EVALUATION OF ITERATIVE ANALYTICAL TECHNIQUES FOR INTERPLANETARY ORBITER MISSIONS

S.P. Parvathi and R. V. Ramanan#*

* Research scholar; #Adjunct Professor
Department of Aerospace Engineering,
Indian Institute of Space science and Technology,
Thiruvananthapuram, India.

ABSTRACT

In interplanetary orbiter missions, a specified arrival parking orbit in terms of closest approach altitude and inclination must be achieved. For a fixed departure date and a flight duration, there are four distinct transfer trajectory options for an interplanetary orbiter missions. The conventional patched conic method does not identify multiple distinct design options. In this paper, an iterative patched conic method is introduced. An iteration on the patch points at the mean sphere of influences along with an analytical tuning strategy helps to obtain the distinct design options. A design analysis tool is developed using the proposed method. The performance of the proposed iterative patched conic method is compared with the iterative pseudostate method. The deviations in the target parameters such as closest approach altitude and the related time, obtained upon numerical propagation of the designs under similar force model, are compared. The benefits derived while attempting numerical refinement of these analytical designs are quantified.

Index Terms— patched conic, pseudostate, trans correction maneuver, orbiter mission

1. INTRODUCTION

In 1925, Hohmann published a book which laid the foundation for the interplanetary mission design. The Hohmann transfer is essentially a heliocentric transfer from a departure planet to an arrival planet considering the planets as point masses. The planetary orbits are assumed as circular and coplanar. He concluded that the minimum delta-V transfer occurs at 180 degree transfer angle and with two impulses, one at the departure for transplanetary injection (TPI) and the other one for arrival parking orbit insertion (POI) [1]. Analytical solutions were derived through the application of conservation laws of angular momentum and energy [2]. But these solutions provide only a rough estimate of how to reach the arrival planet. Also, the Hohmann transfer design does not provide the direction of the departure velocity. The patched-conic approximation has thus been developed as a more accurate solution for the mission design. This method takes into account the gravitational forces due to the target planets also but one at a time basis. The spacecraft experiences the gravitational pull of the planets within the respective mean sphere of influence (MSI). Outside the MSI, only Sun acts on the spacecraft. Thus, the patched conic approximation subdivides the transfer trajectory into three distinct trajectories and patches them together at the MSI to create a single transfer trajectory. Different strategies are employed for patching the velocity discontinuities at the MSI such that the velocity at the exit of one phase is the same as the velocity at the entry of the next phase. However, the patched-conic approximation is still limited in that it only considers the gravity of one celestial body at a time. To overcome this limitation, the overlapped conic method has been introduced by Wilson Jr. in 1970 [3]. In this method, a larger MSI is considered for each planet, called the pseudosphere. Within the pseudosphere, both the Sun and the planet acts on the spacecraft and outside the pseudosphere, only Sun acts on the spacecraft.

Further, for a fixed departure date and flight duration, there are two hyperbolic trajectory options corresponding to the ascending or descending geometry that contains the excess velocity vector of the departure phase. Similarly, there are two options for arrival phase also. Each of the departure hyperbolic trajectory option can be mapped to each of the arrival hyperbolic trajectory option. This process of mapping results in four distinct departure excess velocity vectors that correspond to four distinct transfer trajectory design options. The conventional patched conic method does not distinguish these excess velocity vectors. In this paper, an iterative patched conic technique is introduced which can clearly distinguish the four excess velocity vectors at departure. The high sensitivity of the departure conditions or the initial guess is discussed in detail. Due to patched conic assumptions, the design obtained results in large deviations in the target parameters such as closest approach altitude and the related time. To overcome the deviations in the target conditions, an analytical iterative pseudostate technique is discussed [4]. This method reduces the deviation in target conditions to a great extent when compared with the conventional designs and is demonstrated in this paper.

2. METHODS OF TRAJECTORY DESIGN

The interplanetary trajectory design aims at finding the initial guess or the departure parking orbit characteristics for the given departure date and flight duration, which will transfer the spacecraft to the arrival parking orbit with the specified periapsis altitude and inclination. The patched conic method splits the interplanetary transfer trajectory into three two-body trajectories. This includes (i) a planetocentric departure hyperbolic orbit, (ii) an elliptic orbit about the Sun after escaping the departure planet's MSI, and (iii) a planetocentric arrival hyperbolic orbit. The velocity discontinuities are patched at the MSI using some strategy. Bell and Clarke [5] used the differential correction method to modify the excess velocity vectors such that target conditions are met at the arrival phase. In Reference [6], the velocity at the end of geocentric phase and the velocity at the beginning of the heliocentric phase are synchronized using Newton-Raphson method. Ramanan and Adimurthy [7] used an analytical tuning strategy to modify the hyperbolic orbit characteristics such that excess velocity vectors are obtained at the MSI and subsequently, using the one-step pseudostate technique, target conditions are achieved.

