Technische Universität Braunschweig

Institute of Space Systems

Spacecraft Formation Control using Analytical Integration of GVE

ICATT 2016

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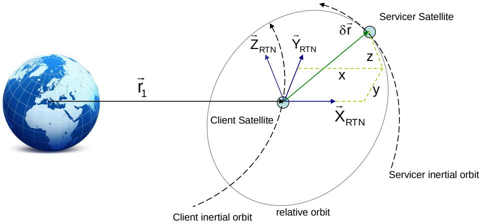
Contents

Motivation

Idea: derive a formation geometry guaranteeing

- minimum collision risk (passive safety)
- minimum number of correction maneuvers (passive stability).



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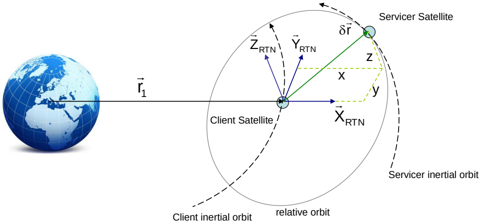

Challenge for active debris removal

- Uncertainties in along-track

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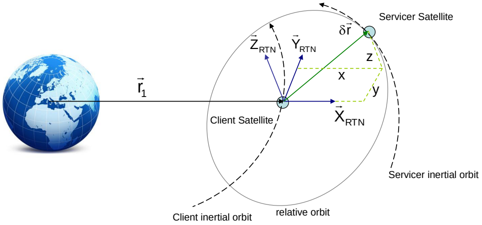


Challenge for active debris removal

- Uncertainties in along-track

$\Longrightarrow$ Separation in RN plane for safe formation

Formation description

Relative orbital elements

$$\delta\alpha = \begin{pmatrix} \delta a \\ \delta\lambda \\ \delta e_x \\ \delta e_y \\ \delta i_x \\ \delta i_y \end{pmatrix} = \begin{pmatrix} (a_2 - a_1)\, a_1^{-1} \\ (u_2 - u_1) + (\Omega_2 - \Omega_1) \cos i_1 \\ e_{x_2} - e_{x_1} \\ e_{y_2} - e_{y_1} \\ i_2 - i_1 \\ (\Omega_2 - \Omega_1) \sin i_1 \end{pmatrix} ,$$

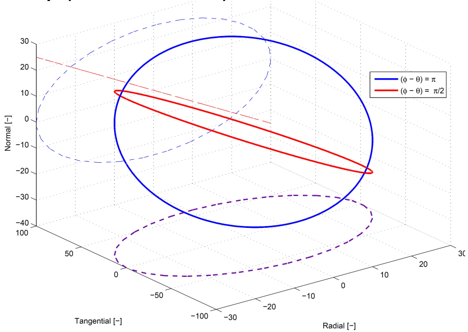
with $u = M + \omega$, $e_x = e \cos\omega$ and $e_y = e \sin\omega$.

E/I polar notation

$\delta\mathbf{e} = \delta e(\cos\phi \ \ \sin\phi)^{\mathrm{T}}$ and $\delta\mathbf{i} = \delta i(\cos\theta \ \ \sin\theta)^{\mathrm{T}}$

Eccentricity/Inclination Separation

- Relative trajectory (without Drift)

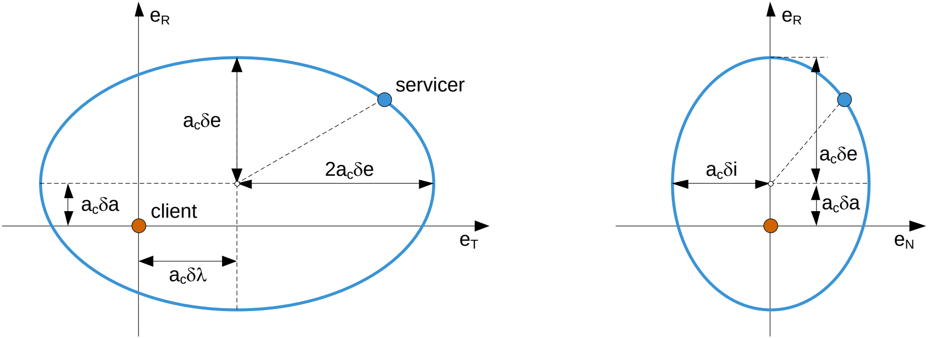


E/I separation

- safe formation for $\delta\mathbf{e} \parallel \delta\mathbf{i}$
- $\delta\boldsymbol{\alpha}_{\text{nom}} = \left(\delta a_{\text{nom}} \quad \delta\lambda_{\text{nom}} \quad 0 \quad -\|\delta\mathbf{e}_{\text{nom}}\| \quad 0 \quad +\|\delta\mathbf{i}_{\text{nom}}\|\right)^{\mathrm{T}}$

