Capitalizing on Relative Motion in Electrostatic Detumble of Axi-Symmetric GEO Debris

Trevor Bennett
Graduate Research Assistant
University of Colorado Boulder

Hanspeter Schaub
Professor
University of Colorado Boulder

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Space Utilization!
Debris Removal, Satellite Servicing
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Debris Removal, Satellite Servicing

- Booster Upper Stage
- Solar Panels and HAMR Objects
- Serviced Satellite
- Antennas and Flexible Structures
**Conventional Grapple**

Current docking techniques or arm capture can only achieve slow rotation rates.
**Target Capture Techniques**

**Space Net**

**Grapple**

**Debris Thrower**

**Tethered Capture**
Subject to tether dynamics and deployment challenges.
Target Capture Techniques

Space Net

Grapple

Ion Shepard

Debris Thrower
Opportunities exist for many space-based applications of electrostatic actuation. Concepts have been explored as early as 1960s.
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Electrostatic Actuation Applications

Opportunities exist for many space-based applications of electrostatic actuation. Concepts have been explored as early as 1960s.

Debris Re/De-Orbiting


Tethered and Inflatable Structures


Electrostatic Actuation

Consider a GEO debris object
Electrostatic Actuation

Consider a GEO debris object
Consider a GEO debris object

Touchlessly actuate objects separated by dozens of meters. Proposed:

Electrostatic Detumble of GEO Debris
Charge Distribution and the Multi-Sphere Method (MSM)

Courtesy of Tech-X Corporation
Charge Distribution and the Multi-Sphere Method (MSM)

FEM vs. MSM

MSM is a lumped charge representation

Charge Distribution and the Multi-Sphere Method (MSM)

**FEM vs. MSM**

MSM is a lumped charge representation

Maxwell FEM

Charge Distribution and the Multi-Sphere Method (MSM)

MSM is a lumped charge representation

FEM vs. MSM

Maxwell FEM

Surface MSM

Charge Distribution and the Multi-Sphere Method (MSM)

MSM is a lumped charge representation

Surface MSM

Finite element methods require minutes to solve the charge for a particular configuration.

Electrostatic Detumble Equations of Motion

Projection Angle
$$\Phi = \arccos \left( \hat{b}_1 \cdot (-\hat{r}) \right)$$

Torque Axis
$$\hat{e}_L = \hat{b}_1 \times -\hat{r}$$
Electrostatic Detumble
Equations of Motion

Projection Angle
\[ \Phi = \arccos \left( \hat{b}_1 \cdot (-\hat{r}) \right) \]

Torque Axis
\[ \hat{e}_L = \hat{b}_1 \times -\hat{r} \]

\[ I\dot{\omega} + \omega \times I\omega = L \]

New Basis Equations of Motion

\[ I_a \dot{\omega}_1 = 0 \]
\[ I_t \dot{\eta} - I_a \omega_1 \dot{\Phi} \sin \Phi = 0 \]
\[ I_t \left( \dot{\Phi} \sin \Phi - \eta^2 \frac{\cos \Phi}{\sin^2 \Phi} \right) + I_a \omega_1 \eta = L \]
\[ L = -L\hat{e}_L = -f(\phi) \sum_{m=1}^{n} \gamma_m g_m (\Phi) \hat{e}_L \]
**Linearized Relative Orbit Elements**

**Clohessy-Wiltshire (CW) Equations**

\[
x(t) = A_1 \cos(nt) - A_2 \sin(nt) + x_{off}
\]

\[
y(t) = -2A_1 \sin(nt) - 2A_2 \cos(nt) - \frac{3}{2}ntx_{off} + y_{off}
\]

\[
z(t) = B_1 \cos(nt) - B_2 \sin(nt)
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\]

\[
X_{\text{NS}} = (A_1, A_2, x_{\text{off}}, y_{\text{off}}, B_1, B_2)
\]

The collection of CW invariants become the state vector for relative motion. This new state vector becomes the Linearized Relative Orbit Element (LROE) state.
**Linearized Relative Orbit Elements**

**Clohessy-Wiltshire (CW) Equations**

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The collection of CW invariants become the state vector for relative motion. This new state vector becomes the Linearized Relative Orbit Element (LROE) state.

**Lagrange Brackets** provides evolution of the invariants given a perturbation acceleration.

\[ \dot{\mathbf{X}}_{NS} = \frac{1}{n} \begin{bmatrix} -\sin(nt) & -2 \cos(nt) & 0 \\ -\cos(nt) & 2 \sin(nt) & 0 \\ 0 & 2 & 0 \\ -2 & 3nt & 0 \end{bmatrix} \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} \]

\[ \dot{\mathbf{X}}_k = \mathbf{F}(\mathbf{X}(t_k), t_k) = B(\mathbf{X}(t_k), t_k) \mathbf{a}_d \]
Linearized Relative Orbit Elements

**Clohessy-Wiltshire (CW) Equations**

\[
\begin{align*}
    x(t) &= A_1 \cos(nt) - A_2 \sin(nt) + x_{\text{off}} \\
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    z(t) &= B_1 \cos(nt) - B_2 \sin(nt)
\end{align*}
\]

\[X_{\text{NS}} = (A_1, A_2, x_{\text{off}}, y_{\text{off}}, B_1, B_2)\]

The collection of CW invariants become the state vector for relative motion. This new state vector becomes the Linearized Relative Orbit Element (LROE) state.

**LROEs can be propagated with any acceleration rotated into the relative orbit frame.**

\[\dot{X}_{\text{NS}} = \frac{1}{n} \begin{bmatrix}
    -\sin(nt) & -2\cos(nt) & 0 \\
    -\cos(nt) & 2\sin(nt) & 0 \\
    0 & 2 & 0 \\
    -2 & 3nt & 0 \\
    0 & 0 & -\sin(nt) \\
    0 & 0 & -\cos(nt)
\end{bmatrix} \begin{bmatrix}
    a_x \\
    a_y \\
    a_z
\end{bmatrix}
\]

**Lagrange Brackets** provides evolution of the invariants given a perturbation acceleration.

\[\dot{X}_k = F(X(t_k), t_k) = B(X(t_k), t_k) a_d\]
Optimization Approach

\[
J = \sum_{i=0}^{N} \left( -1000 \ln[|r_i| - r^* + 1] - 10 \ln \left[ \frac{|r_i \cdot H_i|}{|r_i| \|H_i\|} + 1 \right] \right)
\]
A movie will be included here.
Numerical Simulations

- parameters, conditions, time, etc.
Optimized State Provides a Performance Increase

A movie will be included here.
Conclusions and Future Work

Conclusions:

• The choice of relative orbit provides substantial increase/decrease in detumble performance.

• The Linearized Relative Orbit Element (LROE) approach provides insightful approaches to optimizing the servicer relative orbit.

• Without significant loss in performance, the relative orbit may be selected for operational simplicity using a leader-follower or for operational safety where a safety ellipse orbit is available.

Future Work:

• Expand the relative motion detumble analysis to include objects that are not axisymmetric.
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- The choice of relative orbit provides substantial increase/decrease in detumble performance.

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**Questions?**
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