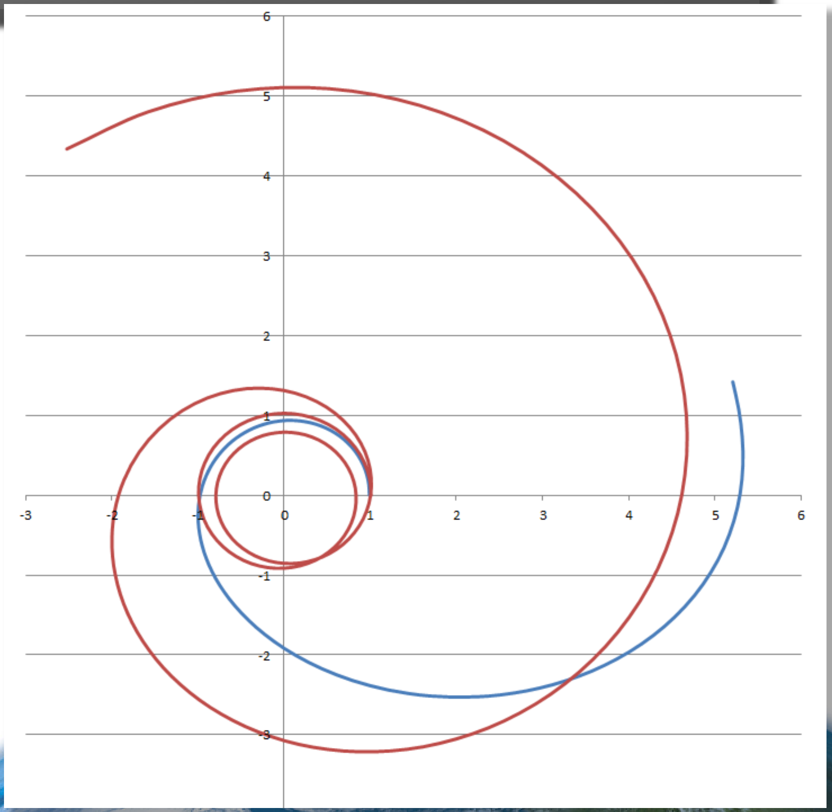
ABOUT COMBINING TISSERAND GRAPH GRAVITY-ASSIST SEQUENCING WITH LOW-THRUST TRAJECTORY OPTIMIZATION



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Knowledge for Tomorrow

International Conference on Astrodynamics Tools and Techniques
6TH ICATT

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Overview

- Introduction
- Tisserand Criterion (TC) and Graphs
- Violation of TC Premises
- Low-thrust Correction Term for TC
- Application on Low-thrust Missions
- Constraints on Search Space
- Optimization Methodology
- Outlook
- Conclusion

Introduction

- Gravity-Assist maneuvers: Enablers for *Voyager*, *Cassini*, *Messenger*, *New Horizons*
- LT propulsion: very efficient
  - => Combining both is attractive

- Strange & Longuski, resp. Labunsky et al.: Tisserand Graphs for sequencing gravity-assists
- Based on Tisserand Criterion (used for comets, GA- effect, energy-based) (TC):

$$\frac{R_{pl}}{a} + 2\sqrt{\frac{a(1-e^2)}{R_{pl}}}\cos i = const \qquad \textbf{(1)}$$

Tisserand Criterion and Graphs (1/2)