The departure and arrival hyperbolic orbit characteristics are derived from the excess velocity vectors. Using the conventional patched conic technique, only one excess velocity vector can be obtained at the MSI of planet. However, there exists four excess velocity vectors at departure that differ from each other marginally and the conventional patched conic method cannot generate them distinctly. The proposed iterative patched conic method generates the four distinct design options that are obtained by mapping each of the two hyperbolic orbit characteristics at departure with each of the arrival hyperbolic orbit characteristics. However, upon numerical propagation under a force model, these designs deviate from the specified target conditions due to patched conic assumptions. To improve the design, an iterative pseudostate technique [4] is discussed. The conventional pseudostate technique, proposed by Wilson Jr., superimposes the gravity of planet over that of the Sun within the pseudosphere, thereby considering the transfer trajectory problem as three body problem at the departure and arrival phases. This technique has been used to solve the gravity assist missions by Byrnes [8]. The State Transition Matrix (STM) has been used to synchronize the periapsis of the approach and exit legs of the flyby trajectory at a specified time. The pseudostate concept has been modified as a multistep technique to design the translunar and cislunar missions by Byrnes and Hooper [9]. They incorporated additional perturbations also in the multistep technique. Sergeyevsky et al. [10] applied the one-step pseudostate technique for direct interplanetary transfers. He assumed rectilinear hyperbolas for the planetocentric departure and arrival phases which removed the complicated computations involving STM. Ramanan [7] used the impact pseudostate technique for the arrival phase of the lunar transfer trajectory and presented an integrated approach to generate the lunar transfer trajectory design. Since the position and velocity vectors are parallel for a rectilinear hyperbola, there arises difficulty in fixing the orientation of the hyperbolic orbit plane. This design results in impact with the planets, which is not desirable in actual mission scenario. A nonimpact algorithm in the context of lunar orbiter mission to generate the transfer trajectory was proposed by Ramanan and Adimurthy [11]. They obtained the lunar parking orbit characteristics based on spherical trigonometry relations and tuned them to achieve the arrival excess velocity vector. The lunar transfer trajectory design obtained using this technique was used as the initial guess for numerical search. The refined design is found to be in the close neighborhood of the pseudostate design [12]. An iterative analytical method based on pseudostate technique for interplanetary orbiter missions has been proposed by Parvathi and Ramanan [4]. An analytical tuning strategy along with iterations upon the pseudostates resulted in reduction in target deviations greatly. This method also clearly distinguishes the four distinct transfer design options.

In this paper, the minimum energy opportunity is obtained through a grid search over the departure dates and flight durations using the conventional patched conic technique. For a fixed departure date and a flight duration, the excess velocity vectors are calculated for the departure and arrival phases from the Lambert problem connecting the planet positions on appropriate dates. The trajectory design is carried out using the iterative analytical methods using patched conic and pseudostate techniques as given in section V. The link between the excess velocity vector and the parking orbit/hyperbolic orbit characteristics is given in section III. Section IV provides a detailed account of the analytical tuning strategy used to modify the hyperbolic orbit characteristics such that excess velocity vector is achieved at the MSI. The relations and the strategy described in the sections III and IV are used in section V.

3. DETERMINATION OF PARKING ORBIT / HYPERBOLIC ORBIT CHARACTERISTICS

The size, shape and inclination of the departure parking orbit are assumed. For an orbiter mission, the periapsis altitude and the inclination of the arrival parking orbit are specified. For minimizing the TPI and POI velocity impulses, it is essential that (i) the parking orbit plane is coplanar with the hyperbolic orbit plane, (ii) the location of TPI/POI is at the periapsis of the parking orbit and (iii) the argument of periapsis (AOP) of the hyperbolic trajectory is the same as the argument of periapsis of the elliptical parking orbit or the argument of latitude of the circular parking orbit. These conditions lead to tangential and horizontal injection of the velocity impulse. The geometry of transfer trajectory is given in Figure 1.

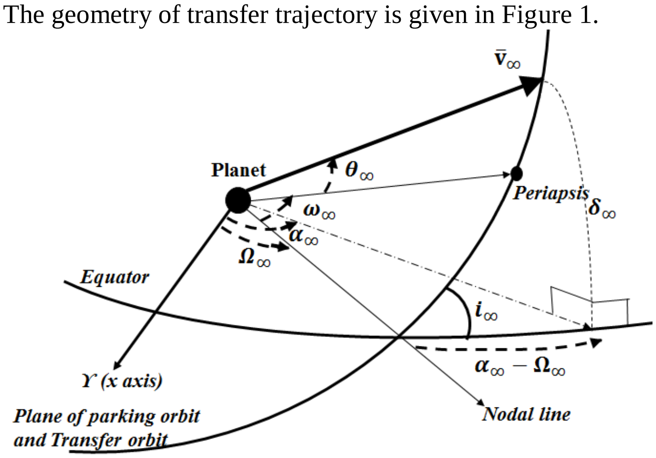

Figure 1. Geometry of the transfer trajectory

The hyperbolic orbit characteristics that achieves the excess velocity vector asymptotically must be obtained and modified using the analytical tuning strategy, given in section IV, to achieve the excess velocity vector at the MSI.
The semi-major axis of the hyperbolic orbit is given by:

