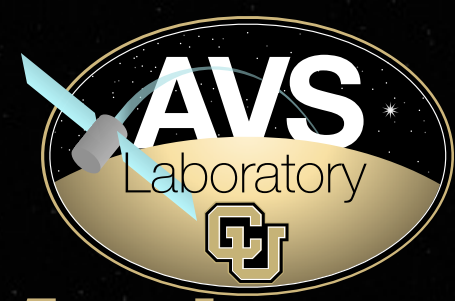




**CCAR**



# Capitalizing on Relative Motion in Electrostatic Detumble of Axi-Symmetric GEO Debris

**Trevor Bennett**  
*Graduate Research Assistant*  
*University of Colorado Boulder*

**Hanspeter Schaub**  
*Professor*  
*University of Colorado Boulder*

*6th International Conference on Astrodynamics Tools and Techniques*  
*ESOC, Darmstadt, Germany*  
*March 14-17, 2016*



# Space Utilization!

Debris Removal, Satellite Servicing





# Space Utilization!

Debris Removal, Satellite Servicing

Booster Upper Stage

Solar Panels and HAMR Objects

Serviced Satellite

Antennas and Flexible Structures





# Space Utilization!

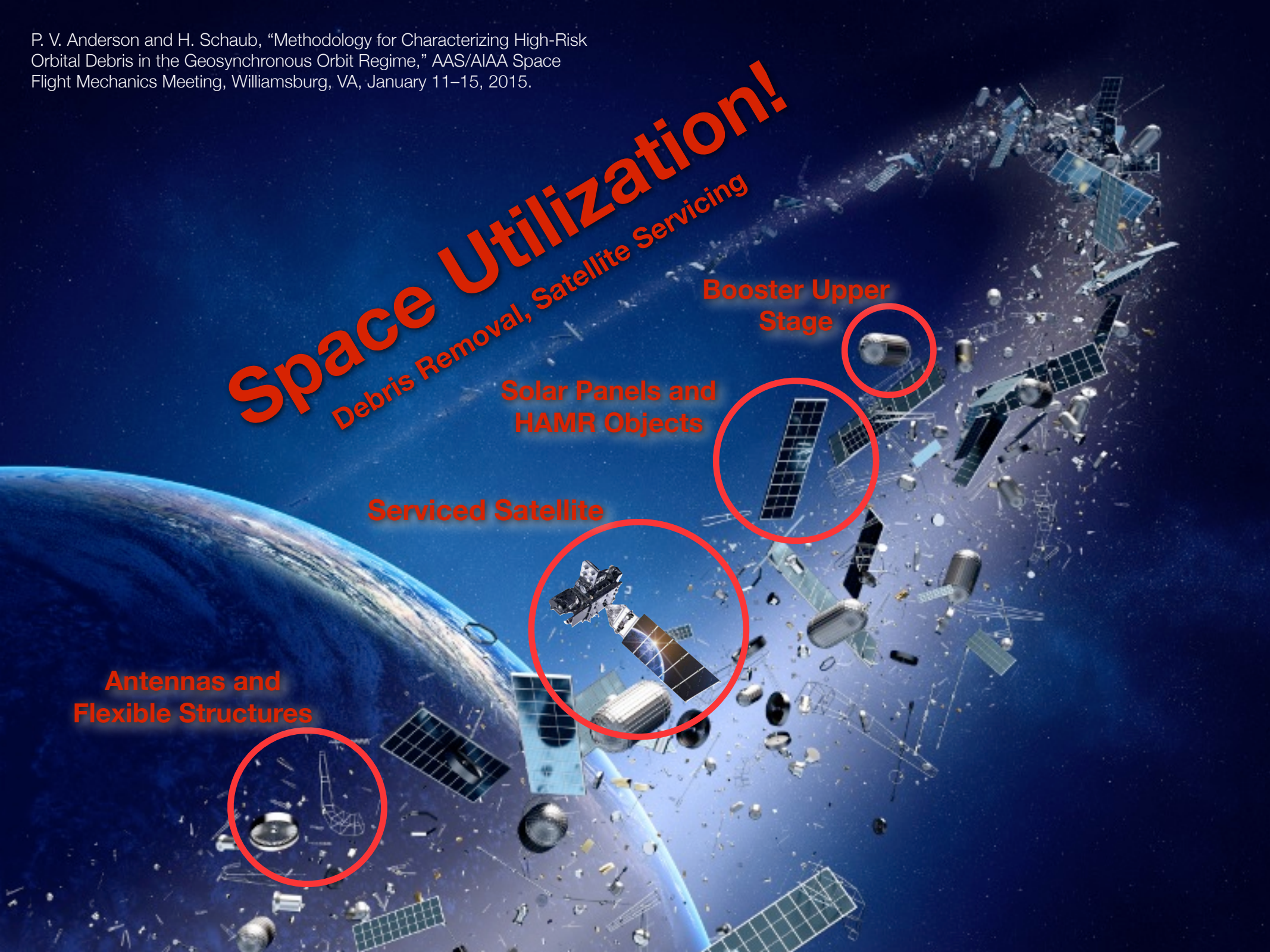