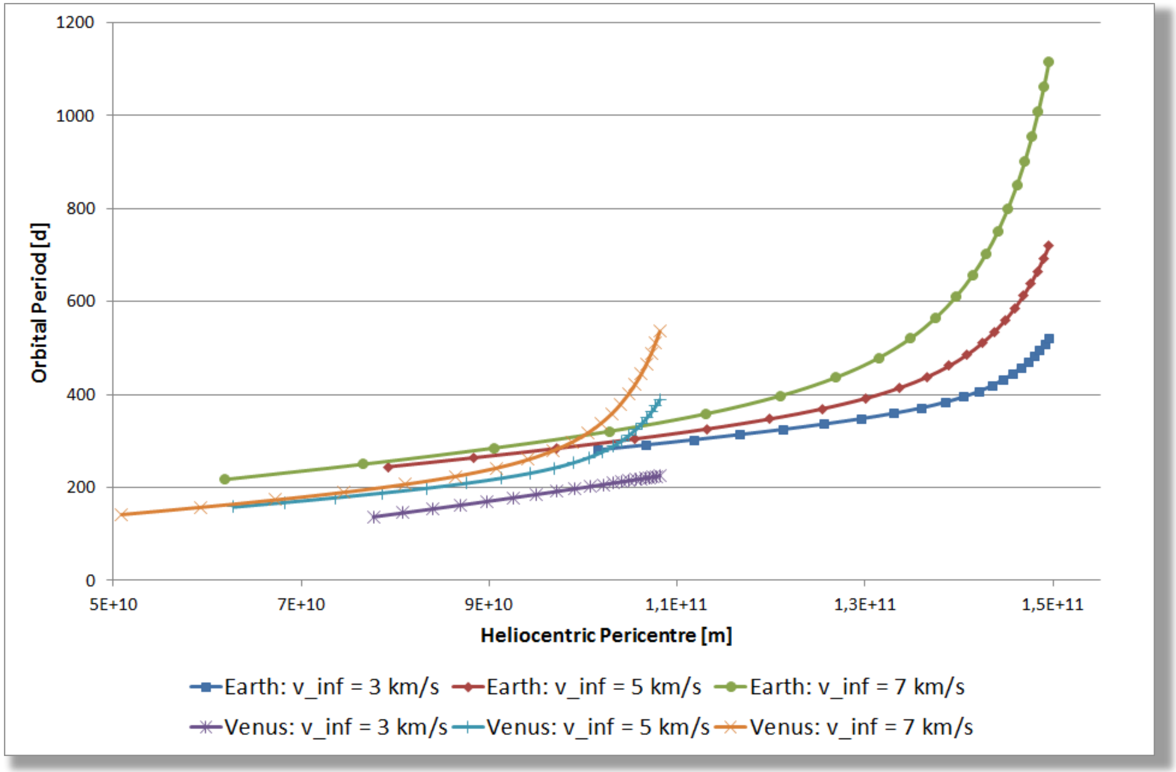

Figure 1: Example of Tisserand Graphs for Earth (right, note the maximum possible heliocentric pericentre being approx. 1AU for the spacecraft) and Venus (left) for various hyperbolic excess velocities (planetcentric). Orbital period (proportional to the semi-major axis, just like the specific orbit energy) as function of the heliocentric pericentre of the spacecraft is given.

Tisserand Criterion and Graphs (2/2)

- Premise: *Restricted, circular three-body system*; gravity only
- Energy relation of relative energy of the motion
- Provide constraints on possible fly-by heliocentric orbits (depending on planetcentric energy, i.e. v_∞)
- Tisserand Graphs are visualisation => mapping of possible Gravity-assist sequences
- Low-thrust violates gravity only premise => what effect do other deviations have? How does the thrust factor in?

Violation of TC Premises (1/3)

- Non-constant spacecraft mass:

typically small for LT

$$\frac{d}{dt}(m \cdot \vec{v}) = \dot{\vec{v}} + \boxed{\frac{\dot{m}}{m}} \cdot \vec{v} = \sum_i \frac{\vec{F}_{g,i}}{m} + \vec{T}, \qquad (7)$$

- Impact of thrust vs. gravity:

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Violation of TC Premises (1/3)