$$a_\infty = -\mu_{(.)}/\mathrm{v}_\infty{}^2 \tag{1}$$

The dot given in equation (1) represents either D (departure planet) or A (arrival planet). The related eccentricity is given by:

$$e_\infty = 1 + (r_{p\infty}\mathrm{v}_\infty{}^2/\mu_{(.)}) \tag{2}$$

From the geometry given in Figure 1, the right ascension of ascending node (RAAN) and the argument of periapsis (AOP) of the hyperbolic transfer trajectory are derived using spherical trigonometric relations [7]. A major assumption used in the derivation is that the planets are spherical and only respective planet act on the spacecraft within its pseudosphere.

$$sin(\alpha_\infty - \Omega_\infty) = tan\,\delta_\infty \,/\, tan\, i_\infty \tag{3}$$

$$sin(u_\infty + \theta_\infty) = sin\,\delta_\infty \,/\, sin\, i_\infty \tag{4}$$

where θ_∞ is given by:

$$\theta_\infty = cos^{-1}(1/e_\infty) \tag{5}$$

and the argument of latitude is given by:

$$u_\infty = \omega_\infty + \nu_{p_\infty} \tag{6}$$

When the injection is at the periapsis, $\nu_{p_\infty} = 0$.
The equations (3) and (4) have two solutions, viz.

$$\Omega_\infty|_1 = \alpha_\infty - sin^{-1}(tan\,\delta_\infty \,/tan\, i_\infty) \tag{7}$$

$$\Omega_\infty|_2 = \alpha_\infty - 180 + sin^{-1}(tan\,\delta_\infty \,/tan\, i_\infty) \tag{8}$$

$$\omega_\infty|_1 = sin^{-1}(sin\,\delta_\infty/sin\, i_\infty) - \theta_\infty \tag{9}$$

$$\omega_\infty|_2 = 180 - sin^{-1}(sin\,\delta_\infty/sin\, i_\infty) - \theta_\infty \tag{10}$$

These solutions correspond to the two possible geometries that result in two hyperbolic orbit planes containing the excess velocity vector, (i) while ascending (ii) while descending. The two geometries at the departure and the arrival planets result in four transfer trajectory design options. The four design options are given in Table 1.

Table 1. Transfer trajectory design options

Transfer trajectory options	Departure	Arrival
Option 1-1	$(\Omega_{\infty_D}|_1, \omega_{\infty_D}|_1)$	$(\Omega_{\infty_A}|_1, \omega_{\infty_A}|_1)$
Option 1-2	$(\Omega_{\infty_D}|_1, \omega_{\infty_D}|_1)$	$(\Omega_{\infty_A}|_2, \omega_{\infty_A}|_2)$
Option 2-1	$(\Omega_{\infty_D}|_2, \omega_{\infty_D}|_2)$	$(\Omega_{\infty_A}|_1, \omega_{\infty_A}|_1)$
Option 2-2	$(\Omega_{\infty_D}|_2, \omega_{\infty_D}|_2)$	$(\Omega_{\infty_A}|_2, \omega_{\infty_A}|_2)$

4. ANALYTICAL TUNING STRATEGY

To achieve the asymptotic excess velocity vector at the MSI/pseudosphere of the planet, the hyperbolic orbit characteristics must be modified. This is done using an analytical tuning strategy adapted from Ref. [11] and is described below.

1. Fix a periapsis ($r_{P\infty}$) and an inclination (i_∞) of the hyperbola which is same as that of the parking orbit. Fix a duration (MSI/sweep-back/sweep-forward duration) for reaching the boundary of the MSI/pseudosphere from the periapsis.
2. Compute the semi-major axis (a_∞) and the related eccentricity (e_∞) of the hyperbola using Eq. (1) and Eq. (2) respectively.
3. Compute two values of RAAN (Ω_∞) and AOP (ω_∞) from Eq. (7) to Eq. (10), and choose one of the four transfer trajectory options.
4. Propagate the hyperbolic characteristics ($a_\infty, e_\infty, i_\infty, \Omega_\infty, \omega_\infty, \nu_\infty = \nu_{p_\infty} = 0$) forward for the sweep-back duration/ backward for sweep-forward duration under the spherical force model. Compute the position vector ($\boldsymbol{r_h}$) and the propagated excess velocity vector ($\mathbf{v_h}$).

