design space prior to higher-fidelity modeling.

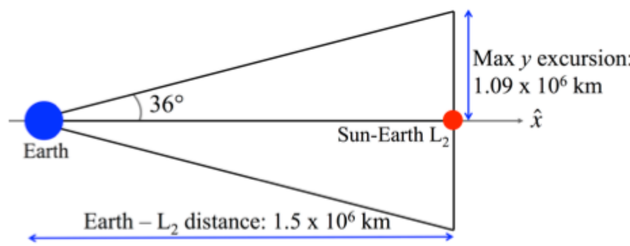

Fig. 11. Translation of maximum SEL2V angle requirement to constraint on y-amplitude of libration point orbit in the Sun-Earth rotating frame.

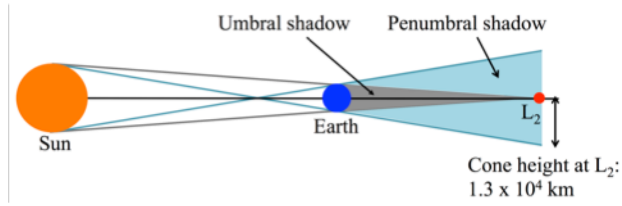

Fig. 12. Translation of Earth shadow avoidance requirement to maximum cone angle constraint.

Within ATD, libration point orbits in the Sun-Earth L_2 region can be generated and examined, along with their potential to satisfy geometrical constraints [14]. Using an ephemeris-level modeling environment to search for and identify an orbit that satisfies these constraints would be challenging, time-consuming and potentially limited in scope. However, directly examining families of periodic orbits from the CR3BP, which are approximately retained in the true ephemeris model, provides valuable guidance into the orbit selection process. Orbits that exist within the L_2 vicinity include the Lyapunov, halo, vertical and axial periodic orbits, along with any associated quasi-periodic motion. In ATD, orbits along each of these families can be explored and evaluated using defined geometrical constraints. First, the Lyapunov family of orbits, which exist solely within the *x-y* plane of a RLP frame, would violate the Earth shadow avoidance constraint every half period. Similarly, vertical orbits, which evolve from small three-dimensional figure-eight shaped orbits near L_2 to large orbits that extend towards L_3, pierce the *x*-axis of an RLP frame and violate the Earth shadow avoidance constraint twice per orbit. Next, axial orbits, which evolve from a planar Lyapunov orbit to a vertical orbit, violate both the Earth shadow avoidance and maximum *z*-amplitude constraints. However, halo orbits with low z-amplitudes in the RLP frame satisfy both WFIRST orbit constraints. In fact, an L_2 halo orbit is leveraged as a candidate science orbit [14]. Nearby quasi-halo orbits may also satisfy the geometrical orbit constraints. An alternative option may include L_2 Lissajous orbits, which are three-dimensional quasi-periodic orbits that regularly pierce the Earth shadow region. While orbit segments may be selected to avoid this region for at least six years in the CR3BP, additional shadow avoidance maneuvers may be required in an ephemeris model.

5.2.2. Transfer Trajectory Design

A transfer trajectory between the Earth and the selected L_2 halo is designed in ATD to actively leverage the underlying natural dynamics of the Sun-Earth system [14]. Within the ATD design environment, trajectory segments are individually generated, clipped and ordered prior to connection using a corrections algorithm. First, the initial LEO, attained via launch from Cape Canaveral and constrained to possess an altitude of 185 km and an inclination of 28.5 degrees, is generated in ATD using the conic import feature. Next, the desired Sun-Earth L_2 halo is input from the orbit selection module and loaded into the main ATD design environment. The selected orbit is automatically analyzed to determine the orbital period, Jacobi constant, and the presence of any stable and unstable modes. Natural motion is then sought to construct a transfer trajectory that connects the LEO to the WFIRST science orbit.

