PROPOSED ALGORITHM FOR ON-BOARD MANOEUVRES CALCULATION *Itziar Barat*⁽¹⁾, *Berthyl Duesmann*⁽²⁾

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ABSTRACT

The orbit control for LEO missions is becoming more and more demanding in terms of manoeuvring. This paper proposes a simplified algorithm in order to calculate on board the drag make-up manoeuvres.

It is not pretended here to give a full solution of the problem, but to show the concept and highlight the issues that need to be considered. This paper aims to be a starting point for future implementations.

The main rationale behind calculating the manoeuvres on-board, is to achieve a very tight orbit control with high accuracy and low operational load.

The proposed algorithm compares the actual time of ascending node crossing with respect to the reference one. The delta-time between them provides information of the current drift with respect to the reference ground-track while its increment allows to determine the altitude of the satellite with respect to the nominal one. When a threshold in the ground-track drift is reached a manoeuvre is triggered based on the current altitude.

Simplicity has been the maxima when deriving the concept, however more complex calculations can be implemented allowing applications for formation flying and low thrust transfer orbits.

Index Terms— Autonomous manoeuvre, orbit maintenance, repeat cycle, formation flying, low thrust.

1. INTRODUCTION

Most of the satellites in LEO orbits follow a repeat cycle, that is a ground-track that repeats after an integer number of days in the case of sun-synchronous orbits or not necessarily integer number of days in the case of no sun-synchronous orbits.

Ideally, if no perturbation other than the earth potential affects the satellite, it would repeat the reference orbit over and over without any manoeuvre being necessary. But in reality mainly due to the atmospheric drag and the third body perturbation (Sun and Moon) the satellite will deviate from its reference orbit. When a defined boundary is reached one or several manoeuvres are necessary to bring the satellite back to its reference position. The most standard way to define a boundary is in terms of ground-track dead band, either along the whole orbit or at the equator and the nodes. For this kind of control based on a ground-track dead-band, two kind of manoeuvres are implemented: Inclination correction manoeuvres, to control the dead-band at the highest latitudes, and drag make-up manoeuvres to control the dead-band at equator.

Inclination correction manoeuvres are very predictable and can be planned well in advance, but drag make-up manoeuvres depend on the orbital decay due to the atmospheric drag, that is extremely difficult to predict.

The classical drag make-up manoeuvre control cycle starts by placing the satellite higher than its nominal altitude and at the eastern boundary of the dead-band at the equator. While the altitude is higher than the nominal one the ground-track drifts towards the western boundary. The initial altitude is selected as when the western boundary is about to be violated, the satellite has already decayed to its nominal altitude, so when the altitude is lower than the nominal one the drift is reversed towards the eastern boundary. When the eastern boundary is about to be violated a new manoeuvre is performed placing the satellite again higher than its nominal altitude and a new manoeuvre cycle starts.



Figure 1 Classical dead-band control at equator

If the dead-band is too tight or the orbital decay too high, the frequency of the manoeuvres to keep the ground-track becomes very high, resulting in a high operational workload. For example, Sentinel-1 has a dead-band of ± 120 meters at equator, resulting in one manoeuvre a week, ref.[1]. In the case of EarthCARE more than the dead-band maintenance, what creates a disadvantage is the required size of the manoeuvres, as they induce vibrations in the satellite, preventing it to be back in the operational science mode during a long time. Smaller and more frequent (~daily) manoeuvres would mitigate this effect.

2. TIME OF ASCENDING NODE CROSSING

One of the characteristics of repeat cycle orbits is that they can be defined by a list of longitude of ascending nodes crossing (ANX) and/or it respective crossing times. If the orbit is perfectly sun-synchronous, the ANX time of each relative orbit will repeat exactly from one cycle to the next. If the orbit is not sun-synchronous, the ANX time will vary for each repeat cycle as a result of the MLST drift. In both cases the reference ANX times can be accurately predicted, upload on board and updated on a regular basis as needed.

The list of reference ANX times or longitudes shall be calculated taking into account the full earth potential and considering the MLST drift, but not the atmospheric drag..

