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POLITÉCNICA

Efficient numerical propagation of planetary close encounters with regularized element methods

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Introduction

- Need for accurate interplanetary orbit propagation:
  - Asteroid impact monitoring
  - Interplanetary mission analysis and design
- Special perturbation techniques satisfy the strictest accuracy requirements

- Challenging case: **close encounters with major bodies**
- Close trajectories diverge after an encounter, *chaotic dynamics*

- Most straightforward numerical method: integration of equations in rectangular coordinates
- Numerical techniques for close encounters: variation of step size/order
- **<u>Is it possible to do better?</u>**

Generalized elements and regularized formulations

Perturbed two-body problem (rectangular coordinates)

$$\frac{d^2\boldsymbol{r}}{dt^2} = -\frac{\mu}{r^3}\boldsymbol{r} + \boldsymbol{F}$$

> "The temptation at this point [...] is to simply integrate the system of differential equations by a numerical technique. This unsophisticated approach throws away all our knowledge of the two-body problem and its integrals." (Bond and Allman, 1996)

- **Elements** are quantities which evolve smoothly if $\boldsymbol{F}$ is small →desirable for numerical integration
- Equations for classical orbital elements are singular $(e = 0, i = 0)$
- Integrate **regularized equations of motion** of alternative element sets

Regularization techniques

- **Regularization:** elimination of singularities from the equations of motion through analytical techniques

- Regularizing the equations has significant numerical advantages

➔ Change of independent variable from physical to fictitious time (generalized Sundman transformation):

$$dt = cr^{\alpha} ds, \ \alpha > 0$$

The *Dromo* family of regularized element methods

- Originally developed in 2006 by Peláez et al. (SDG-UPM) as an orbit propagator for tether applications
- State of the particle conveyed through 7 orbital elements + time/time element
- Equations of motion are almost-fully regularized

Peláez et al., "A special perturbation method in orbital dynamics," CMDA, vol. 97(2), **2006.**	Original formulation
Baù et al., "A new set of integrals of motion to propagate the perturbed two-body problem", CMDA, vol. 116(1), **2013.**	Perturbing potential
Baù et al., "Time elements for enhanced performane of the Dromo orbit propagator", AJ, vol. 148, **2014.**	Time elements
Roa et al., "Orbit propagation in Minkowskian geometry", CMDA, vol. 123(1), **2015.**	Particularization to hyperbolic motion
Baù et al., "Non-singular orbital elements for special perturbations in the two-body problem", MNRAS, vol. 454(3), **2015.**	Particularization to elliptic motion, pert. potential
Baù et al., "New orbital elements for accurate orbit propagation in the Solar System", Proceedings of the 6th ICATT, **2016.**	Particularization to hyperbolic motion, pert. potential

