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Introduction

Starting from the theory introduced and analyzed in the book "***Flexible Spacecraft Dynamics, Control and Guidance***" this presentation shows the problems linked to the optimal orbital transfer with electrical thruster which may be encountered in the following applications:

1. Telecom. missions from GTO or LEO to GEO
2. Navigation missions from LEO or GTO to MEO.
3. LEO and MEO constellation deployment.

Using SOFTT (*Space Optimal Finite Thrust Transfer*) it is possible to face typical problems of the mission analysis for long transfers – even of the order of many hundreds days - computing the optimal finite thrust strategy in order to minimize the transfer time or propellant mass **including the eclipse and the J2 perturbation effects**.

Furthermore new topics have been studied, as multiple launches for constellation deployment, optimal solutions showing a satellite perigee altitude below the Earth radius and multiple extremal solutions of the same problems.

Using SOFTT we have been able to solve these problems with J_2 and eclipse effects. Here below a synthetic explanation of the SOFTT and its main theoretical laws are reported. Furthermore the numerical results and figures obtained during the studies are reported and discussed.

New features are under development as the ***Autonomous Navigation*** that can make the satellite smart and autonomous satellites able to compute on board the optimal trajectory saving cost of the G/S support.

Reference

For this presentation the main reference is represented by Dr. L.Mazzini's book "***Flexible Spacecraft Dynamics, Control and Guidance***" written with the contribution of G.Campolo and distributed by Springer.

The main theoretical references are presented and discussed in this book and in the paper "***Finite thrust orbital transfers***", Acta Astronautica 2014, of the same author.

Springer Aerospace Technology

Leonardo Mazzini

Flexible Spacecraft Dynamics, Control and Guidance

Technologies by Giovanni Campolo

Springer

Mathematical Model

As presented in the L.Mazzini's book SOFTT uses an indirect method based on the Maximmum Principle.

Considering an orbital transfer, the SOFTT calculates the Optimal Control Strategy solving the system of differential equation, the state (X,V,T) evolves as :

$$\frac{dX}{dw} = fB(X, w + X_6)\bar{l}e^{V/V_e}\frac{\sqrt{X_3}}{Y_1^2}$$

$$\frac{dV}{dw} = fe^{V/V_e}\frac{\sqrt{X_3}}{Y_1^2}$$

$$\frac{dT}{dw} = \varepsilon\frac{\sqrt{X_3}}{Y_1^2}$$

Where the controls are f and $\underline{l}$. In this setting we introduce the pre-Hamiltonian $\widetilde{H}$ and the costate L ($L\epsilon R^6, L_V, L_T$), following the standard Maximum Principle method.

$$H = \sup_{f\epsilon[0,\varepsilon],\bar{l}\epsilon|\bar{l}|=1} \widetilde{H}$$

$$H = \varepsilon\kappa(S)S(X, L, L_V, u)e^{V/V_e}\frac{\sqrt{X_3}}{Y_1^2} + \varepsilon L_T\frac{\sqrt{X_3}}{Y_1^2}$$

Where the functions S and $\kappa(S)$ have been defined as

$$S(X, L, L_V, u) = \sqrt{L^T B(X,u)B(X,u)^T L} + L_V$$

$$\kappa(S) = 1, \forall S > 0$$

$$\kappa(S) = 0, \forall S < 0$$

The control action are derived maximizing the pre-Hamiltonian:

