Research Institute of Applied Mechanics and
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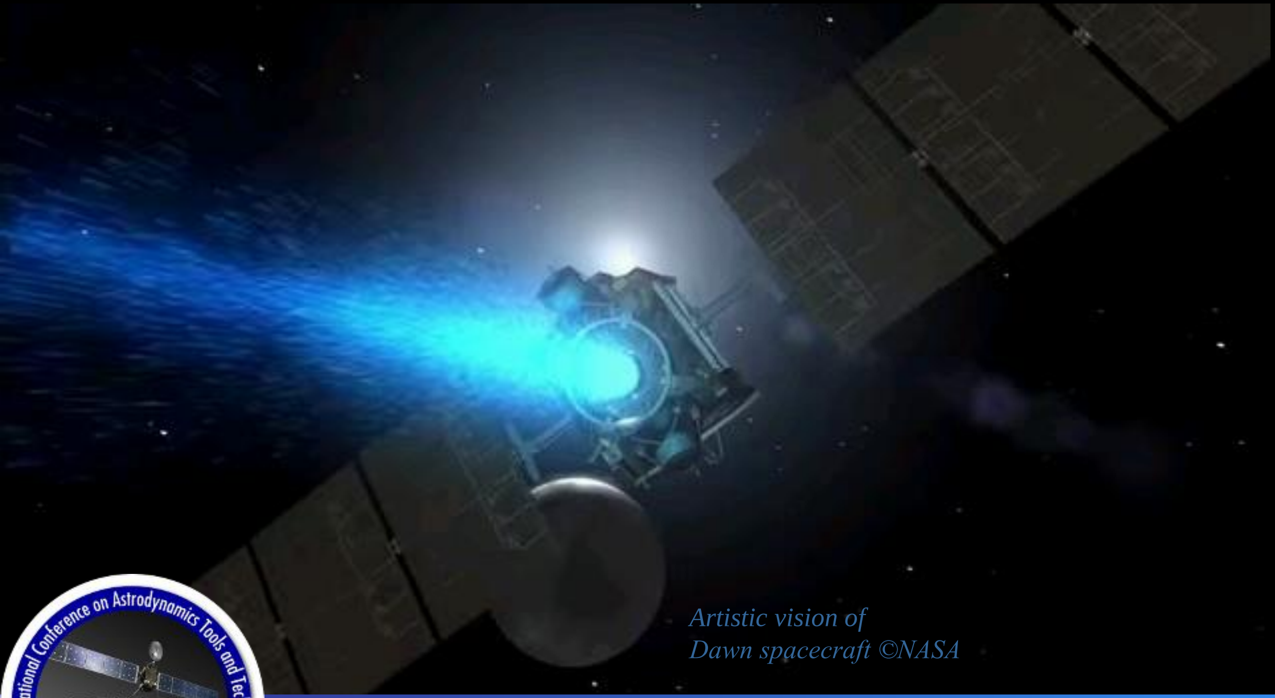
JOINT OPTIMIZATION OF MAIN DESIGN PARAMETERS OF ELECTRIC PROPULSION SYSTEM AND SPACECRAFT TRAJECTORY



*Artistic vision of
Dawn spacecraft ©NASA*

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Low-thrust trajectory optimization: fixed-time problems

Problem	**Minimum propellant**	**Minimum thrust**	**Optimal thrust&power**
Cost function	$m_p \to \min \Leftrightarrow$ $m_{PSFS} = (1+a_t)m_p \to \min$	$T \to \min \Leftrightarrow$ $P \to \min \Leftrightarrow$ $m_{PSPU} = \gamma P = \gamma Tc/2\eta \to \min$	$m_u \to \max \Leftrightarrow$ $m_{PSPU} + m_{PSFS} =$ $= \gamma P + (1+a_t)m_p \to \min$
Pros	- Maximum m_u for fixed T, c, P	- Bounds region of solutions existance for minimum-propellant problem ($T_{\min}(c)$) - Regular problem - Very good diagnostic tool for minimum propellant problem	- Optimal T, c, P are computed together with OCP solving
Cons	- Irregular problem: it is unknown apriori either solution exists or not - Series OCP should be solved to find optimal T, c, P	- Excessive propellant consumption on the minimum-thrust solution	- BVP has increased order (two additional optimality conditions and two additional unknowns)

The dynamic problem: a model of the spacecraft motion with the specified design parameters