Propagation of a “tough” geocentric orbit

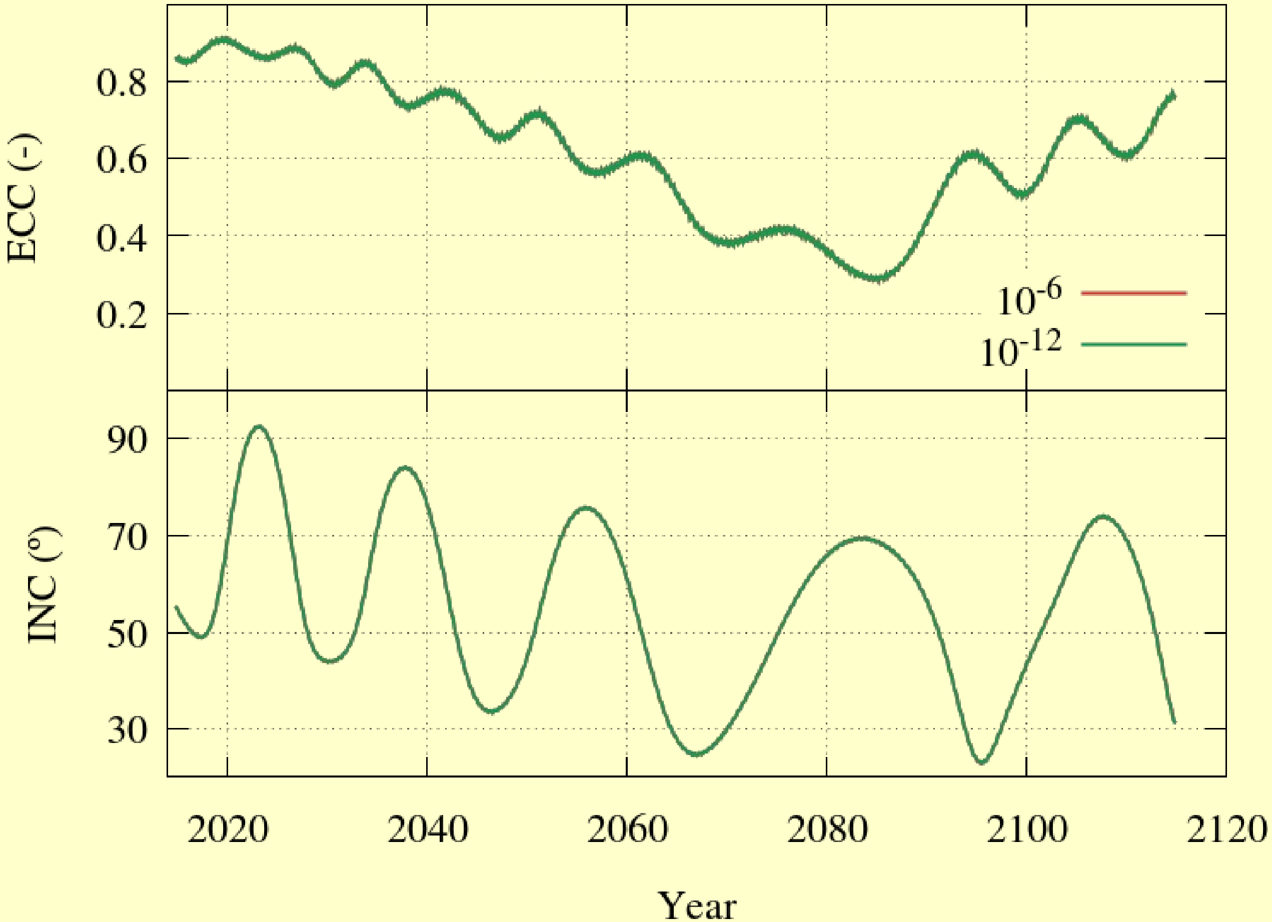
KePASSA 2014 orbit propagation challenge

Initial conditions

SMA (km)	ECC (-)	INC (º)	RAAN (º)	AOP (º)	M0 (º)
106338	0.859	55.1	231.4	257.7	332.6

Accuracy vs. computational time

tol	δr (km)	δε (%)	CPU (s)
10^{-6}	161433	0.01	10.3
10^{-9}	20566	5×10^{-5}	23.0
10^{-11}	501	1.3×10^{-6}	60.0