Why relative orbital elements

- Insight into the formation geometry (E/I separation)



- Maintains decoupling of in-plane and out-of-plane motion
- More accuracy (retaining higher order terms)
- Adoption of Gauss variational equations (GVE)

Gauss variational equations

- $\frac{\mathrm{d}\boldsymbol{\alpha}}{\mathrm{d}t} = \frac{1}{na}\mathbf{D}(\boldsymbol{\alpha}_{osc}) \cdot \boldsymbol{\gamma}$

$$
\begin{aligned}
\frac{\mathrm{d}a}{\mathrm{d}t} &= \frac{2}{n\sqrt{1-e^2}} && \left[e\sin(\nu)\,\gamma_{\mathrm{R}} + (1+e\cos\nu)\,\gamma_{\mathrm{T}}\right] \\
\frac{\mathrm{d}M}{\mathrm{d}t} &= n + \frac{1-e^2}{nae} && \left[\left(\cos\nu - \frac{2e}{1+e\cos\nu}\right)\gamma_{\mathrm{R}} - \left(1+\frac{1}{1+e\cos\nu}\right)\sin\nu\,\gamma_{\mathrm{T}}\right] \\
\frac{\mathrm{d}e}{\mathrm{d}t} &= \frac{\sqrt{1-e^2}}{na} && \left[\sin(\nu)\,\gamma_{\mathrm{R}} + (\cos E + \cos\nu)\,\gamma_{\mathrm{T}}\right] \\
\frac{\mathrm{d}\omega}{\mathrm{d}t} &= \frac{\sqrt{1-e^2}}{nae} && \left[-\cos\nu\,\gamma_{\mathrm{R}} + \left(1+\frac{1}{1+e\cos\nu}\right)\sin\nu\,\gamma_{\mathrm{T}}\right] - \frac{r\sin(\omega+\nu)\cos i}{na^2\sqrt{1-e^2}\sin i}\gamma_{\mathrm{N}} \\
\frac{\mathrm{d}i}{\mathrm{d}t} &= \frac{r\cos(\omega+\nu)}{na^2\sqrt{1-e^2}} && \left[\gamma_{\mathrm{N}}\right] \\
\frac{\mathrm{d}\Omega}{\mathrm{d}t} &= \frac{r\sin(\omega+\nu)}{na^2\sqrt{1-e^2}\sin i} && \left[\gamma_{\mathrm{N}}\right]
\end{aligned}
$$

- The subscript $\gamma_{\bullet}$ indicates the direction of the perturbation acceleration, originated -in this case- from a maneuver

GVE for s/c formation

- Gauss variational equations $\frac{\mathrm{d}\delta\boldsymbol{\alpha}}{\mathrm{d}t} = f(\delta\boldsymbol{\alpha}, \boldsymbol{\gamma}_p)$

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Impulsive thrust

$$\Delta\boldsymbol{v} = \int_{t_\mathrm{M}^-}^{t_\mathrm{M}^+} \boldsymbol{\gamma}_p \mathrm{d}t$$

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Thrust duration

$$\Delta t_{\mathrm{M}} \approx m_2 \|\Delta \boldsymbol{v}_{\mathrm{M}}\| / F_{max}$$

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Finite duration thrust

$$\Delta \boldsymbol{v} = \int_{t_1}^{t_2} \boldsymbol{\gamma}_p \mathrm{d}t$$

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Thrust duration

Δt_M exactly solved

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Thrust duration

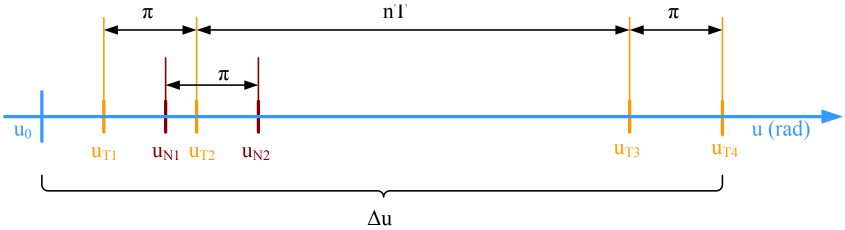
Δt_{M} exactly solved

Aim

Maneuver set to reconfigure the formation into $\delta\boldsymbol{\alpha}_{\mathrm{nom}}$