The problem of optimizing the heliocentric trajectory of the spacecraft with EP in a central gravitational field (fixed-tme problem, approach based on gravity spheres) :

$$\frac{d\mathbf{x}}{dt} = \mathbf{v}$$
$$\frac{d\mathbf{v}}{dt} = \Omega_{\mathbf{x}} + \delta \frac{T}{m} \mathbf{e}_T$$
$$\frac{dm}{dt} = -\delta \frac{T}{c}$$

$$t = t_0 :$$
$$\mathbf{x}(t_0) = \mathbf{x}_0(t_0)$$
$$\left| \mathbf{v}(t_0) - \mathbf{v}_0(t_0) \right| = V_{\infty 0}$$
$$m(t_0) = m_0$$

$$t = t_f = t_0 + \Delta t :$$
$$\mathbf{x}(t_f) = \mathbf{x}_f(t_f)$$
$$\mathbf{v}(t_f) = \mathbf{v}_f(t_f)$$

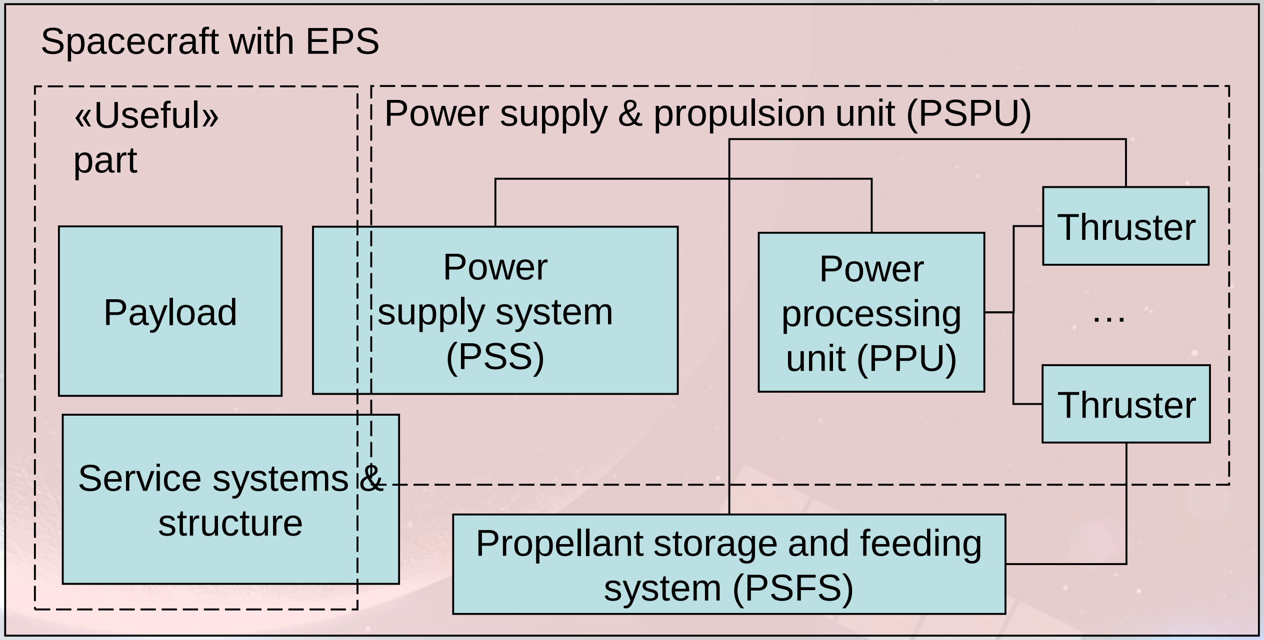
$$\Omega = \mu / r \qquad r = |\mathbf{x}| \qquad T, c, t_0, \Delta t \text{ - } \mathit{fixed}$$

Main trajectory parameters: t_0, Δt, $V_{\infty 0}$, ...

A choice of controls $\delta(t), \mathbf{e}_T(t)$ maximizing final mass of spacecraft:

$$m(t_f) \to \max$$

The parametric problem: simplest model of the mass budget



Initial SC mass: $m_0 = m_\mathrm{u} + m_\mathrm{PSPU} + m_\mathrm{PSFS} = m_\mathrm{u} + \gamma P + (1+a_t)m_p$

PSPU mass: $m_\mathrm{PSPU} = \gamma P = \gamma Tc/2\eta$

PSFS mass: $m_\mathrm{PSFS} = (1+a_t)m_p$

Main design parameters: m_0, η, γ, a_t (fixed),
P, T, c: $P = Tc/2\eta$ (selected)

General formulation of thrust minimization problem

The region of existence of a solutions is bounded by the values of minimum thrust and exhaust velocity.
A certain finite increment in characteristic velocity is required in order to complete the intended flight. This increment may be attained at fixed time $\Delta t = t_f - t_0$ only by applying a sufficient thrust.
Since the allowed propellant consumption is often limited from above:

$$m_p \le m_{p\max} < m_0, \quad \text{or} \quad m(t_f) \ge m_{f\min} < m_0$$

consequently, exhaust velocity should be sufficiently high.