5. Compute $\epsilon = |\mathbf{v_h} - \mathbf{v_\infty}|$. If ϵ is less than a pre-fixed tolerance value, the parking orbit characteristics is achieved, otherwise continue with the following steps.
6. Find the angle (Ψ) between the desired velocity vector ($\mathbf{v}_\infty$) and the propagated velocity vector ($\mathbf{v_h}$).
7. The shift in AOP location is computed as:

$$\Delta\omega = \Psi \tag{11}$$

8. Compute the new location of the periapsis of the hyperbolic trajectory as:

$$\omega_\infty = \omega_\infty \pm \Delta\omega \tag{12}$$

9. Compute the new semi-major axis of the rotated hyperbolic trajectory from the equation:

$$a_\infty = -\,\mu_{(.)} / \left(\mathrm{v}_\infty{}^2 - \frac{2\mu_{(.)}}{r_h} \right) \tag{13}$$

   and the related eccentricity from Eq. 2.
10. Repeat the steps (4) to (9) till the tolerance value for ϵ in the step (5) is achieved.

5. ITERATIVE ANALYTICAL METHODS

The iterative analytical methods can identify the four distinct transfer trajectory design options for interplanetary missions. Two methods are considered to evaluate the distinct design options and the resulting deviations in the target conditions. These iterative analytical methods are based on (i) patched conic technique and (ii) pseudostate technique. The details of iterative pseudostate method is available in Ref. [4] and the iterative patched conic method is described below.

5.1. Iterative patched conic method

The iterative patched conic method determines the distinct excess velocity vectors together with the related hyperbolic orbit characteristics. The three segments of interplanetary transfer considered in the patched conic technique for an example mission, Earth-Mars, is given in Figure 2. Initially, Lambert conic is determined by connecting the planet positions at the departure and arrival dates and the hyperbolic orbit characteristics are computed through steps (1) to (6). Using this Lambert conic, the patch points at the MSI are obtained at the departure and arrival ends through steps (7) to (10). Later, Lambert conic is determined by connecting the patch point positions for a modified flight duration that excludes the MSI durations.

The iteration on the patch point results in four distinct transfer trajectory options.

1. Fix the periapsis altitude and inclination of the departure and arrival hyperbolas.
2. Fix the departure date and flight duration (t_{FD}) of the minimum opportunity transfer. Fix the MSI duration for the departure and arrival planets as t_D and t_A days.

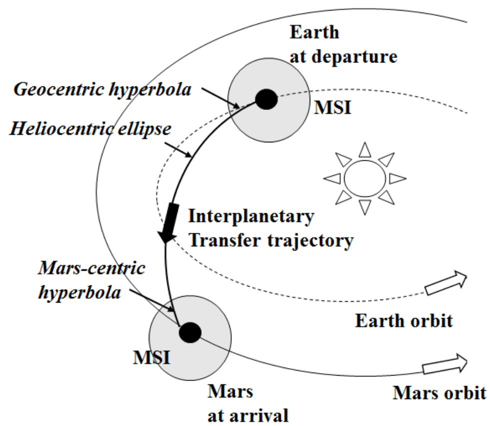

Figure 2. Patched conic approximation for Earth-Mars interplanetary transfer.

3. Set the heliocentric states of the departure planet on the departure date ($\boldsymbol{R_D}, \boldsymbol{V_D}$) and the arrival planet on the arrival date ($\boldsymbol{R_A}, \boldsymbol{V_A}$).
4. Determine the heliocentric Lambert conic connecting the position vectors ($\boldsymbol{R_D}, \boldsymbol{R_A}$) for the given flight duration (t_{FD}). Compute the corresponding heliocentric velocity vectors of the spacecraft in the transfer trajectory: $\boldsymbol{V_{pc_D}}$ and $\boldsymbol{V_{pc_A}}$.
5. Compute the planetocentric excess velocity vectors of the spacecraft at the departure and arrival planets that is, $\mathbf{v}_{\infty\boldsymbol{D}}$ and $\mathbf{v}_{\infty\boldsymbol{A}}$ respectively.

$$\mathbf{v}_{\infty\boldsymbol{D}} = |\boldsymbol{V_{pc_D}} - \boldsymbol{V_D}| \tag{14}$$

$$\mathbf{v}_{\infty\boldsymbol{A}} = |\boldsymbol{V_{pc_A}} - \boldsymbol{V_A}| \tag{15}$$

6. From the excess velocity vectors, find the characteristics ($a_\infty, e_\infty, i_\infty, \Omega_\infty, \omega_\infty, \nu_\infty = \nu_{p_\infty} = 0$) of departure and arrival hyperbolas using the procedure described in section III. One of the four options is chosen for modification such that the target parameters are achieved.
   The following steps make the departure hyperbolic orbit characteristics distinct from the other options. The process can be applied for other options as well.
7. Propagate the departure hyperbolic orbit characteristics forward for the corresponding prefixed MSI duration (t_D) and find the planetocentric velocity vector ($\mathbf{v_{h_D}}$). Similarly, propagate the arrival hyperbolic orbit characteristics backward from APO periapsis for the

prefixed MSI duration (t_A) and find the arrival planetocentric velocity vector ($\mathbf{v}_{\mathbf{h}_A}$).

