JOINT OPTIMIZATION OF MAIN DESIGN PARAMETERS OF ELECTRIC PROPULSION SYSTEM AND SPACECRAFT TRAJECTORY

Alexey Ivanyukhin, Viacheslav Petukhov*,Min Thein***

*Research Institute of Applied Mechanics and Electrodynamics of the Moscow Aviation Institute (RIAME MAI), ** Moscow Aviation Institute (MAI), Moscow, Russia
e-mail: ivanyukhin.a@yandex.ru, vgpetukhov@mail.ru, minnntheino@gmail.com

ABSTRACT

The problems of joint optimization of the main design parameters of electric propulsion, power supply systems and trajectory of the spacecraft is considered in two formulations: minimum-thrust problem and maximum payload mass problem under the optimal parameters of the electric propulsion system (thrust, exhaust velocity, power). The first problem allows solving one of the fundamental problems in optimization of trajectories for the spacecraft with finite thrust related with the existence of the solution which is one of the reasons for the complexity of the constructing robust and efficient numerical optimization technique. Indeed, if the numerical scheme does not converge to a solution then the real reason is unknown since it can be either the absence of a solution or the numerical scheme failure. Therefore, identification of the boundary of the solution region is an actual problem. In this way, solving the minimum thrust problem gives necessary assessment. The minimum thrust problem is formulated similar to any other low thrust trajectory optimization problem but the functional to minimize is the thrust. It is supposed that the thrust can be either maximal or zero, the maximal thrust magnitude is constant along the trajectory and its direction is unconstrained. The transfer time is fixed. The second problem is directly related to the optimization of the parameters of the propulsion system in the region of the existence of solutions. It is well known that for every space transportation operation there is an optimal value of specific impulse corresponding to the minimum total mass of electric propulsion system, power supply system providing electric propulsion operation and propellant. It is easy to show that there is also an optimal value of electric power of the electric propulsion system that being associated with the growth of the required characteristic velocity while reducing the thrust. It is obvious that the optimal values of specific impulse and electric power of the main electric propulsion system can be found by joint optimization of the trajectory and design parameters. An approximated design model of the spacecraft equipped with main electric propulsion system and the Pontryagin's maximum principle are used for optimization. The necessary conditions for the optimality of specific impulse and electric power of the electric propulsion system are derived. Numerical examples of the joint optimization for the interplanetary spacecraft trajectory are presented.

***Index Terms*—** electric propulsion system, trajectory optimization, optimization of design parameters

1. INTRODUCTION

The use of electric propulsion system (EPS) with a high specific impulse compared with traditional chemical rocket engines can significantly increase the efficiency of space transportation operations by a significant reduction in the propellant mass. That leads to the prospect of using electric propulsion for performing orbital and interplanetary missions.The use of EPS for such flights was impeded by a number of technical difficulties. However the number of completed projects of the spacecrafts (SC) using EPS as the main propulsion systems has been constantly increasing since the 1990s when the first practical experience of real transport operations in the near and outer space was obtained. These completed projects include series of geostationary satellites based on Boeing-702HP / 702SP platforms and geostationary spacecrafts AEHF (USA), "Express-AM5/AM6" (Russia) and Artemis (ESA), remote sensing spacecraft EgyptSat-2 (developed in Russia), automatic interplanetary spacecraft DeepSpace-1 and Dawn (USA), Hayabusa-1/2 (Japan), lunar spacecraft SMART-1 (European Space Agency).

Nowadays several new projects for near-earth and interplanetary spacecrafts with electric propulsion systems are being prepared for realization. Moreover, the use of electric propulsion for flights to target orbits or for interplanetary flights is regarded as one of the most promising options for the majority of the spacecraft being designed. The effectiveness of the use of electric propulsion for performing space transportation operations is determined by the level of technology being used and by the optimal choice of the main design parameters of electric propulsion.

