A Bethe-Heitler 5D Polarized Photon-to-e^+e^- Pair Conversion Event Generator

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`http://llr.in2p3.fr/~dbernard/polar/harpo-t-p.html`

Talk Layout

- The context: HARPO: high-performance γ-ray astronomy and linear polarimetry with conversion to e^+e^- pairs with a gas TPC

telescope performance studies	NIM A 701 (2013) 225
polarimeter performance studies	NIM A 729 (2013) 765
cosmic-rays TPC tracker characterization	NIM A 718 (2013) 395
polarized γ-ray beam data-taking campaign	PoS (ICRC2015) 1016
high dilution factor polarimetry on beam demonstrated	SPIE (2016) 99052R
Recent Summary	SciNeGHE2016 arXiv:1702.08429

- Event generator:
  - Past achievements: a VEGAS based generator

Exact, 5D, polarized, Event generator	NIM A 729 (2013) 765
Event generator comparison	Astropart. Phys. 88 (2017) 60

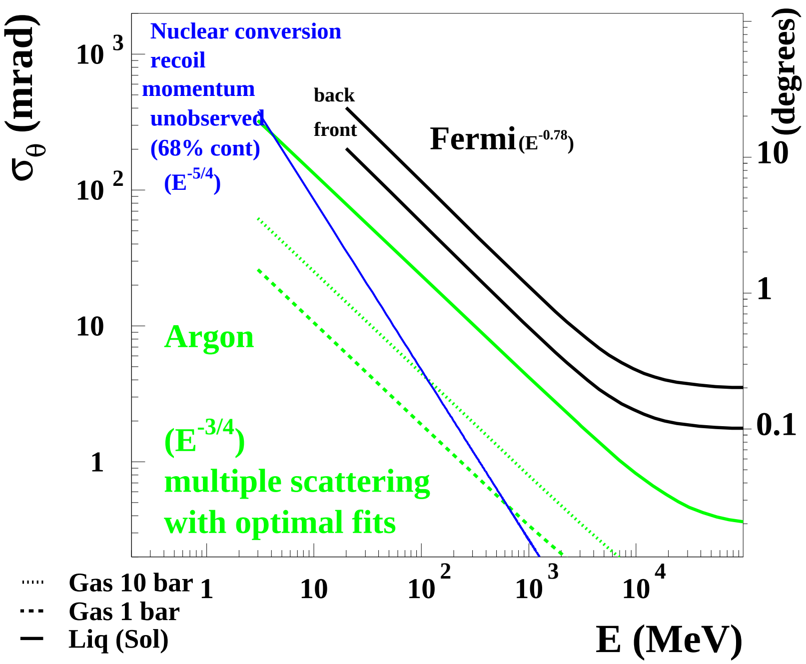
  - Present activities: towards VEGAS free generation
  - Perspectives for the future: change it to a private G4EmModel
    - Donation as a public physics model if agreeable by the G4 Coll.

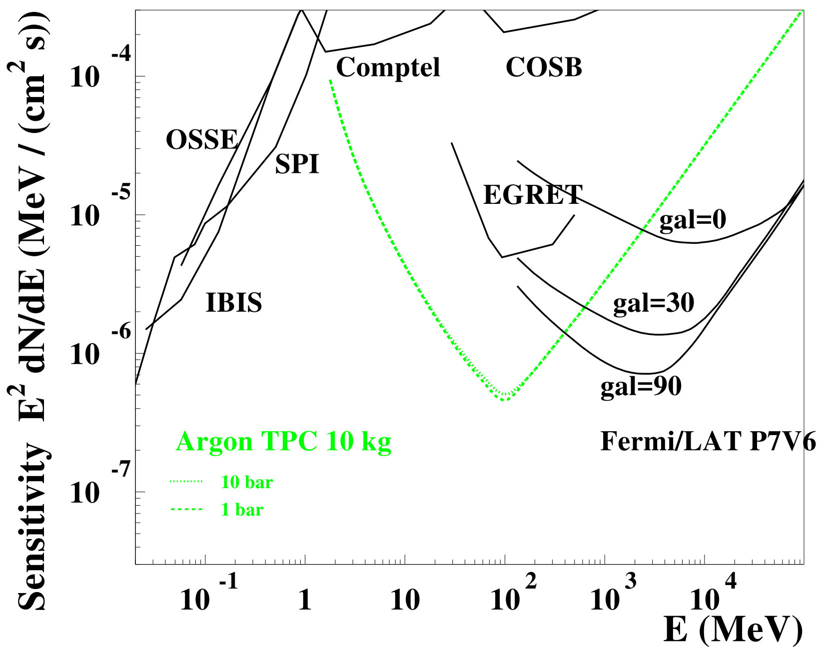
HARPO: angular resolution and sensitivity