$$l = \frac{B(X,u)^T L}{\sqrt{L^T B(X,u)B(X,u)^T L}}$$

$$f = \varepsilon\kappa(S)$$

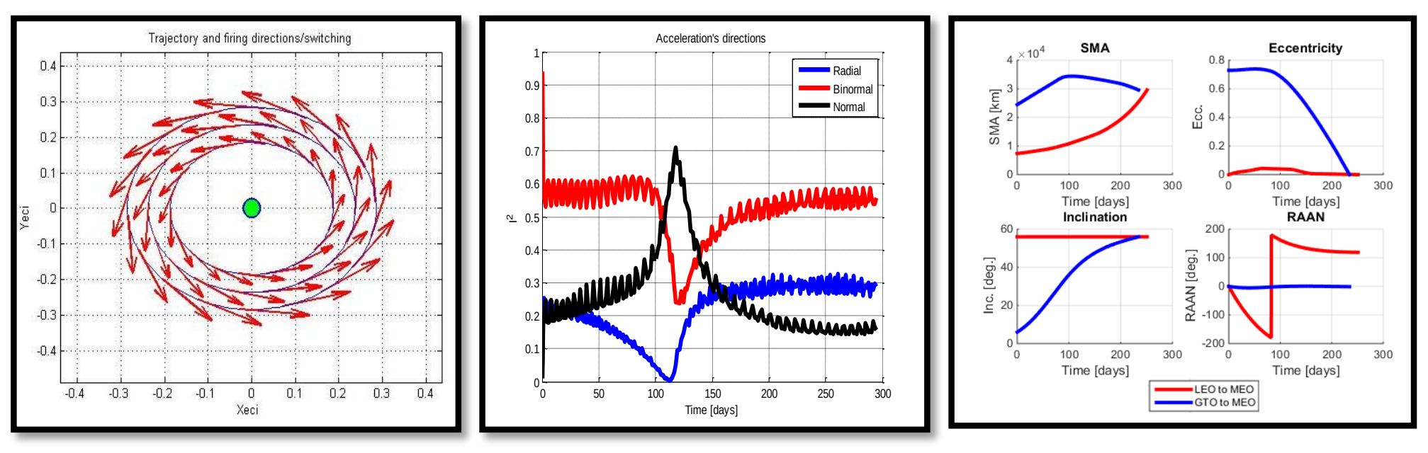
SOFTT Results

With SOFTT is possible to face every orbital transfer and to compute the Optimal Thrust Strategy in order to minimize the Trasnfer Time or the Mass Consumption. The main orbital transfer results are reported in the table below.

Mission	a in [km]	a fin [km]	I in [km]	I fin [km]	Time [days]	DV [m/s]	DM [kg]	DVimp [m/s]	DMimp [kg]
LEO to MEO	7378	29601	56	56	230	3 681	252	3 301	1 012
LEO to GEO	7378	42166	0	0	264	4 276	289	3 654	1 067
LEO to IGSO	7378	42166	56	60	265	4 301	290	3 644	1 065
GTO to MEO	24475	29601	6	56	216	3 441	237	2 425	842
GTO to GEO	24475	42166	6	0	142	2 200	156	1 490	596
GTO to IGSO	24475	42166	6	60	213	3 390	234	2 491	857

SOFTT main orbital transfer results.

After the Optimal Thrust Strategy calculation is always possible to do the complete mission analysis with the SOFTT output. The evolution of the controls and the Keplerian parameters are only an example of the relevant SOFTT outputs. Examples of the evolution of the control and of the Keplerian parameters are reported in the following figures.



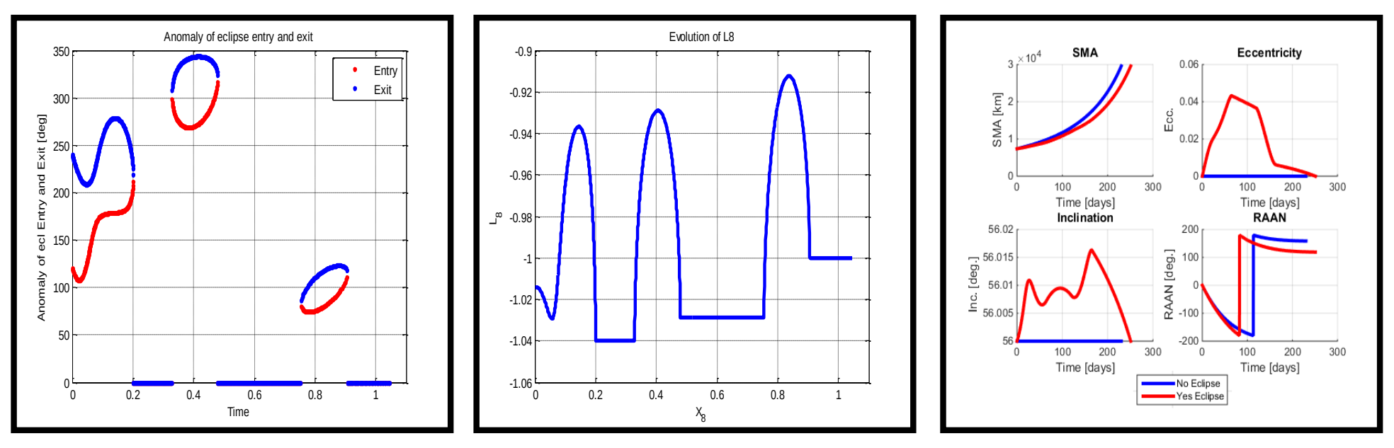
Some SOFTT outputs: trajectory (left), controls (central) and Keplerians evolution (right).