Being considered from the point of view of the transportation problem analysis the level of technology is determined by the total efficiency of EPS η - the ratio of the mechanical power of the plasma beam to the electric power consumed by EPS; specific mass of the power supply and propulsion unit (PSPU) γ - the ratio of the total mass of the electric propulsion system and its onboard power supply system (PSS) to the electrical power consumed by the electric propulsion system; the relative mass of the propellant storage and feeding system (PSFS) a_t - the ratio of the final mass of PSFS to the propellant mass used by the electric propulsion system. The main design parameters of the electric propulsion system are: thrust *T*, specific impulse *c* and power consumption *P*. Although the acceptable range of the main design parameters of the electric propulsion system has technical constraints, the calculation of their optimal values is required to justify the choice of the type, the mode of operation and the number of thrusters in the main electric propulsion system and the characteristics of the power supply system of the spacecraft being designed. In addition, optimization of the main design parameters for typical space transportation operations is the ground for the choice of their values in the design of new thrusters.

2. THE DYNAMIC PROBLEM: A MODEL OF THE SPACECRAFT MOTION WITH THE SPECIFIED DESIGN PARAMETERS

In a typical low thrust trajectory optimization the thrust *T* and the specific impulse *c* are given. The problem of optimizing the heliocentric trajectory of the spacecraft with EPS in a central Newtonian gravitational field is considered [1; 2]. Equations of motion of the spacecraft with EPS in an gravitational field with force function Ω are:

$$\begin{aligned} \frac{d\mathbf{x}}{dt} &= \mathbf{v}, \\ \frac{d\mathbf{v}}{dt} &= \Omega_{\mathbf{x}} + \delta \frac{T}{m}\mathbf{e}_T, \\ \frac{dm}{dt} &= -\delta\frac{T}{c}, \end{aligned} \tag{1}$$

where $\mathbf{x}$ – heliocentric position vector of the spacecraft, $\mathbf{v}$ – heliocentric velocity vector of the spacecraft, δ – engine on/off function equal to 1 when the engine is turned on and to 0 when the engine is turned off, m – mass of the spacecraft, $\mathbf{e}_T$ – unit vector of thrust direction, t – time.

Usually, the movement of the spacecraft outside the gravity spheres of the planets is considered in the central Newtonian gravitational field with the force function $\Omega = \mu/r$, where μ – gravitational parameter of Sun, and $r = |\mathbf{x}|$ – heliocentric distance of the spacecraft.

Initial conditions of the spacecraft's motion on the heliocentric trajectory at $t = t_0$ are:

$$\begin{aligned} &\mathbf{x}(t_0) = \mathbf{x}_0(t_0), \\ &|\mathbf{v}(t_0) - \mathbf{v}_0(t_0)| = V_{\infty 0}, \\ &m(t_0) = m_0, \end{aligned} \tag{2}$$

where $\mathbf{x}_0$ and $\mathbf{v}_0$ – vectors of the heliocentric position and velocity of the departure planet, $V_{\infty 0}$ – initial hyperbolic excess velocity of the spacecraft, m_0 – the spacecraft's mass at the time of the exit from the gravity sphere of the departure planet. As typical final conditions of the spacecraft's motion on heliocentric trajectory on its arrival to the destination planet at $t = t_f = t_0 + \Delta t$ the conditions of the approach to it with zero hyperbolic excess velocity are considered:

$$\begin{aligned} &\mathbf{x}(t_f) = \mathbf{x}_f(t_f), \\ &\mathbf{v}(t_f) = \mathbf{v}_f(t_f), \end{aligned} \tag{3}$$

where Δt – heliocentric flight time, $\mathbf{x}_f$ and $\mathbf{v}_f$ – vectors of the heliocentric position and velocity of the destination planet.

The purpose of heliocentric trajectory optimization problems of the spacecraft with EPS in this case is a choice of control functions $\delta(t), \mathbf{e}_T(t)$ to minimization of the following cost function:

$$J \to \min \tag{4}$$

(usually, minimize propellant consumption) for the specified design parameters $T, c, t_0, \Delta t$.

3. THE PARAMETRIC PROBLEM: THE SIMPLE MODEL OF THE SPACECRAFT DESIGN

SC with the electric propulsion system can be represented as a set of “useful” part and PSPU with the PSFS (Figure 1). In the “useful” part of the SC all the elements responsible for the implementation of its objectives on the target orbit will be included, namely the payload with the power supply system, other service systems and the spacecraft structural elements, to ensure the proper functioning of the payload on the target orbit. The PSPU includes the part of the power supply system which is necessary only for the operation of the electric propulsion system, power processing unit (PPU) of the electric propulsion system, thrusters and the part of the service systems and the spacecraft's structure necessary only for the operation of the electric propulsion system, including interunit cables and pipelines. The PSFS includes propellant tanks for the electric propulsion system and propellant feeding units.