- For $\gamma \to e^+e^-$, the sensitivity wall is mainly an angular resolution wall.

single photon angular resolution
$0.27 \oplus 0.27 = 0.38°$ @ $100\,\mathrm{MeV}$

point-like source sensitivity
(à la Fermi, 3 year, 5σ, $> 10\gamma$ ..)





Nucl. Instrum. Meth. A 701 (2013) 225

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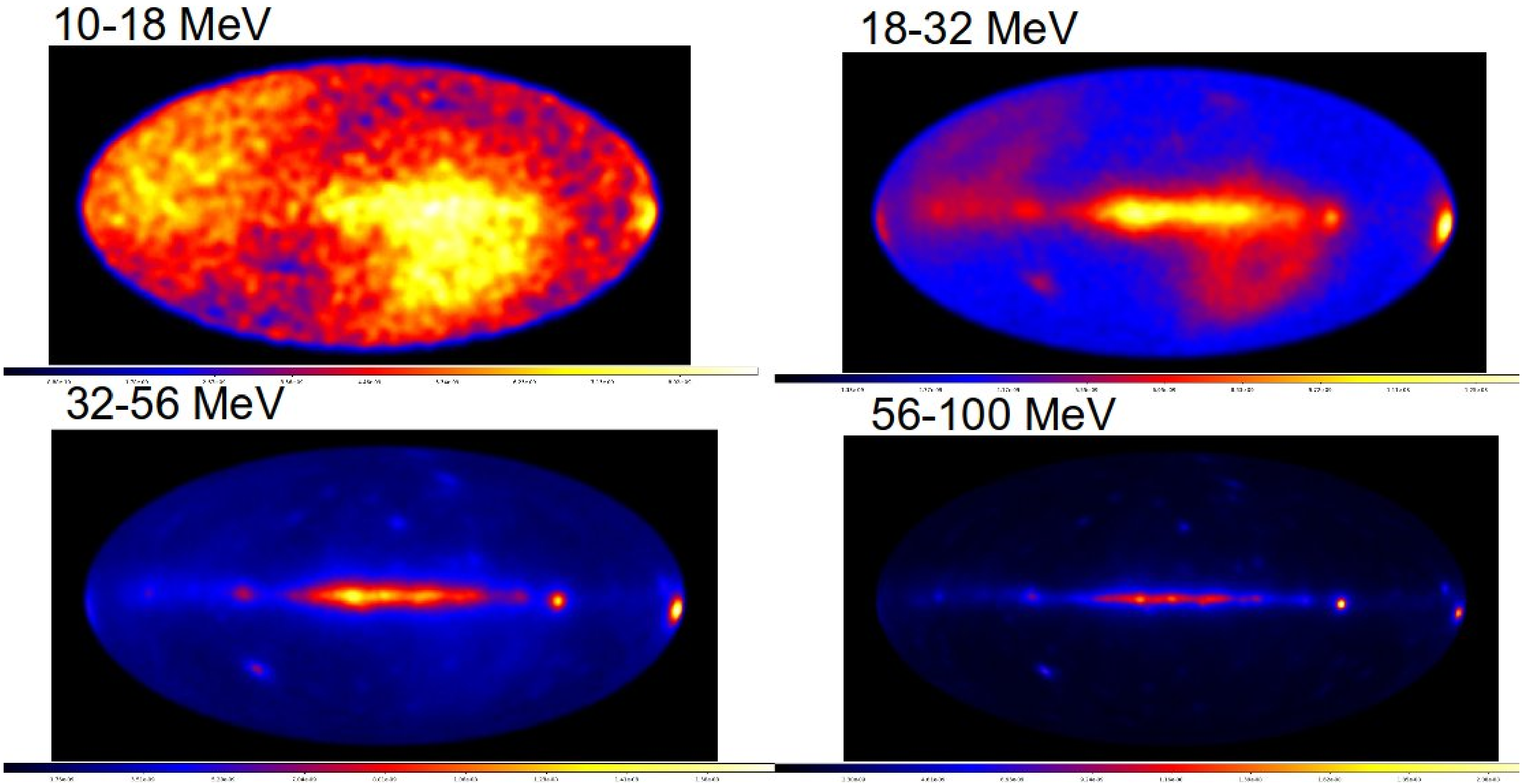
Angular resolution and sensitivity



