Asteroid Rendezvous Uncertainty Propagation

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Motivation

Rendezvous With Asteroid

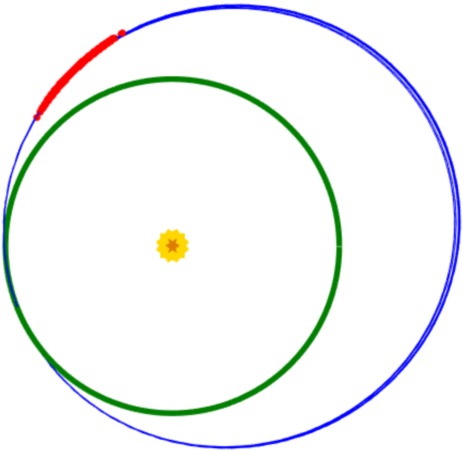
Quantify the Uncertainty of a Rendezvous

- Determine Mission Success
- Seek to Quantify or Reduce Risks and Costs
- Uncertainty Quantification Can Lead to Robust Optimization
- Sensitivity of States With Respect to Inputs

State and Uncertainty Estimation

Non-linear Propagation

- Long propagation times
- Large initial uncertainty
- Tend to yield non-Gaussian posterior PDFs
- Reacquire an object
- Desire for non-intrusive approach
  - legacy software



Traditional Astrodynamic UQ

Established Techniques

- Monte Carlo
- Linearization and the state transition matrix (STM)
  - Astrodynamics Community
- Unscented Transform (UT)
  - Switching Over

These Methods Have Drawbacks

- Convergence rate of MC is slow
- STM, as well as UT, rely on Gaussian distribution assumption

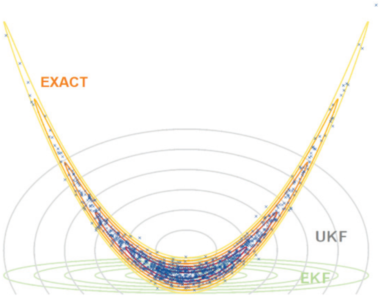


Image credit: NRC - Continuing Kepler's Quest

Therefore, more robust methods must be considered

Proposed Astrodynamic UQ

Methods in Development

- Polynomial Chaos Expansions (PCE)
- Gaussian Mixtures
- State Transition Tensors (STT)
- Differential Algebra (DA)

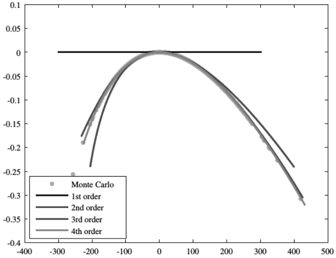


Image credit: Fujimoto, et al. (2012)

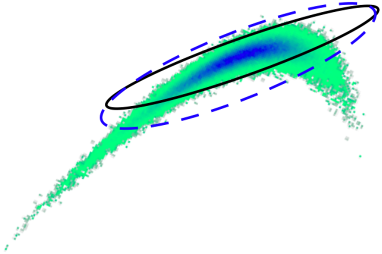


Image credit: Jones, et al. (2013)

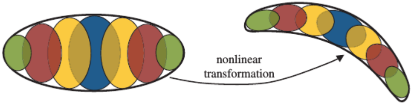


Image credit: Horwood, et al. (2011)

Proposed Astrodynamic UQ - Properties

- PCE benefit from surrogate properties and fast convergence rate
- GMM can leverage existing filters and Gaussian techniques
- STT and DA methods reduce the computation burden

Without mitigation techniques, PCE and Gaussian Mixtures suffer from the curse of dimensionality

- Computation time increases exponentially with respect to input dimensions d
- Resulting in increased computation time or dimension truncation

STT Must Solve for Multiple Differential Equations, While DA is an Intrusive Method

Separated Representations

Separated Representation

Premise: Decompose a multi-variate function into a linear combination of the products of uni-variate functions

$$q\left(\xi_1,\ldots,\xi_d\right)=\sum_{l=1}^{r} s^l u_1^l\left(\xi_1\right) u_2^l\left(\xi_2\right)\cdots u_d^l\left(\xi_d\right)+\mathcal{O}\left(\epsilon\right)$$

- r is the separation rank
- $u_i^l\left(\xi_i\right)$ are the unknown uni-variate functions/factors
- Computation cost dominated by relatively few MC propagations