- Noı … typically small for LT

- Imp

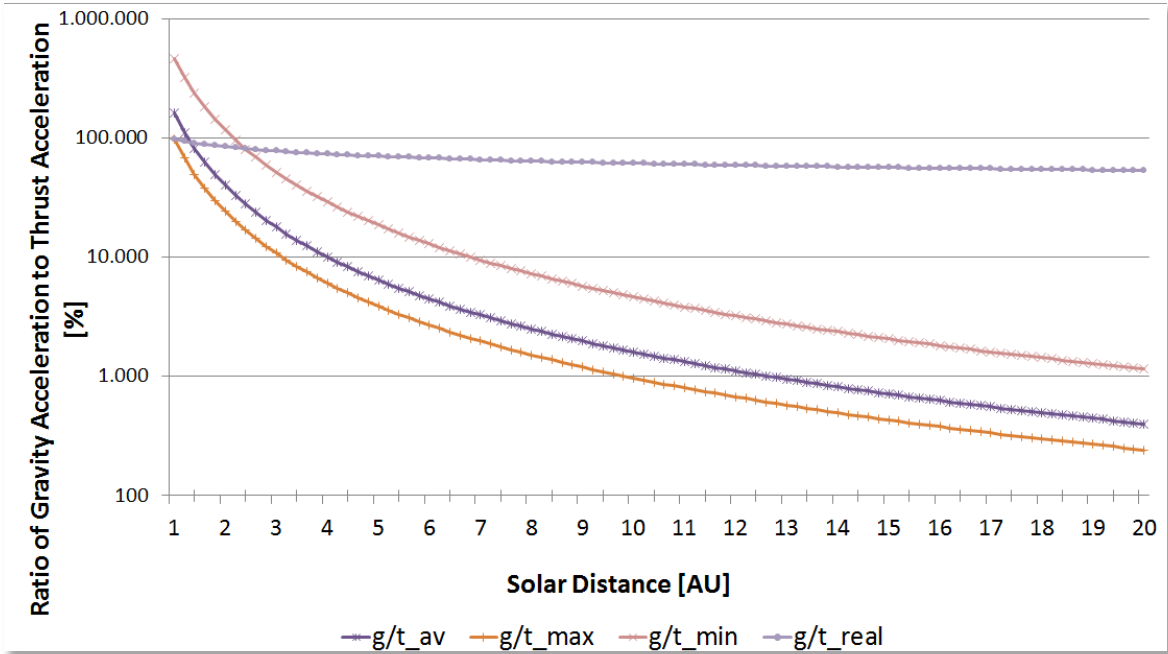

Figure 2: Ratio (logarithmic) of gravity to thrust acceleration as function of solar distance of a sample spacecraft for three cases (av: average thrust of *Dawn*, 55 mN; max: maximum thrust if *Dawn*, 91 mN; min: minimum thrust of *Dawn*, 19 mN; real: realistic thrust drop-off due to power reduction, no cut-out considered).

Violation of TC Premises (2/3)

- Non-circular planetary orbits:
  - Numerical experiments,
  - Varying S/C semimajor-axis,
  - Resulting Tisserand change in elliptical/ circular models

$$T_\Delta = \frac{\left|T_{circular} - T_{elliptical}\right|}{T_{elliptical}} \cdot 100\% \qquad (8)$$

  - Δ of up to 25%

Violation of TC Premises (2/3)

- Non-circular planetary orbits:

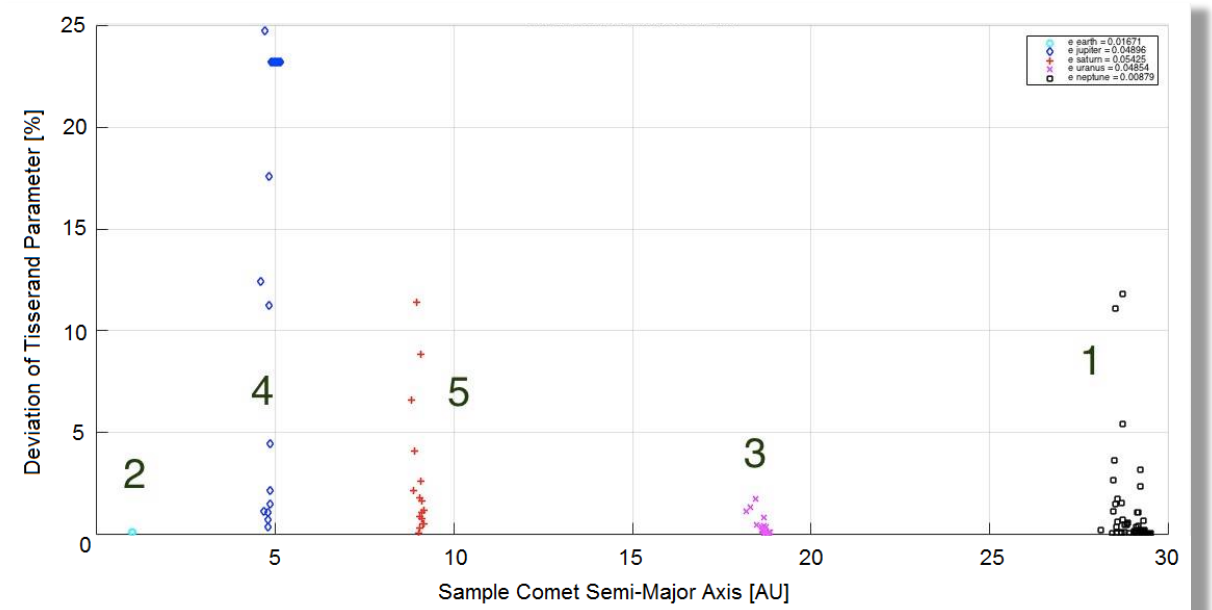

Figure 3: Deviations of the Tisserand Parameter over Sample Comet Semi-Major Axis, related to the Eccentricity of the respective planet.