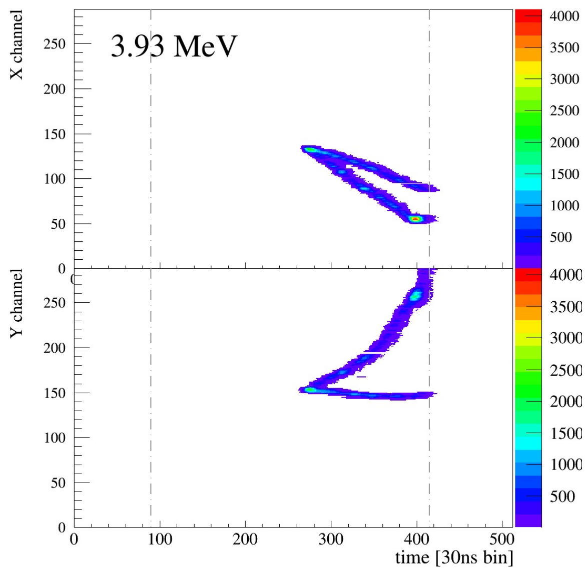
"Fermi-LAT below 100 MeV (Pass8 data)", Julie McEnery,

"e-ASTROGAM workshop: the extreme Universe", Padova Feb-March 2017

HARPO: Linear polarimetry

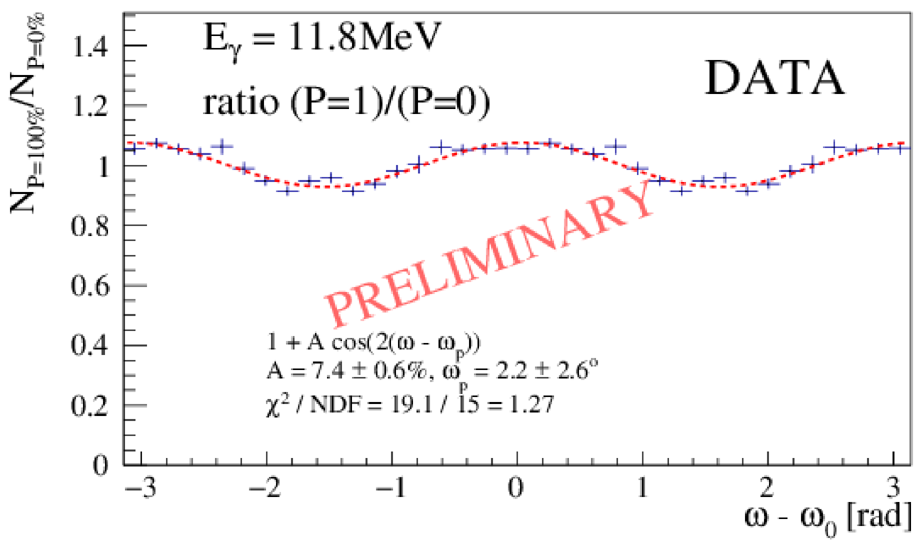
- Polarimetry major diagnostic at low energies (radio - optics - X)
- Missing $E > 1\,\mathrm{MeV}$
- Would enable decipher leptonic/hadronic Blazar models, point to emission region in pulsars, tag the transition energy for magnetars, extend Lorentz invariance violation (LIV) searches sensitivity ...
- Needs large statistics $\rightarrow$ needs low energies

