Spacecraft Formation Control using Analytical Integration of GVE

ICATT 2016

Mohamed Khalil Ben Larbi, 14.-17.03.2016
Contents

- Introduction
  - Motivation
  - Why ROE

- Gauss’ variational equations
  - GVE
  - Control scheme

- Results and evaluation
  - Formation reconfiguration and keeping

- Conclusion and outlook
Motivation

Idea: derive a formation geometry guaranteeing

- minimum collision risk (passive safety)
- minimum number of correction maneuvers (passive stability).
Motivation

Idea: derive a formation geometry guaranteeing

- minimum collision risk (passive safety)
- minimum number of correction maneuvers (passive stability).

Challenge for active debris removal

- Uncertainties in along-track
Motivation

**Idea: derive a formation geometry guaranteeing**

- minimum collision risk (passive safety)
- minimum number of correction maneuvers (passive stability).

**Challenge for active debris removal**

- Uncertainties in along-track

$\implies$ Separation in RN plane for safe formation
Formation description

Relative orbital elements

\[
\delta \alpha = \begin{pmatrix}
\delta a \\
\delta \lambda \\
\delta e_x \\
\delta e_y \\
\delta i_x \\
\delta i_y
\end{pmatrix} = \begin{pmatrix}
(a_2 - a_1) a_1^{-1} \\
(u_2 - u_1) + (\Omega_2 - \Omega_1) \cos i_1 \\
e_{x_2} - e_{x_1} \\
e_{y_2} - e_{y_1} \\
i_2 - i_1 \\
(\Omega_2 - \Omega_1) \sin i_1
\end{pmatrix},
\]

with \( u = M + \omega \), \( e_x = e \cos \omega \) and \( e_y = e \sin \omega \).

E/I polar notation

\[
\delta e = \delta e (\cos \phi \ \sin \phi)^T \quad \text{and} \quad \delta i = \delta i (\cos \theta \ \sin \theta)^T
\]
Eccentricity/Inclination Separation

- Relative trajectory (without Drift)

\[ \delta \alpha_{\text{nom}} = \begin{pmatrix} \delta a_{\text{nom}} & \delta \lambda_{\text{nom}} & 0 & -||\delta e_{\text{nom}}|| & 0 & +||\delta i_{\text{nom}}|| \end{pmatrix}^T \]
Why relative orbital elements

- Insight into the formation geometry (E/I separation)
- Maintains decoupling of in-plane and out-of-plane motion
- More accuracy (retaining higher order terms)
- Adoption of Gauss variational equations (GVE)

Gauss' variational equations results
Conclusion and outlook
Gauss variational equations

\[
\frac{d\alpha}{dt} = \frac{1}{na} \mathbf{D}(\alpha_{osc}) \cdot \gamma
\]

- \[\frac{da}{dt} = \frac{2}{n\sqrt{1-e^2}}\]
- \[\frac{dM}{dt} = n + \frac{1-e^2}{nae}\]
- \[\frac{de}{dt} = \frac{\sqrt{1-e^2}}{na}\]
- \[\frac{d\omega}{dt} = \frac{\sqrt{1-e^2}}{nae}\]
- \[\frac{di}{dt} = \frac{r \cos(\omega + \nu)}{na^2 \sqrt{1-e^2}}\]
- \[\frac{d\Omega}{dt} = \frac{r \sin(\omega + \nu)}{na^2 \sqrt{1-e^2} \sin i}\]

\[\left[ e \sin(\nu) \gamma_R + (1 + e \cos \nu) \gamma_T \right]\]
\[\left[ \cos \nu - \frac{2e}{1 + e \cos \nu} \right] \gamma_R - \left( 1 + \frac{1}{1 + e \cos \nu} \right) \sin \nu \gamma_T\]
\[\left[ \sin(\nu) \gamma_R + \cos E + \cos \nu \gamma_T \right]\]
\[\left[ -\cos \nu \gamma_R + \left( 1 + \frac{1}{1 + e \cos \nu} \right) \sin \nu \gamma_T \right] - \frac{r \sin(\omega + \nu) \cos i}{na^2 \sqrt{1-e^2} \sin i} \gamma_N\]

- The subscript \(\gamma\) indicates the direction of the perturbation acceleration, originated - in this case - from a maneuver
GVE for s/c formation

- Gauss variational equations \( \frac{d\delta \alpha}{dt} = f(\delta \alpha, \gamma_p) \)
GVE for s/c formation

- Gauss variational equations \( \frac{d\delta \alpha}{dt} = f(\delta \alpha, \gamma_p) \)

Impulsive thrust

\[ \Delta v = \int_{t^-_M}^{t^+_M} \gamma_p \, dt \]
GVE for s/c formation

- Gauss variational equations
  \[ \frac{d\delta \alpha}{dt} = f(\delta \alpha, \gamma_p) \]

Impulsive thrust

\[ \Delta v = \int_{t_M^-}^{t_M^+} \gamma_p dt \]

\[ \Downarrow \]

Thrust duration

\[ \Delta t_M \approx m_2 \| \Delta v_M \| / F_{max} \]
GVE for s/c formation

- Gauss variational equations \( \frac{d\delta \alpha}{dt} = f(\delta \alpha, \gamma_p) \)

**Impulsive thrust**

\[
\Delta v = \int_{t^-_M}^{t^+_M} \gamma_p dt
\]

\[\downarrow\]