Debris Removal, Satellite Servicing

Booster Upper Stage

Solar Panels and HAMR Objects

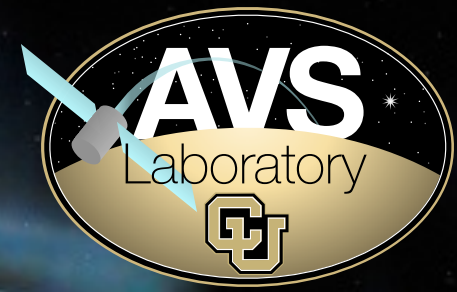
Serviced Satellite

Antennas and Flexible Structures

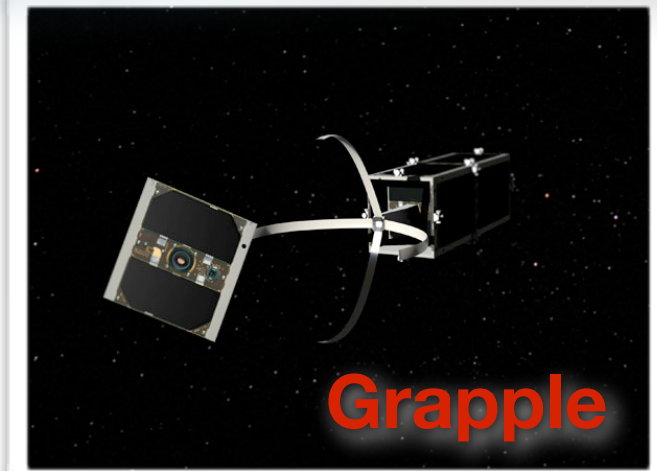




# Target Capture Techniques

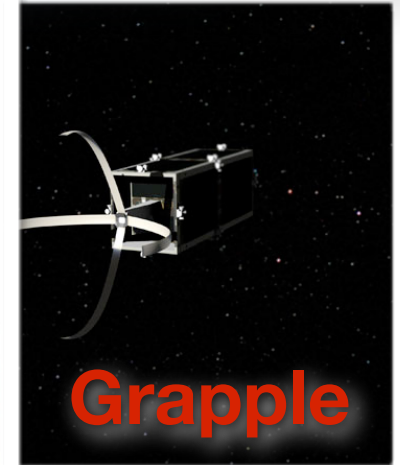
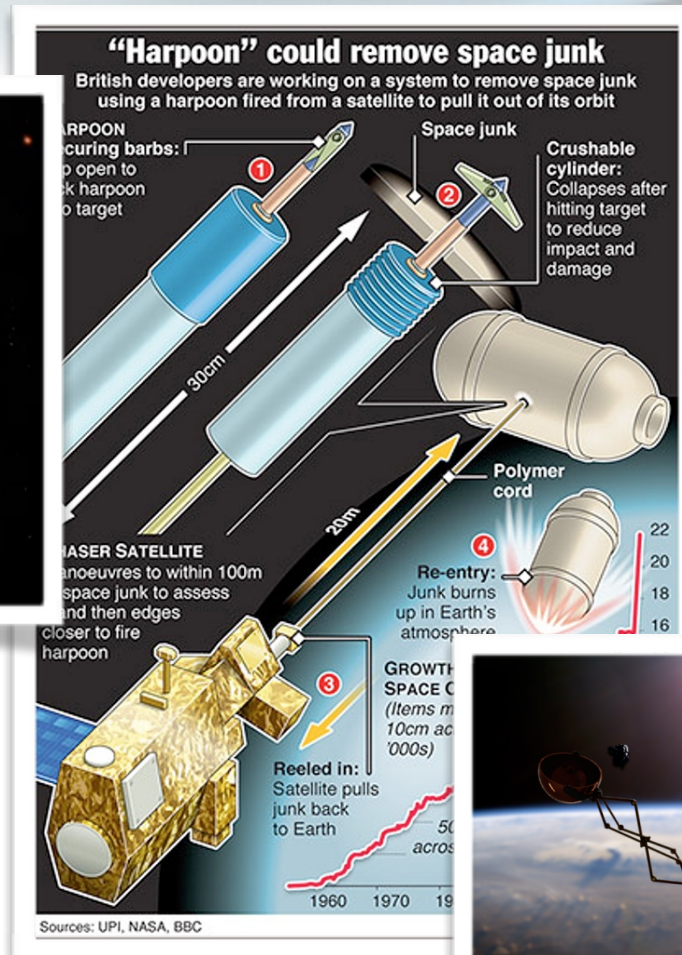
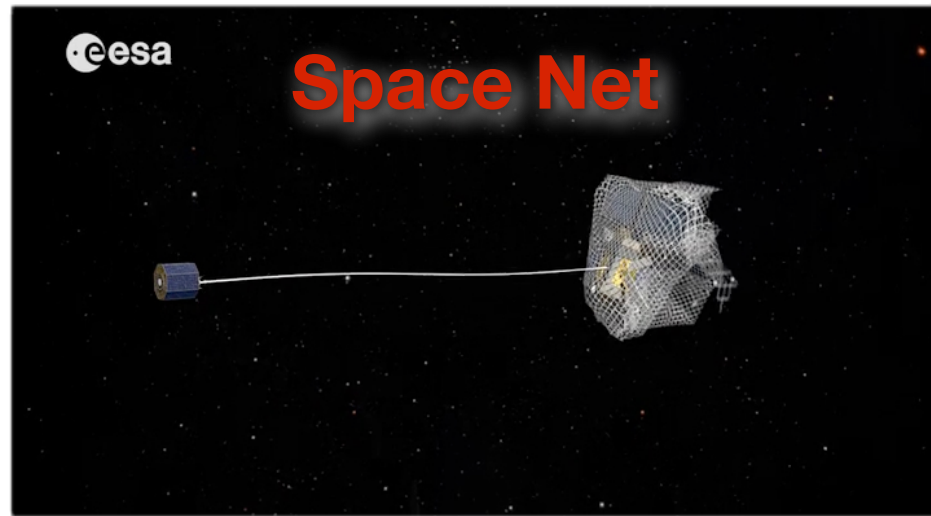
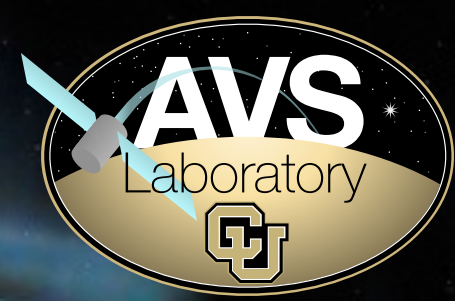


**Conventional Grapple**  
*Current docking techniques or  
arm capture can only achieve  
slow rotation rates.*





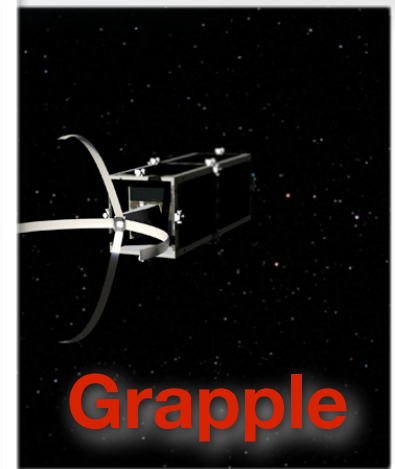
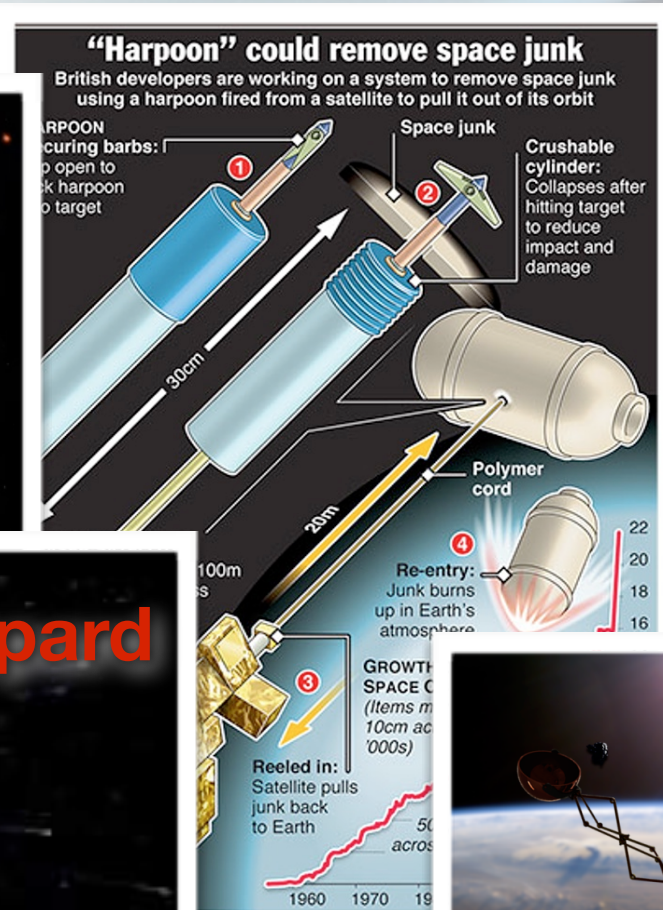
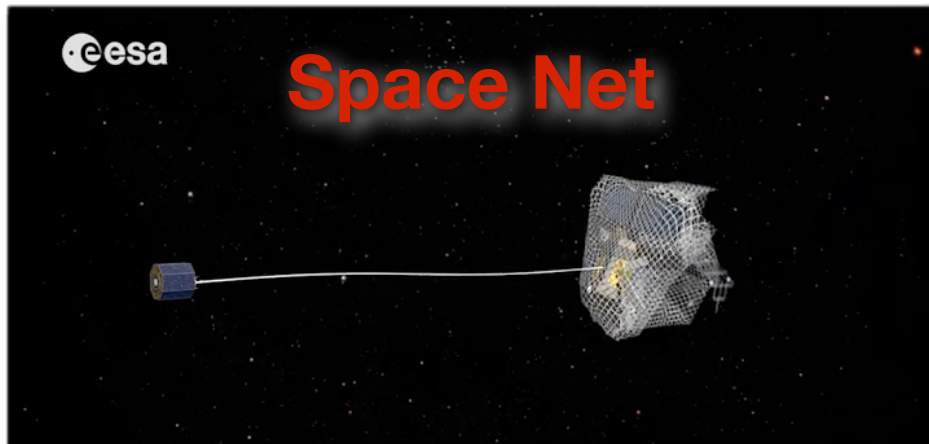
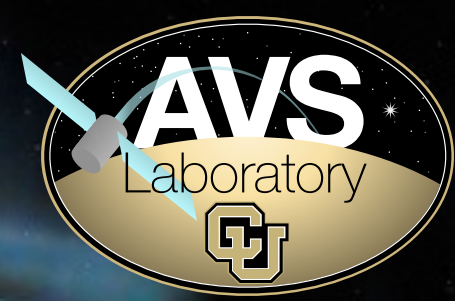
# Target Capture Techniques