The mass of the PSPU can be described as $m_{pspu} = \gamma P = \gamma T c / 2\eta$, and the mass of the filled PSFS – as $m_{PSFS} = (1+a_t) m_p$.

Within the presented simple model of mass budget the initial mass of the spacecraft is determined by the sum of the masses of the “useful” part m_u, the mass of the PSPU and the PSFS:

$$m_0 = m_u + m_{\text{PSPU}} + m_{\text{PSFS}} = m_u + \gamma P + (1+a_t)m_p,$$

where, m_p – the propellant mass of the electric propulsion system.

The main parameters m_0, η, γ, a_t are assumed to be fixed and parameters P, T, c are selected. Taking into consideration the relationship $P = Tc/2\eta$ only two parameters from the set of the selected design parameters P, T and c are independent.

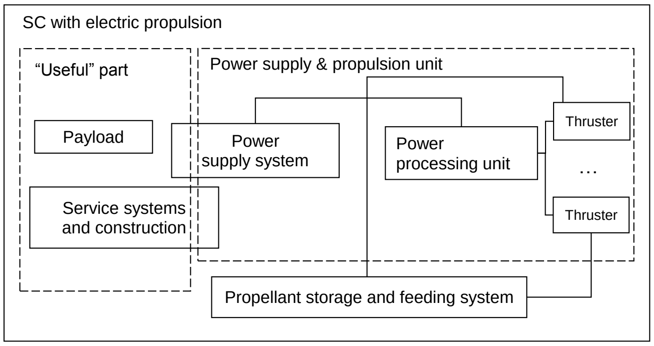

Fig. 1 – The simple model of the spacecraft with the electric propulsion system

4. THRUST MINIMIZATION PROBLEM

The region of existence of a solution to problem (1)–(4) is bounded by the values of minimum thrust and exhaust velocity. Indeed, a certain finite increment in characteristic velocity is required in order to complete the intended flight. This increment may be attained at fixed time $\Delta t = t_f - t_0$ only by applying a sufficient thrust acceleration (and, consequently, sufficient thrust). Since the allowed propellant consumption is often limited from above:

$$m_p \le m_{p\max} < m_0, \quad \text{or} \quad m(t_f) \ge m_{f\min} < m_0 \qquad (5)$$

consequently, exhaust velocity should be sufficiently high.

In fact, each exhaust velocity $c \in [c_{\min};\infty]$ has its corresponding minimum thrust magnitude $T_{\min}$, and a solution to optimal control problem (1)–(4) exists if:

$$T(c) \ge T_{\min}(c),\ c \in [c_{min};\infty]. \qquad (6)$$

We can formulate the problem of minimizing the thrust as follows [5, 6]. It is required to choose optimal $\delta(t), \mathbf{e}_T(t), T$ in order to minimize the cost function

$$J = T \to \min, \qquad (7)$$

for the dynamical system (1) with the boundary conditions (2), (3), (5).

Add to the system (1) formal differential equations for the fixed value of the thrust

$$\frac{dT}{dt} = 0. \qquad (8)$$

Applying the formalism of Pontryagin's maximum principle to optimal control problem (1)-(3), (5), (7), (8), we obtain the Hamiltonian

$$H = \mathbf{p}_\mathbf{x}^\mathbf{T}\mathbf{v} + \mathbf{p}_\mathbf{v}^\mathbf{T}\Omega_\mathbf{x} + \delta T\psi\ ,$$

and the optimal control for the considered problem:

$$\mathbf{e}_T = \frac{\mathbf{p}_\mathbf{v}}{p_v},\ \delta = \begin{cases} 1, \psi > 0, \\ 0, \psi < 0, \end{cases} \qquad (9)$$

where $\psi = \dfrac{p_v}{m} - \dfrac{p_m}{c}$ is switching function.

System (1) should be supplemented by a system of differential equations for the adjoint variables:

$$\frac{d\mathbf{p}_\mathbf{x}}{dt} = -\Omega_{\mathbf{xx}}\mathbf{p}_\mathbf{v}, \quad \frac{dp_m}{dt} = \delta T\frac{p_v}{m^2}, \quad \frac{d\mathbf{p}_\mathbf{v}}{dt} = -\mathbf{p}_\mathbf{x}, \quad \frac{dp_T}{dt} = -\delta\psi. \qquad (10)$$

The system (1) and (10) must satisfy the boundary conditions (2), (3), (5) complemented by the transversality conditions.