8. The desired and the propagated excess velocity vectors that is., $\mathbf{v}_{\infty_D}$ and $\mathbf{v}_{\mathbf{h}_D}$ are matched at the MSI of the departure planet using the tuning strategy described in section IV. The tuning of desired and propagated excess velocity vectors at the arrival planet is also carried out using the same strategy.
9. Obtain the position vectors at the MSI of the departure ($\boldsymbol{r}_{h_D}$) and the arrival ($\boldsymbol{r}_{h_A}$) planets by propagating the respective updated/tuned hyperbolic orbit characteristics.
10. The planetocentric position vectors of the patch points at MSI are transformed into heliocentric position vectors.

$$\boldsymbol{R}_D^{MSI} = \boldsymbol{R}_D^{t_D} + \boldsymbol{r}_{h_D} \tag{16}$$

$$\boldsymbol{R}_A^{MSI} = \boldsymbol{R}_A^{t_A} + \boldsymbol{r}_{h_A} \tag{17}$$

where $\boldsymbol{R}_D^{t_D}$ is the position vector of the departure planet after t_D days since departure epoch and $\boldsymbol{R}_D^{MSI}$ is the position vector of the patch point at the MSI of the departure planet. Similarly in Eqn. (17), $\boldsymbol{R}_A^{t_A}$ is the position vector of the arrival planet before t_A days from the arrival epoch and $\boldsymbol{R}_A^{MSI}$ is the position vector of the patch point at the MSI of the arrival planet.

11. Determine Lambert conic connecting the heliocentric patch point positions for a flight duration defined by ($t_{FD} - t_D - t_A$). The heliocentric velocity vectors at the patch points in the Lambert conic are $\boldsymbol{V}_{pc_D}^{MSI}$ and $\boldsymbol{V}_{pc_A}^{MSI}$ respectievely.
12. Compute the planetocentric velocity vectors of the spacecraft at the patch points that is, $\mathbf{v}_{\infty_D}$ and $\mathbf{v}_{\infty_A}$ respectively,

$$\mathbf{v}_{\infty_D} = |\boldsymbol{V}_{pc_D}^{MSI} - \boldsymbol{V}_D^{t_D}| \tag{18}$$

$$\mathbf{v}_{\infty_A} = |\boldsymbol{V}_{pc_A}^{MSI} - \boldsymbol{V}_A^{t_A}| \tag{19}$$

where $\boldsymbol{V}_D^{t_D}$ is the velocity vector of the departure planet after t_D days since departure epoch and $\boldsymbol{V}_A^{t_A}$ is the velocity vector of the arrival planet before t_A days from the arrival epoch.

13. From the excess velocity vectors, find the characteristics ($a_\infty, e_\infty, i_\infty, \Omega_\infty, \omega_\infty, \nu_\infty = \nu_{p_\infty} = 0$) of departure and arrival hyperbolas.
14. Repeat the steps (7) to (10) and find the difference between two successive position vectors of the patch points at both the departure and arrival planets. If the differences are less than the predefined small values, then the transfer trajectory design is obtained. Otherwise the steps (11) to (14) are repeated.

At the end of these steps, the transfer trajectory design for one of the options is obtained. We can apply these steps for the other three design options.

5.2. Iterative pseudostate method

The iterative analytical pseudostate method is available in Ref [4] and so, not given herein.

6. RESULTS

The design analysis tools based on the iterative analytical methods, described in the previous sections, have been used to analyze the distinct design for minimum energy Earth-Mars orbiter mission in 2018. A typical minimum energy opportunity, based on grid search, is obtained using the conventional patched conic technique. A search over a range of departure dates and flight durations results in a minimum energy opportunity of 12 May 2018 0 hours UTC with a flight duration 204 days. The earth parking orbit is assumed as 300 x 25,000 km and inclination of 75 degree with respect to the Earth equator and equinox of J2000. The mars parking orbit is assumed as 300 km circular and inclination of 75 degree with respect to the Mars equator and equinox of J2000 [13]. The MSI/pseudosphere duration for Earth and Mars are considered as 3 and 2 days respectively. The DE 405 JPL ephemeris has been used to find the position of planets.

