Evaluation of Iterative Analytical techniques for Interplanetary Orbiter Missions

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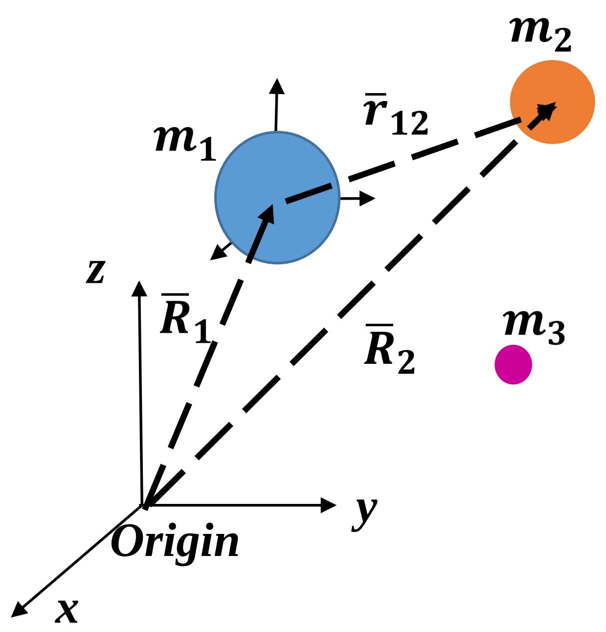
Contents

N-body equations of motion

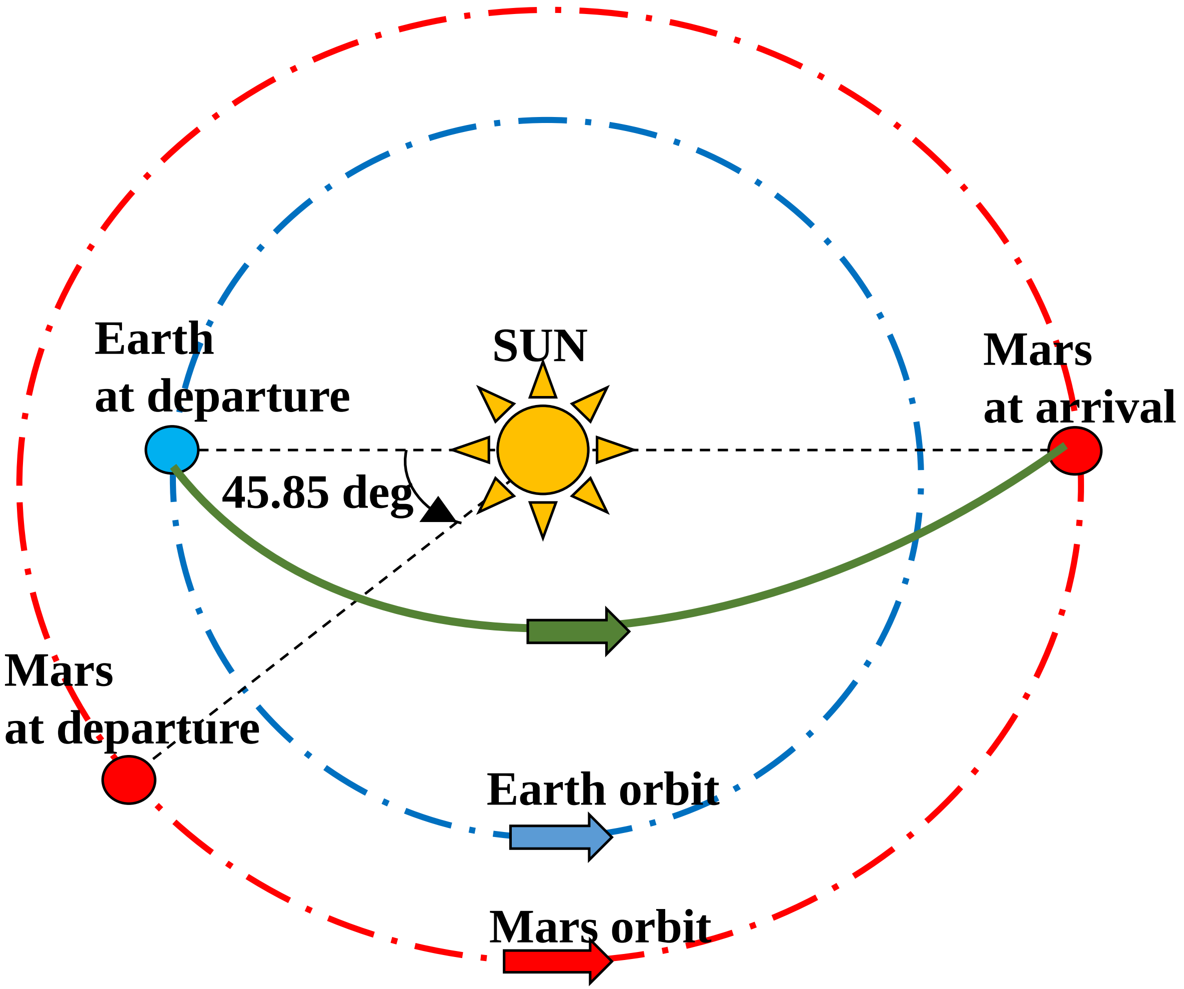


- Equations of motion of body m_2 w.r.t primary body m_1;

$$\ddot{\bar{r}}_{12} = -G(m_1 + m_2)\frac{\bar{r}_{12}}{r_{12}^3} + Gm_3\left(\frac{\bar{r}_{23}}{r_{23}^3} - \frac{\bar{r}_{13}}{r_{13}^3}\right) + G\sum_{j=4}^{N} m_j\left(\frac{\bar{r}_{2j}}{r_{2j}^3} - \frac{\bar{r}_{1j}}{r_{1j}^3}\right)$$

- No Closed form solution.
- Solution by numerical integration of the equations of motion (Cowell's method).
- Numerical integration (NI) requires precise initial conditions with epoch.
- Numerical search for initial conditions and epoch: complex and expensive.
- Quick solution by analytical methods.