Compute optimal $\delta(t), \mathbf{e}_T(t), T, c,$ minimizing useful mass of spacecraft

$$J = T \to \min,$$

$$\frac{d\mathbf{x}}{dt} = \mathbf{v}$$

$$\frac{d\mathbf{v}}{dt} = \Omega_{\mathbf{x}} + \delta \frac{T}{m} \mathbf{e}_T$$

$$\frac{dm}{dt} = -\delta \frac{T}{c}$$

$t = t_0:$

$$\mathbf{x}(t_0) = \mathbf{x}_0(t_0)$$

$$|\mathbf{v}(t_0) - \mathbf{v}_0(t_0)| = V_{\infty 0}$$

$$m(t_0) = m_0$$

$t = t_f = t_0 + \Delta t:$

$$\mathbf{x}(t_f) = \mathbf{x}_f(t_f)$$

$$\mathbf{v}(t_f) = \mathbf{v}_f(t_f)$$

Boundary value problem of PMP

Smoothed thrustig function

$$\delta = \frac{1}{2}\left(\frac{\psi}{|\psi| + \varepsilon} + 1\right)$$

ε - the small regularizing parameter

Differential equations of optimal motion

$$\frac{d\mathbf{x}}{dt} = \mathbf{v} \qquad \frac{d\mathbf{p_x}}{dt} = -\Omega_{\mathbf{xx}}\mathbf{p_v}$$

$$\frac{d\mathbf{v}}{dt} = \Omega_{\mathbf{x}} + \delta\frac{T}{m}\mathbf{e}_T \qquad \frac{d\mathbf{p_v}}{dt} = -\mathbf{p_x}$$

$$\frac{dm}{dt} = -\delta\frac{T}{c} \qquad \frac{dp_m}{dt} = \delta T\frac{p_v}{m^2}$$

$$\frac{dT}{dt} = 0 \qquad \frac{dp_T}{dt} = -\delta\psi$$

Initial conditions

$$\mathbf{x}(t_0) = \mathbf{x}_0(t_0)$$

$$\mathbf{v}(t_0) = \mathbf{v}_0(t_0) + V_{\infty 0}\frac{\mathbf{p_v}(t_0)}{p_v(t_0)}$$

$$m(t_0) = m_0$$

$$p_T(t_0) = 0$$

Final conditions

$$\mathbf{x}(t_f) = \mathbf{x}_f(t_f)$$

$$\mathbf{v}(t_f) = \mathbf{v}_f(t_f)$$

$$\begin{cases} p_m(t_f) = 0,\ m(t_f) > m_{f\min} \\ p_m(t_f) > 0,\ m(t_f) = m_{f\min} \end{cases}$$