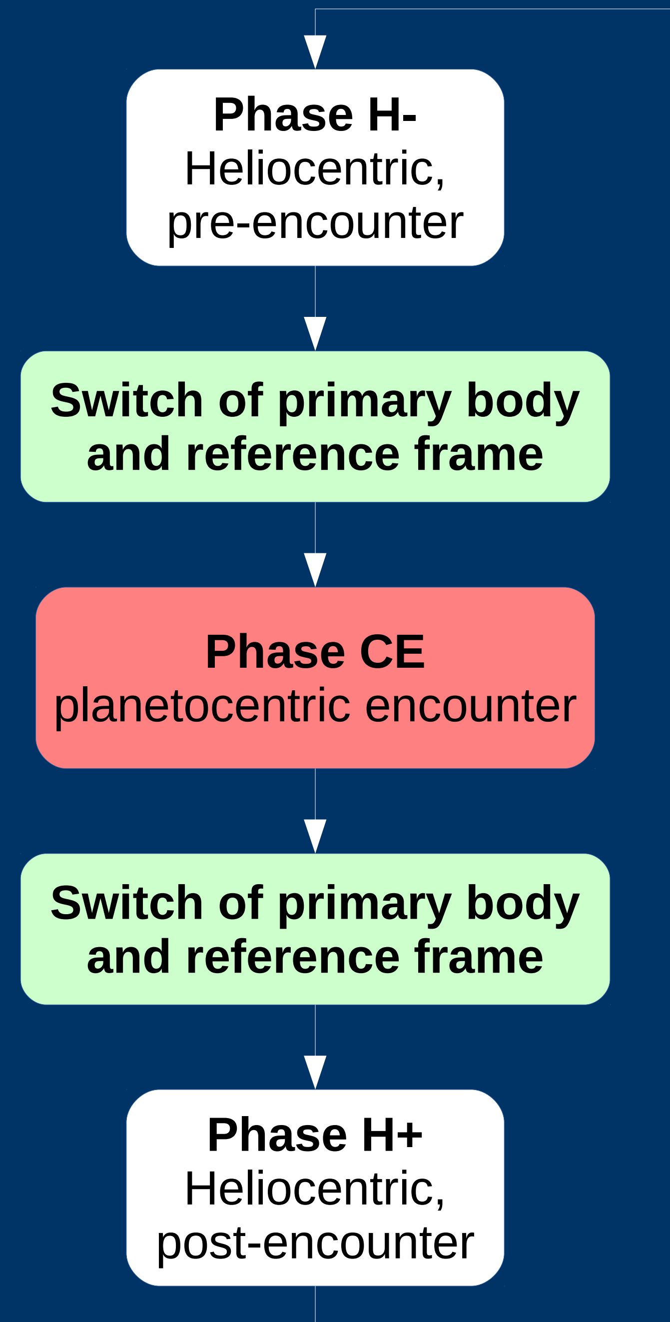


Propagation methodology

Online Trajectory Matching algorithm

- Element methods perform well for **weak perturbations**
- Close encounters: strong perturbations
- **Online Trajectory Matching:** decompose the trajectory into three weakly-perturbed phases
- Change phase at a **pre-defined planetocentric distance,** the switch radius R_{sw}

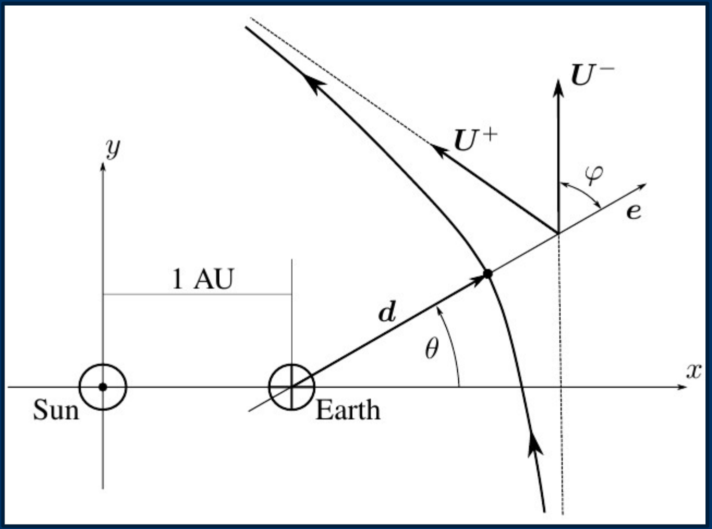
KEY QUESTION:
where to switch?



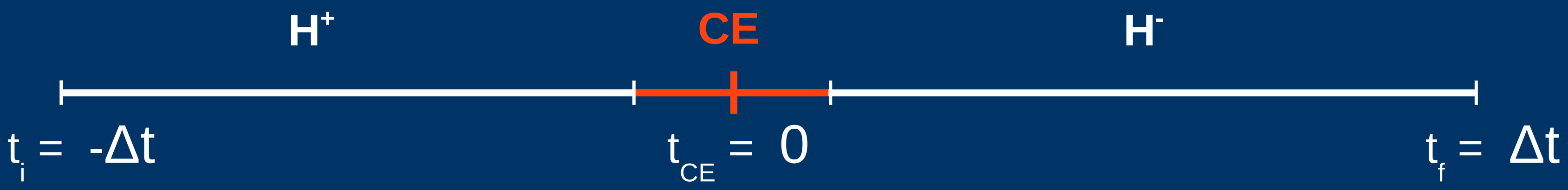
Numerical analysis

Benchmark problem

- **Performance assessment:** large-scale numerical simulations in the planar, circular restricted Sun-Earth 3BP
- **Parametrization of close encounters:**
  - d, minimum approach distance
  - e, encounter eccentricity
  - θ, Sun-Earth-particle angle



- Heliocentric initial conditions are computed by backwards propagation $\Delta t = 6$ months before the encounter
- Total duration of the propagation: $2\Delta t = 1$ year



Numerical analysis (2)