Since the selected Sun-Earth L_2 halo is unstable, it possesses both stable and unstable manifolds that can be incorporated into the transfer trajectory design process within ATD to reduce the maneuver cost [14]. Using the manifold generation tool within the CR3BP Design module, 50 trajectories along the stable manifold are integrated backwards in time for a duration of 250 days as portrayed in Figure 13. In this figure, the Sun-Earth L_2 halo is colored blue while trajectories along the stable manifold are displayed in green. To incorporate a stable manifold trajectory into the transfer design, a segment that achieves a close pass to the previously generated LEO is sought. One candidate stable manifold trajectory is selected and appears in red in Figure 13. This trajectory is clipped to enable connection between the LEO and the Sun-Earth L_2 halo. The clipped stable manifold segment is incorporated into the transfer trajectory design process to deliver a spacecraft from the Earth vicinity to the Sun-Earth L_2 halo used during WFIRST scientific observations.

Each of the selected segments are combined and discretized within the CR3BP Design module within ATD to form an initial guess for an end-to-end trajectory [14]. Once the LEO, stable manifold and Sun-Earth L_2 halo segments are ordered correctly, additional revolutions of the halo are added to achieve the design mission lifetime. The resulting initial guess is then discretized and stored in a MATLAB file for corrections. First, the initial guess is corrected within the CR3BP Corrections module. Additional constraints are added to the trajectory, including the altitude of the LEO, and the periodicity and energy of the halo orbit. Allowable maneuver locations are also selected at the beginning and end of the stable manifold trajectory to enable connections to the initial and final orbits. The resulting end-to-end trajectory that is continuous in the CR3BP is then loaded into the Ephemeris Orbit Corrections module. In this environment, central and perturbing bodies can be selected (e.g. Earth, Sun, Moon) as well the initial epoch. In this ephemeris model, a multiple shooting algorithm corrects the trajectory subject to any user-selected constraints and maneuvers. The resulting end-to-end trajectory for WFIRST, constructed within ATD for an initial epoch of January 1, 2024, is displayed in Figure 14. In addition to saving this trajectory as a .mat file for further analysis, the ATD export function produces a script to propagate and target the computed trajectory within GMAT. In fact, when loading the baseline WFIRST trajectory from Figure 14 into GMAT, the resulting trajectory, depicted in

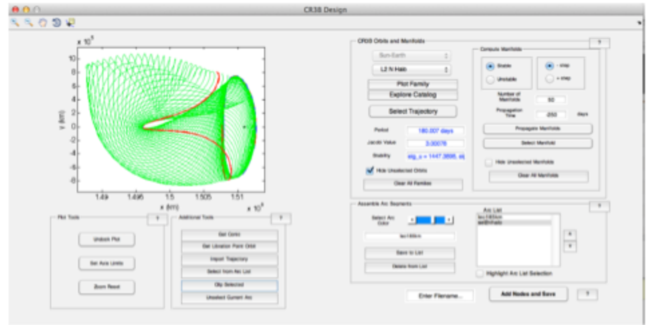

Fig. 13. SEL2 manifold surface (green) propagated in ATD with selected trajectory (red).

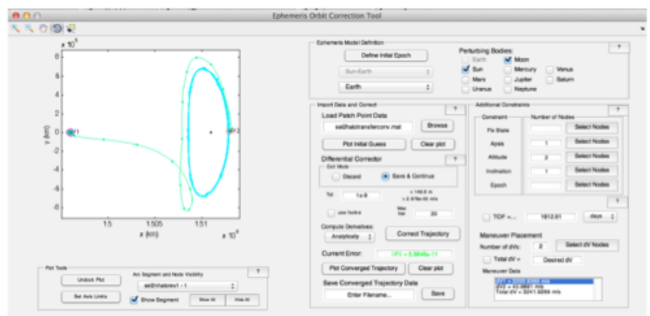

Fig. 14. End-to-end trajectory for WFIRST within an ephemeris model, designed using ATD.