(x, t) and (y, t) signal maps
$4\,\mathrm{MeV}$ γ in $2.1\,\mathrm{bar}$ Ar-iC4H10 95-5%



SPIE (2016) 99052R

azimuthal angle distribution
$11.8\,\mathrm{MeV}$ γ in $2.1\,\mathrm{bar}$ Ar-iC4H10 95-5%



SciNeGHE (2016) arXiv:1702.08429

An exact, 5D, Polarized Generator

- 5D final state: $\varphi_+, \theta_+, \varphi_-, \theta_-, x_+ \equiv E_+/E_\gamma$
- BASES / SPRING version of the VEGAS method, S. Kawabata, Comp. Phys. Comm. 88, 309 (1995).
- Differential cross section either from:
  - Feyman diagrams (important for low energy triplet) HELAS, H. Murayama, KEK-91-11.
  - Bethe-Heitler approximation (2 dominant diagrams only)
- Exact:
  - no low energy approximation
  - no small angle approximation
  - 5D differential Xsection sampled, no product of 1D differential Xsections
  - strict energy-momentum conservation
- Polarized:
  - linearly polarized photons
  - unpolarized photons
  - partially polarized photons
- Target: nucleus or electron (triplet conversion)
- Atomic electron field screening: form factor $F(q^2)$, coherent (nuclear), incoherent (triplet)
- (2012 - 2013): extensive validations for high-energy photons, $E = 4 - 400, \mathrm{MeV}$

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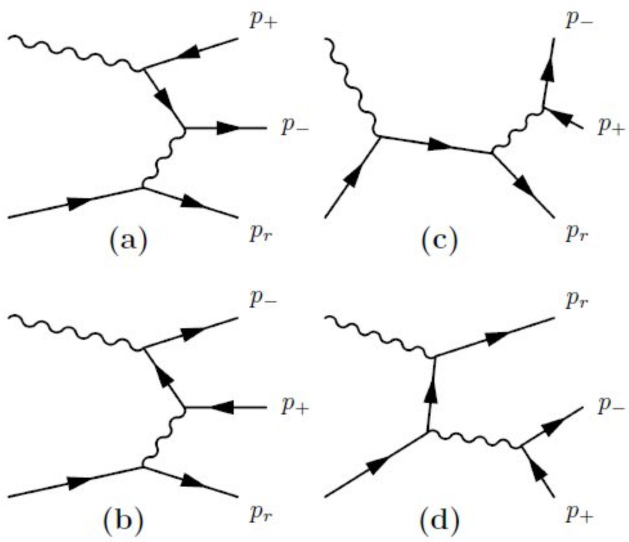
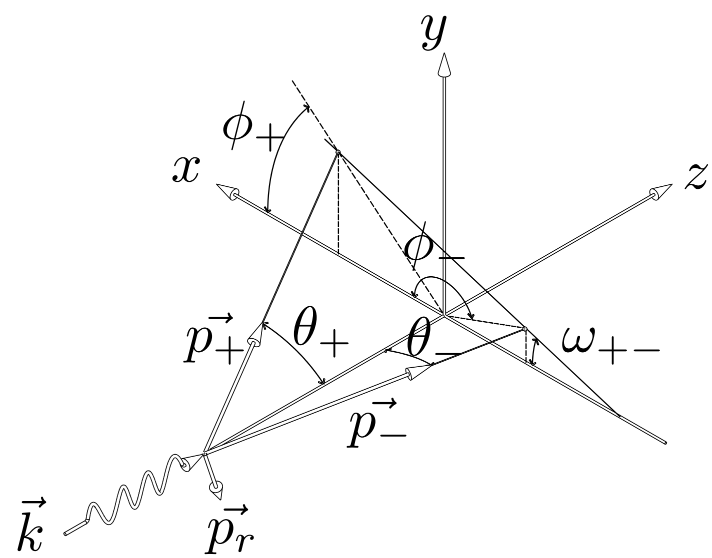
A full (5D) exact (down to threshold) polarized evt generator