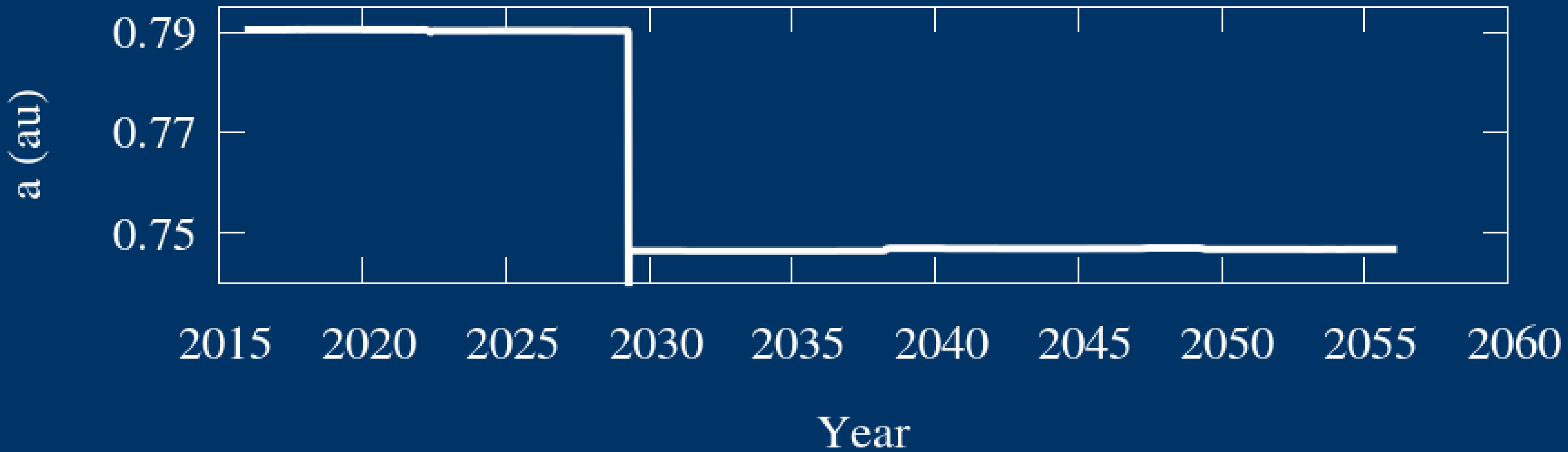
Formulations and solver characteristics

- **Comparison of propagation approaches:**
  - Cowell's method (EoMs in rectangular coordinates), in heliocentric frame
  - *Dromo* regularized element methods with Online Trajectory Matching
  - Different formulations used in each phase
- **Solver:** LSODAR subroutine from ODEPACK
  - Multistep, implicit variable step-size and order numerical scheme
  - Adams-Moulton scheme and Backward Differentiation Formulas
  - Automatic root-finding capability (important for regularized formulations)
- **Reference solution:** computed in quadruple precision at very strict tolerance on the local truncation error, Cowell's method

Numerical analysis (3)