Figure 15, possesses similar characteristics and maneuver estimates. Additional forces such as drag, SRP, and higher-order gravitational contributions can be included in GMAT, producing an operational-level modeling environment. The candidate transfer and science orbit can also be exported from GMAT into Analytical Graphics, Inc.'s Systems Tool Kit (STK) for verification within an operationally-proven modeling environment. Using the produced ephemeris data, along with a mission sequence and differential corrector, a similar trajectory is produced within STK. The resulting trajectory, depicted in green and red in Figure 16 and 17 closely resembles the transfer designed within ATD, colored magenta. Accordingly, a trajectory for the WFIRST mission is rapidly designed using ATD to leverage known dynamical structures that satisfy geometrical mission constraints. These selected trajectory segments are combined to produce an end-to-end trajectory that connects a low Earth orbit to the final science orbit [14]. The designed trajectory is transitioned to higher-fidelity models and imported to operational-level modeling software.

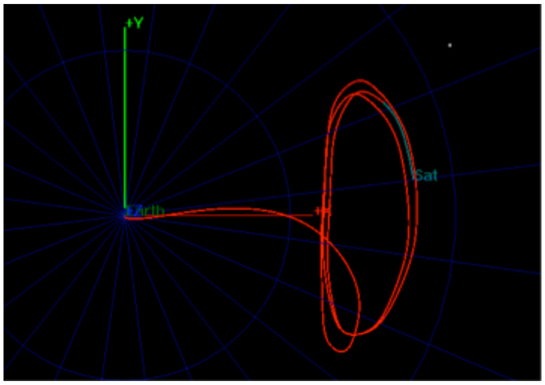

Fig. 15. Transfer trajectory propagated and differentially corrected in GMAT.

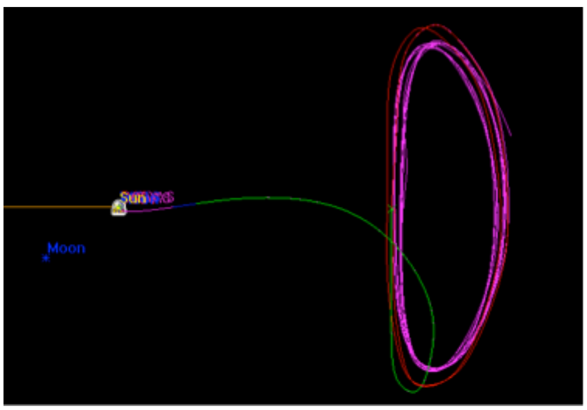

Fig. 16. Trajectory generated within ATD ephemeris (magenta) and STK (green and red), viewed in RLP frame.

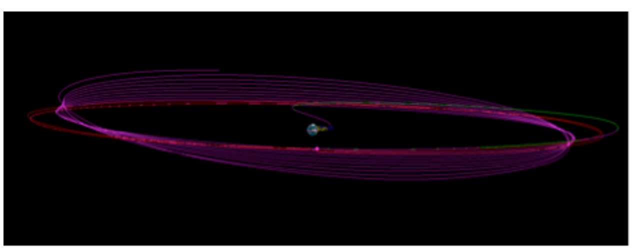

Fig. 17. Trajectory generated within ATD ephemeris (magenta) and STK (green and red) overlaid, as viewed looking down the Earth to L_2 line.

6. CONCLUDING REMARKS

ATD, a graphical design environment developed by NASA GSFC and Purdue University, along with operationally-proven modeling software such as GMAT, provides the mission designer with the capability to design complex trajectories within multi-body systems. An interactive design environment that leverages well-known solutions with the CR3BP enables an exploration of the trajectory design space, along with guidance into redesign for contingency studies. This design environment is demonstrated using two mission examples: the Lunar IceCube CubeSat mission and WFIRST. The Lunar IceCube mission, which is subject to constraints and uncertainties in its deployment state and a limited propulsive capability, benefits from the use of techniques from dynamical systems theory. Although feasible point solutions can be identified using operational-level modeling software, a dynamical systems approach supplies insight into the sensitivity of these paths and regions of availability for similar transfers. Such analysis is valuable for spacecraft that are unable to implement large corrective maneuvers to remain on a reference path. For Lunar IceCube, a flexible design process is constructed that enables rapid trajectory redesign to mitigate state uncertainties, orbit determination errors, and maneuver execution errors, as well as an understanding of the trajectory design space. Once a set of feasible connections has been identified, a corrections scheme may be applied to produce an end-to-end trajectory in operational-level software. In addition, the WFIRST trajectory design process leverages ATD to accurately and efficiently generate a transfer and science orbit that satisfies the mission requirements. In fact, ATD is used for orbit selection and manifold generation to produce a low cost transfer. The resulting trajectory is imported to higher-fidelity software such as GMAT and STK for further analysis. In each scenario, the use of ATD enables rapid and well-informed trajectory design that can provide solutions for further exploration in operational-level modeling tools.