Eclipse Management

As seen in the L.Mazzini's paper "***Finite thrust orbital transfers***", Acta Astronautica 2014, the plasmic thrusters use a significant electric power, therefore the satellite design often requires the engine to be switched off during the eclipses, in order to avoid oversizing the battery accumulator system. The constraint to switch off the motor, during eclipses or when the battery level is low, introduces a mixed state-control inequality constraint. We introduce this constraint using an Eclipse Function which is negative when there is an eclipse and positive when there is no eclipse.

During the eclipse a discontinuity in the costate evolution exist at the entry and exit anomaly. This problem of the "***Costate Jump***" was faced and solved with the introduction of the δ eclipse parameter that provides a new eclipse optimization.

The following figures show examples of the SOFTT output in order to study the Eclipse effects during an orbital transfer.



SOFTT outputs for Eclipse: Entry/Exit eclipse anomly (left), Costate Jump (central) and Keplerians evolution w/o eclipse (right).

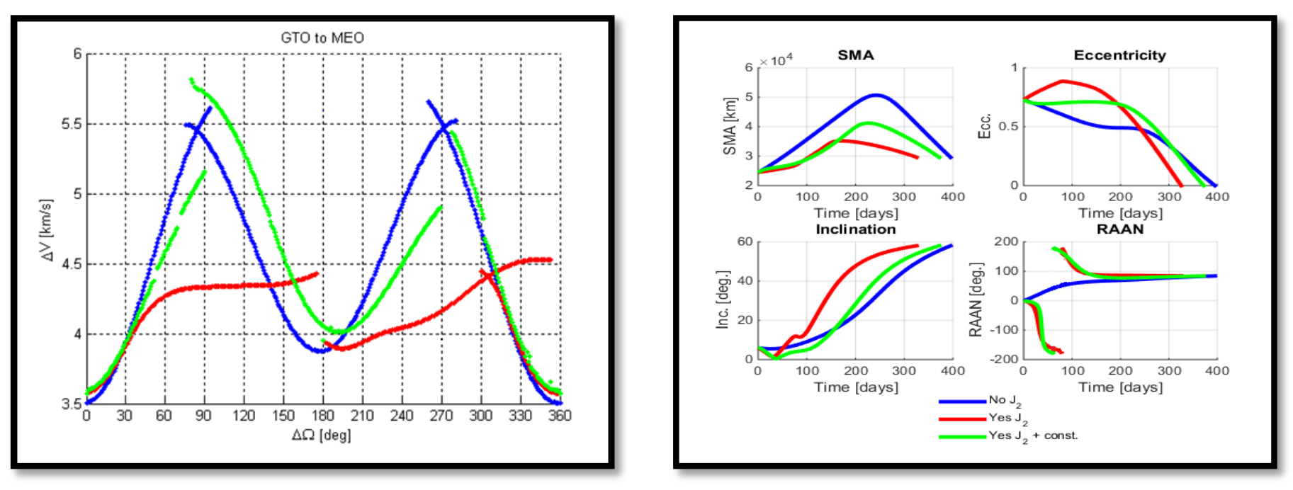
J2 Management

The use of low thrust allows to reduce the launcher performance required to reach the final orbit or to use a single launch to deliver multiple satellites in different orbital planes. The second case brings to study the evolution of the delta-V cost as function as the change of the final right ascension node.

During this study we have found that starting from the same initial and final condition it's possible to obtain two different solutions.

Furthermore including the J2 perturbation effects a new difficulty emerges for transfers requiring a big plane change from highly elliptical orbits may bring the optimal solution perigee to unrealistic values below the Earth radius. In order to solve this problem, the "***Perigee Altitude Constraint*** " was introduced. Using this new constraint the SOFTT computational time increases, but viable optimal solutions are found without problems.