Extensive Background

- Chemistry, data mining, imaging, etc
- Doostan & Iaccarino 07,09. Nouy 07,10,11,12,13. Koromskij & Schwab 10. Cances et al. 11. Kressner & Tobler 11. Doostan et al. 12,13,14. Beylkin et al. 09

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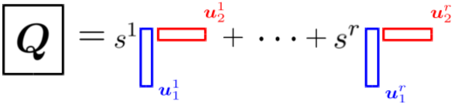
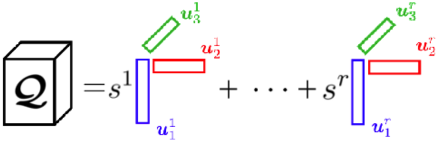
Connections With SVD

Singular-value decomposition (SVD):



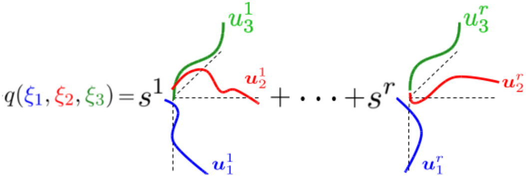
$$Q \approx \sum_{l=1}^{r} s^l \boldsymbol{u}_1^{l^T} \boldsymbol{u}_2^l$$

Separated approx: generalization of matrix SVD to tensors:



$$\mathcal{Q} \approx \sum_{l=1}^{r} s^l \boldsymbol{u}_1^l \otimes \boldsymbol{u}_2^l \otimes \boldsymbol{u}_3^l$$

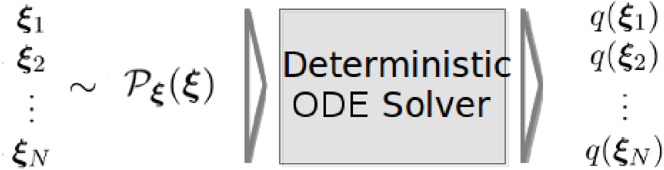
Functions:



$$q \approx \sum_{l=1}^{r} s^l u_1^l(\xi_1) u_2^l(\xi_2) u_3^l(\xi_3)$$

A Non-Intrusive Implementation

Problem Set Up: Given n random samples



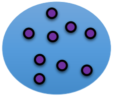
reconstruct a low-rank separated representation:

$$\hat{q}\left(\boldsymbol{\xi}\right) = \sum_{l=1}^{r} s^l u_1^l\left(\xi_1\right) u_2^l\left(\xi_2\right) \cdots u_d^l\left(\xi_d\right) \text{ s.t. } \|q - \hat{q}\|_D \leq \epsilon$$

where,

$$\|\hat{q}\|_D^2 := \frac{1}{N} \sum_{j=1}^{N} \hat{q}\left(\boldsymbol{\xi}_j\right)^2$$

Traditional Monte Carlo



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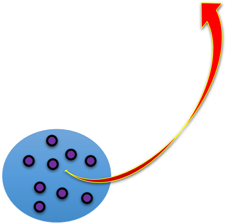

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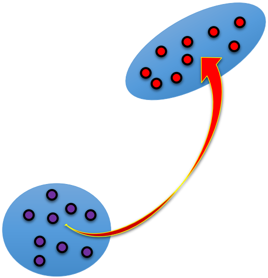

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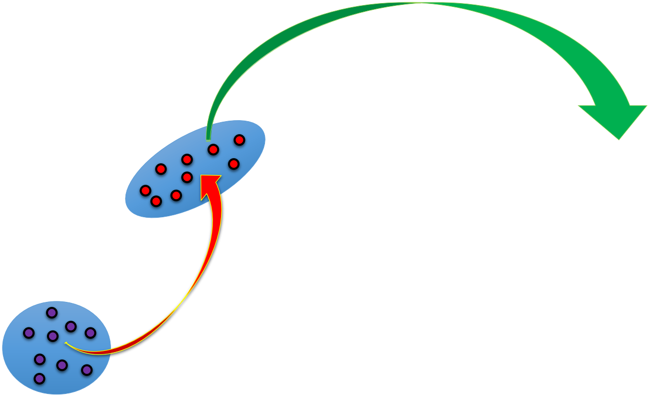