The complete system of conditions at the initial point of the trajectory (at $t = t_0$) is given by:

$$\mathbf{x}(t_0) = \mathbf{x}_0(t_0), \qquad m(t_0) = m_0,$$
$$\mathbf{v}(t_0) = \mathbf{v}_0(t_0) + V_{\infty 0}\frac{\mathbf{p}_\mathbf{v}(t_0)}{p_v(t_0)}, \quad p_T(t_0) = 0. \qquad (11)$$

The final point of the trajectory ($t = t_f = t_0 + \Delta t$) is described as

$$\mathbf{x}(t_f) = \mathbf{x}_f(t_f), \quad \mathbf{v}(t_f) = \mathbf{v}_f(t_f), \quad \begin{cases} p_m(t_f) = 0,\ m(t_f) > m_{f\min}, \\ p_m(t_f) > 0,\ m(t_f) = m_{f\min}, \end{cases} \quad p_T(t_f) = \text{const} < 0. \qquad (12)$$

From the transversality conditions (12) follows that the problem is divided into two cases:

1) final mass greater than the minimum allowable

$$\forall t \in [t_0, t_1] \quad p_m(t) \le 0, \quad \Psi > 0 \quad \Rightarrow \quad \delta \equiv 1,$$

In this case, due to the non-negativity of the right-hand side of the differential equation for the adjoint variable to the mass (10) and its nullification at the end point follows that p_m always is not positive and the switching function always non-negative and there will not be passive segment of trajectory;

2) final mass equals to minimum allowable mass. In this case, the optimal control is determined by the (9) and the trajectory may include the passive segments.

Thus, the problem of the thrust minimization problem is reduced to the calculation of 8 unknown parameters $\left(\mathbf{p}_\mathbf{x}(t_0), \mathbf{p}_\mathbf{v}(t_0), p_m(t_0), T\right)$ of the boundary value problem (1), (9)-(12).

In other words, minimum condition for T is following:

$$\int_{t_0}^{t_f} \delta\psi = \text{const} > 0.$$

Positive constant can be chosen arbitrarily and serves for the normalization of adjoints variables.

5. THE JOINT OPTIMIZATION PROBLEM

The purpose of the joint optimization of the trajectory and the main design parameters of the electric propulsion system is to maximize the "useful" mass of the spacecraft $m_u = m_0 - \gamma P - (1 + a_t) m_p$, where the propellant mass can be described as $m_p = m_0 - m(t_f) = \int_{t_0}^{t_f} \delta \frac{T}{c} dt$.

Moreover, taking into account that the specific impulse can be represented as $c = \frac{2\eta P}{T}$ maximization of m_u is equivalent to minimization of the following cost function

$$J = \int_{t_0}^{t_f} \left(\alpha P + \delta \frac{T^2}{2\eta P} \right) dt \to \min, \tag{13}$$

where $\alpha = \frac{\gamma}{\Delta t \cdot (1 + a_t)}$.

In the system of differential equations of the spacecraft motion (1) the specific impulse c is expressed via the thrust T and the electrical power of the electric propulsion system P, as well as formal differential equations for the fixed values of the thrust and the electric power of the electric propulsion system are added:

$$\frac{d\mathbf{x}}{dt} = \mathbf{v}, \quad \frac{d\mathbf{v}}{dt} = \Omega_{\mathbf{x}} + \delta \frac{T}{m} \mathbf{e}_T, \quad \frac{dm}{dt} = -\delta \frac{T^2}{2\eta P}, \quad \frac{dT}{dt} = 0, \quad \frac{dP}{dt} = 0. \tag{14}$$

Thus, it is required to choose optimal $\delta(t), \mathbf{e}_T(t), T, P$ in order to minimize the cost function (13) for the dynamical system (14) with the boundary conditions (2) and (3).