**Tethered Capture**  
*Subject to tether dynamics and deployment challenges.*

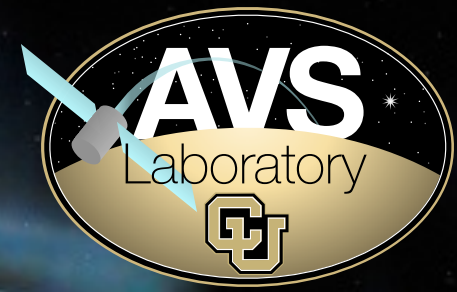


# Target Capture Techniques





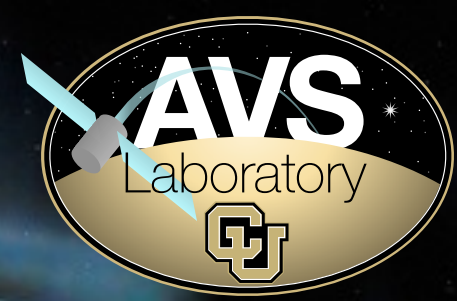
# Electrostatic Actuation Applications



*Opportunities exist for **many** space-based applications of electrostatic actuation. Concepts have been explored as early as **1960s**.*

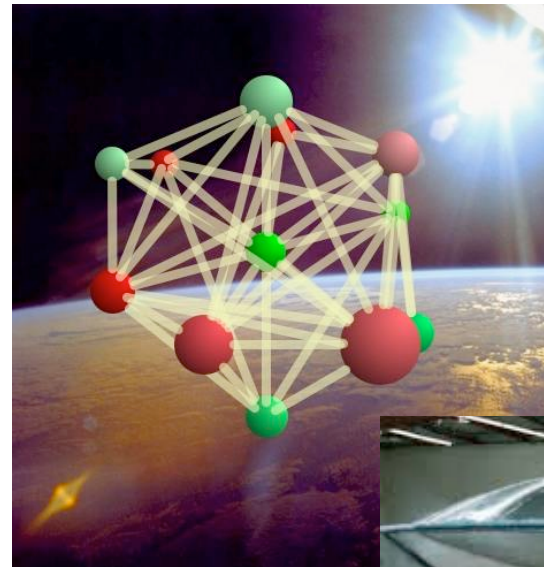


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C. R. Seubert and H. Schaub, "Tethered Coulomb Structures: Prospects and Challenges," Journal of Astronautical Sciences, Vol. 57, Nos. 1-2, Jan.-June 2009. doi:10.1007/BF03321508



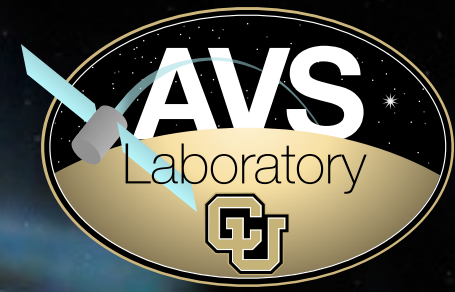
**Tethered and Inflatable Structures**



J. H. Cover, W. Knauer, and H. A. Maurer, "Lightweight Reflecting Structures Utilizing Electrostatic Inflation", US Patent 3,546,706, October 1966

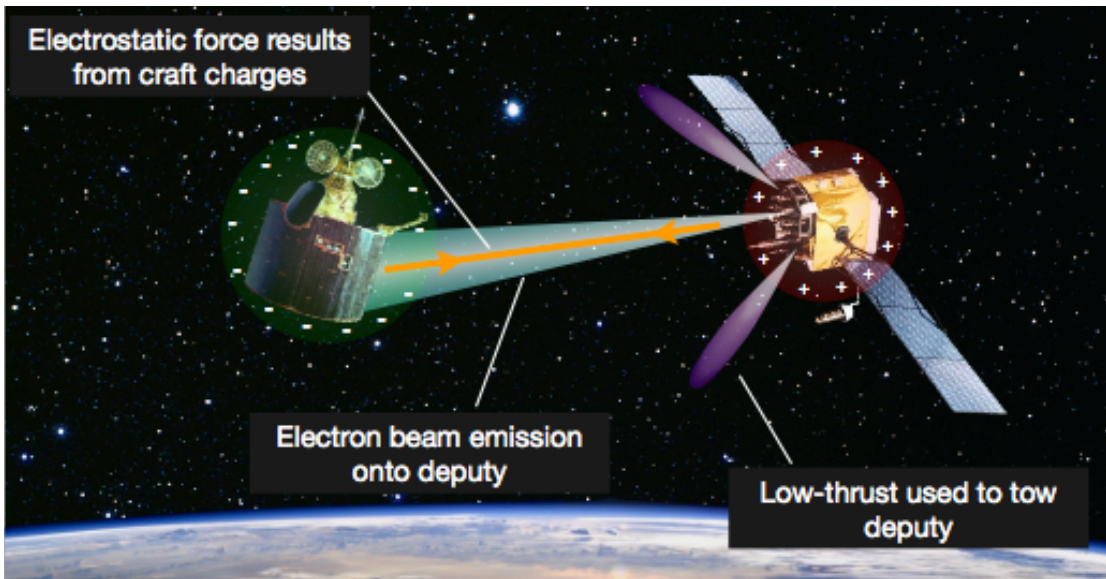


# Electrostatic Actuation Applications



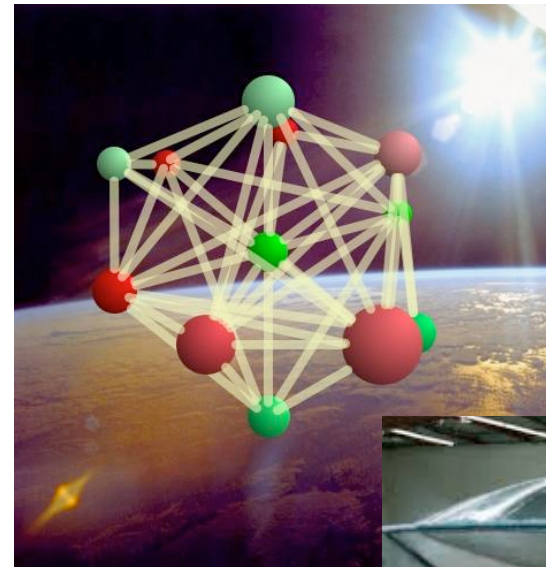
*Opportunities exist for **many** space-based applications of electrostatic actuation. Concepts have been explored as early as **1960s**.*

## Debris Re/De-Orbiting



E. A. Hogan and H. Schaub, "Space Debris Reorbiting Using Electrostatic Actuation," AAS Guidance, Navigation and Control Conference, Breckenridge, February 3–8, 2012.

C. R. Seubert and H. Schaub, "Tethered Coulomb Structures: Prospects and Challenges," Journal of Astronautical Sciences, Vol. 57, Nos. 1-2, Jan.-June 2009. doi:10.1007/BF03321508



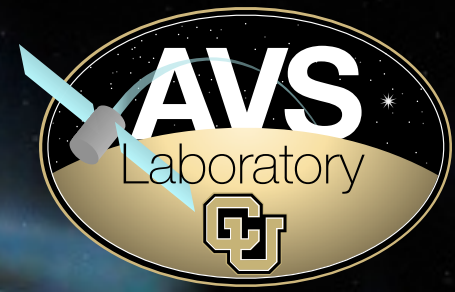
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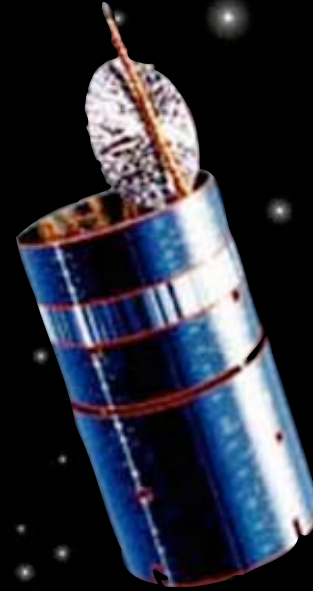
J. H. Cover, W. Knauer, and H. A. Maurer, "Lightweight Reflecting Structures Utilizing Electrostatic Inflation", US Patent 3,546,706, October 1966



