APPLICATION OF KALMAN FILTERS IN ORBIT DETERMINATION: A LITERATURE SURVEY

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ABSTRACT

This paper was narrowed down to significant contributions in the area of Kalman filtering applications for orbit determination. Research work from 1967 to 2013 is included in this paper. It was observed that examples from 1967 demonstrate the importance of pre-flight parametric studies when orbit determination estimations are carried out in short periods. Development of the extended Kalman filter requires some proper selection of parameters that define the probability density of the initial state vector, and any other parameters required by modifications to the extended Kalman filter algorithm. This paper can be used as a foundation for further investigations in the area of orbit prediction.

1. INTRODUCTION

This paper discusses the application of the Kalman filter towards precise real-time orbit prediction. Research shows that extended Kalman filters can be made to work in real-data situations, contrary to what some professionals in the aerospace industry believe. A popular technique for orbit determination is the Kalman filter. The study of the dynamics of bodies in interplanetary or interstellar space is in general referred to as astrodynamics. Within this discipline there are two major divisions. The first is the movement of the body's center of mass, referred to as kinematics (also celestial mechanics or orbit dynamics). The second is the movement of the body around its center of mass, referred to as attitude dynamics.

The Kalman filter is a data processing algorithm used due to its excellent recursive properties, and small computational requirements. It can be applied to linear systems exhibiting Gaussian error statistics. Some important omissions are the continuous-time algorithms [1] [2]. Also the Chandrasekhar type algorithms reported by Morf and Kailath and Lindquist [3] [4]. The main reason for omitting these continuous-time algorithms is that such algorithms are heavily dependent upon integration methods for their accuracy and numerical stability. It was assumed best not to try and compare the continuous and discrete algorithms in terms of numerical accuracy. The Chandrasekar-type algorithms were omitted because they are not directly applicable to time-varying problems and, thus, do not seem to be computationally competitive with our other algorithms for this class of problems.

The algorithms selected for study include the Conventional Kalman Filter, Joseph stabalized Kalman filter [5] [6]. Conventional Kalman Filter with lower bounding and the Potter-Schmidt square root filter [6]. Finally the Bierman-Thornton factorization filter is also discussed [7] [8]. Part of the 1977 Mariner Jupiter-Saturn (MJS) mission model shown in Equation 1 was used to analyze aforementioned filtering techniques.

$$\boldsymbol{X}_{i+1} = \boldsymbol{\Phi}_i \boldsymbol{X}_i + \boldsymbol{B}_i \boldsymbol{\omega}_i \tag{1}$$

Equation 1 is a dynamic model. An observation can be found using Equation 2.

$$\boldsymbol{X}_{i+1} = \boldsymbol{\Phi}_i \boldsymbol{X}_i + \boldsymbol{B}_i \boldsymbol{\omega}_i \tag{2}$$

Where,

$$\boldsymbol{X}_i = \boldsymbol{X}(t_i) \in \boldsymbol{R}^{\boldsymbol{n}}$$
$$\boldsymbol{z}_i = \boldsymbol{z}(t_i) \in \boldsymbol{R}^{\boldsymbol{m}}$$
$$\boldsymbol{\Phi}_i = \boldsymbol{\Phi}(t_{i+1}, t_i)$$
$$\boldsymbol{A}_i = \boldsymbol{A}(t_i)$$

Disturbances $\{\boldsymbol{\omega}_j\}$ and $\{\upsilon_i\}$ are independent Gaussian random sequences with zero means $\boldsymbol{Q}_i$ and covariances $\boldsymbol{R}_i$ respectively. The measurement and time updates are computed as shown in Equations 3 and 4 respectively. The measurement and time updates are computed as shown in Equations 3 and 4 respectively.

$$\widehat{\boldsymbol{X}}_i = \widetilde{\boldsymbol{X}}_i + \boldsymbol{K}_i(\boldsymbol{z}_i - \boldsymbol{A}_i \widetilde{\boldsymbol{X}}_i) \tag{3}$$
$$\widetilde{\boldsymbol{X}}_i = \boldsymbol{\Phi}_i \widehat{\boldsymbol{X}}_i \tag{4}$$

Where $\widetilde{\boldsymbol{X}}_i = \boldsymbol{X}(t_i|t_{i-1})$ is the one-stage predicted estimate of $\widetilde{\boldsymbol{X}}_i$ and $\widehat{\boldsymbol{X}}_i = \boldsymbol{X}(t_i|t_{i-1})$ is the filter estimate. The algorithms differ principally in the way in which they compute the Kalman gains $\{\boldsymbol{K}_i\}$ and these methods depend on either estimate error covariance or covariance factor recursions.