Geometry for minimum energy transfer



Hohmann transfer

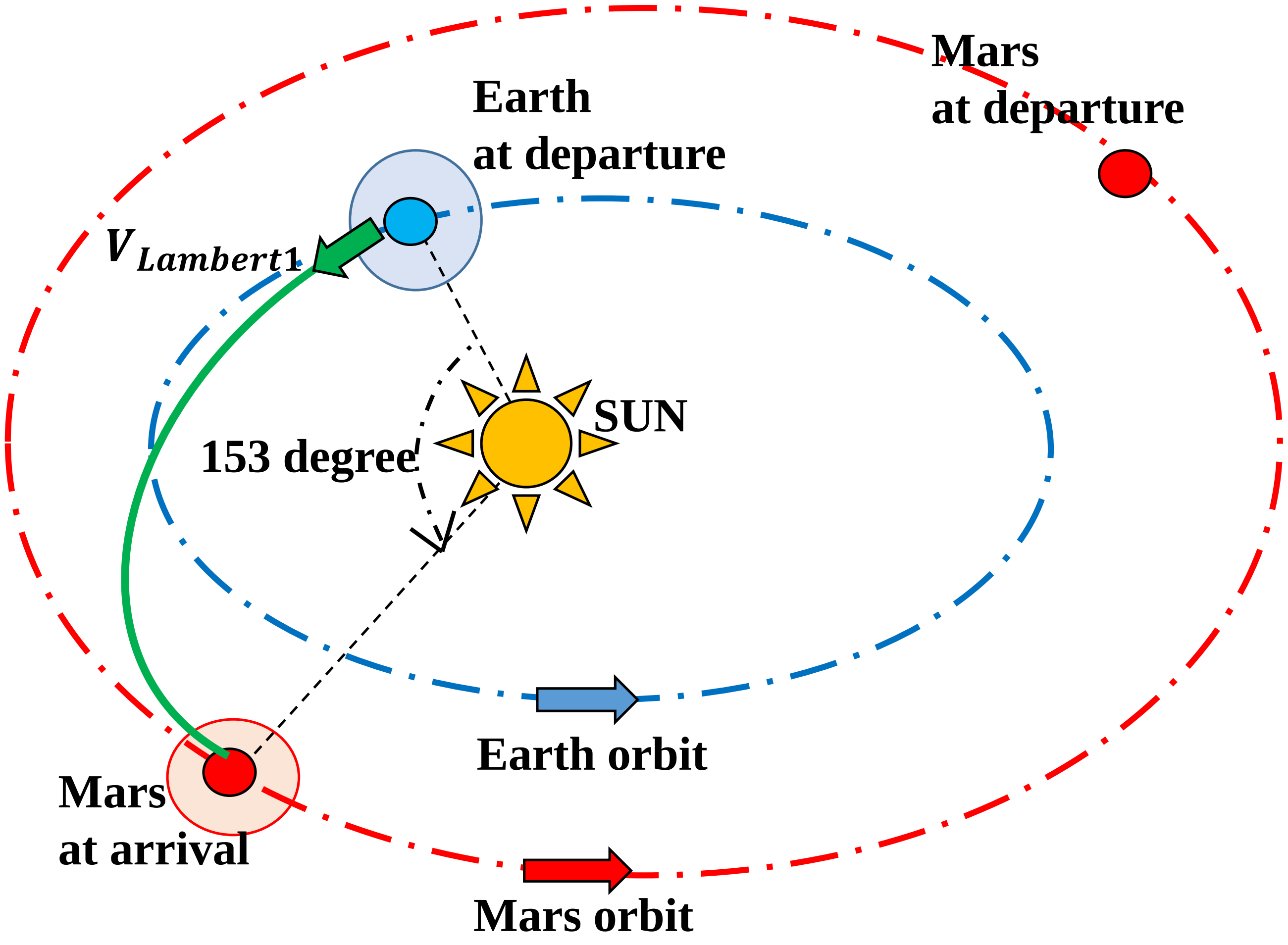
Coplanar circular planet orbits around SUN.

- Hohmann opportunity:
  - Transfer angle ~ 180 degree
  - Initial phase angle ~ 46 degree
  - TPI impulse = 1367.65 m/s
  - POI impulse = 2889.73 m/s
  - Flight duration = 256 days
  - SMA_transfer = 187.5 million km