# Electrostatic Actuation

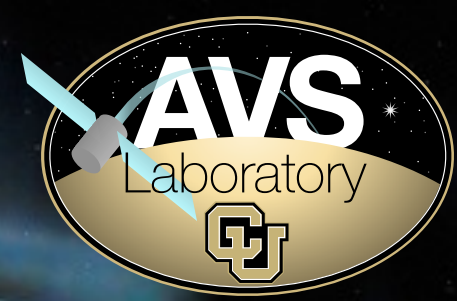


Consider a GEO debris object

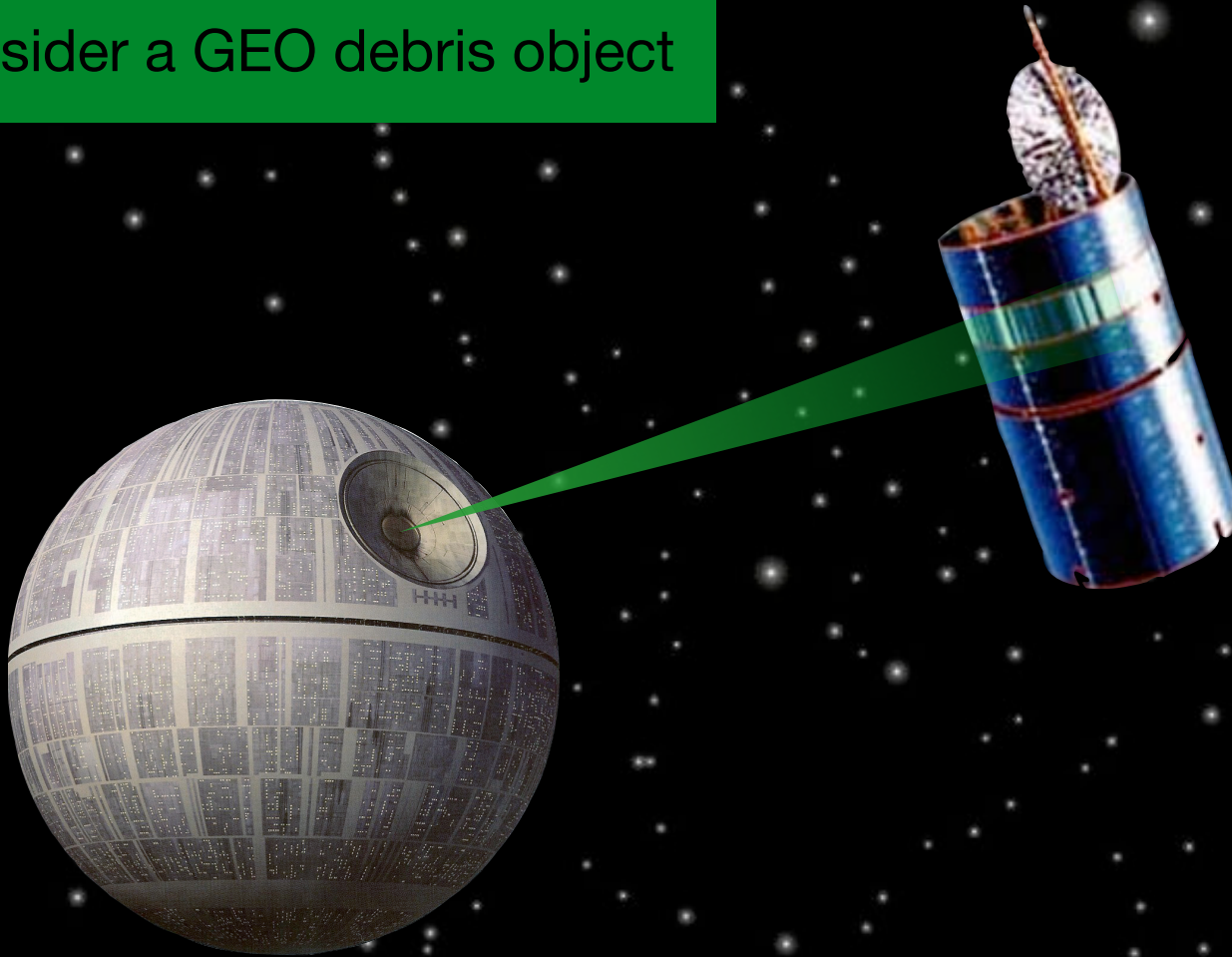




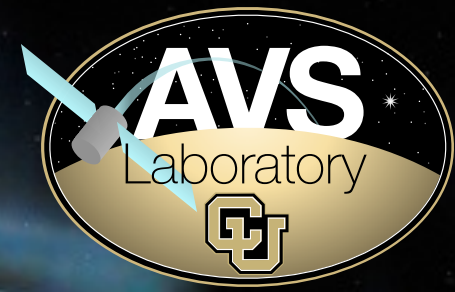
# Electrostatic Actuation



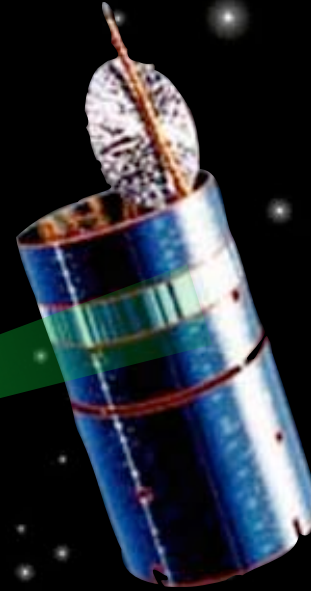
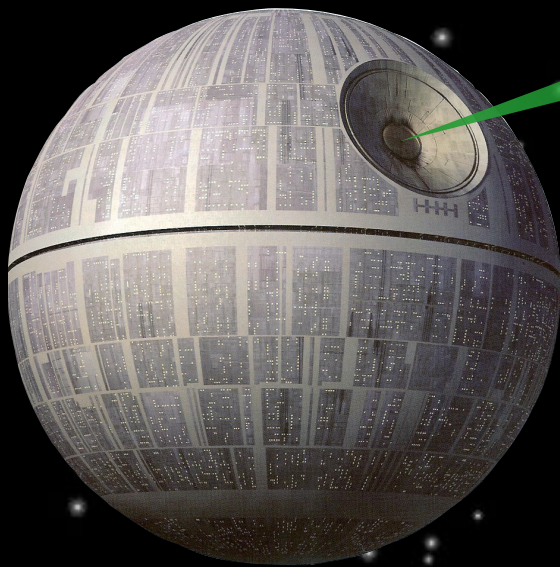
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# Electrostatic Actuation



Consider a GEO debris object

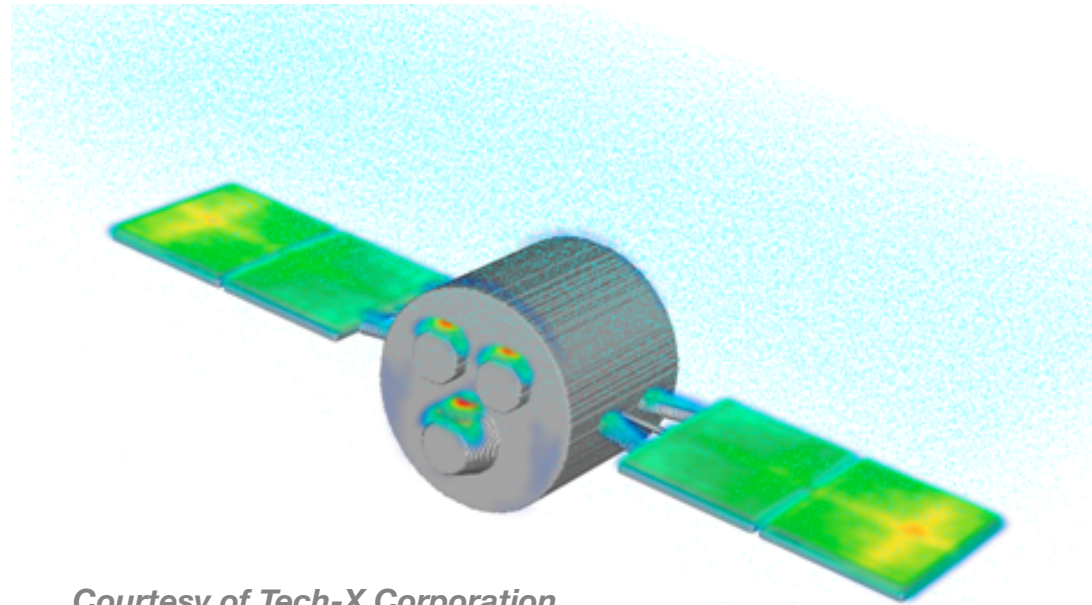
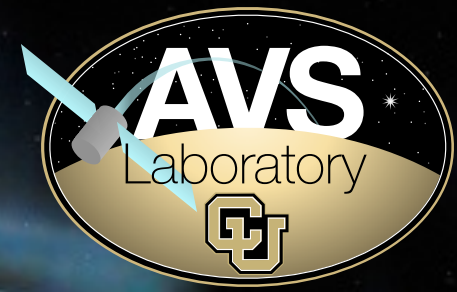