The following figures show examples of the SOFTT output in order to study the Perigee Altitude value during orbital trasnfer with big delta-RAAN.



SOFTT outputs for J2 effects: ΔV vs Δ RAAN (left) and Keplerians evolution (right) w/o J2 effects.

Minimum Velocity

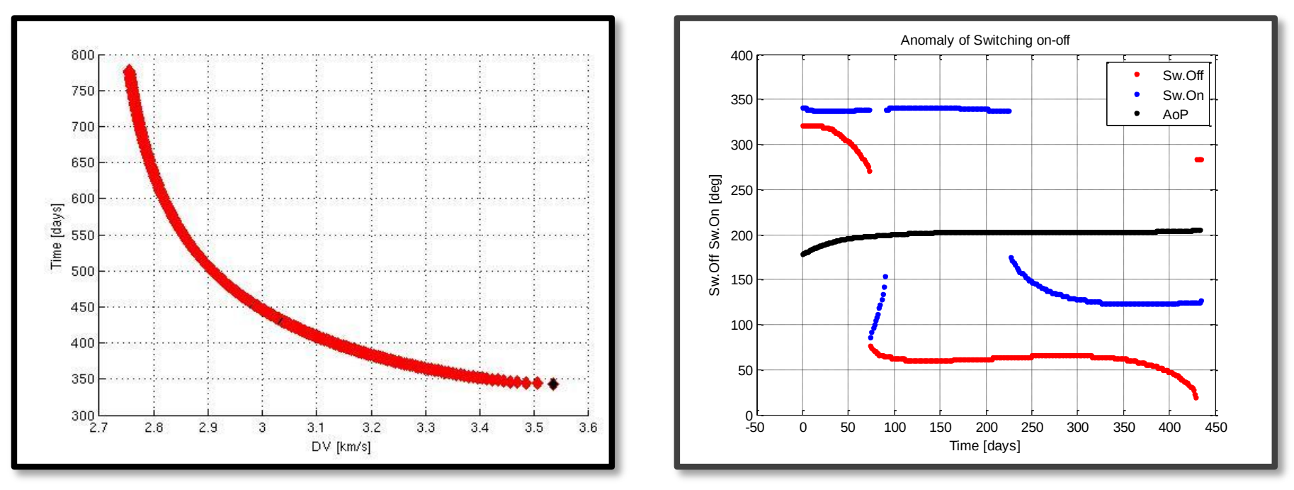
Additional features of SOFTT have been introduced to answer to concrete mission analysis needs as the Telecom Satellites delta-V optimization.

In all classic finite thrust analysis the minimized quantity is the time, this approach is not always satisfactory. In order to minimize the delta-V mission cost, a new Hamiltonian has been introduced in SOFTT where the dV is minimized for a fixed transfer time.

Increasing the available transfer time SOFTT optimizes the transfer delta-V cost, until to the minimum value delta-V corresponding to the impulsive solution is reached.

In particular changing the boundary condition and the initial costate with special expressions, SOFTT can compute the optimization of the ΔV mission cost studying the Switching Function which provides the thrusters switching on/off phases during the orbital transfer.

The following figures show examples of the SOFTT output in order to study the solution at the Vmin problem.



SOFTT outputs for Vmin solution: Time vs ΔV (left) and Switchin On/Off anomalies (right).