- **Only velocity magnitude is obtained, not the direction.**

†	Earth parking orbit	300 x 25,000 km
†	Mars parking orbit	300 km circular

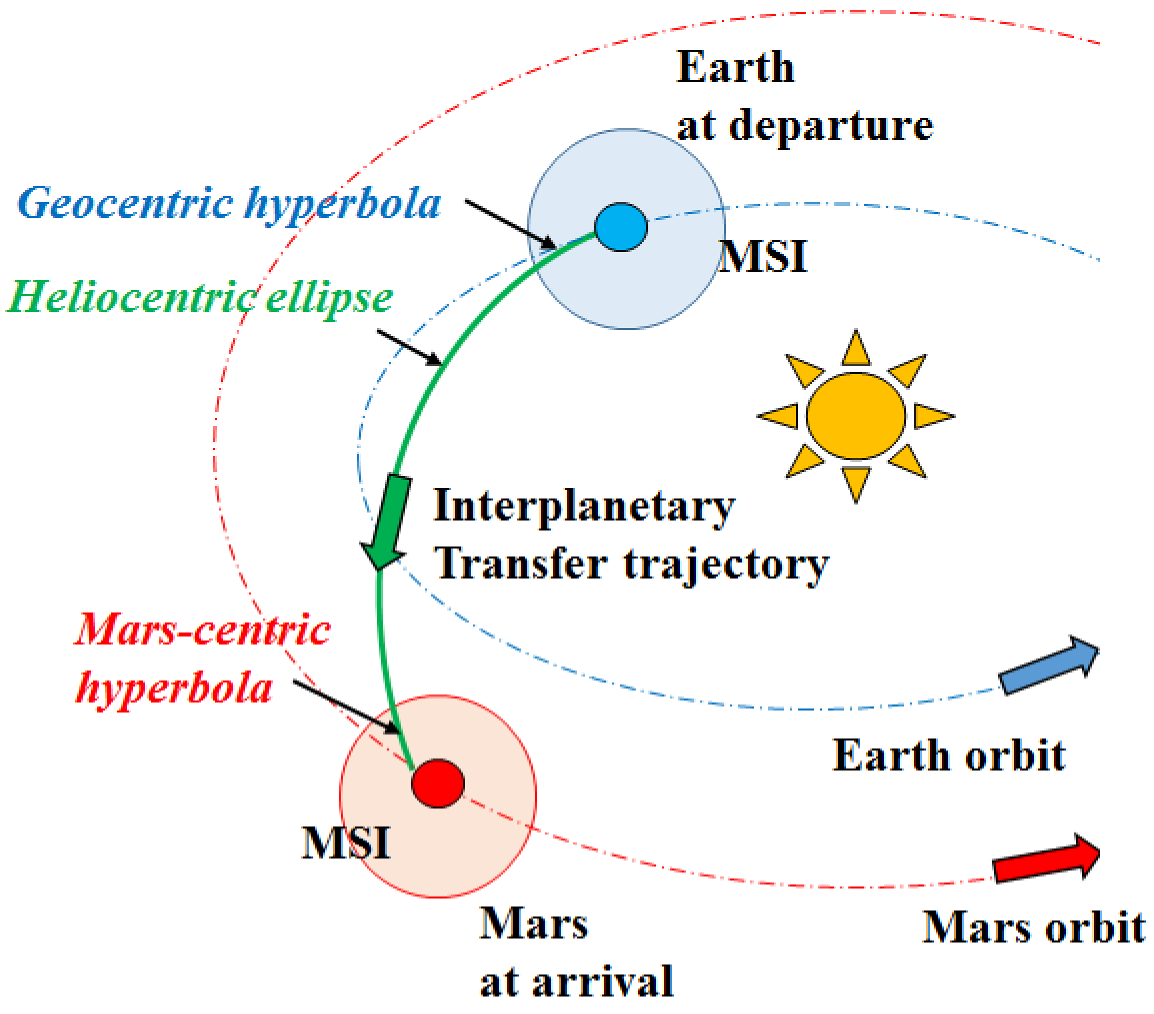
Conventional patched conic method



- Earth parking orbit 300 x 25,000 km
- Mars parking orbit 300 km circular
- Determination of transfer trajectory through Lambert.
- $\boldsymbol{v_\infty = V_{Lambert} - V_{planet}}$
- The asymptotic excess velocity ($\boldsymbol{v_\infty}$) is used to find the hyperbolic characteristics and the velocity impulses.

- Minimum energy opportunity
  - Dep. Date ~ 12 May 2018
  - Flight duration = 204 days
  - Transfer angle ~ 153 degree
  - TPI impulse = 1355.22 m/s
  - POI impulse = 2248.21 m/s
  - SMA_transfer = 182.71 million km