7. ACKNOWLEDGEMENTS

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8. REFERENCES

[1] A. Haapala, K.C. Howell, and D.C. Folta, "Incorporating the Evolution of Multi-Body Orbits into the Trajectory Trade Space and Design Process," *65th International Astronautical Congress*, Toronto, Canada, October 2014.

[2] D. J. Conway, and S.P. Hughes, "The General Mission Analysis Tool (GMAT): Current Features and Adding Custom Functionality," *4th International Conference on Astrodynamical Tools and Techniques*, Madrid, Spain, May 2010.

[3] S. Hughes, *General Mission Analysis Tool. Available*: gmatcentral.org. Software package, 2016.

[4] D.C. Folta, N. Bosanac, A. Cox, and K.C. Howell, "The Lunar Icecube Mission Design: Construction of Feasible Transfer Trajectories With A Constrained Departure", *AAS/AIAA Space Flight Mechanics Meeting*, Napa, CA, February 2016.

[5] NASA WFIRST Website: http://wfirst.gsfc.nasa.gov/science/sdt_public/WFIRST-AFTA_SDT_Report_150310_Final.pdf, Accessed February 2016.

[6] D.C. Folta, M. Woodward, T. Sweetser, and S. Broschart, "Design and Implementation of The Artemis Lunar Transfer Using Multi-Body Dynamics" *AAS/AIAA Astrodynamics Conference*, Anchorage, AK, August 2011.

[7] D.C Folta, and M. Beckman, "Libration Orbit Mission Design: Applications of Numerical and Dynamical Methods", *Libration Point Orbits and Applications*, Girona, Spain, June 2002.

[8] A. Haapala, M. Vaquero, T. Pavlak, K.C. Howell, and D.C. Folta, "Trajectory Selection Strategy for Tours in the Earth-Moon System," *AAS/AIAA Astrodynamics Specialist Conference*, Hilton Head, SC, August 2013.

[9] K. Howell, B. Barden, and M. Lo, "Application of Dynamical System Theory to Trajectory Design for a Libration Point Mission", *Journal of the Astronautical Sciences*, Vol. 45 No. 2, 1997, pp. 161-178.

[10] J.J. Guzman, D. S. Cooley, K.C. Howell, and D.C. Folta, "Trajectory Design Strategies that Incorporate Invariant Manifolds and Swingby", *AIAA/AAS Astrodynamics Conference*, Boston, MA, August 1998.

[11] A.F. Haapala, and K.C. Howell, "Representations of Higher-Dimensional Poincaré Maps with Application to Spacecraft Trajectory Design," *63rd International Astronautical Congress*, Naples, Italy, October 2012.

[12] D.C. Folta, N. Bosanac, D. Guzzetti, and K.C. Howell, "An Earth Moon System Trajectory Design Reference Catalog", *Acta Astronautica*, Vol 110, May-June 2015, pp. 341-353.

[13] D. Guzzetti, N. Bosanac, A. Haapala, K.C. Howell, and D.C Folta, "Rapid Trajectory Design in the Earth-Moon Ephemeris System via an Interactive Catalog of Periodic and Quasi-Periodic Orbits," *66th International Astronautical Congress*, Jerusalem, Israel, October 2015.

[14] N. Bosanac, K.C. Howell, Internal Document, Purdue University and NASA Goddard Space Flight Center, "ATD Application: WFIRST," July, 2015.