Maneuver sequence

- Maneuver sequence 4T2N



- solve GVE for $\Delta \boldsymbol{v}_{\mathrm{M}} \Longrightarrow \Delta \boldsymbol{v}_{\mathrm{M}} = f(\Delta\delta\boldsymbol{\alpha}, u_0, \Delta u)$

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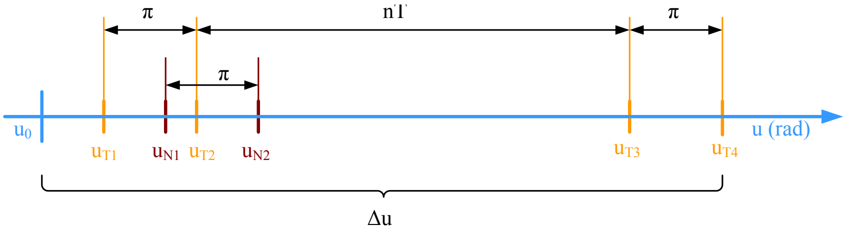

- solve GVE for $\Delta \boldsymbol{v}_{\mathrm{M}} \Longrightarrow \Delta \boldsymbol{v}_{\mathrm{M}} = f(\Delta\delta\boldsymbol{\alpha}, u_0, \Delta u)$

Major challenge

Compute the intermediate alteration $\Delta\delta a_{\mathrm{I}} = f(\Delta\delta\boldsymbol{\alpha}, u_0, \Delta u)$

Impulsive & finite-duration planning

Impulsive thrust (IT)

solve GVE for Δv_{M}

Finite-duration thrust (FDT)

solve integrated GVE for Δt_{M}

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IT major challenge

$\Delta\delta a_{\mathrm{I}} = f(\Delta\delta\alpha, u_0, \Delta u)$ analytically resolvable

Finite-duration thrust (FDT)

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FDT major challenge

$\Delta\delta a_{\mathrm{I}} = f(\Delta\delta\alpha, u_0, \Delta u)$ analytically not resolvable

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$\Downarrow$

Approximation

$\Delta\delta a_{\mathrm{I}}$ solution from IT

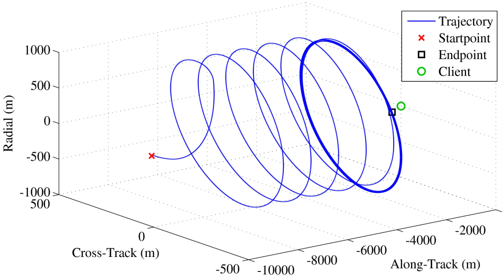
Computation

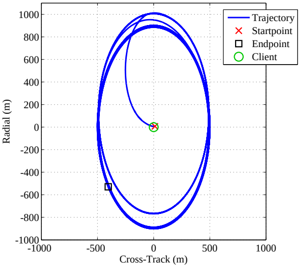
$\Delta\delta a_{\mathrm{I}}$ numerical iteration

Formation reconfiguration and keeping

Tabelle 1: Initial and final configurations

	$a\delta a$	$a\delta\lambda$	$a\delta e_x$	δe_y	$a\delta i_x$	$a\delta i_y$
Initial(m)	29	-10000	0	0	0	0
Final (m)	0	-2000	0	-900	0	-500

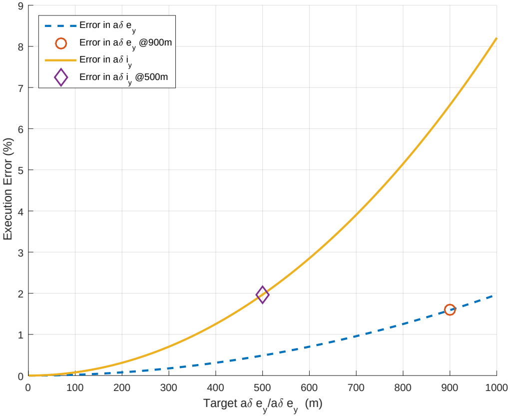




Error assessment

- Inserting Δt_{M} from IT into integrated GVE

$\Rightarrow$ analytical assessment of formation error induced through impulsive planning



Conclusion

Summary

- GVE for finite duration maneuver derived with several possible applications
- FDT and IT control scheme using 4T2N maneuvers

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Outlook

- Verification via high fidelity simulation with high risk debris objects
- Inclusion of perturbations and estimation uncertainties.
- Assessment of required computational power and suitability as on-board solution