Autonomous Navigation

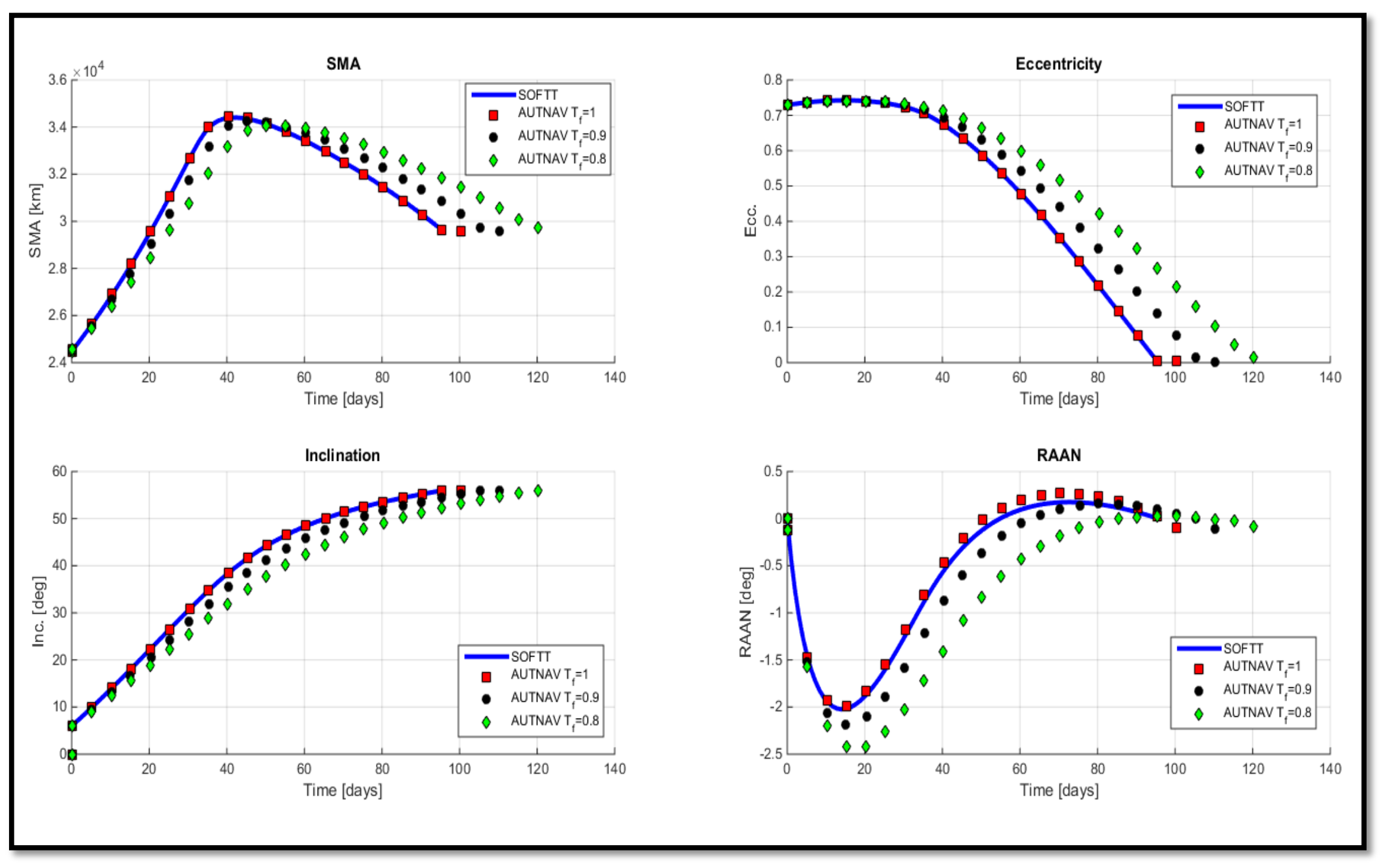
Additional features of SOFTT are under development to answer to concrete mission needs as the Satellite Autonomous Navigation : the objective is the autonomous on board management of the Plasmic Motor optimal firing direction and times during LEOP and Station Keeping

The ground segment cost to support an orbital transfer can last 6 months to 1 year can be quite relevant (range of few M€ for a 6months transfer). Furthermore during the orbital transfer the real trajectory can be quite different respect to the optimal one due to perturbations or contingencies PPS underperformances, therefore the optimal path must be realigned periodically. The SOFTT core can provide the AUTNAV package that will allow to design «smart» satellites able to compute the optimal transfer trajectory reducing the ground station contacts and commands.

The satellite computes periodically and saves the Optimal Solution in the computer memory. The satellite use this database during the flight to compute firing direction. Every time period (which may be few orbital periods), the satellite measures its real position and start a new calculation of the actual optimal thrust strategy.

During our first approach we obtained very interesting results, shown in the figures close to the text.

AUTNAV additional feature to be developed will be a the collision avoidance algorithm particularly useful for the constellation case.



SOFTT outputs for the Autonomous Navigation: Evolution of the Keplerian parameters SOFTT vs AUTNAV with thrust lower than nominal.