Iterative patched conic method

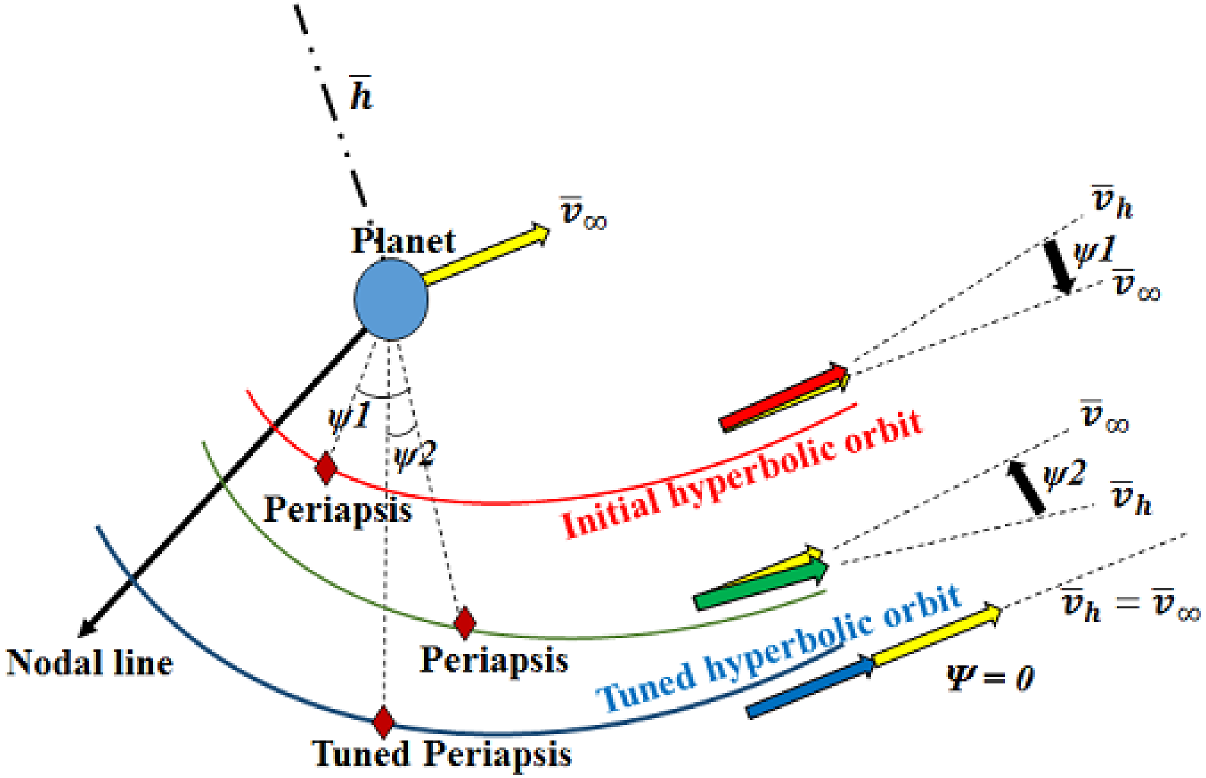


Interplanetary transfer geometry used in patched conic method

- The asymptotic $\boldsymbol{v}_{\infty}$ is tuned such that it occurs at the exit point of MSI.
- Determination of hyperbolic orbit characteristics that achieves the $\boldsymbol{v}_{\infty}$ at MSI.
- Iteration on the patch points at the MSI.
- Four distinct trajectory options.

- **Iterative patched conic technique + Analytical tuning = four distinct trajectory options.**

Analytical tuning strategy



Orbital RAAN and inclination does not change.

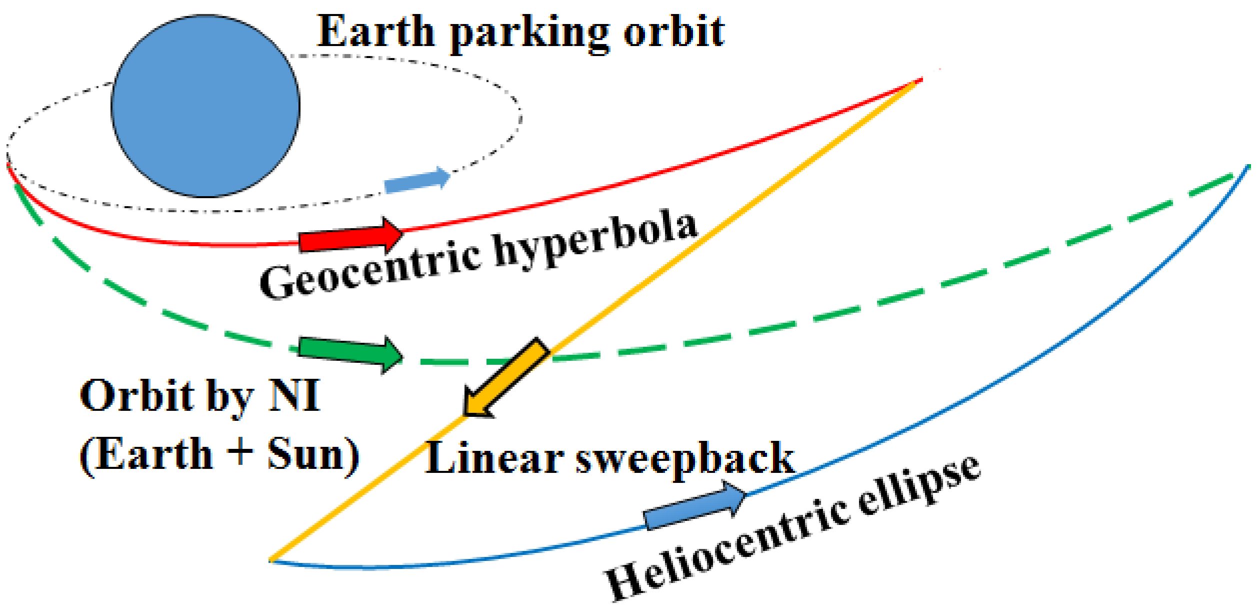
Analytical tuning of the planetocentric hyperbola.

- The asymptotic $\bar{v}_\infty$ must occur at pseudosphere exit/entry.
- Achieved using analytical tuning strategy.

Steps:

- Determination of initial hyperbolic orbit characteristics from the asymptotic velocity vector $\bar{v}_\infty$.
- Keplerian propagation from the periapsis for the pre-fixed duration to find $\bar{v}_h$.
- Rotation of orbital plane about the angular momentum vector to minimize ψ.
- Reshape the hyperbolic orbit to match the asymptotic velocity vector in magnitude.

Pseudostate technique
(Wilson Jr.)