Touchlessly actuate objects separated by dozens of meters. Proposed:

***Electrostatic Detumble of GEO Debris***

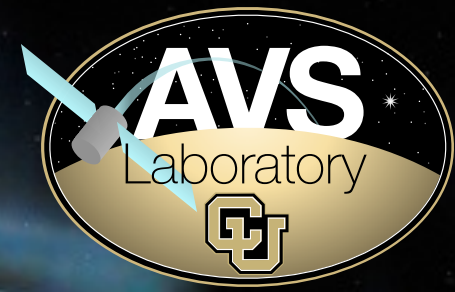


# Charge Distribution and the Multi-Sphere Method (MSM)

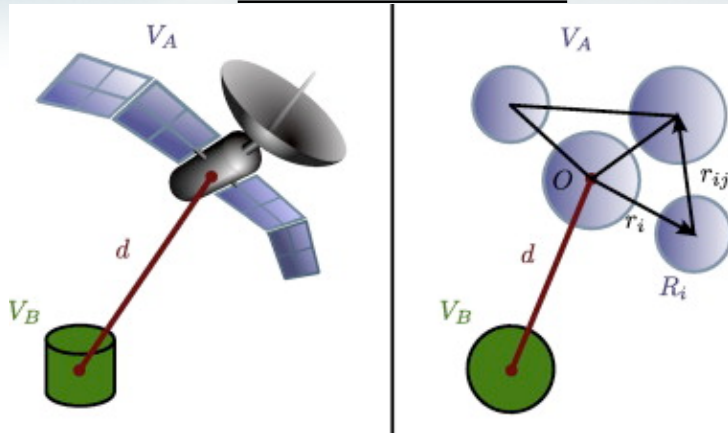


*Courtesy of Tech-X Corporation*

# Charge Distribution and the Multi-Sphere Method (MSM)



## FEM vs. MSM

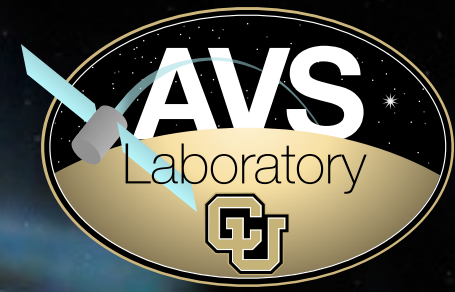


*MSM is a lumped charge representation*

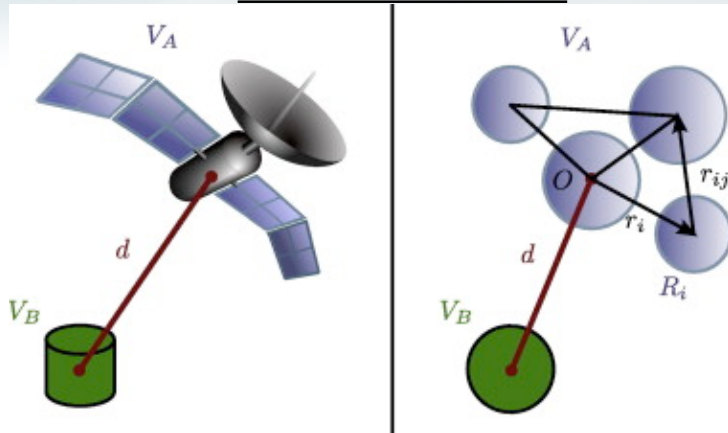
D. Stevenson, and H. Schaub, Optimization of Sphere Population for Electrostatic Multi Sphere Model, 12th Spacecraft Charging Technology Conference, Kitakyushu, Japan, May 14–18, 2012



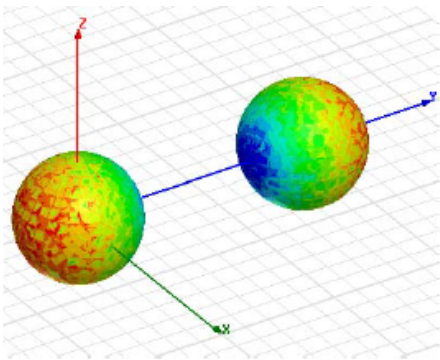
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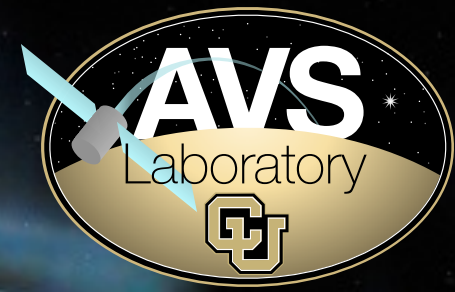
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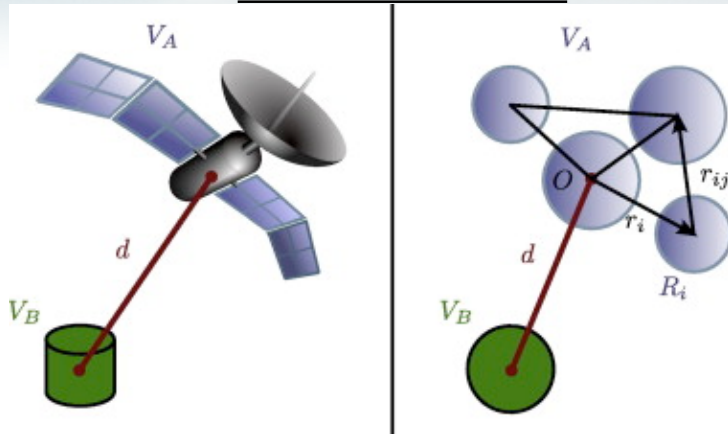
**Maxwell FEM**

D. Stevenson, and H. Schaub, Optimization of Sphere Population for Electrostatic Multi Sphere Model, 12th Spacecraft Charging Technology Conference, Kitakyushu, Japan, May 14–18, 2012

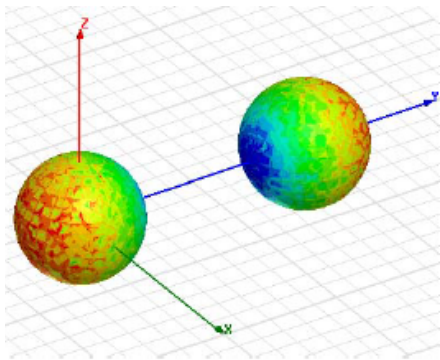
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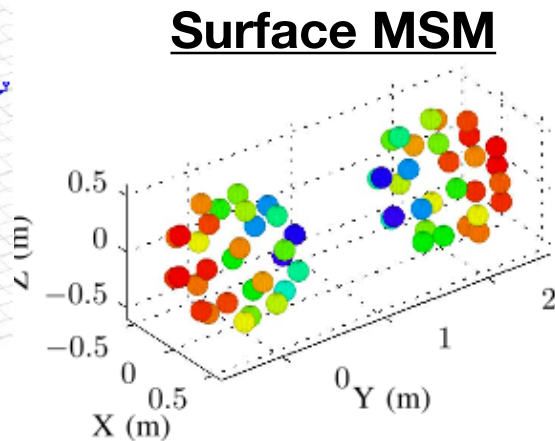
## FEM vs. MSM