Applying the formalism of Pontryagin's maximum principle to the Lagrange optimal control problem (13), (14), we obtain the Hamiltonian

$$H = -\alpha P + \mathbf{p}_\mathbf{x}^\mathbf{T} \mathbf{v} + \mathbf{p}_\mathbf{v}^\mathbf{T} \Omega_\mathbf{x} + \delta T \psi,$$

and the optimal control for the considered problem:

$$\mathbf{e}_T = \frac{\mathbf{p}_\mathbf{v}}{p_v}, \quad \delta = \begin{cases} 1, \psi > 0, \\ 0, \psi < 0, \end{cases} \tag{15}$$

where $\psi = \frac{p_v}{m} - \frac{T \cdot (p_m + 1)}{2\eta P}$ is switching function.

System (14) should be supplemented by a system of differential equations for the adjoint variables:

$$\frac{d\mathbf{p}_\mathbf{x}}{dt} = -\Omega_{\mathbf{xx}} \mathbf{p}_\mathbf{v}, \quad \frac{d\mathbf{p}_\mathbf{v}}{dt} = -\mathbf{p}_\mathbf{x}, \quad \frac{dp_m}{dt} = \delta T \frac{p_v}{m^2}, \quad \frac{dp_T}{dt} = -\delta\psi + \delta T \frac{p_m + 1}{2\eta P}, \quad \frac{dp_P}{dt} = \alpha - \delta T^2 \frac{p_m + 1}{2\eta P^2}. \tag{16}$$

The system (14) and (17) must satisfy the boundary conditions (2), (3) complemented by the transversality conditions.

The complete system of conditions at the initial point of the trajectory (at $t = t_0$) is given by:

$$\mathbf{x}(t_0) = \mathbf{x}_0(t_0), \quad \mathbf{v}(t_0) = \mathbf{v}_0(t_0) + V_{\infty 0} \frac{\mathbf{p}_\mathbf{v}(t_0)}{p_v(t_0)}, \quad m(t_0) = m_0, \quad p_T(t_0) = 0, \quad p_P(t_0) = 0, \tag{17}$$

The final point of the trajectory ($t = t_f = t_0 + \Delta t$) is described as

$$\mathbf{x}(t_f) = \mathbf{x}_f(t_f), \quad \mathbf{v}(t_f) = \mathbf{v}_f(t_f), \quad p_m(t_f) = 0, \quad p_T(t_f) = 0, \quad p_P(t_f) = 0. \tag{18}$$

Thus, the problem of the joint optimization of the trajectory and the main design parameters of the electric propulsion system is reduced to the calculation of 9 unknown parameters $(\mathbf{p}_\mathbf{x}(t_0), \mathbf{p}_\mathbf{v}(t_0), p_m(t_0), T, P)$ of the boundary value problem (14), (15), (16)-(18).

In other words, T and P optimality conditions are following:

$$\int_{t_0}^{t_f} \left(-\delta\psi + \delta T \frac{p_m + 1}{2\eta P} \right) = 0, \quad \int_{t_0}^{t_f} \left(\alpha - \delta T^2 \frac{p_m + 1}{2\eta P^2} \right) = 0.$$

6. METHOD FOR SOLVING THE BOUNDARY VALUE PROBLEM

The following algorithm for calculating the boundary of the region of existence of a solution to the problem of a flight between two planets that is to be completed in a certain interval of time with thrust switching [5, 6]:

1) the optimum trajectory of a flight that is to be completed in a given interval of time with an power-limited engine with set boundary conditions is calculated. Practically regular methods presented in [3] may be used to solve this problem;
2) the problem of the minimization of a constant thrust acceleration (special case with infinite exhaust velocity without mass flow) is solved using the method of

continuation from the optimum trajectory with an power-limited engine;

3) the trajectory with minimum thrust acceleration is continued into a trajectory with minimum thrust and minimum allowable exhaust velocity, in the process of continuation, the dependence of minimum thrust on the exhaust velocity is calculated, this dependence represents the desired boundary of the region of existence of a solution to the problem with thrust switching.

The method of continuation on parameter (Newton homotopy) is used to solve the boundary value problem by reducing it to a Cauchy problem [3, 4]. The solution of the power-limited trajectory optimization problem [3] is used as the initial approximation for the unknown parameters of the boundary value problem. The initial guess value of the thrust is assumed to be the average value of the thrust on the solution to the power-limited problem. To regularize the continuation process for solving the boundary value problem with a discontinuous control we use the natural smoothing function of the thrust proposed in [4]:

$$\delta = \frac{1}{2}\left(\frac{\psi}{|\psi| + \varepsilon(\tau)} + 1\right),$$

where ε - the small regularizing parameter, τ - the parameter of continuation.