- Variables: azimuthal (ϕ_+, ϕ_-) and polar (θ_+, θ_-) angles of e^+ and e^-, and $x_+ \equiv E_+/E$





- Diagrams
  - (a), (b) dominant, either for nuclear, or triplet at high-energy.
  - in addition, for triplet, 4 additional "exchange" diagrams.

Bethe-Heitler differential cross section: 1

- Linearly polarized gamma rays: $(c = 1, \quad \hbar = 1, \quad E = \hbar\omega = \omega)$

$$\mathrm{d}\sigma = \frac{-2\alpha Z^2 r_0^2 m^2}{(2\pi)^2 \omega^3} \mathrm{d}E_+ \mathrm{d}\Omega_+ \mathrm{d}\Omega_- \frac{|p_-||p_+|}{|\vec{q}|^4} \left[\left(2E_+ \frac{p_- \sin\theta_- \cos(\psi+\phi)}{E_- - p_- \cos\theta_-} + 2E_- \frac{p_+ \sin\theta_+ \cos\psi}{E_+ - p_+ \cos\theta_+} \right)^2 \right.$$

$$-q^2 \left(\frac{p_- \sin\theta_- \cos(\psi+\phi)}{E_- - p_- \cos\theta_-} - \frac{p_+ \sin\theta_+ \cos\psi}{E_+ - p_+ \cos\theta_+} \right)^2$$

$$\left. -\omega^2 \frac{(p_+ \sin\theta_+)^2 + (p_- \sin\theta_-)^2 + 2p_+ p_- \sin\theta_+ \sin\theta_- \cos\phi}{(E_- - p_- \cos\theta_-)(E_+ - p_+ \cos\theta_+)} \right]$$

with: $|\vec{q}|^2 = |\vec{p_+} + \vec{p_-} - \vec{k}|^2,$ $\quad \psi \equiv \varphi^+$ and $\psi + \phi \equiv \varphi^-$

"On the Polarization of High Energy Bremsstrahlung and of High Energy Pairs", M. May, Phys. Rev. 84, 265 (1951).
Correction by a factor of 2: Jauch and Rohrlich, *The theory of photons and electrons* (Springer Verlag, 1976).

- Non polarized gamma rays:

$$\frac{-\alpha Z^2 r_0^2 m^2}{(2\pi)^2 \omega^3} \mathrm{d}E_+ \mathrm{d}\Omega_+ \mathrm{d}\Omega_- \frac{|p_-||p_+|}{|\vec{q}|^4} \left[\left(\frac{p_+ \sin\theta_+}{E_+ - p_+ \cos\theta_+} \right)^2 (4E_-^2 - q^2) + \left(\frac{p_- \sin\theta_-}{E_- - p_- \cos\theta_-} \right)^2 (4E_+^2 - q^2) + \right.$$

$$\left. \frac{2p_+ p_- \sin\theta_+ \sin\theta_- \cos\phi}{(E_- - p_- \cos\theta_-)(E_+ - p_+ \cos\theta_+)} (4E_+ E_- + q^2 - 2\omega^2) - 2\omega^2 \frac{(p_+ \sin\theta_+)^2 + (p_- \sin\theta_-)^2}{(E_+ - p_+ \cos\theta_+)(E_- - p_- \cos\theta_-)} \right]$$

"The quantum theory of radiation", W. Heitler, 1954.