Multiconic approach

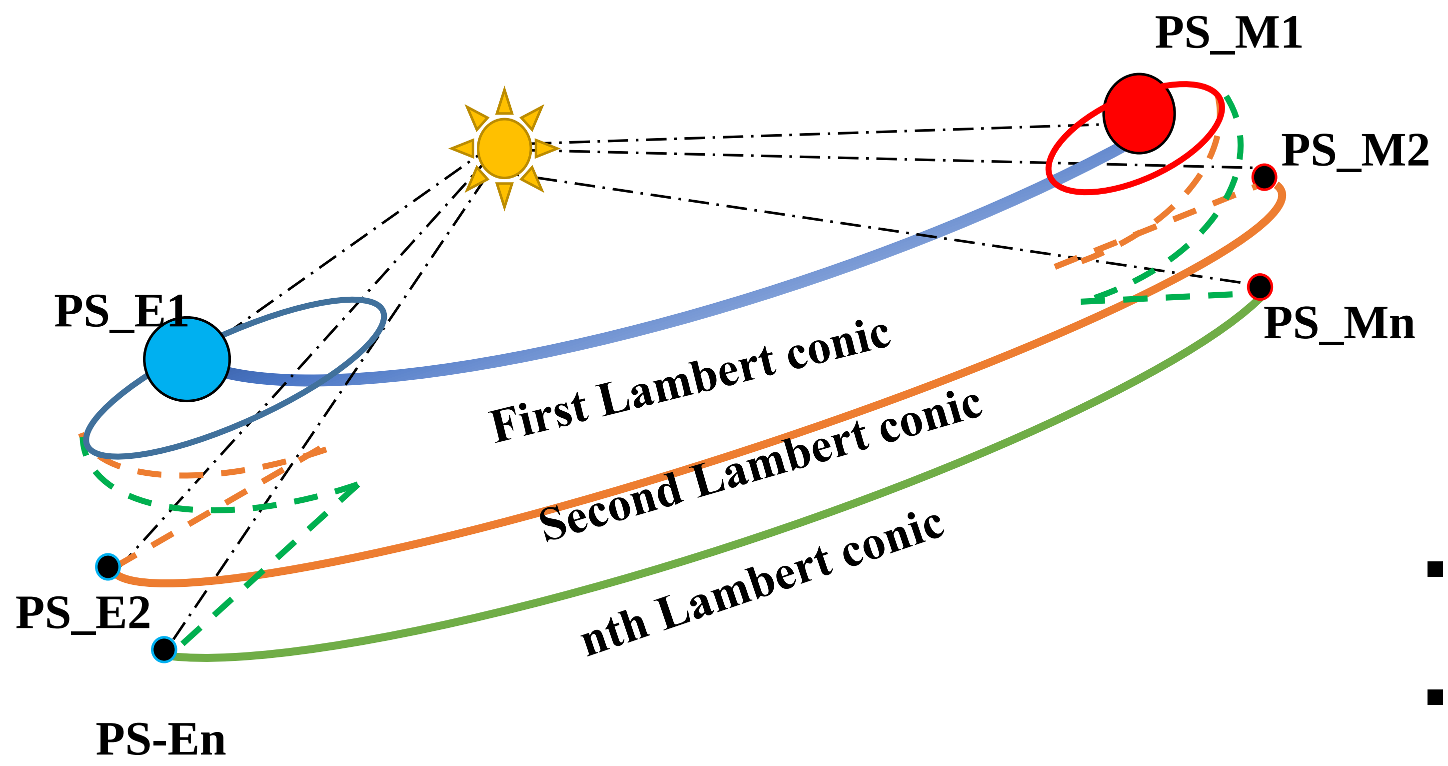
- Within the pseudosphere of a planet, Gravity of the Sun (primary body) and the planet (secondary body) acts on the spacecraft.
- Outside the pseudosphere, Only Sun acts on the spacecraft.

- Multiconic orbit (3-body problem) split into:

I. Geocentric hyperbola (Keplerian)
II. Linear *sweepback* for the same *duration*.
III. Heliocentric ellipse (Keplerian)

- The state thus obtained is equivalent to the State obtained using numerical integration under Earth + Sun.

Iterative pseudostate technique



Iterative method for the transfer trajectory using pseudostate technique.

- ***To account for the planet's gravity within the pseudosphere in addition to Sun***

- The location of periapsis in dep. hyperbola changes with analytical tuning.
- The orientation of the dep. hyperbolic plane changes with pseudostate iteration.