Violation of TC Premises (3/3)

- Small distance between S/C and planet:

$$C_j = \frac{\mu_1}{a} + \frac{\mu_2}{R_2} + 2\sqrt{a\,(1-e^2)}\cos(I)$$

- Same order of magnitude:

$$R_2 \overset{!}{\leq} = \frac{m_2}{m_1} a,$$

- Sample calculations for all planets (varying *e* and *a*, with R_2 = SOI): Largest error for Jupiter (e = 0.99), 1.5%
    - => no noticable error

Low-thrust Correction Term for TC (1/2)

- Tisserand Criterion is energy relation
- Basic goal of orbital maneuver: adapt specific orbital energy to mission target => for high-energy missions gravity-assist/ low-thrust
- Deviation in specific energy is not neglectable => correction term needed

- Including thrust into equations of motion, transformation yields:

$$C_j^* = C_j + 2\int \vec{V} \cdot \vec{T}\, dt.$$

original integral (C_j)

Modification based on thrust acceleration ($2\int \vec{V} \cdot \vec{T}\, dt$)

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Correction Term (2/2)

- For two points in time:

$$\frac{1}{2\,a_2} + \sqrt{a_2\,(1 - {e_2}^2)}\cos i_2 = \frac{1}{2\,a_1} + \sqrt{a_1\,(1 - {e_1}^2)}\cos i_1 + 2\int_{t_1}^{t_2} \vec{V} \cdot \vec{T}\,dt$$

- Depending on assumptions (e.g. tangential thrust, circular velocity, etc.) more simplifications possible
- No state quantity, path needs to be known

Application on Low-thrust Missions (1/2)

- Correction Term changes graph
- Dependence on flight path => mission specific, no a priori information
- Thrust combines orbital states not linked by Tisserand Graphs

Variables and Constraints

- Variables can be "global" (overall mission) or "local" (mission segment)
- GA-partner is one variable, others: ToF, Launch Date, δ, v_∞…
- GA-partner is discrete and sensitive: matching required

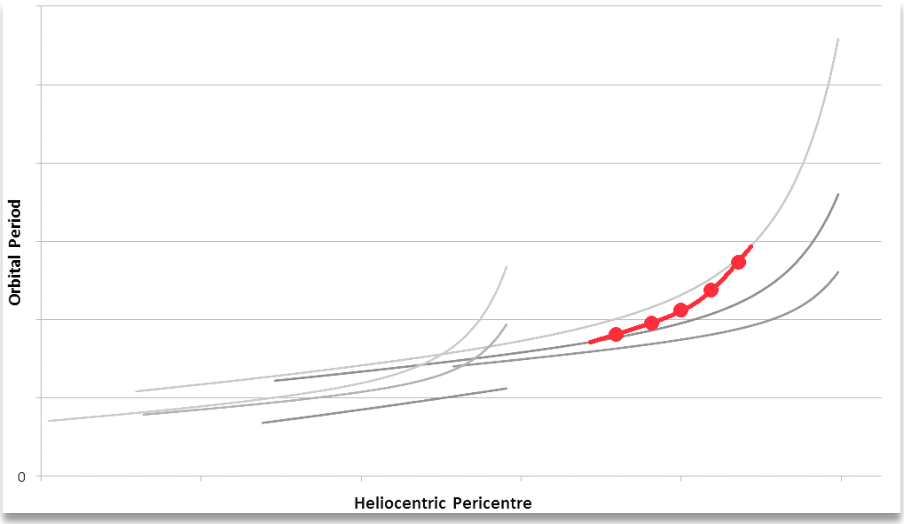


Figure 3: Example graphs (grey) for orbital period as function of heliocentric pericenter and a possible transfer between the respective energy conditions (as represented by the graphs) via a thrusting scheme (red, dotted line).