Table 2. Excess velocity vectors from the conventional patched conic method

Parameters	Conventional patched conic method
$v_{\infty D}$(km/s)	2.7891
$\alpha_{\infty D}$(deg)	321.4262
$\delta_{\infty D}$(deg)	-36.8551
$v_{\infty A}$(km/s)	2.9621
$\alpha_{\infty A}$(deg)	245.6645
$\delta_{\infty A}$(deg)	9.2562

The trajectory design is done using different analytical methods. The departure and arrival excess velocity vectors obtained from the conventional patched conic method is given in Table 2. This method cannot identify the distinct design options due to the absence of an iterative procedure connecting the departure and arrival phases. The proposed iterative method based on patched conic technique identifies the four distinct design options and the corresponding excess velocity vectors are given in Table 3. The iterative pseudostate technique also identifies the four distinct design options and Table 4 lists the corresponding excess velocity vectors from Ref [4]. The related hyperbolic characteristics obtained from the conventional patched conic method is given in Table 5. Note that the design options 1-1 and 1-2 result in entirely different arrival hyperbolas even if the departure conditions are exactly the same. Similarly the design options 1-1 and 2-1 result in the same arrival hyperbolas even if the departure conditions are entirely different. This is not feasible and is due to the inability of the method to distinguish the design options.

Table 3. Excess velocity vectors from the proposed iterative patched conic method

Parameters	Iterative patched conic method			
	Option 1-1	Option 1-2	Option 2-1	Option 2-2
$v_{\infty D}$(km/s)	2.7826	2.7839	2.7779	2.7791
$\propto_{\infty D}$(deg)	321.65	321.68	321.53	321.55
$\delta_{\infty D}$(deg)	-37.205	-37.285	-36.842	-36.923
$v_{\infty A}$(km/s)	2.9602	2.9598	2.9614	2.9610
$\propto_{\infty A}$(deg)	245.51	245.49	245.60	245.56
$\delta_{\infty A}$(deg)	9.5035	9.5434	9.2193	9.2594

Table 4. Excess velocity vectors from the iterative pseudostate method [4]

Parameters	Iterative pseudostate method			
	Option 1-1	Option 1-2	Option 2-1	Option 2-2
$v_{\infty D}$(km/s)	2.7893	2.7906	2.7847	2.7859
$\propto_{\infty D}$(deg)	321.921	321.944	321.785	321.808
$\delta_{\infty D}$(deg)	-37.1345	-37.2146	-36.764	-36.8449
$v_{\infty A}$(km/s)	2.9609	2.9605	2.9622	2.9618
$\propto_{\infty A}$(deg)	245.516	245.484	245.599	245.567
$\delta_{\infty A}$(deg)	9.5043	9.5446	9.2203	9.2607

The hyperbolic orbit characteristics obtained from the proposed iterative patched conic method is given in Table 6. Note that the design options are distinct. There is marginal but still significant difference in TPI angles viz., departure RAAN and AOP, between the design options 1-1 and 1-2 (0.057 degree and 0.072 degree respectively). These design options result in entirely different arrival hyperbolas. Similar trend is observed for the design options 2-1 and 2-2. This reflects the high sensitivity of the arrival conditions to the departure angles.

Table 5. Design options obtained from the conventional patched conic method

Parameters	Option 1-1	Option 1-2	Option 2-1	Option 2-2
$a_{\infty D}$ (km)	-51239.9	-51239.9	-51239.9	-51239.9
$e_{\infty D}$	1.13033	1.13033	1.13033	1.13033
$i_{\infty D}$ (deg)	75	75	75	75
$\Omega_{\infty D}$ (deg)	333.0131	333.0131	129.8392	129.8392
$\omega_{\infty D}$(deg)	169.3999	169.3999	64.5845	64.5845
$a_{\infty A}$(km)	-4881.1	-4881.1	-4881.1	-4881.1
$e_{\infty A}$	1.75744	1.75744	1.75744	1.75744
$i_{\infty A}$ (deg)	75	75	75	75
$\Omega_{\infty A}$(deg)	68.1673	243.1616	68.1673	243.1616
$\omega_{\infty A}$(deg)	115.0951	314.2668	115.0951	314.2668

Table 6. Design options obtained from the proposed iterative patched conic method

Parameters	Option 1-1	Option 1-2	Option 2-1	Option 2-2
$a_{\infty D}$ (km)	-58965.7	-58904.1	-59206.2	-59145.1
$e_{\infty D}$	1.11325	1.11337	1.11279	1.11291
$i_{\infty D}$ (deg)	75	75	75	75
$\Omega_{\infty D}$ (deg)	333.3889	333.4465	129.9454	129.9341
$\omega_{\infty D}$(deg)	167.378	167.305	64.457	64.554
$a_{\infty A}$ (km)	-4980.0	-4981.3	-4975.7	-4977.1
$e_{\infty A}$	1.74240	1.74220	1.74304	1.74284
$i_{\infty A}$ (deg)	75	75	75	75
$\Omega_{\infty A}$(deg)	68.0878	242.9041	68.0925	243.0652
$\omega_{\infty A}$(deg)	115.178	314.908	115.458	314.599