Performance metrics

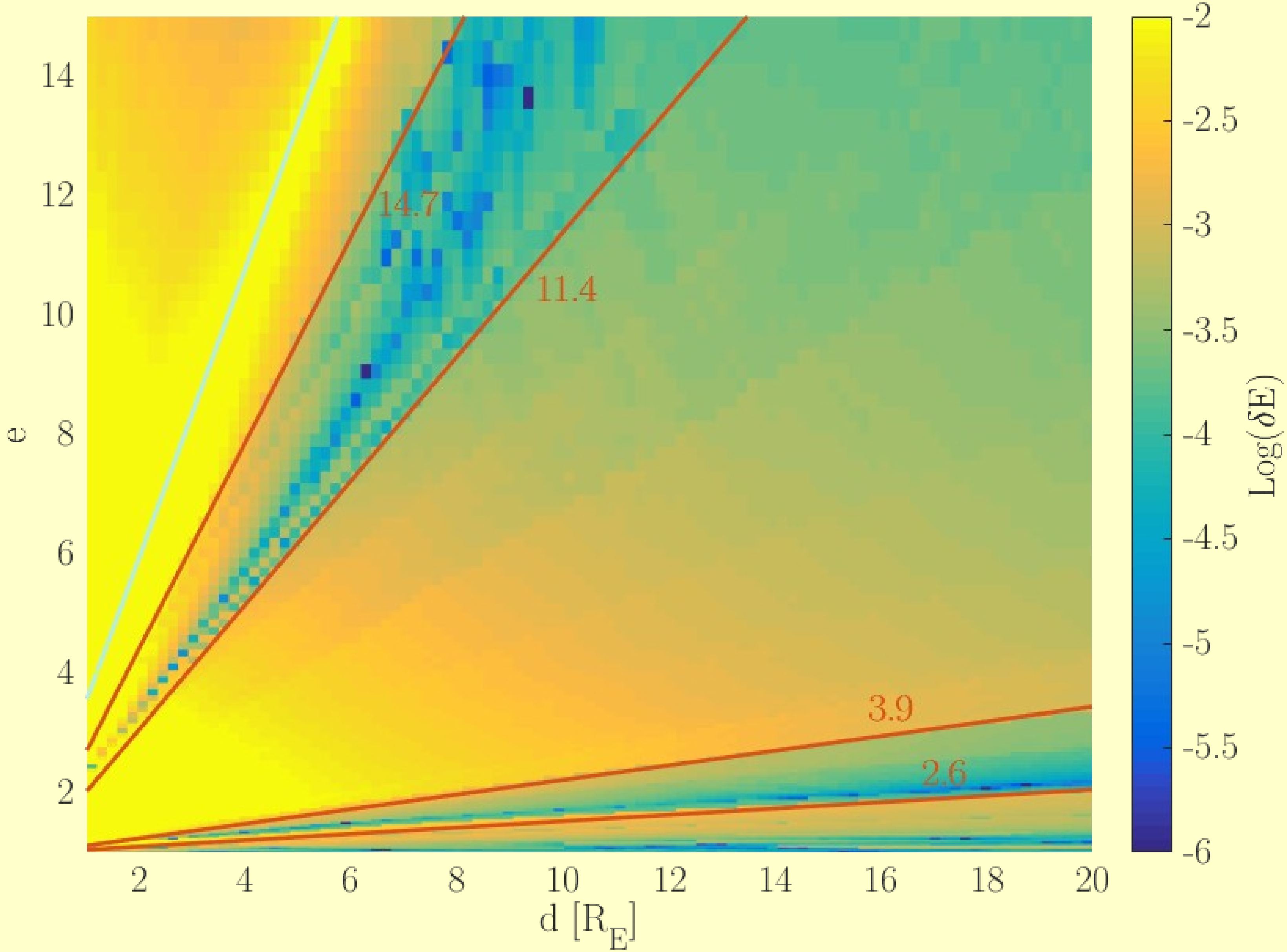
- Close encounters generally modify the heliocentric semi-major axis



- Accurate estimation of the SMA (equiv. period) is mandatory:
  - resonant returns for NEAs may generate chaotic dynamics (Valsecchi et al., 2003)
  - divergence of numerical error
- **Error metric:** relative energy difference, $\delta\varepsilon = |(\varepsilon_{\mathrm{t}} - \varepsilon_{\mathrm{r}})/\varepsilon_{\mathrm{r}})|$
- **Computational cost metric:** No. of right-hand side evaluations