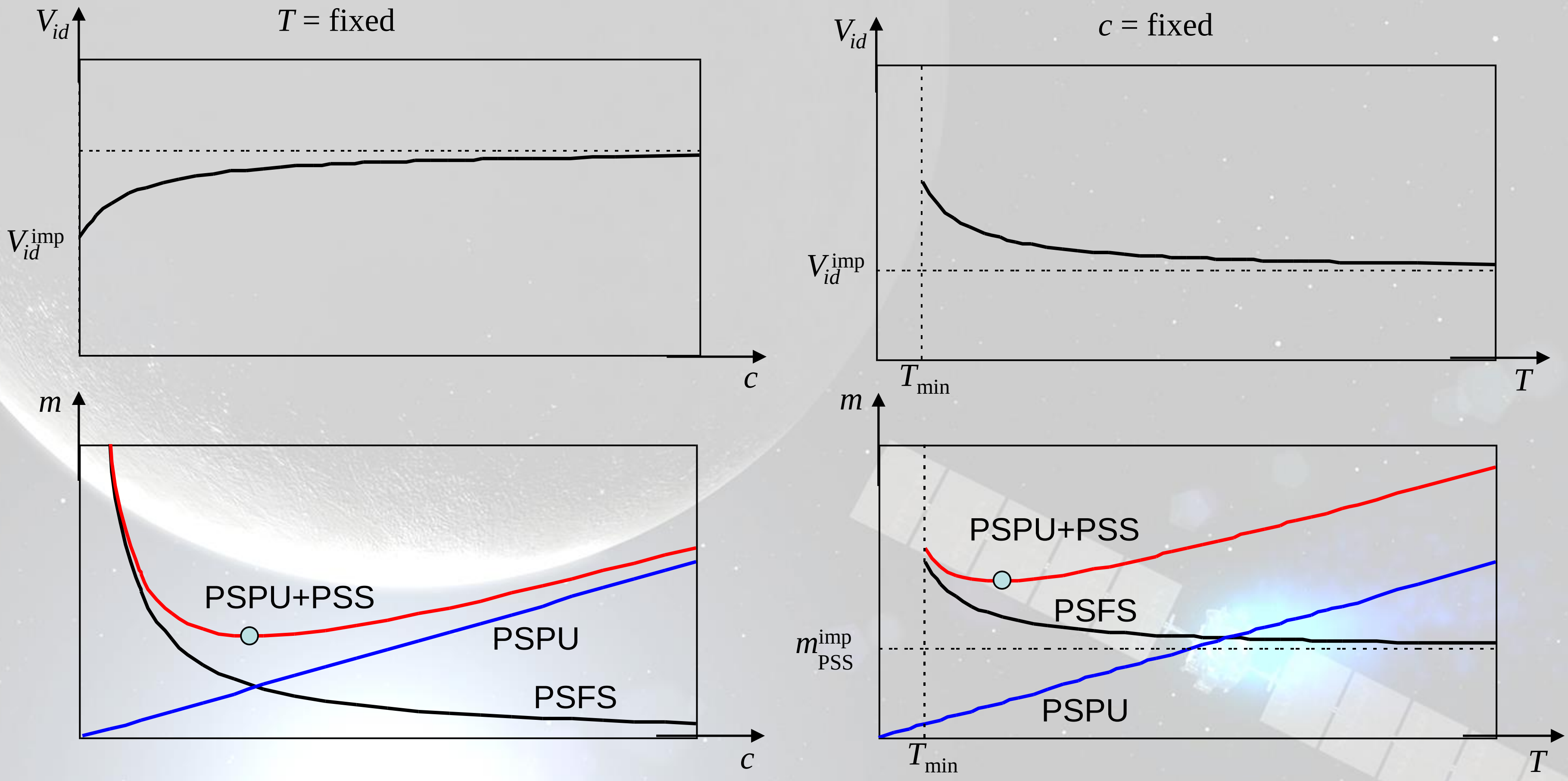
$$p_T(t_f) = \text{const} < 0$$

Unknown BVP parameters

$$\left(\mathbf{p_x}(t_0), \mathbf{p_v}(t_0), p_m(t_0), T\right)$$

Necessary conditions of thrust minimality

Minimum of PSPU+PSS mass wrt. specific impulse and thrust



$$m_{PSPU} = \gamma Tc/2\eta$$

$$m_{PSFS} = (1+a_t)m_p = (1+a_t)\exp(-V_{id}/c)$$

General formulation of joint optimization problem

Compute optimal $\delta(t), \mathbf{e}_T(t), T, c,$ maximizing useful mass of spacecraft

$$m_u = m_0 - \gamma P - (1 + a_t) m_p$$

where $P = \dfrac{Tc}{2\eta}$ - electrical power of EPS

$$m_p = m_0 - m(t_f) = \int_{t_0}^{t_f} \delta \frac{T}{c} dt$$ - active propellant mass

$$\frac{d\mathbf{x}}{dt} = \mathbf{v}$$

$$\frac{d\mathbf{v}}{dt} = \Omega_{\mathbf{x}} + \delta \frac{T}{m} \mathbf{e}_T$$

$$\frac{dm}{dt} = -\delta \frac{T}{c}$$

$t = t_0 :$

$$\mathbf{x}(t_0) = \mathbf{x}_0(t_0)$$

$$|\mathbf{v}(t_0) - \mathbf{v}_0(t_0)| = V_{\infty 0}$$

$$m(t_0) = m_0$$

$t = t_f = t_0 + \Delta t :$

$$\mathbf{x}(t_f) = \mathbf{x}_f(t_f)$$

$$\mathbf{v}(t_f) = \mathbf{v}_f(t_f)$$

Considering T & P optimization instead T & c optimization

Dependency of specific impulse on power and thrust $$c = \frac{2\eta P}{T}$$

Equations of motion

$$\frac{d\mathbf{x}}{dt} = \mathbf{v}$$

$$\frac{d\mathbf{v}}{dt} = \Omega_{\mathbf{x}} + \delta \frac{T}{m}\mathbf{e}_T$$