Results in four distinct transfer trajectory design options

Hyperbolic trajectory characteristics from the excess velocity vector

- Using spherical trigonometry

Semi major axis of the hyperbola $a_\infty = -\mu/{v_\infty}^2$

Related eccentricity $e_\infty = 1 + (r_p\, {v_\infty}^2/\mu)$

RAAN $sin(\alpha_\infty - \Omega_\infty) = tan\,\delta_\infty / tan\, i$

AOP $sin(u_\infty + \theta_\infty) = sin\,\delta_\infty / sin\, i$

where $u_\infty = \omega_\infty + \nu_p \qquad \nu_p = 0$

$\theta_\infty = cos^{-1}(1/e_\infty)$

-Continued-

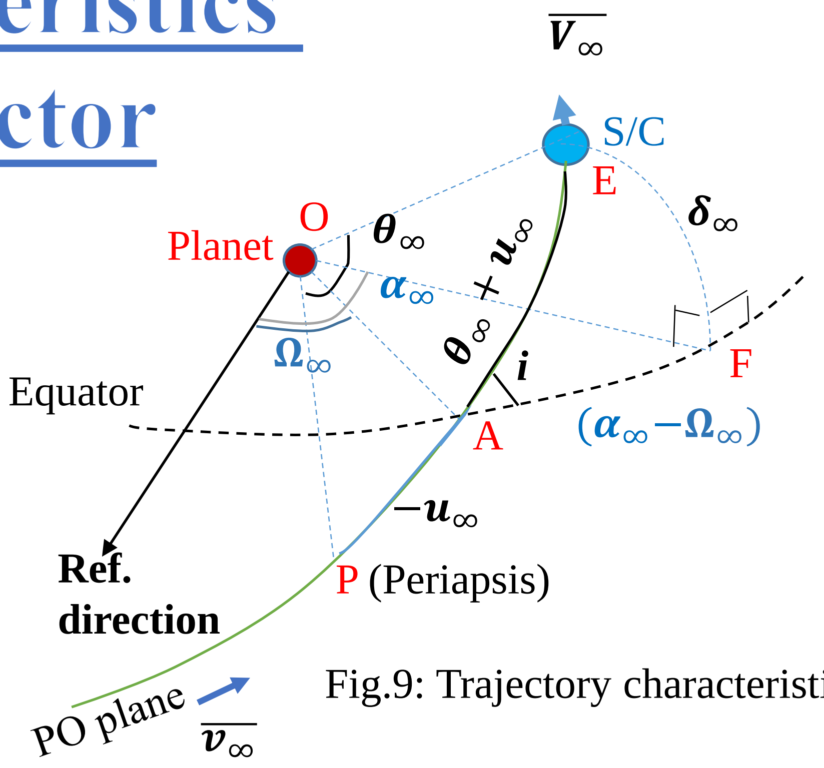
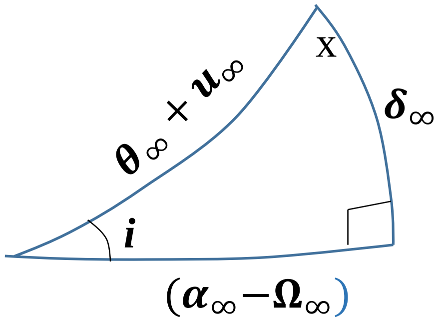
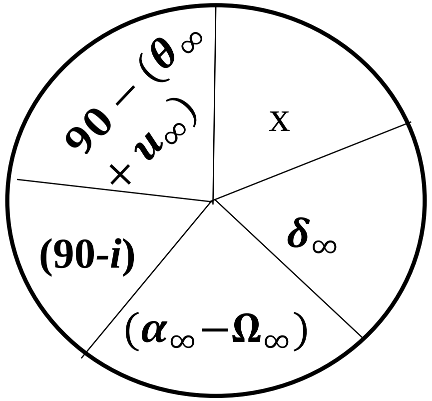


Fig.9: Trajectory characteristics





Napier's 5-parts formula in Spherical trigonometry

Parking orbit characteristics

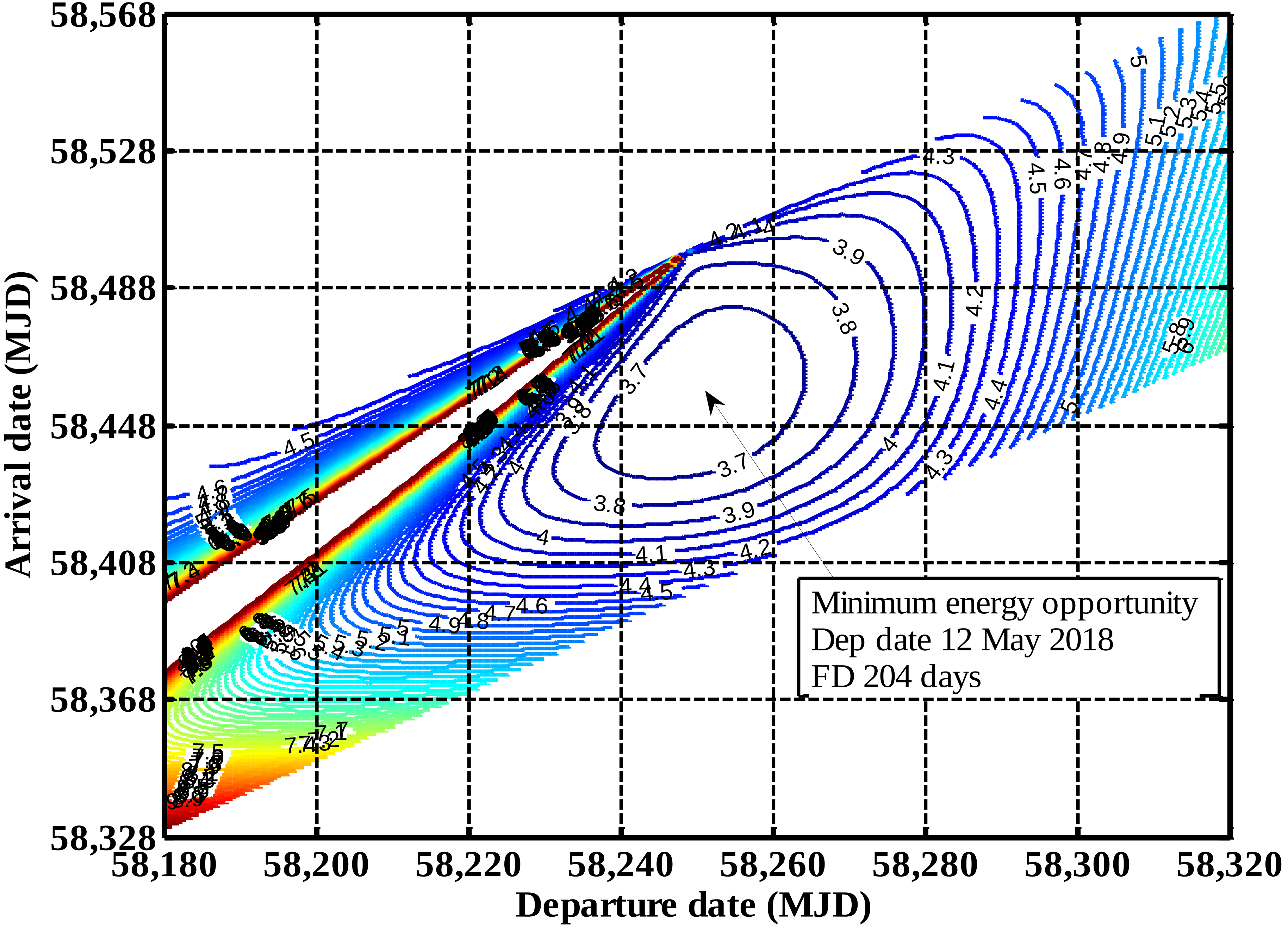
- Inclination of the hyperbolic trajectory is same as that of the parking orbit,
  - To ensure coplanar transfer
  - Minimizes energy.