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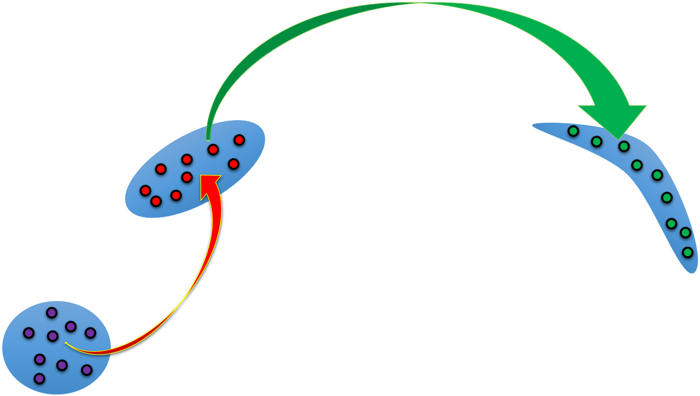
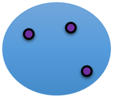


Separated Approach



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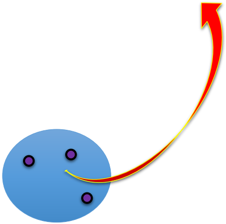

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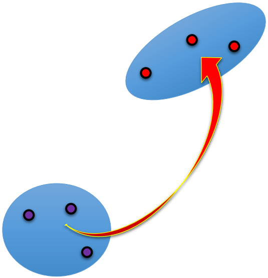

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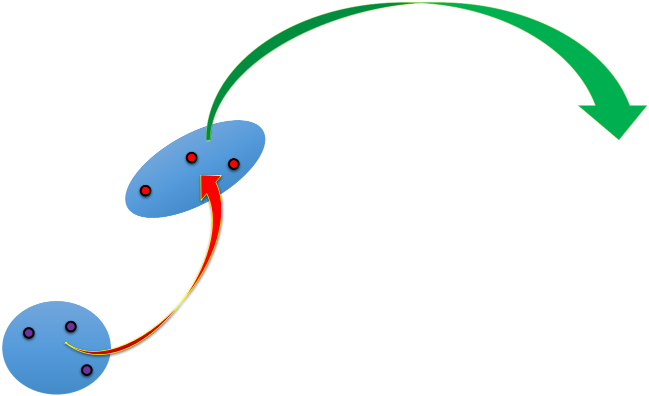

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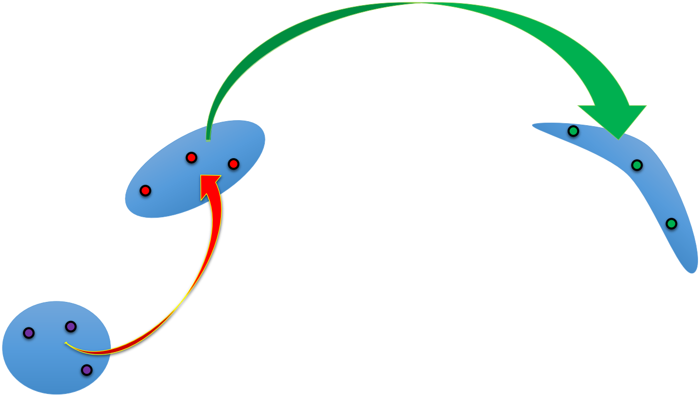

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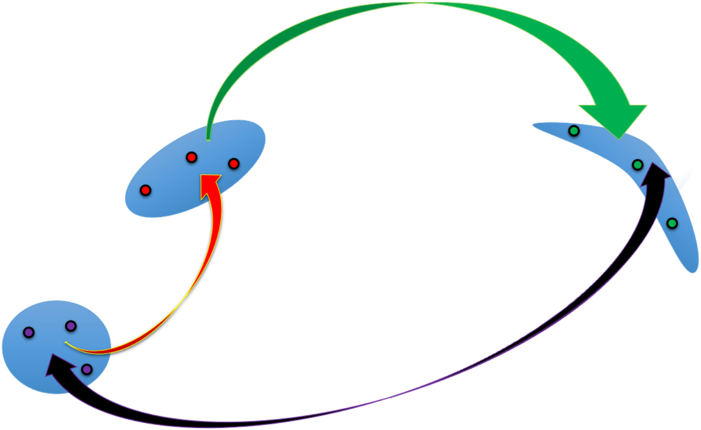

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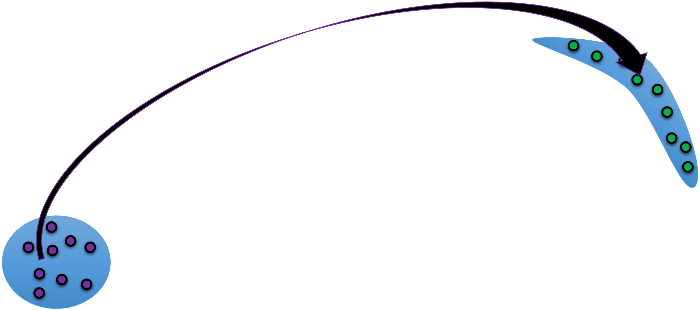