$$\frac{dm}{dt} = -\delta \frac{T^2}{2\eta P}$$

$$\frac{dT}{dt} = 0$$

$$\frac{dP}{dt} = 0$$

Cost function

$$J = \int_{t_0}^{t_f} \left(\frac{\gamma P}{\Delta t \cdot (1 + a_t)} + \delta \frac{T^2}{2\eta P} \right) dt = \int_{t_0}^{t_f} \left(\alpha P + \delta \frac{T^2}{2\eta P} \right) dt$$

Hamiltonian

$$H = -\alpha P + \mathbf{p}_{\mathbf{x}}^{\mathbf{T}}\mathbf{v} + \mathbf{p}_{\mathbf{v}}^{\mathbf{T}}\Omega_{\mathbf{x}} + \delta T \psi$$

Optimal control

$$\mathbf{e}_T = \frac{\mathbf{p}_{\mathbf{v}}}{p_v} \qquad \delta = \begin{cases} 1, \psi > 0 \\ 0, \psi < 0 \end{cases} \qquad \psi = \frac{p_v}{m} - \frac{T \cdot (p_m + 1)}{2\eta P}$$

Boundary value problem of PMP

Smoothed thrustig function

$$\delta = \frac{1}{2}\left(\frac{\psi}{|\psi| + \varepsilon(\tau)} + 1\right)$$

Differential equations of optimal motion

$$\frac{d\mathbf{x}}{dt} = \mathbf{v}$$

$$\frac{d\mathbf{v}}{dt} = \Omega_{\mathbf{x}} + \delta \frac{T}{m} \frac{\mathbf{p_v}}{p_v}$$

$$\frac{dm}{dt} = -\delta \frac{T^2}{2\eta P}$$

$$\frac{dT}{dt} = 0$$

$$\frac{dP}{dt} = 0$$

$$\frac{d\mathbf{p_x}}{dt} = -\Omega_{\mathbf{xx}} \mathbf{p_v}$$

$$\frac{d\mathbf{p_v}}{dt} = -\mathbf{p_x}$$

$$\frac{dp_m}{dt} = \delta T \frac{p_v}{m^2}$$

$$\frac{dp_T}{dt} = -\delta\psi + \delta T \frac{p_m + 1}{2\eta P}$$

$$\frac{dp_P}{dt} = \alpha - \delta T^2 \frac{p_m + 1}{2\eta P^2}$$

Initial conditions

$t = t_0$:

$$\mathbf{x}(t_0) = \mathbf{x}_0(t_0)$$

$$\mathbf{v}(t_0) = \mathbf{v}_0(t_0) + V_{\infty 0} \frac{\mathbf{p_v}(t_0)}{p_v(t_0)}$$

$$m(t_0) = m_0$$

$$p_T(t_0) = 0$$

$$p_P(t_0) = 0$$

Final conditions

$t = t_f = t_0 + \Delta t$:

$$\mathbf{x}(t_f) = \mathbf{x}_f(t_f)$$

$$\mathbf{v}(t_f) = \mathbf{v}_f(t_f)$$

$$p_m(t_f) = 0$$

$$p_T(t_f) = 0$$

$$p_P(t_f) = 0$$

Unknown BVP parameters

$$\left(\mathbf{p_x}(t_0), \mathbf{p_v}(t_0), p_m(t_0), T, P\right)$$

Necessary conditions of thrust and power optimality

BVP solving

Additional optimality conditions:

$$\int_{t_0}^{t_f} \left(-\delta\psi + \delta T \frac{p_m + 1}{2\eta P} \right) = 0,$$

$$\int_{t_0}^{t_f} \left(\alpha - \delta T^2 \frac{p_m + 1}{2\eta P^2} \right) = 0.$$

$$\alpha = \frac{\gamma}{\Delta t \cdot (1 + a_t)}$$

The method of continuation on parameter (Newton homotopy) is used to solve the boundary value problem by reducing it to a Cauchy problem.