Application on Low-thrust Missions (2/2)

Repercussions on Search Algorithms

- # of variables is function of # of GAs => no common structure if not set for given population ⬄ Evolutionary Algorithms cannot be used
- Interdependency of variables/ large sensitivity (e.g. GA-partner influences what ToF is reasonable) ⬄ not all variables can be recombined

=> EAs not universally suited; multi-start as alternative

Constraints on Search Space

Planet Partner Pool

- GA-partner is variable
- "jumping" inward and outward extensively is unlikely to produce solutions with acceptable ToF and Δ*v*

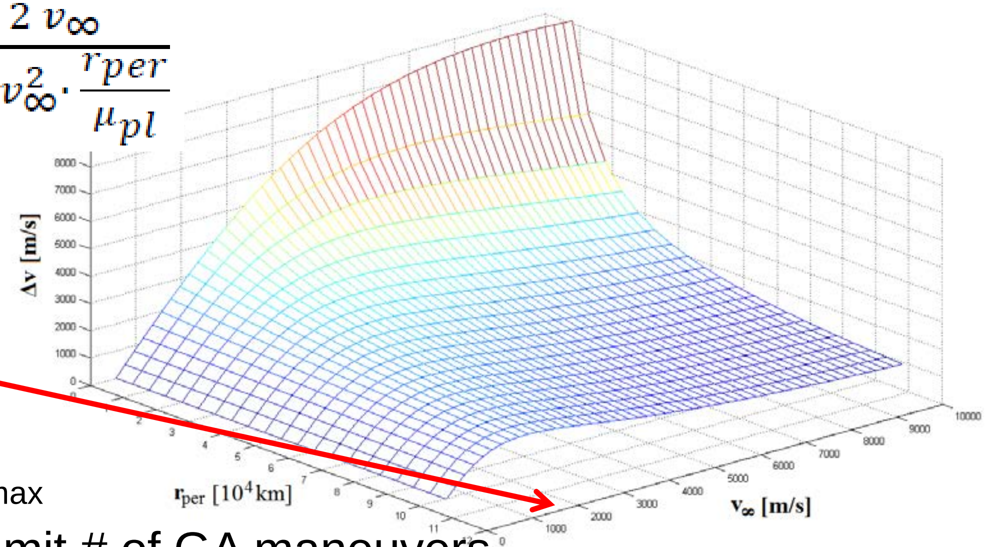
=> restrict possible „next" partner to favor „forward progression"

$$\Delta v = \frac{2\, v_\infty}{1+v_\infty^2 \cdot \frac{r_{per}}{\mu_{pl}}}$$

Maximum ΔV

- Analysis of Δ*v*-function:

$$\Delta v_{max} = \frac{1}{\sqrt{k}} = \sqrt{\frac{\mu_{pl}}{r_{per}}} = v_{\infty,ex}$$



- => knowledge of r_{per} => Δv_{max}
- use areas around Δv_{max} to limit # of GA maneuvers

Figure 4: The Δ*v* gain of a gravity-assist maneuver as function of hyperbolic pericenter distance r_{per} and hyperbolic excess velocity v_∞ for the example of an Earth fly-by (μ_{pl} = 3.98 · 1014 m³/s²).

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Optimization Methodology

- To have solutions with similar structure => # of GAs similar within population (but varies from 1 to maximum allowed number)
- TC is used for modelling turning of velocity (but not mapping of mission)
- „global“ variables are recombined, „local“ not
- Shape-based model for trajectory (fast ⇔ large # of evaluations possible)

i) Optimize no-GA benchmark

ii) Create population for each number of GAs (1 to maximum)

iii) Optimize segments

iv) Model GAs with TC

v) Optimize solutions within population

--- repeat until stopping criterion is reached ---

vi) Compare solutions with non-GA benchmark

Outlook

- Currently: Implementation as code
- Convergence of Evolutionary Algorithms is tested
- Constraints' effect is tested
- Overall convergence is tested
- Verification via optimizing missions with known trajectory's (e.g. *Dawn*)

Conclusion

- Thorough discussion on using Tisserand Criterion for low-thrust
- Presentation of correction term to adapt for energy changes by thrust
- Modified Tisserand Criterion is no state variable ⇔ no a priori knowledge about mission obtainable
- Constraints on search space presented, based on maximum obtainable Δv gain and the pool of GA partners
- Methodology for avoiding issues due to problem variables structure proposed, based on benchmarking with no-GA trajectories
- Next step: testing methodology

Thank ***YOU*** *for your attention!*

And please see the paper for more details!

Questions?

Image: NASA