Table 7 shows the target conditions achieved upon numerical propagation of these design options under a force model. The force model used in this paper includes Sun and Earth for 3 days, Sun alone for 199 days, and Sun and Mars for 2 days. The closest approach altitude desired is 300 km, however with the assumptions involved in iterative patched conic method, the achievable CAA ranges from 192,798 to 212,644 km. To account for the modelling errors, trans correction maneuvers (TCM) are required for these designs to achieve the desired target conditions. The TCM requirement for iterative patched conic design is about 11-12 m/s is and is applied after 13 days from the Earth departure. The estimated TCM magnitudes are given in Table 7.

Table 7. Achievable accuracies from the patched conic methods

Parameters	Proposed iterative patched conic design			
	Option 1-1	Option 1-2	Option 2-1	Option 2-2
CAA (km)	192,798	197,674	207,562	212,644
T_p (UTC) DD/MM/YYYY HH:MM:SS	2 Dec 2018 10:06:53.608	2 Dec 2018 10:06:17.249	2 Dec 2018 12:09:31.693	2 Dec 2018 12:09:02.922
TCM (m/s)	12	11.61	11.64	11.61

The hyperbolic orbit characteristics obtained from the iterative pseudostate technique is given in Table 8. The target conditions achieved upon numerical propagation of the pseudostate designs under the same force model is given in Table 9. The CAA achieved ranges from 1,965 to 11,457 km. The difference in TPI angles between the iterative patched and iterative pseudostate methods for option 1-1 are 0.24 deg (RAAN) and 0.14 deg (AOP). These differences are large and results in improved CAA upon numerical propagation, that is, from 192,798 km to 1,965 km. The time of periapsis is deviated by about 10 hours in the case of iterative patched conic design whereas the deviation is about 2 hours for the iterative pseudostate design (cf. Table 7 and 9). The target inclination is not achieved since the improved pseudostate design still needs refinement. The TCM required for the pseudostate design is found to be less than 1 m/s to achieve the desired CAA and inclination at the specified periapsis time and is given in Table 9 [4]. The importance of the precise initial conditions for interplanetary transfer missions is illustrated. The numerical refinement of the iterative patched conic design takes about 420 seconds in an Intel core i5 machine whereas the iterative pseudostate design takes about 7 seconds.

The TPI and POI velocity impulses required for Earth-Mars 2018 orbiter mission from different analytical designs are given in Tables 10 and 11. The difference in total velocity impulse between different designs is marginal. Hence, it is evident that there is no considerable increase in the required velocity impulse to reach Mars with an improved closest approach altitude of about 1,965 km from 212,644 km.

Table 8. Design options obtained from the iterative pseudostate method [4]

Parameters	Option 1-1	Option 1-2	Option 2-1	Option 2-2
$a_{\infty D}$ (km)	-58625.7	-58564.6	-58859.5	-58799.1
$e_{\infty D}$	1.11391	1.11403	1.11345	1.11357
$i_{\infty D}$ (deg)	75	75	75	75
$\Omega_{\infty D}$ (deg)	333.627	333.685	130.236	130.225
$\omega_{\infty D}$(deg)	167.521	167.448	64.444	64.542
$a_{\infty A}$(km)	-4977.4	-4978.7	-4973.1	-4974.5
$e_{\infty A}$	1.74279	1.74259	1.74343	1.74323
$i_{\infty A}$ (deg)	75	75	75	75
$\Omega_{\infty A}$(deg)	68.086	242.901	68.091	243.062
$\omega_{\infty A}$(deg)	115.168	314.900	115.447	314.591

Table 9. Achievable accuracies from the iterative pseudostate method [4]

Parameters	Iterative pseudostate design			
	Option 1-1	Option 1-2	Option 2-1	Option 2-2
CAA (km)	1,965	4,727	7,084	11,457
T_p (UTC) DD/MM/YYYY HH:MM:SS	2 Dec 2018 01:36:00.766	2 Dec 2018 01:42:27.149	2 Dec 2018 02:37:05.208	2 Dec 2018 02:43:55.109
TCM (m/s)	Less than 1 m/s			

Table 10. Velocity impulses from the patched conic designs

Velocity impulses (m/s)	Conventional patched conic design	Design from the proposed iterative patched conic method			
		Option 1-1	Option 1-2	Option 2-1	Option 2-2
TPI	1355.22	1309.93	1310.25	1308.71	1309.02
POI	2248.21	2232.78	2232.57	2233.44	2233.23

Table 11. Velocity impulses from the iterative pseudostate designs [4]