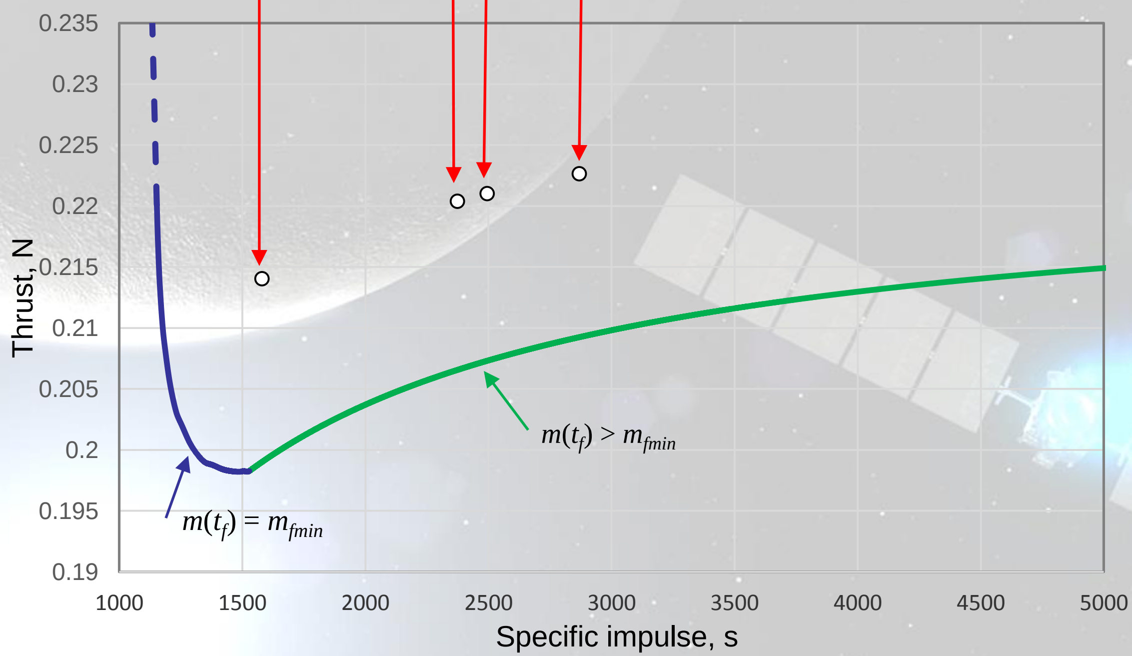
The solution of the power-limited trajectory optimization problem is used as the initial approximation for the unknown parameters of the boundary value problem.

The initial value of the thrust is assumed to be the average value of the thrust on the solution to the power-limited problem.

Numerical example: trajectrory to Mars (1/5)

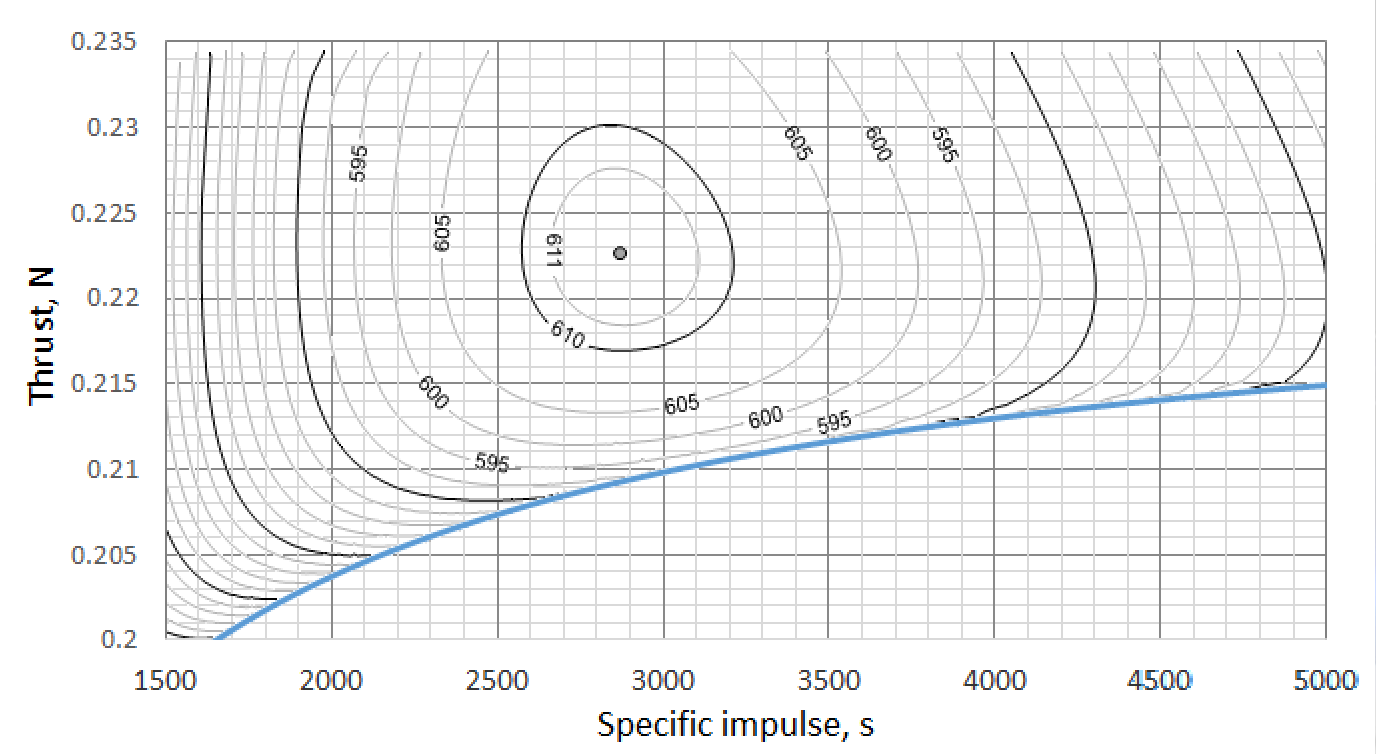
η	γ	a_t	c_{opt}	T_{opt}
--	kg/kW	--	sec	mN
0.7	40	0.1	2869	222.66
0.5	40	0.1	2374	220.39
0.5	80	0.1	1580	214.05
0.5	40	0.2	2494	221.02