2. CONVENTIONAL KALMAN FILTER

The gain, measurement update and time update respectively for the conventional Kalman filter are given in Equations 5, 6, and 7 respectively.

$$\boldsymbol{K}_i = \boldsymbol{P}_i \boldsymbol{A}_i{}^{\boldsymbol{T}} (\boldsymbol{A}_i \boldsymbol{P}_i \boldsymbol{A}_i{}^{\boldsymbol{T}} + \boldsymbol{r}_i)^{-1} \tag{5}$$

$$\widehat{\boldsymbol{P}}_i = \boldsymbol{P}_i - \boldsymbol{K}_i (\boldsymbol{P}_i \boldsymbol{A}_i{}^{\boldsymbol{T}})^{\boldsymbol{T}} \tag{6}$$

$$\boldsymbol{P}_{i+1} = \boldsymbol{\Phi}_i \widehat{\boldsymbol{P}}_i \boldsymbol{\Phi}_i{}^{\boldsymbol{T}} - \boldsymbol{B}_i \boldsymbol{Q}_i \boldsymbol{B}_i{}^{\boldsymbol{T}} \tag{7}$$

Where $\boldsymbol{P}_i$ and $\widehat{\boldsymbol{P}}_i$ are the one step predicted and filter estimate error covariance matrices $\boldsymbol{P}_i = \boldsymbol{E}\left[(\boldsymbol{X}_i - \widetilde{\boldsymbol{X}}_i)(\boldsymbol{X}_i - \widetilde{\boldsymbol{X}}_i)^T\right]$ and $\widehat{\boldsymbol{P}}_i = \boldsymbol{E}\left[(\boldsymbol{X}_i - \widehat{\boldsymbol{X}}_i)(\boldsymbol{X}_i - \widehat{\boldsymbol{X}}_i)^T\right]$. All of the terms in Equations 5 through 7 are time dependent, as indicated by the subscripts. Although time dependence can be suppressed to simplify notation. The measurement error covariance in Equation 5 is represented by a scalar r. Vector outer products are used where the algorithm can be so arranged, and symmetry of the covariance matrix is preserved by computing only the upper triangular non-redundant entries. An exception to this principle of non-redundant computation is the formulation of Joseph's stabilized algorithm.

3. MEASUREMENT UPDATE IN JOSEPH'S STABILIZED FILTER

Symmetry is exploited in Equation 8 because K is computed using Equation 5. The matrix $\widehat{\boldsymbol{P}}$ can be obtained from Equation 9 by arranging the computations as indicated by the parentheses, using vector outer products and computing only the upper triangular elements.

$$\overline{\boldsymbol{P}} = \boldsymbol{P} - \boldsymbol{K}(\boldsymbol{P}\boldsymbol{A}^{\boldsymbol{T}})^{\boldsymbol{T}} \tag{8}$$

$$\widehat{\boldsymbol{P}} = \overline{\boldsymbol{P}} - (\overline{\boldsymbol{P}}\boldsymbol{A}^{\boldsymbol{T}})\boldsymbol{K}^{\boldsymbol{T}} + (\boldsymbol{K}\boldsymbol{r})\boldsymbol{K}^{\boldsymbol{T}} \tag{9}$$

Significantly more accurate results can be obtained, when all n^2 elements of the first term of Equation 9 are computed and the off-diagonal elements were averaged. It should be noted that factorizing $\boldsymbol{K}^{\boldsymbol{T}}$ in Equation 9 tends to create numeric instability. The fact that numerical results are sensitive to such details is indicative of the algorithms numerical instability.

4. CONVENTIONAL KALMAN FILTER WITH LOWER BOUNDING

In Equation 10 and 11 $M(j,j) = \rho_{min}^2\, \widehat{\boldsymbol{P}}(i,i)\, \widehat{\boldsymbol{P}}(j,j)\ and\ i = 1, 2, \dots j-1$. Then n components of σ_{min} and the correlations ρ_{min} are chosen by deduction.

$$\widehat{\boldsymbol{P}}(j,j) = max[\overline{\boldsymbol{P}}(j,j), \sigma_{min}^2(j)] \quad j = 1, 2, \dots n \tag{10}$$

$$\widehat{\boldsymbol{P}}(i,j) = \begin{cases} \overline{\boldsymbol{P}}(i,j)\ if\ \overline{\boldsymbol{P}}^2\,(i,j) < M(i,j) \\ SGN(\overline{\boldsymbol{P}}(i,j))\ \sqrt{M(i,j)}\ otherwise \end{cases} \tag{11}$$

This formulation is typical of the techniques that are used to prevent the computed covariance from having diagonals (variances) that are too small, or negative, and correlations that are too large [9]. Such formulations are, to be sure, not optimal and the computed $\widehat{\boldsymbol{P}}$ is generally not the actual estimate error covariance. Research shows that choosing the bounds σ_{min} and ρ_{min} is difficult; their appropriate values are obtained from lengthy simulation studies.