Bethe-Heitler differential cross section: 2

- That is: $\mathrm{d}\sigma = \Phi(X_u + P \times X_p)\mathrm{d}E_+\mathrm{d}\Omega_+\mathrm{d}\Omega_-$ with $\Phi = \frac{-\alpha Z^2 r_0^2 m^2}{(2\pi)^2\omega^3}\frac{|p_-||p_+|}{|\vec{q}|^4}$

- P the polarization fraction of the incident photon,

- And:

$$X_u = \left[\left(\frac{p_+\sin\theta_+}{E_+ - p_+\cos\theta_+}\right)^2(4E_-^2 - q^2) + \left(\frac{p_-\sin\theta_-}{E_- - p_-\cos\theta_-}\right)^2(4E_+^2 - q^2) + \frac{2p_+p_-\sin\theta_+\sin\theta_-\cos\phi}{(E_- - p_-\cos\theta_-)(E_+ - p_+\cos\theta_+)}(4E_+E_- + q^2 - 2\omega^2) - 2\omega^2\frac{(p_+\sin\theta_+)^2 + (p_-\sin\theta_-)^2}{(E_+ - p_+\cos\theta_+)(E_- - p_-\cos\theta_-)}\right]$$

$$X_p = \cos 2(\phi+\psi)(4E_+^2 - q^2)\left(\frac{p_-\sin\theta_-}{E_- - p_-\cos\theta_-}\right)^2 + \cos 2\psi(4E_-^2 - q^2)\left(\frac{p_+\sin\theta_+}{E_+ - p_+\cos\theta_+}\right)^2 + 2\cos(\phi+2\psi)(4E_+E_- + q^2)\frac{p_-\sin\theta_- p_+\sin\theta_+}{(E_- - p_-\cos\theta_-)(E_+ - p_+\cos\theta_+)}$$

- Partial screening effect of (other) atomic electrons parametrized by form factor (fn of q)

Gears

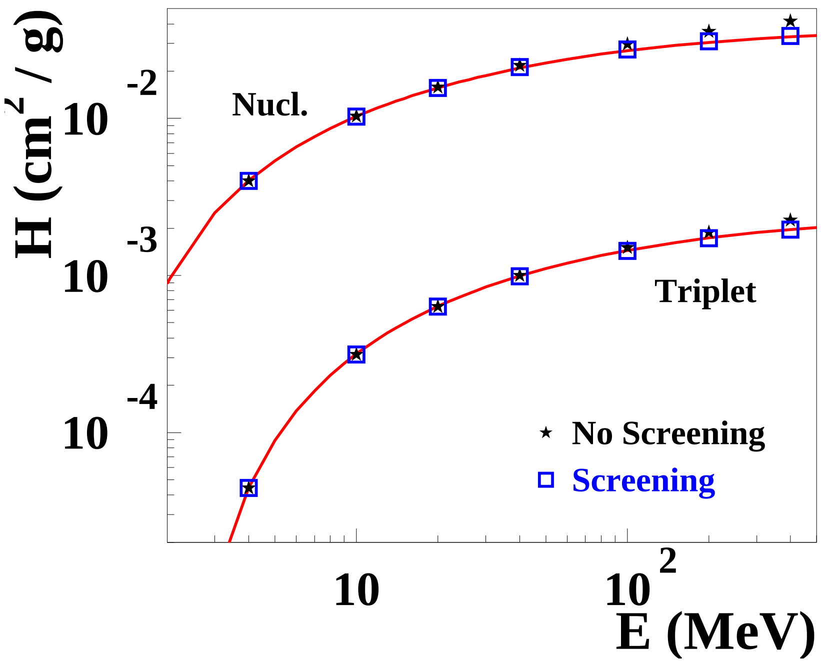
- Variables that are actually generated:

variable	related to	
$\lambda(1)$	recoil polar angle	in the CMS
$\lambda(2)$	recoil azimutal angle	
$\lambda(3)$	pair Invariant Mass(MeV)	
$\lambda(4)$	positron polar angle	in the pair CMS frame
$\lambda(5)$	positron azimutal angle	in the pair CMS frame

- Output:
  - e^+, e^- and recoil 4-vectors

CMS : center of mass system

Benchmark: 0: cross sections (argon)



- Comparison of the total mass attenuation coefficients (argon; with and without screening) with the data from NIST
- NIST data include all known effects (screening, Coulomb corrections ..)