2.1. Ground-track drift determination

Nowadays, it is possible to calculate on board the actual time at ANX with high precision, being an accuracy of less than five meters certainly achievable.

Calculating the difference from the reference to the actual ANX time is a basic operation that can be easily performed on board.

$$\Delta T_{anx} = T_{anx} - T_{anx \, ref} \quad (1)$$

The amount of information that can be derived from the ANX time difference by means of basic operations is sufficient to implement automatic on-board manoeuvres.

By just multiplying for the Earth velocity at equator the ground track drift is obtained

$$\Delta \lambda = \lambda - \lambda_{ref} = \Delta T_{anx} \cdot \omega_{\oplus} \quad (2)$$

If the satellite crosses the ascending node earlier than the reference, the ANX time difference, ΔT_{anx} will be negative. With this notation a positive $\Delta \lambda$ represents an actual ground-track Eastern than the reference one while a negative $\Delta \lambda$ represents an actual ground-track Western than the reference one.

Alternatively, the actual longitude of ascending node can be computed directly from the state vector at ANX. The advantage of this option is that the reference orbit is defined in terms of ANX longitudes instead of ANX times, and consequently independent of MLST drift. The drawback evidently is that more computation effort is needed onboard.

In both cases the position of the satellite with respect to its reference ground-track is known, and an alert can be triggered when the eastern boundary is about to be violated.

2.1. Altitude determination

The ground-track drift rate is directly proportional to the altitude of the satellite, to be more precise, it is related to the difference between the actual altitude and the reference altitude that would comply with the reference ANX longitudes and/or times. We will review this concept later, now we will focus on how to relate the ground track-drift with the altitude difference.

It is well know that the distance form one ANX to the consecutive one can be expressed as

$$\lambda_{n+1} - \lambda_n = (\omega_{\oplus} - \Omega) \cdot P_{\Omega} \quad (3)$$

And the variation of this distance as function of a semimajor axis increment can be expressed according to ref.[2]

$$\Delta(\lambda_{n+1} - \lambda_n) = \left[\left(\omega_{\oplus} - \dot{\Omega} \right) \cdot \frac{\partial P_{\Omega}}{\partial sma} - \frac{\partial \dot{\Omega}}{\partial sma} \cdot P_{\Omega} \right] \Delta sma \quad (4)$$

Where

$$\frac{\partial P_{\Omega}}{\partial sma} \cong 3\pi \sqrt{\frac{sma}{\mu}} \left[1 + \frac{J_2}{2} \left(\frac{R_{\oplus}}{sma} \right)^2 (4\cos^2(i) - 1) \right] (5)$$

And

$$\frac{\partial \dot{\Omega}}{\partial sma} \cong -\frac{7}{2} \frac{\dot{\Omega}}{sma} (6)$$

Therefore if we notate

$$\left(\omega_{\oplus} - \dot{\Omega}\right) \cdot \frac{\partial P_{\Omega}}{\partial sma} = k_1 \quad (7)$$
$$-\frac{\partial \dot{\Omega}}{\partial sma} \cdot P_{\Omega} = k_2 \quad (8)$$

Equation (4) can be written as

$$\Delta(\lambda_{n+1} - \lambda_n) = (k_1 + k_2) \cdot \Delta sma \quad (9)$$

Then, if the distance between two actual consecutives ANX's is compared with the distance of the corresponding reference ANX's, the difference between the actual and the

reference semi-major axis, $\Delta a = sma - sma_{ref}$, can be calculated. Mathematically it can be expressed as

$$(\lambda_{n+1} - \lambda_n) - (\lambda_{n+1\,ref} - \lambda_{n\,ref}) = \Delta\lambda_{(n+1)} - \Delta\lambda_n \quad (10)$$
$$\Delta\lambda_{(n+1)} - \Delta\lambda_n = (k_1 + k_2) \cdot \Delta a \quad (11)$$

Note that the resulting Δa is the average difference between the reference and actual semi-major axis over the whole orbit, from node n to node n+1. In a more generic way it can be also defined instead over one orbit, over m orbits as:

$$\Delta\lambda_{(n+m)} - \Delta\lambda_n = (k_1 + k_2) \cdot \Delta a/m$$
(11a)

Then the resulting Δa would be the average difference between the reference and actual semi-major axis over the m orbits, from node n to node n+m.