A Non-Intrusive Implementation

Spectral Decomposition of Factors

$$u_i^l(\xi_i) = \sum_{p=0}^{P} c_{i,p}^l \psi_p(\xi_i)$$

where,

$$\int_{\Gamma_i} \psi_j(\xi_i)\,\psi_k(\xi_i)\,\rho(\xi_i)\,d\xi_i = \delta_{jk}$$

Discrete Approximation

$$\left\{c_{i,p}^l\right\} = \underset{\left\{\hat{c}_{i,p}^l\right\}}{\arg\ \min} \left\| q(\cdot) - \sum_{l=1}^{r} s^l u_1^l(\cdot)\, u_2^l(\cdot) \cdots u_d^l(\cdot) \right\|_D$$

Computation Cost

Linear Scalability: Required Number of Solution Samples

$$\hat{q}\left(\boldsymbol{\xi}\right) = \sum_{l=1}^{r} s^l u_1^l\left(\xi_1\right) u_2^l\left(\xi_2\right) \cdots u_d^l\left(\xi_d\right)$$

Number of unknowns $= r \cdot d \cdot P$

$$N \sim \mathcal{O}\left(r \cdot d \cdot P\right)$$

Total Computation Time is Quadratic With Respect to d When ALS is Applied

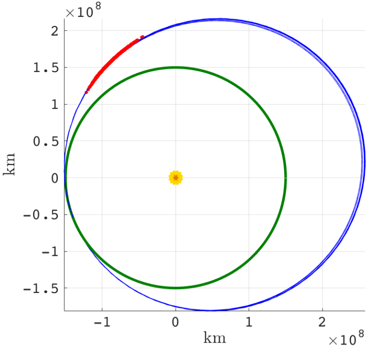
$$C_d \sim \mathcal{O}\left(K \cdot r^2 \cdot d \cdot P^2 \left(N + S\right)\right)$$

This cost should be small when compared to the number of required MC propagations

Analysis

Distribution Characteristics

- Nominal Trajectory and Maneuver Found Using Lambert Solver
- Uncertainty in asteroid 2006 DN orbital elements and interceptor initial state
- Error in magnitude and direction of interceptor maneuver at epoch
- Propagated for 1088 days, Dormand-Prince (5)4 Integrator
- Estimate heliocentric Cartesian coordinates and velocity



Results compared to one million MC samples

Random Inputs

	Mean	STD
a (AU)	1.38013	$3.0097e-04$
e	0.27859	$1.5878e-04$
inc (deg)	0.26764	$1.3974e-04$
ω (deg)	101.24110	$4.3343e-03$
Ω (deg)	96.62356	$6.6975e-03$
M (deg)	8.69171	0.72173

	Mean	STD
x (AU)	-0.93808	$1.33691e-09$
y (AU)	-0.35197	$1.33691e-09$
z (AU)	$1.9736e-05$	$1.33691e-09$
$\dot{x}$ (km/s)	9.99105	$8e-03$
$\dot{y}$ (km/s)	-28.00263	$8e-03$
$\dot{z}$ (km/s)	$2.1797e-04$	$8e-03$
$\overline{\Delta V}_x$ (km/s)	1.51305	0.01513
$\overline{\Delta V}_y$ (km/s)	-3.48573	0.03485
$\overline{\Delta V}_z$ (km/s)	-0.05830	$5.830e-04$
θ (deg)	0	1
ϕ (deg)	0	π

SR Results ($d = 15$)

15 random inputs required 1200 samples, $r = 8$, and $P = 3$

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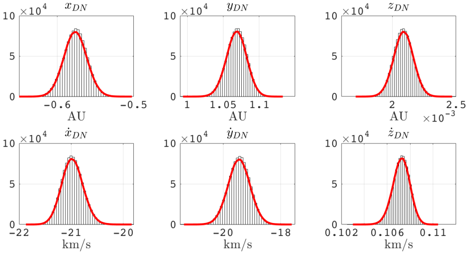