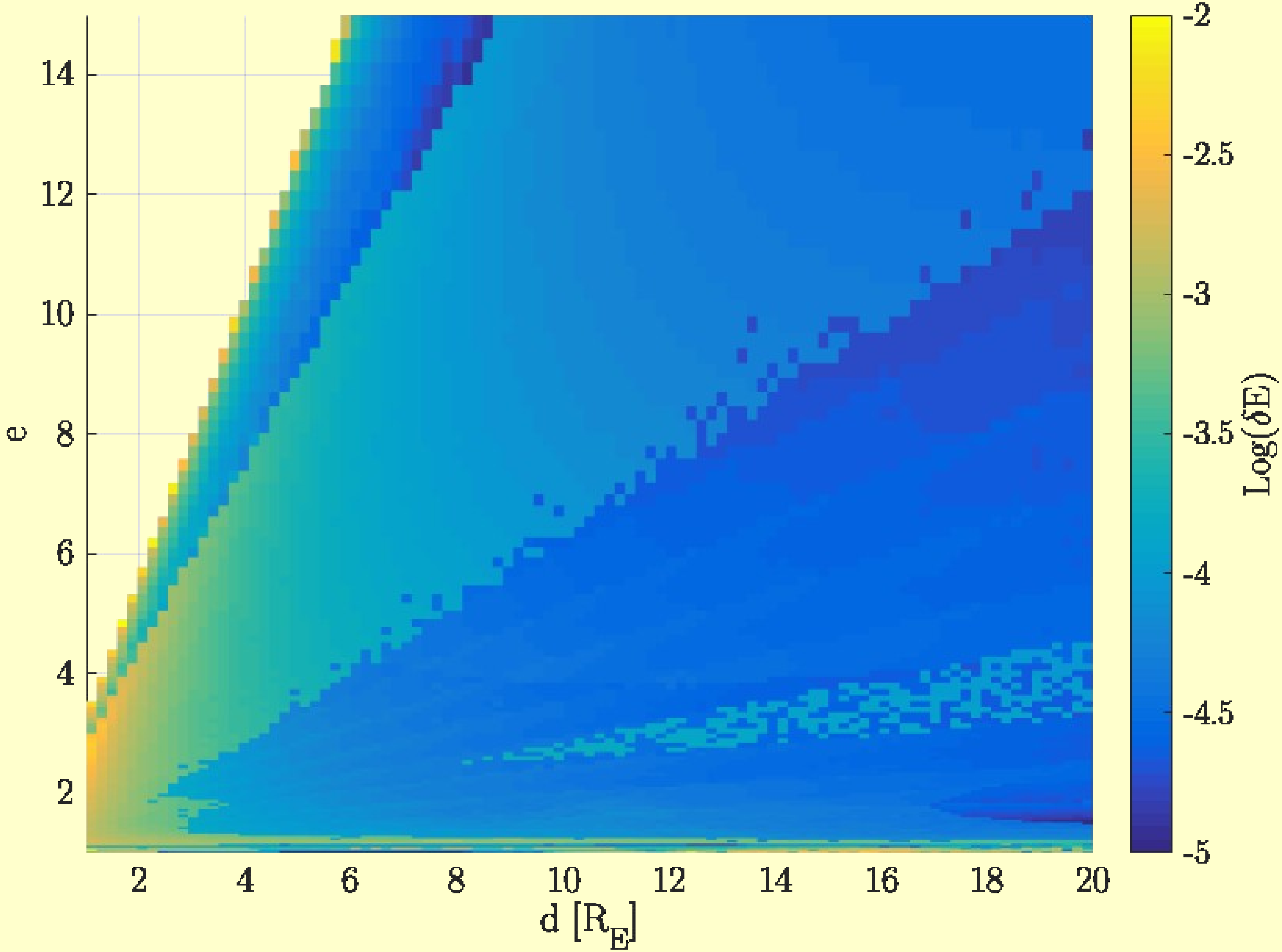
Propagation performance

Cowell's method, heliocentric frame, θ = 0º



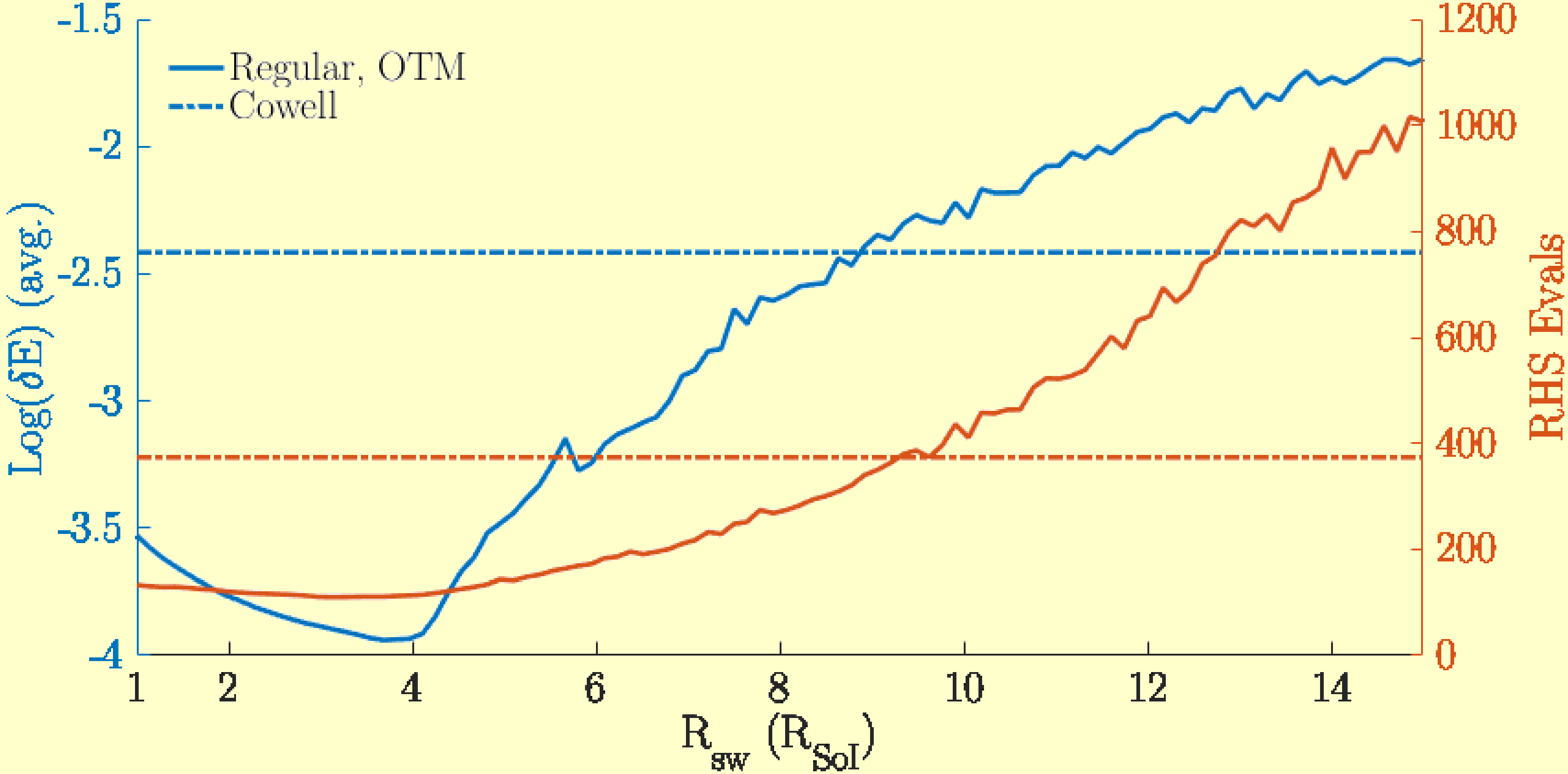
Propagation performance (2)