7. NUMERICAL EXAMPLES

Numerical examples of the use of this approach in case of the joint optimization of the heliocentric trajectory and the main design parameters of the electric propulsion system for missions to Mars are given below.

The problem of optimizing the trajectory of the spacecraft with the electric propulsion system to Mars with the date of the departure from the Earth 20/04/2035 with zero hyperbolic excess velocity, flight duration Δt = 350 days, the initial spacecraft mass m_0 = 1000 kg and the limit on the final mass $m(t_f) \geq 600$ kg is presented.

Table 1 shows the results of the joint optimization of the trajectory to Mars and the main design parameters of the electric propulsion system for different values of the design parameters η, γ, и a_t, characterizing the level of technology being used. The reduction of the efficiency of the electric propulsion system or the increase in the relative mass of PSFS reduces the optimal value of the specific impulse and increases the optimal electrical power of the electric propulsion system. The increase in the specific mass of the PSPU reduces the optimum values of the specific impulse and the electrical power. Of course, the maximum "useful" mass of the spacecraft is achieved at the point of maximum efficiency of the electric propulsion system and minimum values of the specific mass of the PSPU and of the relative mass of PSFS.

Table 1 – The results of the joint optimization of the flight trajectory to Mars and the main design parameters of the electric propulsion system for different values of the design parameters η, γ и a_t, characterizing the level of technology being used

№	1	2	3	4
η	0.7	0.5	0.5	0.5
γ, kg/kW	40	40	80	40
a_t	0.1	0.1	0.1	0.2
c, s	2869	2374	1580	2494
T, mN	0.22266	0.22039	0.21405	0.22102
$m(t_f)$, kg	809.95	775.11	681.77	784.69
m_p, kg	190.05	224.89	318.23	215.31
m_{pspu} , kg	388.04	452.61	615.33	474.61
m_u, kg	611.96	547.39	384.67	525.39
P, W	4474	5131	3316	5406

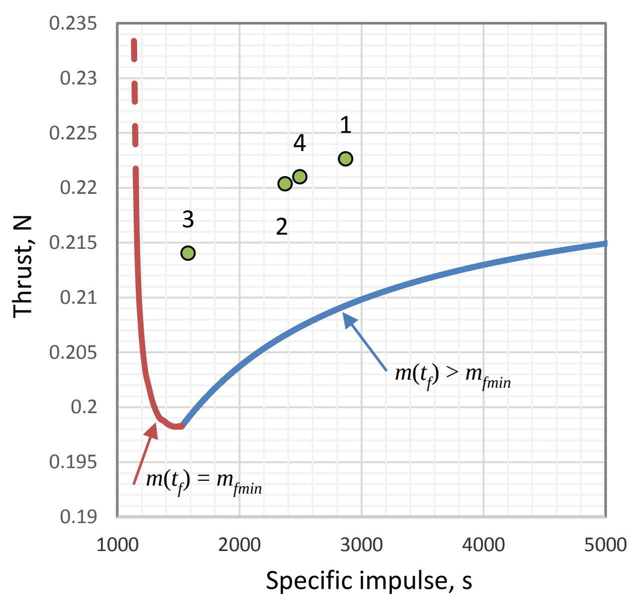

Fig. 2 – Region of existence of a solution and the optimal values of the specific impulse and the thrust of the electric propulsion system (markers) for different values of the design parameters characterizing the level of the technology being used.

In figure 2 square markers present the optimal results of the main design parameters of the electric propulsion system on the plane of the specific impulse – the thrust. The numbers of the markers correspond to the variant from table 1. Thick line in the figure shows the dependence of the minimum thrust from the specific impulse which bounds the region of existing solutions below. The curve for the minimum thrust was obtained by using the procedure described in [5, 6].

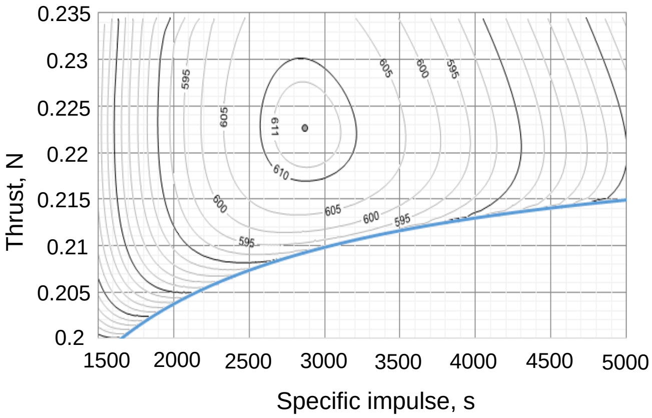

Fig. 3 – Isolines of the useful mass.