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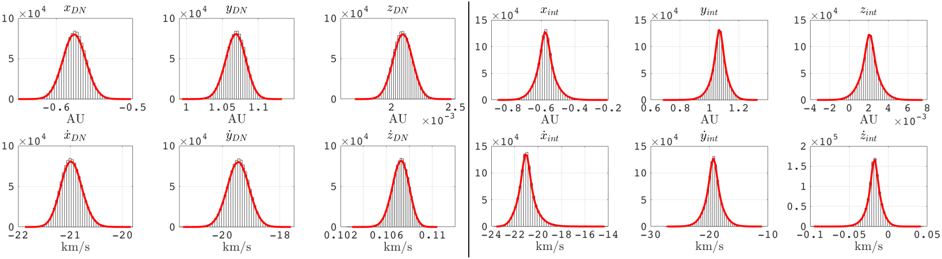

SR Results ($d = 15$)

	Rel. Mean	Rel. STD
x_{DN}	3.5e-05	7.9e-04
y_{DN}	1.6e-05	1.4e-03
z_{DN}	4.4e-05	9.4e-04
$\dot{x}_{DN}$	1.7e-06	2.6e-04
$\dot{y}_{DN}$	1.0e-05	9.1e-04
$\dot{z}_{DN}$	7.9e-06	4.2e-03
x_{int}	6.5e-05	5.7e-04
y_{int}	9.9e-06	2.2e-03
z_{int}	1.0e-04	4.3e-04
$\dot{x}_{int}$	3.6e-05	4.7e-04
$\dot{y}_{int}$	3.2e-05	2.2e-04
$\dot{z}_{int}$	5.9e-04	2.0e-03

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Sensitivity Analysis

	Quantities of Interest					
Inputs	x_{DN}	y_{DN}	z_{DN}	$\dot{x}_{DN}$	$\dot{y}_{DN}$	$\dot{z}_{DN}$
a	~ 0	~ 0	~ 0	~ 0	~ 0	~ 0
e	~ 0	~ 0	~ 0	~ 0	~ 0	~ 0
inc	~ 0	~ 0	~ 0	~ 0	~ 0	~ 0
ω	~ 0	~ 0	~ 0	~ 0	~ 0	~ 0
Ω	~ 0	~ 0	~ 0	~ 0	~ 0	~ 0
M	0.9	0.9	0.9	0.9	0.9	0.9

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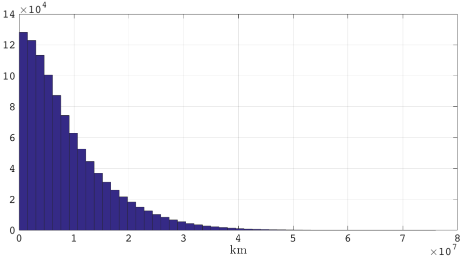
Sensitivity Analysis

	Quantities of Interest					
Inputs	x_{DN}	y_{DN}	z_{DN}	$\dot{x}_{DN}$	$\dot{y}_{DN}$	$\dot{z}_{DN}$
a	~ 0	~ 0	~ 0	~ 0	~ 0	~ 0
e	~ 0	~ 0	~ 0	~ 0	~ 0	~ 0
inc	~ 0	~ 0	~ 0	~ 0	~ 0	~ 0
ω	~ 0	~ 0	~ 0	~ 0	~ 0	~ 0
Ω	~ 0	~ 0	~ 0	~ 0	~ 0	~ 0
M	0.9	0.9	0.9	0.9	0.9	0.9

	Quantities of Interest					
Inputs	x_{int}	y_{int}	z_{int}	$\dot{x}_{int}$	$\dot{y}_{int}$	$\dot{z}_{int}$
x_{int}	~ 0	~ 0	~ 0	~ 0	~ 0	~ 0
y_{int}	~ 0	~ 0	~ 0	~ 0	~ 0	~ 0
z_{int}	~ 0	~ 0	~ 0	~ 0	~ 0	~ 0
$\dot{x}_{int}$	~ 0	~ 0	~ 0	~ 0	~ 0	~ 0
$\dot{y}_{int}$	~ 0	~ 0	~ 0	~ 0	~ 0	~ 0
$\dot{z}_{int}$	~ 0	~ 0	~ 0	~ 0	~ 0	~ 0
$\lvert\boldsymbol{\Delta V}\rvert$	0.8	0.9	0.9	0.9	0.8	0.8
θ	~ 0	~ 0	~ 0	~ 0	~ 0	~ 0
ϕ	0.8	0.8	0.9	0.8	0.8	0.9