Regularized element methods with OTM, θ = 0º, R_{sw} = 4 R_{Sol}



Propagation performance (3)

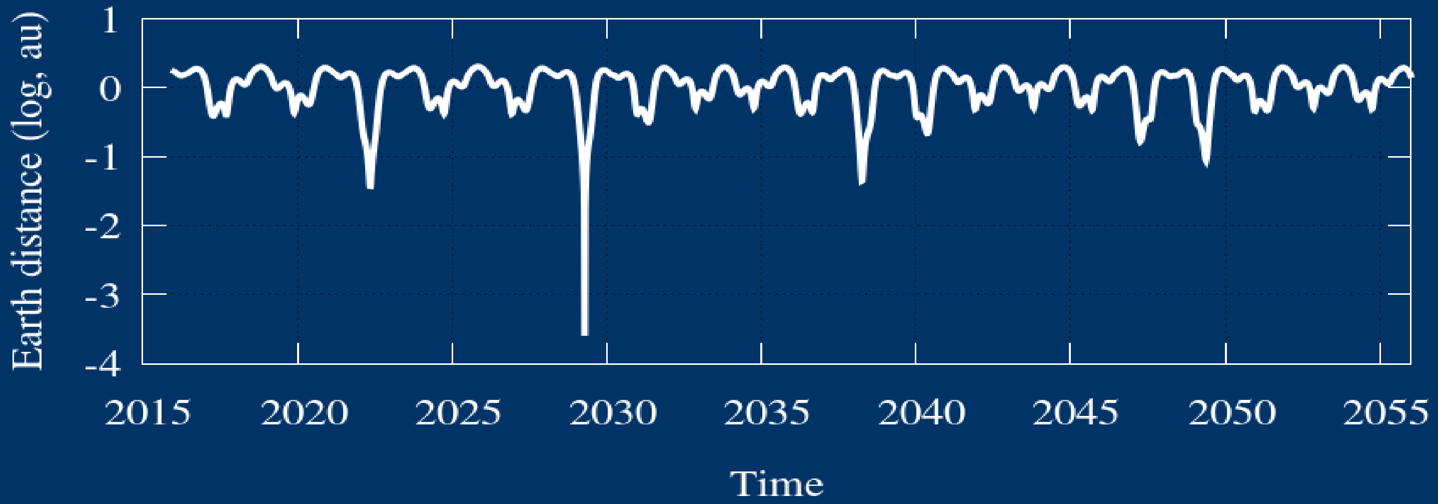
Where to switch? Impact of the switch radius on performance



Optimal switch radius: reduction of the average error of at least 1 order of magnitude with respect to Cowell's method, at <u>half</u> of the computational cost

Apophis test case

- **Application of the regularized+OTM methods:** orbit propagation of NEA 99942 Apophis
- Simplified physical model (Sun-Earth CR3BP)
- Propagation from 2016 to 2056, with a deep close encounter in 2029