Velocity impulses (m/s)	Design from the iterative pseudostate method			
	Option 1-1	Option 1-2	Option 2-1	Option 2-2
TPI	1311.68	1311.99	1310.48	1310.79
POI	2233.18	2232.98	2233.84	2233.63

7. CONCLUSIONS

The proposed iterative patched conic method is efficient in identifying the four distinct transfer trajectory design options available for a given departure date and a flight duration. A Fortan 95 code has been developed for design analysis using the proposed method. The numerical propagation of the proposed design achieves a closest approach altitude of 191,798 to 212,644 km. A trans correction maneuver of about 12 m/s is required for the design to meet the desired target conditions. This indicates the high sensitivity of the arrival conditions to the initial departure conditions. The proposed design is compared with the iterative pseudostate design. The latter design achieves the target conditions with a TCM less than 1 m/s. The reduction in deviation of closest approach altitude is about 98% and the related time is 95%. The iterative patched conic design requires about 420 seconds for numerical refinement while the iterative pseudostate method requires less than 7 seconds. The iterative pseudostate method provides a better initial guess and takes much less time for numerical refinement. However, the iterative patched conic method can be used as an alternative design analysis tool considering its simplicity in formulation that involves two-body models. Both these iterative analytical methods generate better designs when compared to the conventional patched conic method.

8. REFERENCES

[1] Richard H. Battin. An Introduction to the Mathematics and Methods of Astrodynamics, Revised Edition. American Institute of Aeronautics and Astronautics, Inc. Reston, VA., 1999.

[2] Hanspeter Schaub and John L. Junkins. Analytical Mechanics of Space Systems. American Institute of Aeronautics and Astronautics, Inc. Reston, VA., 2003.

[3] S.W. Wilson, Jr., "A Pseudostate Theory for the Approximation of Three-Body Trajectories." AIAA Paper No. 70-1061, AAS/AIAA Astrodynamics Conference, Santa Barbara, California, Aug. 19-21, 1970.

[4] S.P. Parvathi and R.V. Ramanan, "Multiple design options for interplanetary orbiter missions using pseudostate technique", presented in 26th AAS/AIAA Space Flight Mechanics Meeting, Napa, California, February 2016.

[5]. V.C. Clarke, Jr., W.E. Bollman, et al., " Design parameters for Ballistic Interplanetary trajectories, Part II: One-way transfers to Mercury and Jupiter.", NASA Technical report 32-77, Jet Propulsion laboratory, Pasadena, CA, Jan 15, 1966.

[6]. J.W. Cornelisse, "Trajectory analysis for interplanetary missions." ESA Journal. Vol. 2, No. 2, 1978, pp. 131–144.

[7]. R.V. Ramanan, " Integrated Algorithm for Lunar Transfer Trajectories using a Pseudostate Technique", Journal of Guidance, Control and Dynamics, Vol.25, No.5, Sep.-Oct., 2002.
doi: 10.2514/2.4968.

[8]. D.V. Byrnes, "Application of Pseudostate Theory to the Three-Body Lambert Problem", Journal of Astronautical Sciences, Vol. 37, No.3, July-Sep. 1989, p.p. 221-232.

[9]. D.V. Byrnes and H.L. Hooper, "Multiconic: A Fast and Accurate Method of Computing Space Flight Trajectories", AIAA paper No.70-1062, AAS/AIAA Astrodynamics Conference, Santa Barbara, California, Aug. 19-21, 1970.

[10]. A.B. Sergeyevsky, D.V. Byrnes and L.A. D'Amario, "Application of the Rectilinear Impact Pseudostate Method to Modeling of Third Body Effects of Interplanetary Trajectories", AIAA-83-0015, AIAA 21st Aerospace Sciences Meeting, January 10-13, 1983.

[11]. R.V. Ramanan and V. Adimurthy, "Nonimpact Lunar Transfer Trajectories Using the Pseudostate Technique", Journal of Guidance, Control and Dynamics, Vol.28, No.2, March-April, 2005.
doi: 10.2514/1.7607.

[12]. V. Adimurthy, R.V. Ramanan, et al., "Launch strategy for Indian Lunar mission and precision injection to the Moon using genetic algorithm", Journal of Earth System Sciences, Vol. 114, No.6, December 2005, pp. 711-716.

[14]. R.H. Battin, "Introduction to the Mathematical Methods of Astrodynamics", AIAA Education series, 1987, AIAA, New York.
doi: 10.2514/4.861543.

[15]. B. A. Archinal, M. F. A'Hearn, E. Bowell, et. al, "Report of the IAU Working Group on Cartographic Coordinates and Rotational Elements: 2009," Celestial Mechanics & Dynamical Astronomy, vol. 109, no. 2, February, 101-135. doi 10.1007/s10569-010-9320-4.