Rendezvous Distance

One-to-One Comparison of One Million Samples



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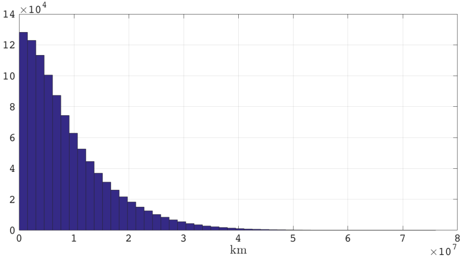

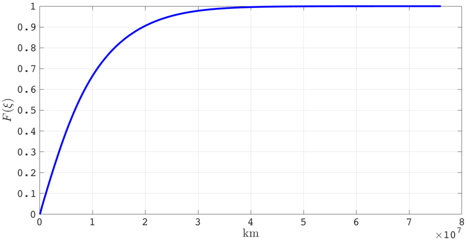

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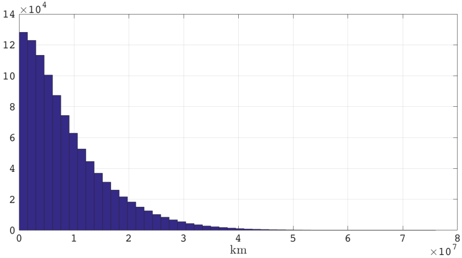

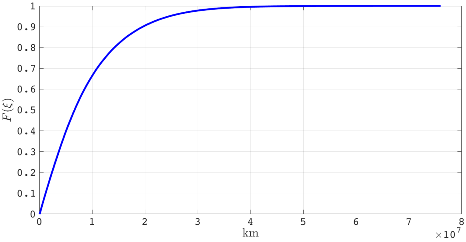

Minimum Distance Was Approximately 4400 km

Summary

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- Rendezvous PDF too sparse for target distance
  - Different approach such as all-to-all

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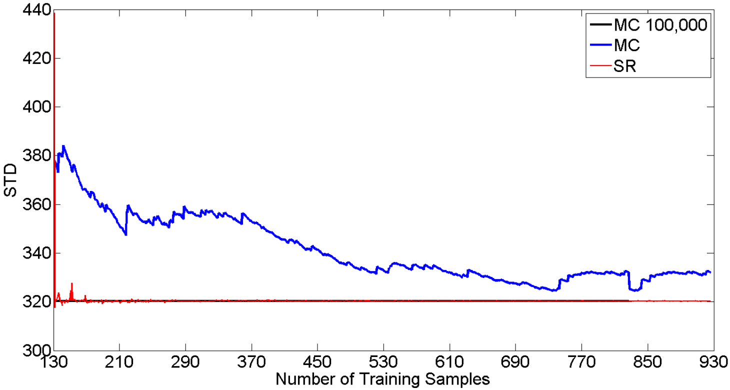
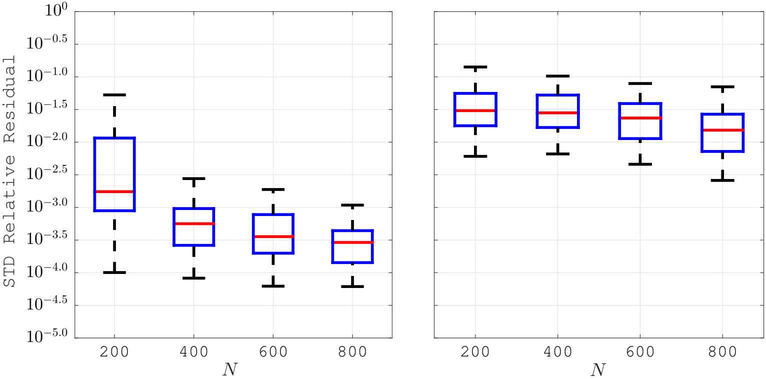
- Non-linear propagation of uncertainty can be expensive or complex
- SR estimates a posterior distribution with a surrogate method
  - With a largely linear cost in d
- Rendezvous PDF too sparse for target distance
  - Different approach such as all-to-all
  - Addition of TCM and optimization

Questions and Comments

Additional Material


440
420
400
380
360
340
320
300
STD
130
210
290
370
450
530
610
690
770
850
930
Number of Training Samples
MC 100,000
MC
SR



10^0
$10^{-0.5}$
$10^{-1.0}$
$10^{-1.5}$
$10^{-2.0}$
$10^{-2.5}$
$10^{-3.0}$
$10^{-3.5}$
$10^{-4.0}$
$10^{-4.5}$
$10^{-5.0}$
STD Relative Residual
200
400
600
800
N
200
400
600
800
N