- The periapsis of the hyperbolic trajectory is same as periapsis of the parking orbit,
  - To ensure tangential and horizontal injection.
  - Minimizes energy.

- The size, shape and the inclination of the parking orbit,
  - Depend on the launch vehicle.

- RAAN and AOP are same as that of the hyperbolic trajectory.

Results and discussion



Total velocity impulse contour chart for 2018 from conventional patched conic technique

Four distinct excess velocity vectors using the proposed iterative patched conic method

Parameters	Iterative patched conic method			
	Option 1-1	Option 1-2	Option 2-1	Option 2-2
$v_{\infty D}$(km/s)	2.7826	2.7839	2.7779	2.7791
$\propto_{\infty D}$(deg)	321.652	321.675	321.527	321.550
$\delta_{\infty D}$(deg)	-37.205	-37.285	-36.842	-36.923
$v_{\infty A}$(km/s)	2.9602	2.9598	2.9614	2.9610
$\propto_{\infty A}$(deg)	245.518	245.4861	245.5999	245.5689
$\delta_{\infty A}$(deg)	9.5035	9.5434	9.2193	9.2594

Four distinct excess velocity vectors using the iterative pseudostate method

Parameters	Iterative pseudostate method			
	Option 11	Option 12	Option 21	Option 22
$v_{\infty D}$(km/s)	2.7893	2.7906	2.7847	2.7859
$\propto_{\infty D}$(deg)	321.9206	321.9437	321.7844	321.8078
$\delta_{\infty D}$(deg)	-37.1345	-37.2146	-36.7642	-36.8449
$v_{\infty A}$(km/s)	2.9609	2.9605	2.9622	2.9618
$\propto_{\infty A}$(deg)	245.5156	245.4841	245.5985	245.5668
$\delta_{\infty A}$(deg)	9.5043	9.5446	9.2203	9.2607

Four distinct design options using the iterative patched conic method

Parameters	Option 1-1	Option 1-2	Option 2-1	Option 2-2
$a_{\infty D}$ (km)	-58965.7364	-58904.0810	-59206.2363	-59145.1884
$e_{\infty D}$	1.1132545	1.113373	1.112794	1.112910
$i_{\infty D}$ (deg)	75	75	75	75
$\Omega_{\infty D}$ (deg)	333.3889	333.4465	129.9454	129.9341
$\omega_{\infty D}$(deg)	167.37819	167.3056	64.4571	64.5547
$a_{\infty A}$ (km)	-4980.0353	-4981.3695	-4975.7226	-4977.1037
$e_{\infty A}$	1.742404	1.74220	1.743047	1.742841
$i_{\infty A}$ (deg)	75	75	75	75
$\Omega_{\infty A}$(deg)	68.0878	242.9041	68.0925	243.0652
$\omega_{\infty A}$(deg)	115.1783	314.9084	115.4580	314.5994

Four distinct design options using the iterative pseudostate method

Parameters	Option 11	Option 12	Option 21	Option 22
$a_{\infty D}$ (km)	-58625.7268	-58564.6827	-58859.5196	-58799.0814
$e_{\infty D}$	1.113911	1.114030	1.113458	1.113575
$i_{\infty D}$ (deg)	75	75	75	75
$\Omega_{\infty D}$ (deg)	333.6273	333.6850	130.2362	130.2252
$\omega_{\infty D}$(deg)	167.5207	167.4482	64.4443	64.5418
$a_{\infty A}$(km)	-4977.4201	-4978.7431	-4973.0997	-4974.4693
$e_{\infty A}$	1.742794	1.742597	1.743439	1.743235
$i_{\infty A}$ (deg)	75	75	75	75
$\Omega_{\infty A}$(deg)	68.0867	242.9018	68.0914	243.0627
$\omega_{\infty A}$(deg)	115.1685	314.9005	115.4479	314.5917

Performance of the analytical designs

Parameters	Proposed iterative patched conic design			
	Option 1-1	Option 1-2	Option 2-1	Option 2-2
CAA (km)	192,798	197,674	207,562	212,644
T_p (UTC) DD/MM/YYYY HH:MM:SS	2 Dec 2018 10:06:53.608	2 Dec 2018 10:06:17.249	2 Dec 2018 12:09:31.693	2 Dec 2018 12:09:02.922
TCM (m/s)	12	11.61	11.64	11.61

Velocity impulses (m/s)	Conventional patched conic design	Design from the proposed iterative patched conic method			
		Option 1-1	Option 1-2	Option 2-1	Option 2-2
TPI	1355.22	1309.93	1310.25	1308.71	1309.02
POI	2248.21	2232.78	2232.57	2233.44	2233.23

Velocity impulses (m/s)	Design from the iterative pseudostate method			
	Option 11	Option 12	Option 21	Option 22
TPI	1311.68	1311.99	1310.48	1310.79
POI	2233.18	2233.98	2233.84	2233.63
Total	3544.86	3545.97	3544.32	3544.42

Comparison of transfer trajectory design by different methods (option11)

