Efficient numerical propagation of planetary close encounters with regularized element methods

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\textsuperscript{6}\textsuperscript{th} International Conference on Astrodynamics Tools and Techniques
Darmstadt, Germany, 14\textsuperscript{th}-17\textsuperscript{th} March 2016

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Introduction

- Need for accurate interplanetary orbit propagation:
  ▶ Asteroid impact monitoring
  ▶ Interplanetary mission analysis and design
- Special perturbation techniques satisfy the strictest accuracy requirements

- Challenging case: close encounters with major bodies
- Close trajectories diverge after an encounter, chaotic dynamics

- Most straightforward numerical method: integration of equations in rectangular coordinates
- Numerical techniques for close encounters: variation of step size/order
- Is it possible to do better?
Generalized elements and regularized formulations

Perturbed two-body problem (rectangular coordinates)

\[
\frac{d^2 \mathbf{r}}{dt^2} = -\frac{\mu}{r^3} \mathbf{r} + \mathbf{F}
\]

“The temptation at this point [...] is to simply integrate the system of differential equations by a numerical technique. This unsophisticated approach throws away all our knowledge of the two-body problem and its integrals.” (Bond and Allman, 1996)

- **Elements** are quantities which evolve smoothly if \( F \) is small → desirable for numerical integration
- Equations for classical orbital elements are singular \((e = 0, i = 0)\)
- Integrate **regularized equations of motion** of alternative element sets
Regularization techniques

- **Regularization**: elimination of singularities from the equations of motion through analytical techniques

- Regularizing the equations has significant numerical advantages

→ Change of independent variable from physical to fictitious time (generalized Sundman transformation):

\[
dt = cr^\alpha \, ds, \quad \alpha > 0
\]
The *Dromo* family of regularized element methods

- Originally developed in 2006 by Peláez et al. (SDG-UPM) as an orbit propagator for tether applications
- State of the particle conveyed through 7 orbital elements + time/time element
- Equations of motion are almost-fully regularized

<table>
<thead>
<tr>
<th>Author(s)</th>
<th>Title</th>
<th>Publication Details</th>
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Propagation of a “tough” geocentric orbit

KePASSA 2014 orbit propagation challenge

Initial conditions

<table>
<thead>
<tr>
<th>SMA (km)</th>
<th>ECC (-)</th>
<th>INC (º)</th>
<th>RAAN (º)</th>
<th>AOP (º)</th>
<th>M0 (º)</th>
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<tbody>
<tr>
<td>106338</td>
<td>0.859</td>
<td>55.1</td>
<td>231.4</td>
<td>257.7</td>
<td>332.6</td>
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Accuracy vs. computational time

<table>
<thead>
<tr>
<th>tol</th>
<th>δr (km)</th>
<th>δε (%)</th>
<th>CPU (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{-6}$</td>
<td>161433</td>
<td>0.01</td>
<td>10.3</td>
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<tr>
<td>$10^{-9}$</td>
<td>20566</td>
<td>$5 \times 10^{-5}$</td>
<td>23.0</td>
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<tr>
<td>$10^{-11}$</td>
<td>501</td>
<td>$1.3 \times 10^{-6}$</td>
<td>60.0</td>
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Propagation methodology

Online Trajectory Matching algorithm

- Element methods perform well for weak perturbations
- Close encounters: strong perturbations
- **Online Trajectory Matching**: decompose the trajectory into three weakly-perturbed phases
- Change phase at a **pre-defined planetocentric distance**, the switch radius $R_{sw}$

**KEY QUESTION:**
where to switch?
Numerical analysis

Benchmark problem

- **Performance assessment:** large-scale numerical simulations in the planar, circular restricted Sun-Earth 3BP

- **Parametrization of close encounters:**
  - $d$, minimum approach distance
  - $e$, encounter eccentricity
  - $\theta$, Sun-Earth-particle angle

- Heliocentric initial conditions are computed by backwards propagation $\Delta t = 6$ months before the encounter

- Total duration of the propagation: $2\Delta t = 1$ year

![Diagram](image)

Efficient numerical propagation of planetary close encounters with regularized element methods
Numerical analysis (2)
Formulations and solver characteristics

• **Comparison of propagation approaches:**
  - Cowell's method (EoMs in rectangular coordinates), in heliocentric frame
  - *Dromo* regularized element methods with Online Trajectory Matching
  - Different formulations used in each phase

• **Solver:** LSODAR subroutine from ODEPACK
  - Multistep, implicit variable step-size and order numerical scheme
  - Adams-Moulton scheme and Backward Differentiation Formulas
  - Automatic root-finding capability (important for regularized formulations)

• **Reference solution:** computed in quadruple precision at very strict tolerance on the local truncation error, Cowell's method
Numerical analysis (3)

Performance metrics

- Close encounters generally modify the heliocentric semi-major axis
  ![Graph](image)
  - Year
  - $a$ (au)
  - 2015 2020 2025 2030 2035 2040 2045 2050 2055 2060

- Accurate estimation of the SMA (equiv. \textit{period}) is mandatory:
  - resonant returns for NEAs may generate chaotic dynamics (Valsecchi et al., 2003)
  - divergence of numerical error

- \textbf{Error metric}: relative energy difference, $\delta\varepsilon = |(\varepsilon_t - \varepsilon_r)/\varepsilon_r|$

- \textbf{Computational cost metric}: No. of right-hand side evaluations
Propagation performance

Cowell's method, heliocentric frame, $\theta = 0^\circ$
Propagation performance (2)
Regularized element methods with OTM, $\theta = 0^\circ$, $R_{sw} = 4 \ R_{Sol}$
Optimal switch radius: reduction of the average error of at least 1 order of magnitude with respect to Cowell's method, at half of the computational cost.
Apophis test case

- **Application of the regularized+OTM methods**: orbit propagation of NEA 99942 Apophis

- Simplified physical model (Sun-Earth CR3BP)

- Propagation from 2016 to 2056, with a deep close encounter in 2029

- Several orders of magnitude increase in accuracy with respect to Cowell's method with a variable order and step-size integrator.

- More details: G. Baù's talk, 14:40 March 16\(^{th}\) in 3.02 Hassium
Conclusions

- Novel regularized element methods are remarkably efficient in propagating weakly-perturbed orbits
- A new algorithm (OTM) increases accuracy of propagation in close encounters
- Three weakly-perturbed phases with distinct dynamics
- Significant improvement of propagation efficiency in the PCR3BP with the new methods
- Outstanding increase in accuracy for the propagation of Apophis' orbit
Future work

• Expanded analysis of results in the PCR3BP:
  ➔ influence of $\theta$ (Sun-Earth-particle angle)
  ➔ comparison with further regularized formulations

• Comparative study between different numerical schemes (variable vs. fixed step-size)

• Improved methods for low-energy encounters and ballistic capture cases

• Publishing of the software in an online repository

Vielen Dank für Ihre Aufmerksamkeit
Separation of dynamical regimes

Impact of the switch distance

- In Online Trajectory Matching, it is important to establish where/when to switch between phases
- … which is the primary body?
- Can the numerical error be minimized by choosing carefully the switch distance?
- Are the existing criteria (Tisserand/Laplace, Hill) sufficient for the purpose?

![Graphs showing Earth, Pre-enc, and Post-enc trajectories for x and y coordinates.](image-url)