5. POTTER-SCHMIDT SQUARE-ROOT FILTER

Here the error covariance matrix is factored as $P = SS^T$ with S square. Measurement updating is accomplished by triangularizing the augmented array $[\Phi S\ BQ^{1/2}]$ by applying an orthogonal transformation from the right. Algorithms details can be found in references [5] and [6].

Formulas for factorization updating are not as compactly represented as are their covariance counterparts. Detailed comparisons in references have shown that factorization algorithms require no more computers storage, are no harder to automate and are competitive computationally with their covariance counterparts [7] [8] [10]. In fact for problems involving moderate numbers of process noise parameters, the U-D factorization filter can be actually more efficient than even the conventional Kalman filter algorithm.

6. BIERMAN-THORNTON U-D FACTORIZATION FILTER

The error covariance matrix is uniquely factored as $P = UDU^T$, with U unit upper triangular and D diagonal. Measurement and time updating algorithms for the U and D factors are derived in references [7] [8].

7. SIMULATION RESULTS OF ORBIT DETERMINATION CASE STUDY OF THE MJS MISSION

In the case study done by Bierman and Thornton, of the five algorithms analyzed the most difficulty occurred with the Kalman stabilized formulation [9]. This is surprising result because the equations appear simple. The researchers stated that there were no programming errors, only that the single precision results were sensitive to the statistics and to the grouping of terms in the computer code [9]. By contrast the single precision factorization results were always consistent. These findings and other results of interest were related by describing results for the basic 19 state filtering problem found in [9].

The statistics given in Table 1 are typical assumptions for the 19 state filtering problem. In orbit determination it is common practice to begin filtering with larger uncertainties in position and velocity. However, to avoid the initialization numerical instability associated with the Kalman algorithms were chosen to use relatively small variances.

Table 1. Summary of statistics used to generate sample data for MJS mission [9].

Variable	Standard Deviation
Position	1000 km
Velocity	100 m/s
Acceleration	10^{-11} km/sec^2
Station	Spin axis – 1 meter
Location Errors	Longitude – 2 meter
	Latitude – 5 meter
GM for Saturn	0.1%
Range	3 meters
Doppler	1 mm/sec (for 1 min. count time)

For this case and others to follow the double precision filters agree to at least 8 digits (and generally to 10 or more).

The single precision programs, however, produce a variety of results. Actual filter performance (accuracies obtained from the error analysis program which evaluates computed gain profiles from the various filter algorithms) for this case is illustrated in Figure 1 which compares actual uncertainties for each of the algorithms.

In Figure 1 the position and velocity uncertainties of the factored single precision algorithms are shown to agree with the double precision references. It is important to note that this consistency was observed in all of the cases studied; i.e. the single precision factorization results always agreed well with the double precision reference cases [9].

The single precision Kalman algorithms on the other hand, exhibit no such numerical stability. Obvious numeric deterioration in the form of negative computed variances, appear at inexplicable times. Negative variances first appear in the conventional Kalman mechanism after four days of

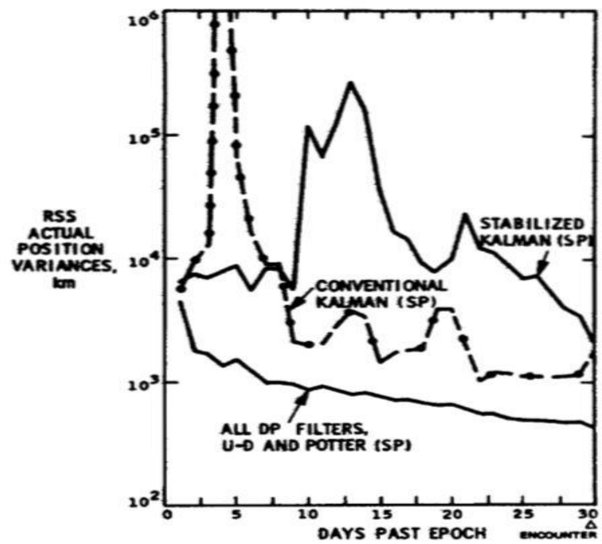

Figure 1 (a). Comparison of Actual Position Uncertainties [5].