By using any of both methods proposed to derive the ground-track drift, the variation of the ANX longitude difference between orbit n and orbit n+1, can be obtained. Either directly measuring the $\Delta\lambda$ at two consecutive orbit ANX's, $\Delta\lambda_{(n+1)} - \Delta\lambda_n$, or by measuring the ΔT_{anx} and using equation (2),

$$\Delta\lambda_{(n+1)} - \Delta\lambda_n = (\Delta T_{anx \ (n+1)} - \Delta T_{anx \ n}) \cdot \omega_{\oplus} \ (12)$$

It is essential for this concept to work that the MLST drift needs has already been taken into account in case of defining the reference orbit in terms of ANX times instead that in longitudes.

As mention before, neither the actual semi-major axis nor the reference one are calculated, but the difference between the actual semi-major axis and the one that complies with the defined ground-track. Any change or drift in the inclination affecting the orbital period will result automatically in an intrinsic readjustment of the semi-major axis offset, in order to comply with the defined reference ground-track taking into account the new inclination conditions.

This is quite an advantage in orbit like GOCE or Cryosat-2, where the inclination is left to drift so in order to maintain the ground-track the reference semi-major axis needs to be constantly adjusted.

The disadvantage comes in orbits where inclination correction manoeuvres are performed, as they will result in a jump on the semi-major axis difference. Luckily this kind of manoeuvres occur no more than three or four times a year and are planned well in advance, thus the necessary measures can be arranged. Also these jumps are much smaller than the semi-major axis decay due to the atmospheric drag, and in most of the cases could be neglected.

It is worthy to mention that this algorithm was already successfully implemented on-ground to control GOCE ground-track, ref.[3]. GOCE was in a non-sun-synchronous orbit, which inclination was let to drift during its mission lifetime. By just defining the ANX's longitudes of the reference orbit and using the above concept, GOCE altitude was readjusted when necessary to compensate both, the error in the drag-free control system and the effect of inclination evolution on the orbital period.

3. OPERATIONAL IMPLEMENTATION

Previous section explained how the difference between the actual ground-track to the reference one and the difference between the actual semi-major axis to the reference one can be known on board by means of very simple calculations,

$$\Delta \lambda = \lambda - \lambda_{ref} = \Delta T_{anx} \cdot \omega_{\oplus} \quad (2)$$
$$\Delta \lambda_{(n+1)} - \Delta \lambda_n = (k_1 + k_2) \cdot \Delta a \quad (11)$$

The method this information is used to trigger and calculate the size of a manoeuvre is open to multiple options.

3.1. Manoeuvre Trigger

The conditions to trigger a manoeuvre seems clear, the difference between the actual ground-track and the reference one has reached a threshold, $\Delta \lambda \ge cte$, and the actual semimajor axis is lower than the reference one, $\Delta a < 0$.

A different approach could be to allocate predefined slots for the manoeuvres and define a criteria to resolve if to perform a manoeuvre in the current slot or wait to the next possible manoeuvre slot.

In both cases, attention should be paid to missions when the inclination drift may cause the reference semi-major axis to decrease faster than the natural semi-major axis decay due to the atmospheric drag.

3.2. Manoeuvre Size

The calculation of the size of the manoeuvre is a more complex problem, when many solutions are possible.

Following the algorithm to determine the manoeuvre size for ground-track control proposed by D.A. Vallado in ref.[2], the new altitude offset is given by:

$$\Delta a_{new} = \sqrt{\frac{4 \cdot P_{\Omega} \cdot Deadband}{(k_1 + k_2)}} \sqrt{\frac{da}{dt}} = k_3 \sqrt{\frac{da}{dt}}$$
(12)

Where $\frac{da}{dt}$ denotes the orbital decay.