*MSM is a lumped charge representation*



**Maxwell FEM**

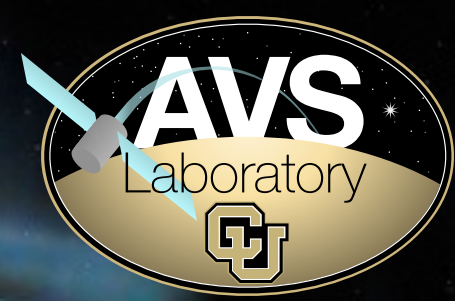


**Surface MSM**

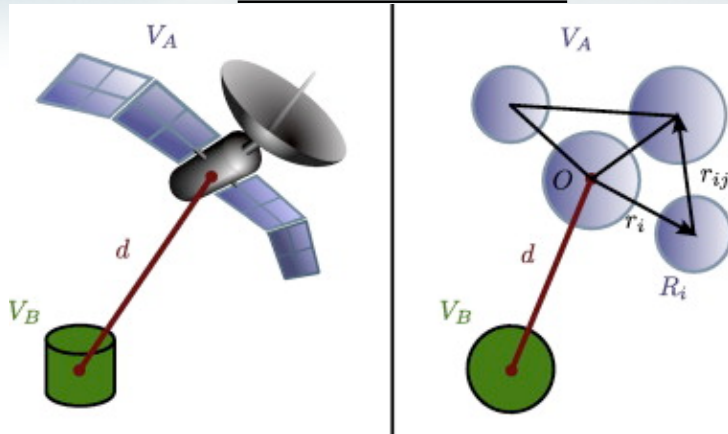
D. Stevenson, and H. Schaub, Optimization of Sphere Population for Electrostatic Multi Sphere Model, 12th Spacecraft Charging Technology Conference, Kitakyushu, Japan, May 14-18, 2012



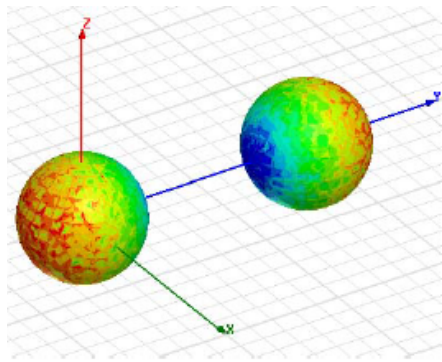
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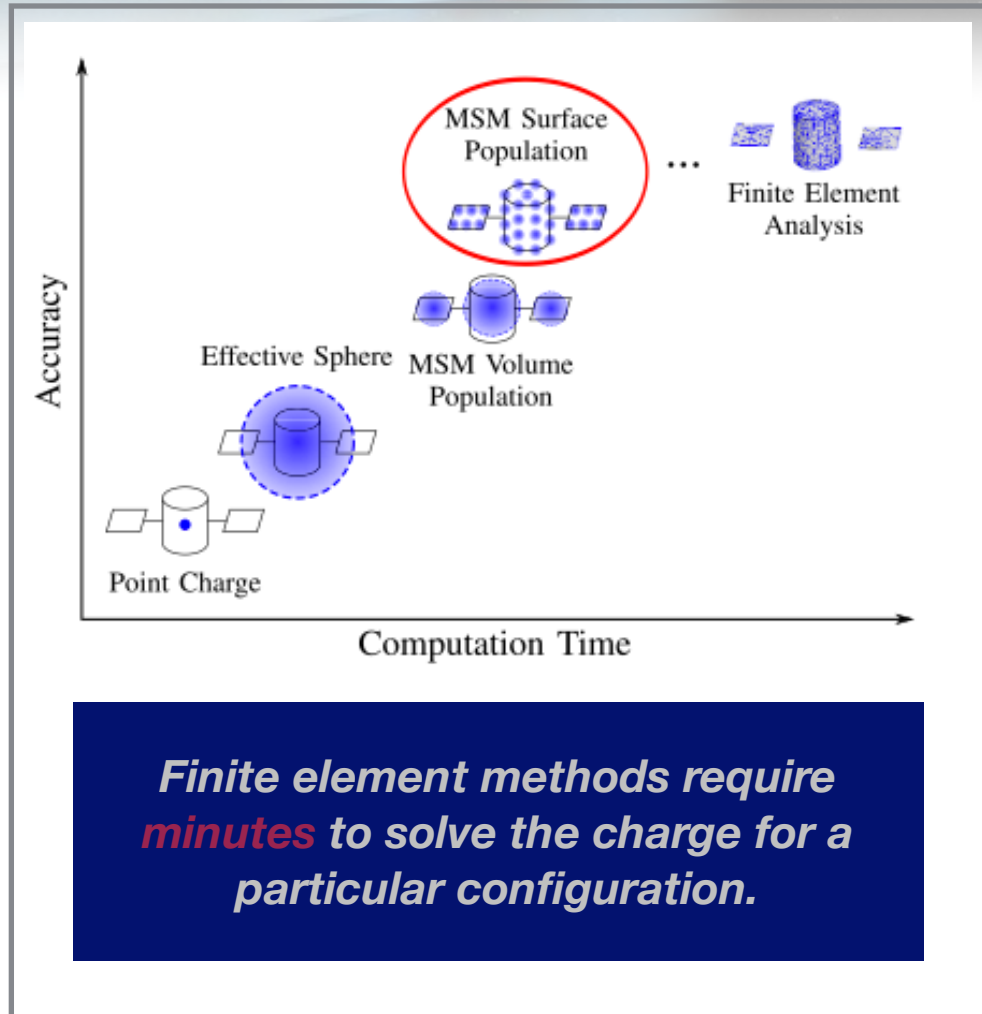
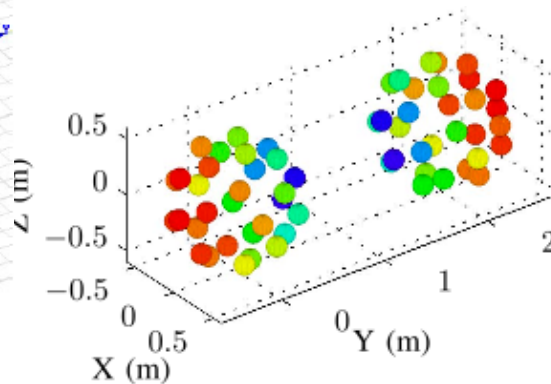


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**Maxwell FEM**

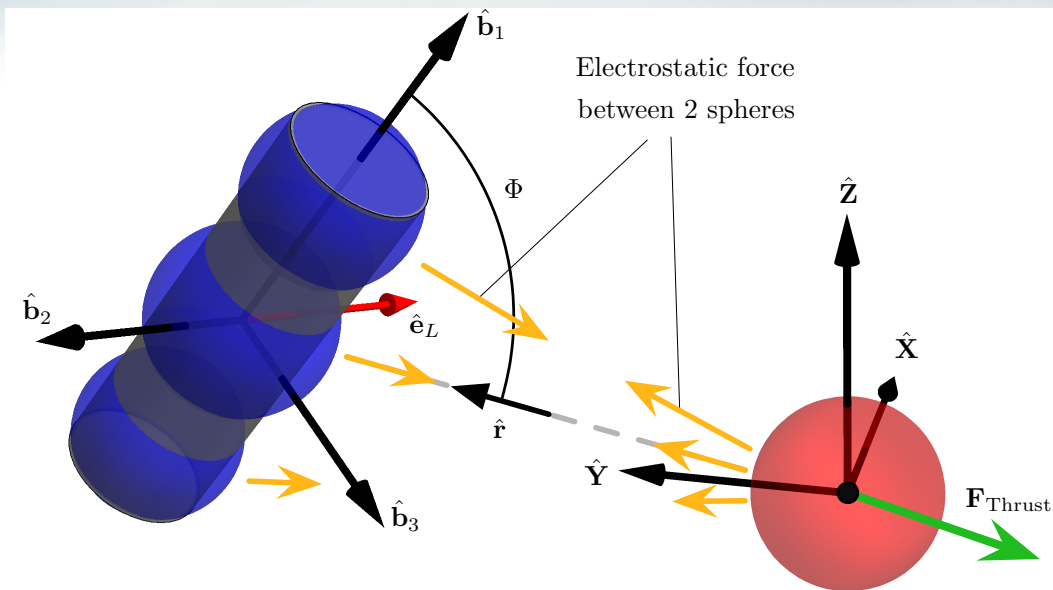
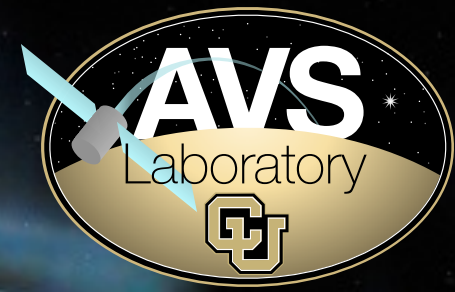
## Surface MSM