filtering and after the days when the stabilized formulation is used. Several other surprising phenomena from Bierman-Thornton study are mentioned below:

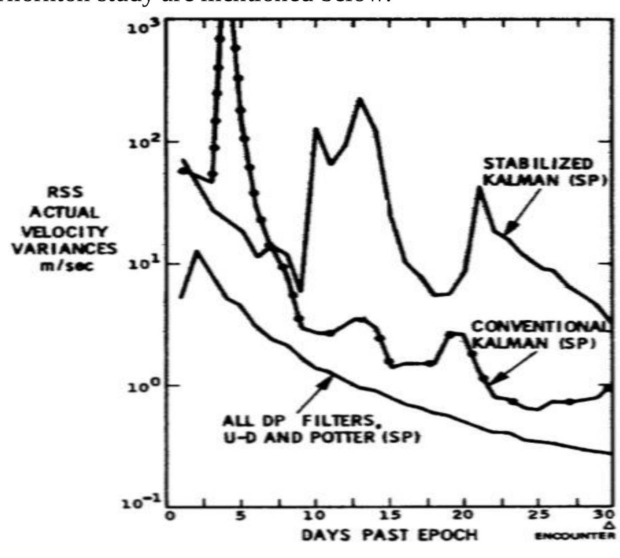

Figure 1 (b). Comparison of Actual Velocity Uncertainties [5].

(a) Both the conventional and stabilized algorithms compute intermittent negative variances. From a total of 607 measurement updates the conventional algorithm computes negative variances 177 times and the stabilized algorithm produced negative variances 69 time.

(b) Bias parameter variances are also intermittently negative. This violates the theoretic monotonicity of constant parameter variances.

(c) The numerical instability discussed here is related to the choice of statistics. However, even in the case of the conventional algorithm it takes more than 48 time and 80 measurement updates before negative computed variances appear.

(d) The appearance of negative diagonal elements in the computed covariance is not necessarily related to filter variances which are tending toward zero.

8. ERRORS ASSOCIATED WTH KALMAN FILTERS IN ORBIT DETERMINATION

The nonstationary linear discrete estimation problem has been numerically tested with various Kalman filter algorithms. In the previous orbit determination case study the errors generally have the following properties:

(a) The errors emphasize the importance of numerics in determining system performance. This is important to show that numeric effects are important both in obtaining accurate results and computing estimates.

(b) Errors show that computer numeric effects mentioned in previous point can cause erroneous predictions based on linear estimation theory. This point is necessary for describing the numerical effects in obtaining computational accuracy.

(c) Errors also show that both the conventional and stabilized Kalman are numerically unreliable.

(d) Errors demonstrate that the Bierman-Thornton U-D factorization filter is computationally efficient and numerically stable.

9. RECENT RESEARCH

More recent research focuses on the Extended Kalman Filter (EKF) based implementation approach to derive an orbit determination solution. Objectives include: using angles-only measurements to reconstruct the satellite orbit, use a combination of angles and range measurements for orbit reconstruction, and evaluate the orbit determination solution accuracy subject to various levels of fidelity for both environmental variations and sensor locations [11].

In the first objective using angles-only techniques for orbit determination means that no range measurements are used. From a general target tracking application perspective, angles-only filtering has been the subject of many on-going research efforts due to the difficulty in the processing solutions. Current research also focuses on nonlinear effects and uncertainty of the derivative terms of the linearized measurement matrix which may become ill-conditioned due to poor geometry of the target to sensor measurements [11].

There is no single solution available that clearly outperforms all other strategies. A series of nonlinear estimators have been proposed over time which for the most part is nonlinear extensions of the Kalman Filter [12]. Those nonlinear filtering techniques include the Extended Kalman Filter [13]; Unscented Kalman Filter [14] and its variants like Divided Difference based approach [15] (i.e., DD1 and DD2); State Dependent Riccati Equation/θ-D Based Filter [16] and others.