Therefore the size of the manoeuvre is the difference of the new target altitude offset and the current one.

The remaining problem is then to estimate the orbital decay. As the main driver of implementing autonomous manoeuvres on board is missions requiring very often manoeuvres, it can be assumed that the atmospheric drag is nearly constant during the interval from one manoeuvre to the next. Consequently the decay occurred in the last hours can be extrapolated to the following ones and used for determining the size of the manoeuvre.

Alternatively a look-up table with the expected decay in the following days may be updated regularly on board.

The problem of predicting the orbital decay is not only related to on-board implementations, but it is also present when manoeuvres are calculated on-ground. The advantage of doing on-board, over doing on-ground is that the longer in future the prediction of the orbital decay is done, the less accurate it is. When a manoeuvre is calculated on-ground, from the moment the prediction is done to the moment the manoeuvre is performance, one or several days have passed. If the manoeuvre is calculated on board it will likely be performed on the very same orbit. Then the loss of accuracy of a simpler orbital decay determination can be compensated by the shorter in time applicability of this decay.

It shall be noted that the same mitigation measurement commonly applied on ground is valid also when the manoeuvres are calculated on board, that is targeting a closer Western boundary, or what it is equivalent, narrower the dead-band control with respect to the required one. If the actual decay is higher than the predicted one, the Eastern boundary will be reached again sooner than expected, and if on the contrary the actual decay is lower than the predicted one, the targeted Western boundary will be trespassed, but if the margin has been properly established, the required Western boundary won't be reached.

Additionally, it needs to be mentioned the accuracy on the orbit maintenance strategy is not reduced with time as the error committed when calculating one manoeuvre is not propagated to the next one. When the Eastern boundary is reached again, the inputs to calculate a new manoeuvre, i.e. ground-track position and altitude offset, are independent of any previous dead-band violation.

3.3. Eccentricity Control

Most of the missions require an eccentricity control around the frozen eccentricity. If the eccentricity vector were not perturbed, by just splitting the manoeuvre in two burns of same size at opposite positions in the orbit the problem would be solved.

But the eccentricity is perturbed, ref[4], and due to other constraints, it is not always possible to find opposite positions in the orbit to perform the manoeuvres.

The pre-selection of the possible slots to perform the manoeuvres and a pre-defined split of the manoeuvres in two burns shall be done in order to take into account the eccentricity control.

3.4. Space Debris

Another aspect to be considered is the space debris. With the classical on-ground manoeuvre calculation, the predicted orbit containing the manoeuvre is screened to reduce the risk of collision, what is not possible if the manoeuvres are calculated on-board.

A new approach needs to be investigated in order to warn for possible collisions. The straight forward would be to screen the reference orbit with a covariance equal to the allowed limits before a manoeuvre is trigger to drift back to the reference orbit.

It is not intended to propose any solution here but to highlight that an space debris screening approach shall be consider when implementing an on-board autonomous manoeuvre strategy.

4. LOW-TRHUST APPLICATIONS

So far it has been considered the case of satellites with chemical propulsion where impulsive manoeuvres are performed, but the same concept of comparing the ANX times or longitudes with respect to a reference can be applied to low thrust missions.

In fact, as mention before, the proposed algorithms were implemented, but on ground, for GOCE mission to adjust its altitude in order to follow a predefined ground-track, ref.[3].

Using an ion engine for compensating the along track non – gravitational forces, GOCE was the first European drag-free mission. Adjustments on its altitude were necessary on a regular basis to correct for the drag-free control errors and due to the evolution of its inclination that was not controlled. The determination of the ground-track drift and altitude offset were done as explained in this paper, only the correction manoeuvres were planed differently to cope on one side with the low-thrust capability and on the other with the drag-free profile of the mission.

Due to operational constrains only one manoeuvre a week was planned, consisting in applying a bias during nearly a day. The manoeuvre was designed to reach the centre of the dead-band in the following week, when a second manoeuvre was re-planned either to reduce the altitude off set to zero and stop the drift or to target again the centre of the groundtrack.

6. REFERENCES

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