*Finite element methods require **minutes** to solve the charge for a particular configuration.*

D. Stevenson, and H. Schaub, Optimization of Sphere Population for Electrostatic Multi Sphere Model, 12th Spacecraft Charging Technology Conference, Kitakyushu, Japan, May 14-18, 2012

# Electrostatic Detumble Equations of Motion



## Projection Angle

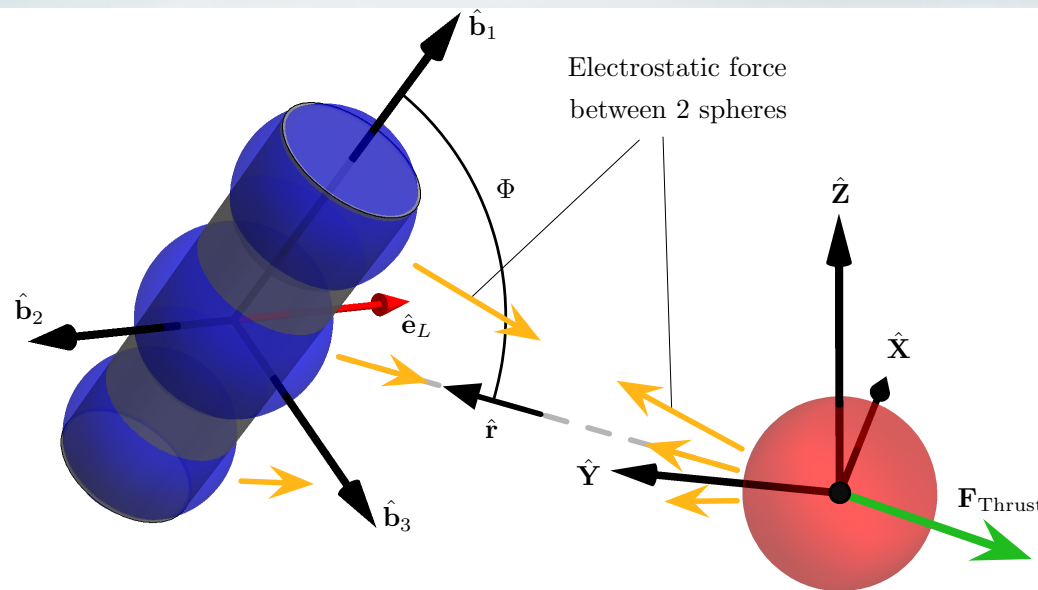
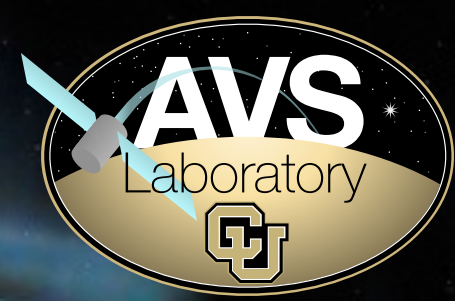
$$\Phi = \arccos \left( \hat{b}_1 \cdot (-\hat{r}) \right)$$

## Torque Axis

$$\hat{e}_L = \hat{b}_1 \times -\hat{r}$$



# Electrostatic Detumble Equations of Motion



## Projection Angle

$$\Phi = \arccos(\hat{\mathbf{b}}_1 \cdot (-\hat{\mathbf{r}}))$$

## Torque Axis

$$\hat{\mathbf{e}}_L = \hat{\mathbf{b}}_1 \times -\hat{\mathbf{r}}$$

$$I\dot{\omega} + \omega \times I\omega = \mathbf{L}$$



## New Basis Equations of Motion

$$I_a \dot{\omega}_1 = 0$$

$$\eta \equiv -\omega_2(\hat{\mathbf{r}} \cdot \hat{\mathbf{b}}_2) - \omega_3(\hat{\mathbf{r}} \cdot \hat{\mathbf{b}}_3)$$

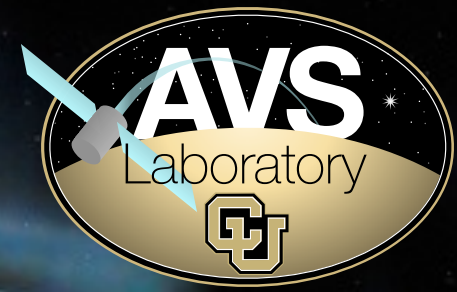
$$I_t \dot{\eta} - I_a \omega_1 \dot{\Phi} \sin \Phi = 0$$

$$\dot{\Phi} \sin \Phi = -\omega_2(\hat{\mathbf{r}} \cdot \hat{\mathbf{b}}_3) + \omega_3(\hat{\mathbf{r}} \cdot \hat{\mathbf{b}}_2)$$

$$I_t \left( \ddot{\Phi} \sin \Phi - \eta^2 \frac{\cos \Phi}{\sin^2 \Phi} \right) + I_a \omega_1 \eta = L$$

$$\mathbf{L} = -L\hat{\mathbf{e}}_L = -f(\phi) \sum_{m=1}^n \gamma_m g_m(\Phi) \hat{\mathbf{e}}_L$$

# Linearized Relative Orbit Elements



## **Clohessy-Wiltshire (CW) Equations**

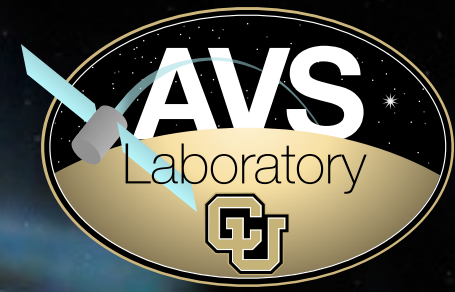
$$x(t) = A_1 \cos(nt) - A_2 \sin(nt) + x_{\text{off}}$$

$$y(t) = -2A_1 \sin(nt) - 2A_2 \cos(nt) - \frac{3}{2}ntx_{\text{off}} + y_{\text{off}}$$

$$z(t) = B_1 \cos(nt) - B_2 \sin(nt)$$



# Linearized Relative Orbit Elements



## Clohessy-Wiltshire (CW) Equations

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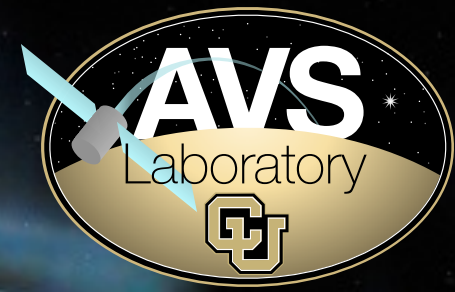
$$z(t) = B_1 \cos(nt) - B_2 \sin(nt)$$



$$\mathbf{X}_{\text{NS}} = (A_1, A_2, x_{\text{off}}, y_{\text{off}}, B_1, B_2)$$

*The collection of **CW invariants** become the **state vector** for relative motion. This new state vector becomes the **Linearized Relative Orbit Element (LROE) state**.*

# Linearized Relative Orbit Elements



## Clohessy-Wiltshire (CW) Equations

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*The collection of **CW invariants** become the **state vector** for relative motion. This new state vector becomes the **Linearized Relative Orbit Element (LROE) state**.*

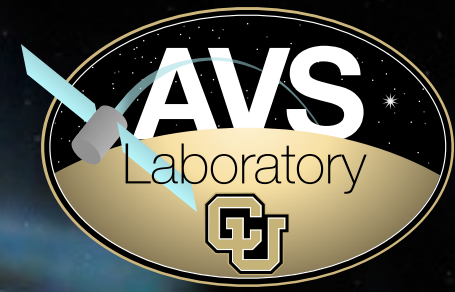
*Lagrange Brackets provides evolution of the invariants given a perturbation acceleration.*

$$\dot{\mathbf{X}}_{\text{NS}} = \frac{1}{n} \underbrace{\begin{bmatrix} -\sin(nt) & -2\cos(nt) & 0 \\ -\cos(nt) & 2\sin(nt) & 0 \\ 0 & 2 & 0 \\ -2 & 3nt & 0 \\ 0 & 0 & -\sin(nt) \\ 0 & 0 & -\cos(nt) \end{bmatrix}}_{B(\mathbf{X},t)} \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}$$

$$\dot{\mathbf{X}}_k = \mathbf{F}(\mathbf{X}(t_k), t_k) = B(\mathbf{X}(t_k), t_k) \mathbf{a}_d$$



# Linearized Relative Orbit Elements



## Clohessy-Wiltshire (CW) Equations

$$x(t) = A_1 \cos(nt) - A_2 \sin(nt) + x_{\text{off}}$$

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$$\mathbf{X}_{\text{NS}} = (A_1, A_2, x_{\text{off}}, y_{\text{off}}, B_1, B_2)$$

The collection of **CW invariants** become the **state vector** for relative motion. This new state vector becomes the **Linearized Relative Orbit Element (LROE) state**.