On figure 2 there are the isolines of the useful mass for the reason 1 from Table 1. The minimal trust is blue colored; as we see, the isolines cross it quite smoothly. Also it is seemed that the maximum of useful mass is very flat. So region of existence with useful mass is quite big (about 0.219-227 N for trust and 2700 – 3100 s for specific impulse), and for mass about 600 kg it increases in a few times.

Table 2 - The optimal values of the main design parameters related to the hyperbolic excess velocity

V_∞, m/s	0	1000	2000	3000
T, mN	222.66	160.77	137.08	137.43
c, s	2869	3206	3214	2997
$m(t_f)$, kg	809.95	849.56	869.71	879.46
m_p, kg	190.05	150.44	130.29	120.54
m_{pspu} , kg	388.04	309.92	266.75	247.99
m_u, kg	611.96	690.08	733.25	752.01
P, W	4474	3611	3086	2885

The results of the joint optimization of the main design parameters of the electric propulsion system and spacecraft trajectory for η = 0.5, γ = 40 kg / kW a_t = 0.1 and for different values of the hyperbolic excess velocity $V_{\infty0}$ are shown in Table 2. The optimal electrical power of the electric propulsion system decreases monotonically with the increase of $V_{\infty0}$, the optimal value of the thrust has the minimum at the $V_{\infty0}$ = 2000 m/s.

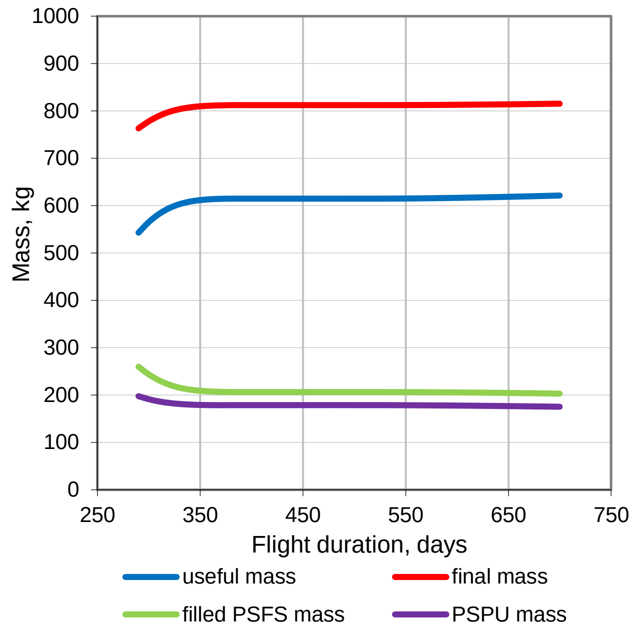

Fig. 4 – The dependence of mass from the flight duration at optimum values of thrust and specific impulse.

Figure 4 and 5 shows the dependence of mass from the flight duration at optimum values of thrust and specific impulse, also the dependence of optimum values of thrust and specific impulse from the flight duration for the following values of the design parameters that determine the level technology being used: efficiency of the electric propulsion system η = 0.7, specific mass of the PSPU γ = 40 kg/kW, the relative mass of PSFS a_t = 0.1.

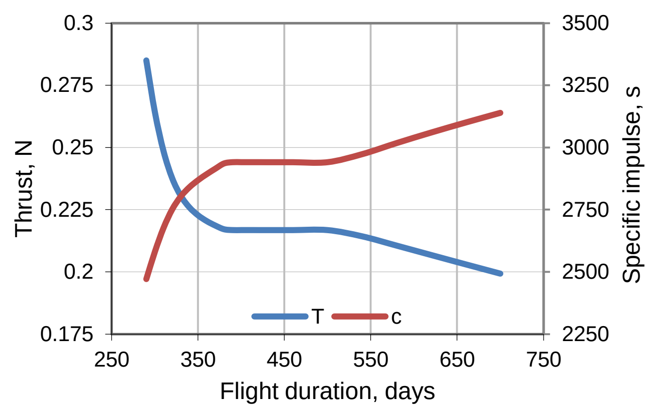

Fig. 5 – The dependence of optimum values of thrust and specific impulse from the flight duration.