m_{fmax} = 600 kg



Region of existence of a solution and the optimal values of the specific impulse and the thrust of the electric propulsion system (markers) for different values of the design parameters characterizing the level of the technology being used

Numerical example: trajectrory to Mars (2/5)



Isolines of the useful mass

Numerical example: trajectrory to Mars (3/5)

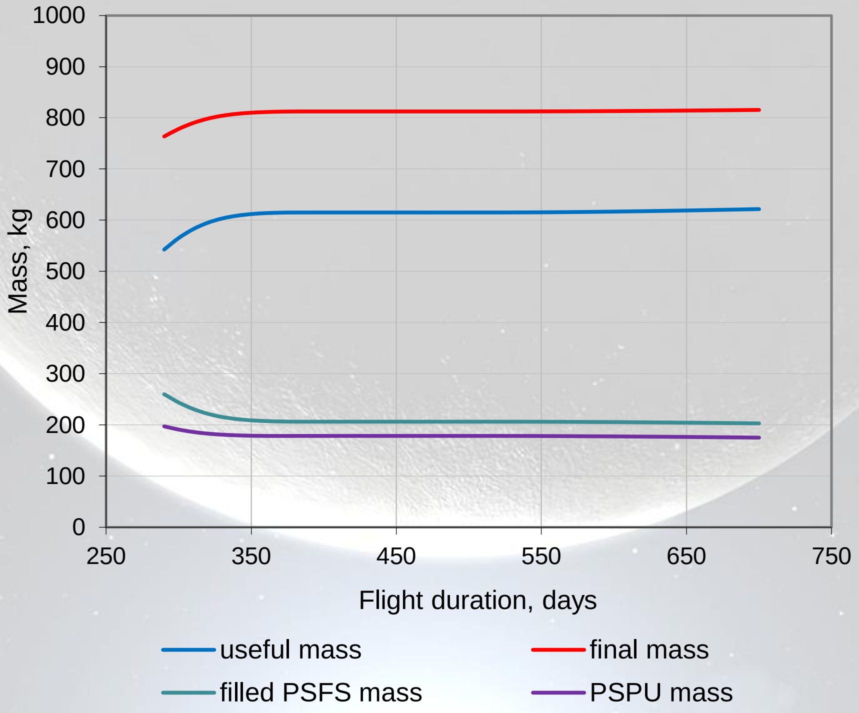
Trajectories with $\mathbf{V}_\infty = 0$

η	γ	a_t	c	T	$m(t_f)$	m_p	m_{PSPU}	m_u	P
--	kg/kW	--	sec	mN	kg	kg	kg	kg	W
0.7	40	0.1	2869	222.66	809.95	190.05	388.04	611.96	4474
0.5	40	0.1	2374	220.39	775.11	224.89	452.61	547.39	5131
0.5	80	0.1	1580	214.05	681.77	318.23	615.33	384.67	3316
0.5	40	0.2	2494	221.02	784.69	215.31	474.61	525.39	5406