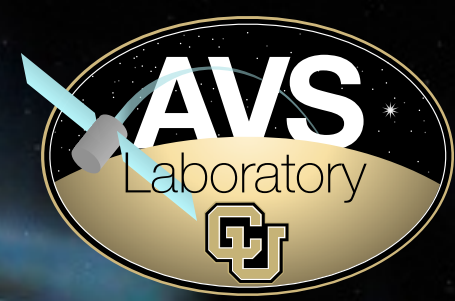
**LROEs can be propagated with any acceleration rotated into the relative orbit frame.**

**Lagrange Brackets** provides evolution of the invariants given a perturbation acceleration.

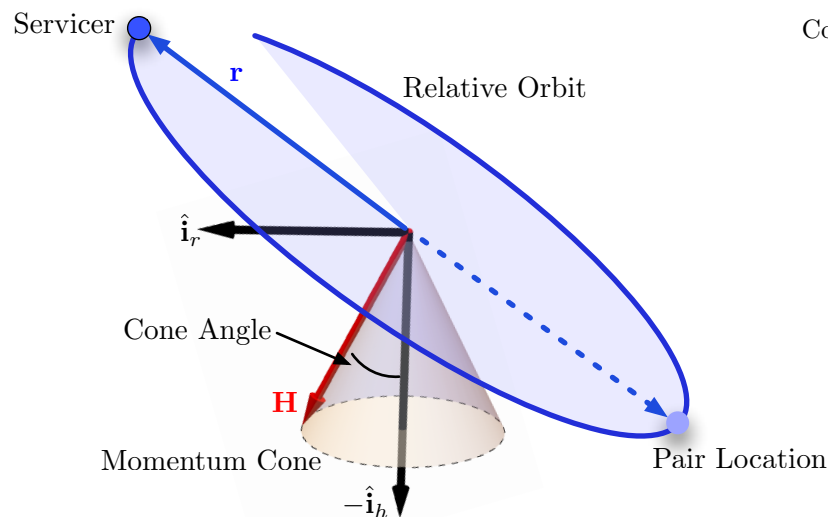
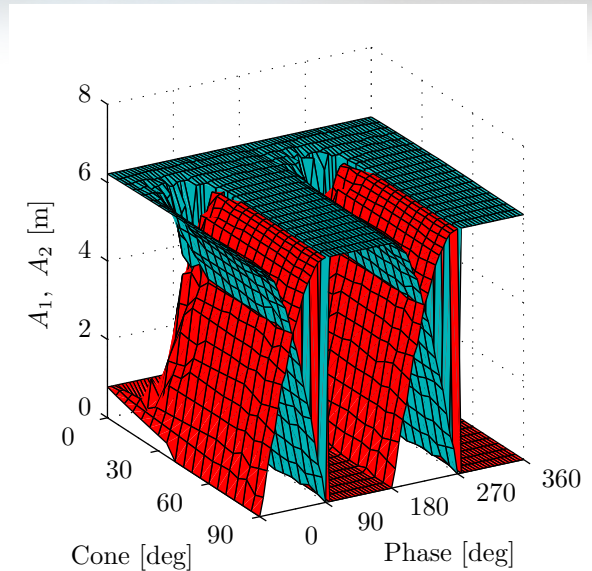
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$$\dot{\mathbf{X}}_k = \mathbf{F}(\mathbf{X}(t_k), t_k) = B(\mathbf{X}(t_k), t_k) \mathbf{a}_d$$

# Optimization Approach

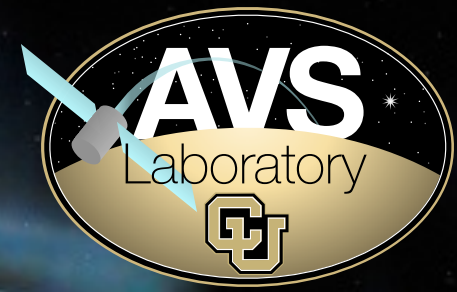


$$J = \sum_{i=0}^N \left( -1000 \ln[|\mathbf{r}_i| - r^* + 1] - 10 \ln \left[ \left| \frac{\mathbf{r}_i \cdot \mathbf{H}_i}{\|\mathbf{r}_i\| \|\mathbf{H}_i\|} \right| + 1 \right] \right)$$



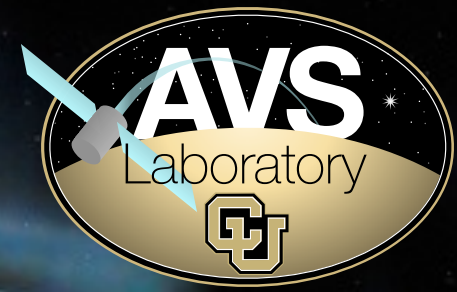


# Optimization Design Output



A movie will be included here.

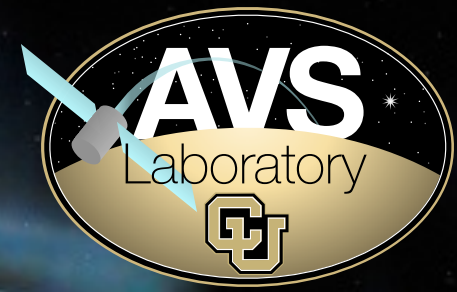
# Numerical Simulations



- parameters, conditions, time, etc.

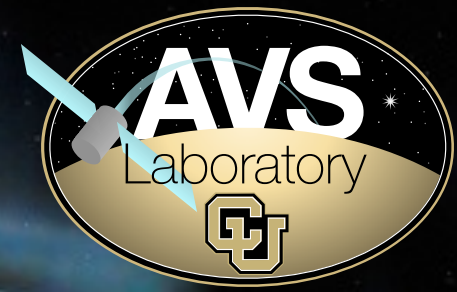


# Optimized State Provides a Performance Increase



A movie will be included here.

# Conclusions and Future Work



## Conclusions:

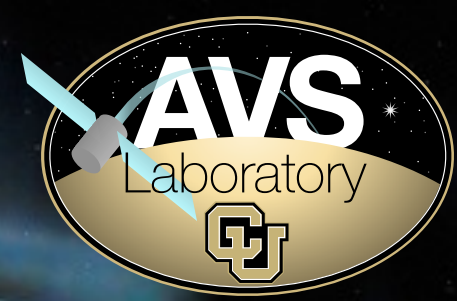
- The choice of relative orbit provides substantial increase/decrease in detumble performance.
- The Linearized Relative Orbit Element (LROE) approach provides insightful approaches to optimizing the servicer relative orbit.
- Without significant loss in performance, the relative orbit may be selected for operational simplicity using a leader-follower or for operational safety where a safety ellipse orbit is available.

## Future Work:

- Expand the relative motion detumble analysis to include objects that are not axisymmetric.



# Conclusions and Future Work



## Conclusions:

- The choice of relative orbit provides substantial increase/decrease in detumble performance.
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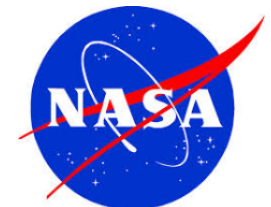
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- Expand the relative motion detumble analysis to include objects that are not axisymmetric.

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*The authors would like to thank the NASA Space Technology Research Fellowship (NSTRF) program, grant number NNX14AL62H, for support of this research.*

# Questions?



# Backup Slides

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*The stuff box...*