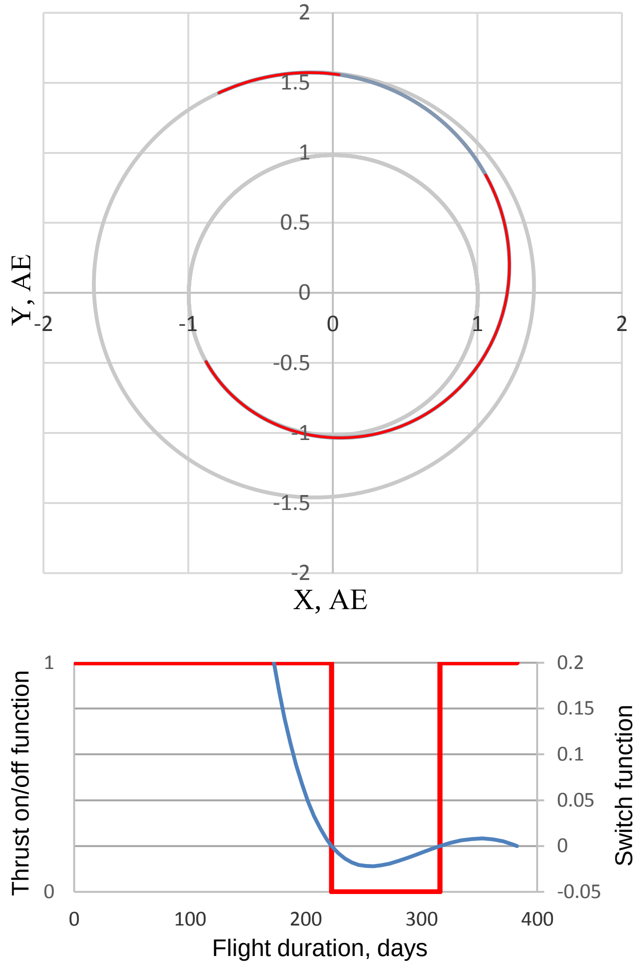

Fig. 6 – The example of the optimal heliocentric trajectory to Mars.

Figure 6 shows the example of the optimal trajectory with the optimal parameters of the electric propulsion system and the optimal flight duration.

As the result of the optimization problem's solution, in addition to the optimal control program, the optimal values for the specific impulse c_{opt} = 2940 s, the thrust T_{opt} = 216.75 mN and the electrical power of the electric propulsion system P_{opt} = 4465 watts are computed. The optimal values for the flight duration Δt_{opt} = 382.534 days. The propellant mass of the electric propulsion system on the optimal trajectory is m_p = 187.6 kg, and the "useful" mass of the spacecraft is m_u = 615.04 kg.

At the top of figure 6 the projection of the optimal trajectory on ecliptic plane is shown (thrusting arcs of the trajectory are indicated by red lines and coasting arcs are indicated by blue lines). Thin lines indicate the orbits of the Earth and Mars. At the bottom of the figure 5 the switch function ψ (blue line) and the engine on/off function δ (red line) related to the time of flight are shown.

8. CONCLUSION

It is demonstrated that the refusal of the separation on the dynamic and parametric parts of the optimization problem can significantly reduce the computation consumption and reduce it to the calculation of the optimal values of the specific impulse, the thrust and the electrical power of the main electric propulsion system of SC.

These results should be considered primarily from the methodological standpoint because the paper suggested approaches only for determining possible ways of solving applied problems of the joint optimization of the main design parameters and the trajectory of the spacecraft with the electric propulsion system.

In particular, in this paper we do not consider the issues related to possible dependence of the electrical power, thrust, specific impulse and efficiency of the electric propulsion system from the spacecraft's heliocentric distance; more complex mass budget models of spacecraft and PSPU and various options for setting the boundary conditions corresponding to the trajectories with gravity assist maneuvers and to round trip flights; the issues of end-to-end optimization of interplanetary trajectories, including planet-centric and heliocentric segments of a trajectory.

The main point of the paper is about the development of the efficient numerical methods for solving the applied problems on the basis of the technique mentioned above.

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