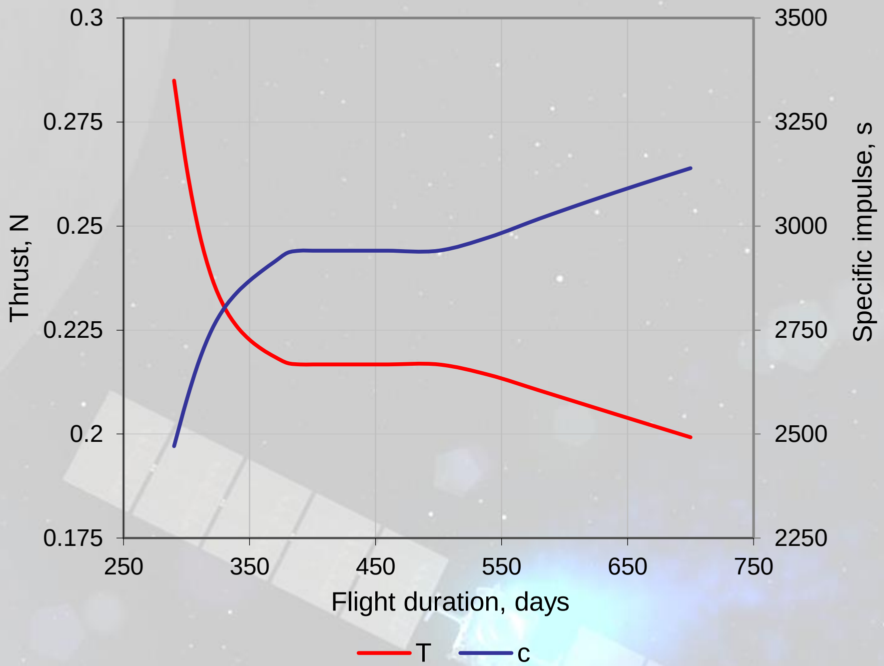
$\mathbf{V}_\infty$ impact

V_∞	T	c	$m(t_f)$	m_p	m_{PSPU}	m_u	P
m/s	mN	s	kg	kg	kg	kg	W
0	222.66	2869	809.95	190.05	388.04	611.96	4474
1000	160.77	3206	849.56	150.44	309.92	690.08	3611
2000	137.08	3214	869.71	130.29	266.75	733.25	3086
3000	137.43	2997	879.46	120.54	247.99	752.01	2885

Numerical example: trajectrory to Mars (4/5)



The dependence of mass from the flight duration at optimum values of thrust and specific impulse



The dependence of optimum values of thrust and specific impulse from the flight duration

Numerical example: trajectrory to Mars (5/5)

Departure date – 20.04.2035

$m_0 = 1000$ kg

$\eta = 0.5$

$\gamma = 40$ kg/kW

$a_t = 0.1$

$\Delta t_{opt} = 382.53$ days

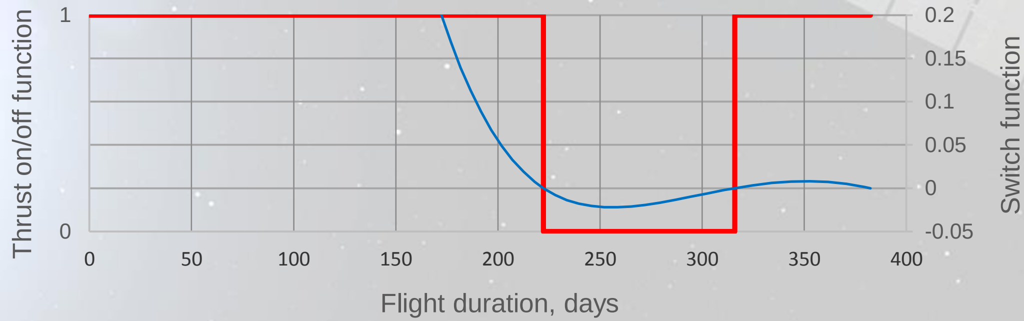
$c_{opt} = 2940$ sec

$T_{opt} = 216.75$ mN

$P_{opt} = 4465$ W

$m_p = 187.6$ kg

$m_u = 615.04$ kg



Conclusion

- It is demonstrated that the refusal of the separation on the dynamic and parametric parts of the optimization problem can significantly reduce the computation consumption.
- Necessary conditions for the optimality are received
- The numerical method for the thrust minimization problem and the joint optimization are developed.

Artistic vision of Dawn spacecraft ©NASA