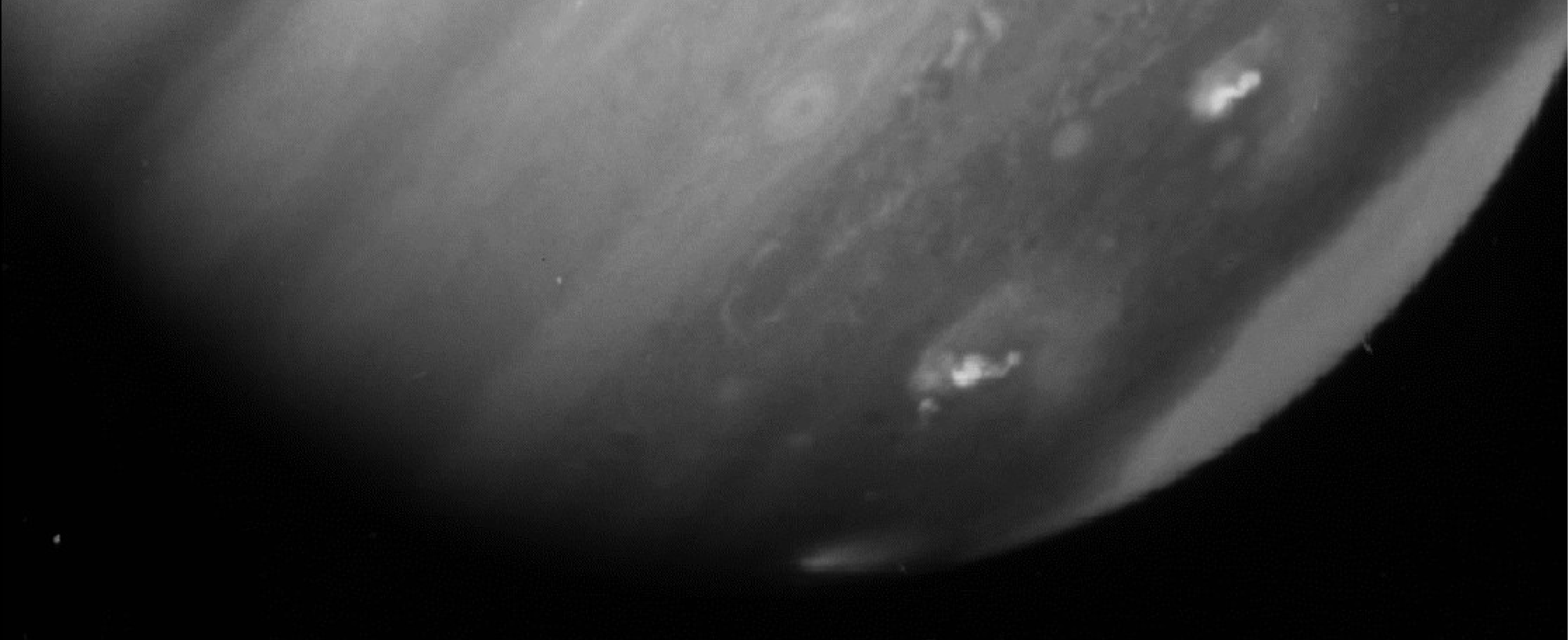
- **Several orders of magnitude increase in accuracy with respect to Cowell's method** with a variable order and step-size integrator**.**
- More details: G. Baù's talk, 14:40 March 16th in 3.02 Hassium

Conclusions

- Novel regularized element methods are remarkably efficient in propagating weakly-perturbed orbits
- A new algorithm (**OTM**) increases accuracy of propagation in close encounters
- Three **weakly-perturbed** phases with distinct dynamics
- **Significant improvement of propagation efficiency** in the PCR3BP with the new methods
- **Outstanding increase in accuracy** for the propagation of Apophis' orbit

Future work

- **Expanded analysis of results in the PCR3BP:**
  - ➔ influence of θ (Sun-Earth-particle angle)
  - ➔ comparison with further regularized formulations
- **Comparative study between different numerical schemes** (variable vs. fixed step-size)
- **Improved methods for low-energy encounters** and ballistic capture cases
- **Publishing of the software in an online repository**



Too close an encounter: impact of comet Shoemaker-Levy 9 on Jupiter as imaged by HST on July 20th 1994. Courtesy NASA/JPL.

Vielen Dank für Ihre Aufmerksamkeit

Separation of dynamical regimes

Impact of the switch distance

- In Online Trajectory Matching, it is important to establish where/when to switch between phases
- … which is the primary body?
- Can the numerical error be **minimized** by choosing carefully the switch distance?
- Are the existing criteria (Tisserand/Laplace, Hill) sufficient for the purpose?