The subject of orbit determination using angles-only measurements is revisited in light of Extended Kalman Filter processing and measurement quality including the relative geometry between the target and sensor platforms. The observability requirement defined as the information captured by sensors was briefly analyzed in [11]. The processing of the Extended Kalman Filter against simulated data, and Orbit Determination Tool Kit processing of actual tracking data both show successful results. As expected, range measurements significantly improve the orbit determination solution accuracy over the angles-only filter solution case. A case using the real tracking data was constructed and run to simulate decreased observability. The results showed a large increase in the smoother position uncertainty.

In nonlinear estimation problems, various filtering algorithms have been developed and applied. There are two methods for nonlinear estimation: sequential estimation and batch estimation. The sequential estimation method predicts and corrects the state vector to produce a better estimation result recursively at each epoch. On the other side, the batch estimation method collects all measurements for a specified period and processes them together to obtain the best estimation result non-recursively at each period. The sequential estimation is commonly used for on-line applications such as on-board spacecraft navigation and real-time orbit determination for satellites because it has advantages in terms of the processing time. However, for off-line applications such as precise orbit determination for satellites, the batch estimation method is generally used because it has the advantage of accurate estimation results. The EKF method and the batch least-squares filter (BLSF) method are the most popular methods of sequential and batch estimation, respectively. EKF and BLSF have been successfully applied to nonlinear estimation problems in simple linearization and approximation nonlinear models.

Linearization in EKF and BLSF starts with the assumption that the reference value is very close to the true value, but this linearization assumption may give unstable solutions when the problem has an inaccurate initial reference condition or sparse or insufficient measurements. To overcome these problems, several estimation algorithms have been developed, such as the unscented Kalman filter (UKF), [17], particle filters (PF) [18] exact nonlinear recursive filters [19] iterative or smoothing filters based on UKF, a batch filter based on unscented transformation (UT) [20] and others.

Since UT does not include any linearization concept, for on-line applications the UKF has been applied and has shown better performance for satellite attitude estimation and orbit determination or for trajectory determination of ballistic missiles [21]. For off-line applications, a batch filter based on UT has been applied and has shown better performance for satellite attitude determination [22] and orbit determination. However, initial covariance and the scaling parameter adjustments in the batch filter based on UT sensitively affect the precision and performance of filtering in some nonlinear problems [23]. The scaling parameters change the distribution of the sigma point in UT. The physical meaning of the scaling parameters is a value that can control the distribution and range limit of the sigma points. Generally, the major factor that determines the formation of sigma points is the scale parameter α [24]. In orbit determination problems, the scaling parameter α of a batch filter based on UT affects the estimation performance [23].

The effects of the scaling parameter α of a batch filter based on UT on satellite attitude determination problems have also been analyzed.6 In that line of research, they showed that adjusting the scaling parameter can improve the estimation performance but that an additional tuning process is required to set an acceptable range of the scaling parameters. This may be why batch filters based on UT are not commonly used for effective nonlinear batch estimations.

To overcome the scaling parameter problem, the PF concept is utilized for a one-dimensional simple batch estimation problem, and the performance of a batch filter based on PF is found to be better than that with a batch filter based on UT [25]. The PF method is a completely nonlinear and non-Gaussian estimation process based on sequential Monte Carlo sampling. It has been widely applied to various real-time estimation problems [19]. PF uses a complete probability density function for measurements and complete nonlinear system dynamics. Furthermore, mathematical modeling is not complicated because the PF method uses system dynamic and measurement noise models in a straightforward manner. Because EKF and UKF use mean and covariance values, which are based on Gaussian assumptions, they cannot give best estimation results when state distribution or measurement noise has non-Gaussian characteristics. In the case of a batch filter based on PF, we can obtain robustness against high nonlinearities due to its use of completely nonlinear dynamics and a non-Gaussian probability density function. Additionally, a batch filter based on PF does not require a heavy tuning process like batch filter based on UT, allowing it to give more robust batch estimation results that can be conveniently obtained.

10. CONCLUSION

A brief literature review on the use of the Kalman filter and its derivatives has been provided. Research done in the past has shown that the conventional and stabilized Kalman filters are numerically unreliable. More recent research involved nonlinear conditions in which batch filters based on Particle Filters and Unscented Transformation performance levels are similar and better than that of Batch Least-Squares Filter. Precise orbit determination using a batch filter based on Particle Filters is more suitable than Batch Least-Squares Filter and a batch filter based on Unscented Transformation for precise orbit determination problems that arise under nonlinear and non-Gaussian conditions. Further research currently being done will shed light on tracking and detection of multiple “resident space objects” whose initial orbit parameters are required for a complete solution. The development of orbit determination techniques will always focus on providing a solution with minimal computation time.

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