**Thrust duration**

\[\Delta t_M \approx m_2 \| \Delta v_M \| / F_{max}\]

**Finite duration thrust**

\[
\Delta v = \int_{t_1}^{t_2} \gamma_p dt
\]
GVE for s/c formation

- Gauss variational equations $\frac{d\delta \alpha}{dt} = f(\delta \alpha, \gamma_p)$

**Impulsive thrust**

\[ \Delta \mathbf{v} = \int_{t_M^-}^{t_M^+} \gamma_p \, dt \]

\[ \Delta t_M \approx m_2 \| \Delta \mathbf{v}_M \| / F_{max} \]

**Finite duration thrust**

\[ \Delta \mathbf{v} = \int_{t_1}^{t_2} \gamma_p \, dt \]

\[ \Delta t_M \text{ exactly solved} \]
GVE for s/c formation

- Gauss variational equations
  \[ \frac{d\delta \alpha}{dt} = f(\delta \alpha, \gamma_p) \]

**Impulsive thrust**

\[ \Delta v = \int_{t_M^-}^{t_M^+} \gamma_p dt \]

\[ \downarrow \]

**Thrust duration**

\[ \Delta t_M \approx m_2 \|\Delta v_M\|/F_{max} \]

**Finite duration thrust**

\[ \Delta v = \int_{t_1}^{t_2} \gamma_p dt \]

\[ \downarrow \]

**Thrust duration**

\[ \Delta t_M \text{ exactly solved} \]

**Aim**

Maneuver set to reconfigure the formation into \( \delta \alpha_{nom} \)
Maneuver sequence

- Maneuver sequence 4T2N

\[ \Delta u \]

- solve GVE for \( \Delta \mathbf{v}_M \implies \Delta \mathbf{v}_M = f(\Delta \delta \alpha, u_0, \Delta u) \]
Maneuver sequence

- Maneuver sequence 4T2N

\[ u_0 \quad u_{T1} \quad u_{N1} \quad u_{T2} \quad u_{N2} \quad u_{T3} \quad u_{T4} \]

\[ \Delta u \]

- solve GVE for \( \Delta v_M \) \( \Rightarrow \) \( \Delta v_M = f(\Delta \delta \alpha, u_0, \Delta u) \)

Major challenge

Compute the intermediate alteration \( \Delta \delta a_I = f(\Delta \delta \alpha, u_0, \Delta u) \)
Impulsive & finite-duration planning

Impulsive thrust (IT)
solve GVE for $\Delta \nu_M$

Finite-duration thrust (FDT)
solve integrated GVE for $\Delta t_M$
Impulsive & finite-duration planning

**Impulsive thrust (IT)**

- solve GVE for $\Delta v_M$

- IT major challenge
  \[ \Delta \delta a_I = f(\Delta \delta \alpha, u_0, \Delta u) \text{ analytically resolvable} \]

**Finite-duration thrust (FDT)**

- solve integrated GVE for $\Delta t_M$

- FDT major challenge
  \[ \Delta \delta a_I = f(\Delta \delta \alpha, u_0, \Delta u) \text{ analytically not resolvable} \]
## Impulsive & finite-duration planning

<table>
<thead>
<tr>
<th>Impulsive thrust (IT)</th>
<th>Finite-duration thrust (FDT)</th>
</tr>
</thead>
<tbody>
<tr>
<td>solve GVE for $\Delta v_M$</td>
<td>solve integrated GVE for $\Delta t_M$</td>
</tr>
</tbody>
</table>

### IT major challenge

$\Delta \delta a_I = f(\Delta \delta \alpha, u_0, \Delta u)$ analytically resolvable

### FDT major challenge

$\Delta \delta a_I = f(\Delta \delta \alpha, u_0, \Delta u)$ analytically not resolvable

### Approximation

$\Delta \delta a_I$ solution from IT

### Computation

$\Delta \delta a_I$ numerical iteration
Formation reconfiguration and keeping

Tabelle 1: Initial and final configurations

<table>
<thead>
<tr>
<th></th>
<th>$a\delta a$</th>
<th>$a\delta \lambda$</th>
<th>$a\delta e_x$</th>
<th>$\delta e_y$</th>
<th>$a\delta i_x$</th>
<th>$a\delta i_y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial (m)</td>
<td>29</td>
<td>$-10000$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Final (m)</td>
<td>0</td>
<td>$-2000$</td>
<td>0</td>
<td>$-900$</td>
<td>0</td>
<td>$-500$</td>
</tr>
</tbody>
</table>

Cross-Track (m) | Radial (m) | Trajectory

14.-17.03.2016 | Mohamed Khalil Ben Larbi | Seite 11
Spacecraft Formation Control using Analytical Integration of GVE
Error assessment

- Inserting $\Delta t_M$ from IT into integrated GVE
  $\Rightarrow$ analytical assessment of formation error induced through impulsive planning
Conclusion

Summary

- GVE for finite duration maneuver derived with several possible applications
- FDT and IT control scheme using 4T2N maneuvers
Conclusion

Summary

- GVE for finite duration maneuver derived with several possible applications
- FDT and IT control scheme using 4T2N maneuvers
Conclusion

Summary
- GVE for finite duration maneuver derived with several possible applications
- FDT and IT control scheme using 4T2N maneuvers

Outlook
- Verification via high fidelity simulation with high risk debris objects
- Inclusion of perturbations and estimation uncertainties.
- Assessment of required computational power and suitability as on-board solution