Parameters	Iterative patched conic design	Iterative Pseudostate method	Numerical method
$a_{\infty D}$ (km)	-58965.7	-58625.7268	-58613.9834
$e_{\infty D}$	1.11325	1.113911	1.113911
$i_{\infty D}$ (deg)	75	75	75
$\Omega_{\infty D}$ (deg)	333.3889	333.6273	333.6176
$\omega_{\infty D}$(deg)	167.378	167.5207	167.5163
$a_{\infty A}$(km)	-4980.0	-4977.4201	-4977.4480
$e_{\infty A}$	1.74240	1.742794	1.742577
$i_{\infty A}$ (deg)	75	75	75
$\Omega_{\infty A}$(deg)	68.0878	68.0867	68.0835
$\omega_{\infty A}$(deg)	115.178	115.1685	115.1746

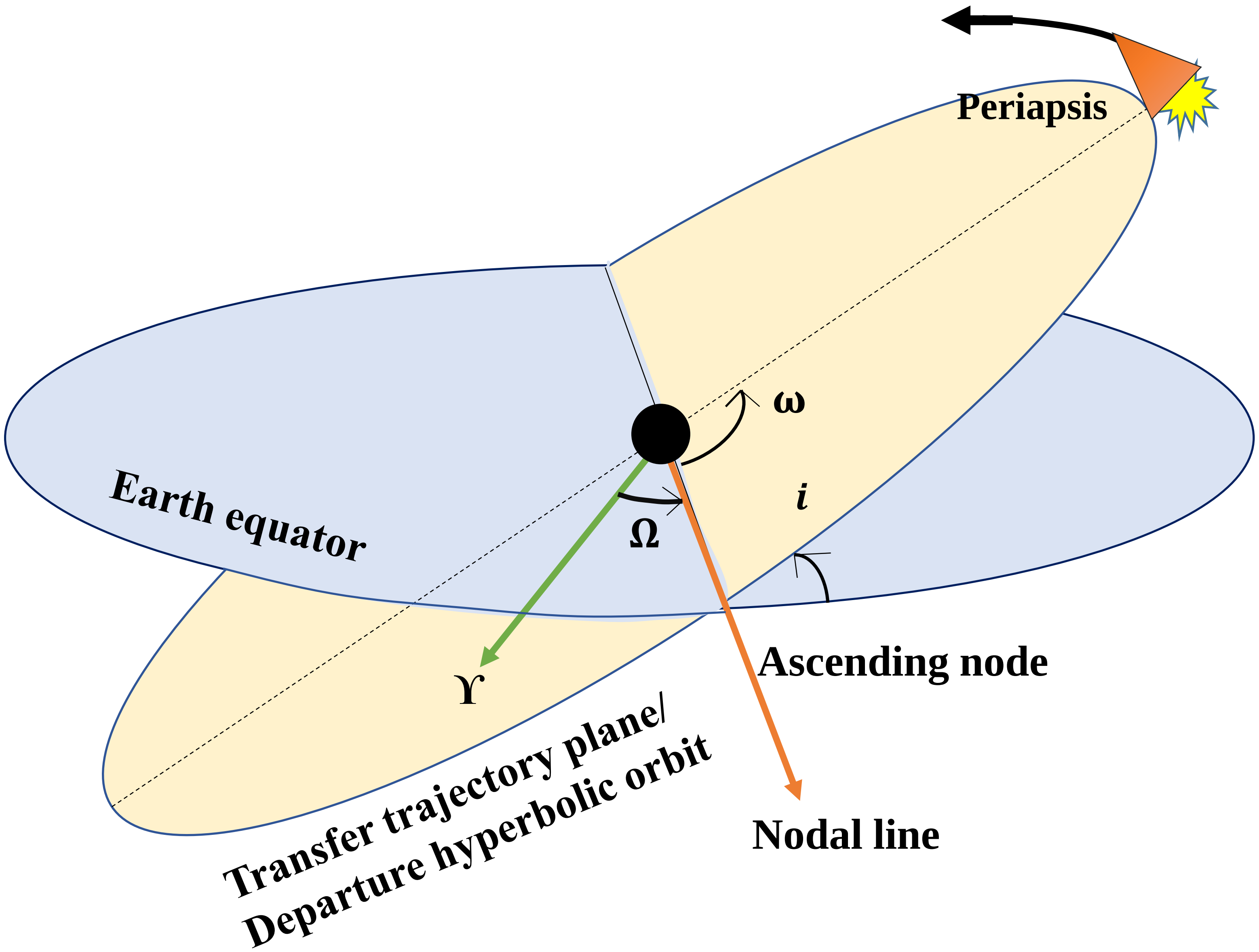
Conclusion

- **The iterative patched conic method can identify the four distinct transfer trajectory design options for a given departure date and flight duration.**
- **The design obtained from the iterative pseudostate method is very close to the numerical design,**
  - **quick mission design and analysis tool.**
  - **Design by proposed method can be used as initial guess for numerical refinement.**
- **Computation time for such analysis using the patched conic method and the pseudostate method are comparable.**
- **The Trans correction maneuver required due to modeling errors is very less with the pseudostate design as compared to that from the conventional design.**

Thankyou !


Constellation: Andromeda
Distance: 350 million light-years
ARP 273

Angles that fix the transfer trajectory plane



Ω	**Right ascension of ascending node (RAAN)**
ω	Argument of periapsis (AOP)
i	inclination

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Force model

Forces	Duration (days)	Remarks
Earth + Sun	t_D	Within the pseudosphere of Earth
Sun	$t_{FD} - t_D - t_A$	Outside the pseudospheres of the planets
Arrival planet + Sun	t_A	Within the pseudosphere of the arrival planet