J. H. Hubbell et al., J. Phys. Chem. Ref. Data 9, 1023 (1980).

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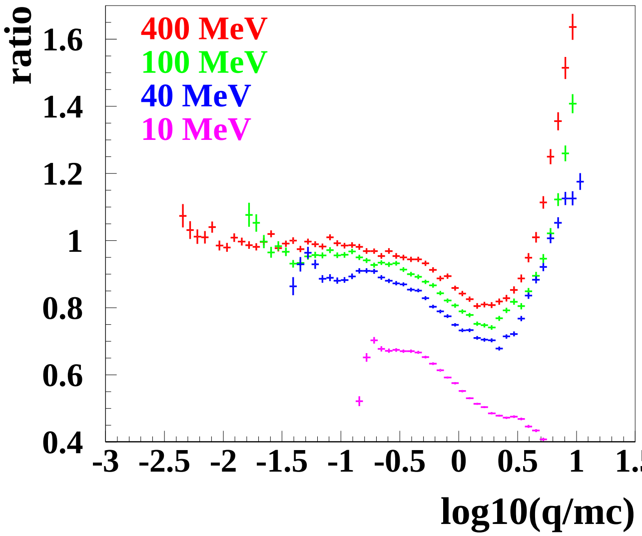
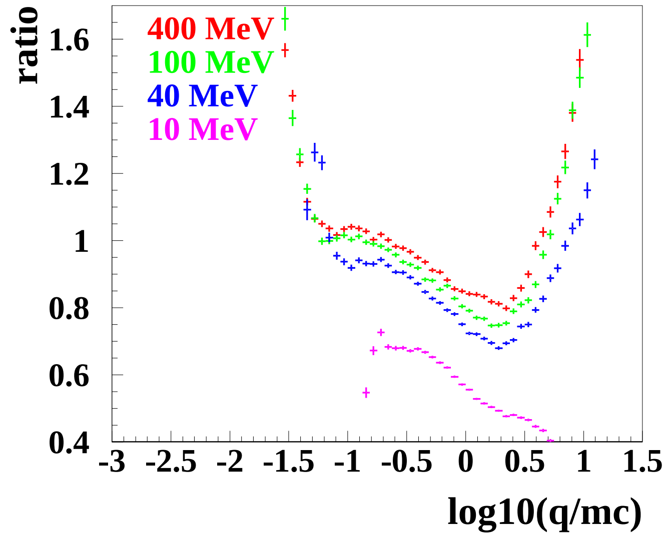
Benchmark: 1: triplet / nuclear q-distribution ratio





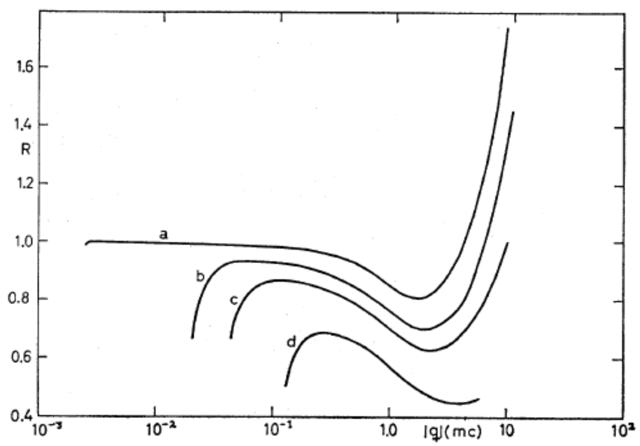


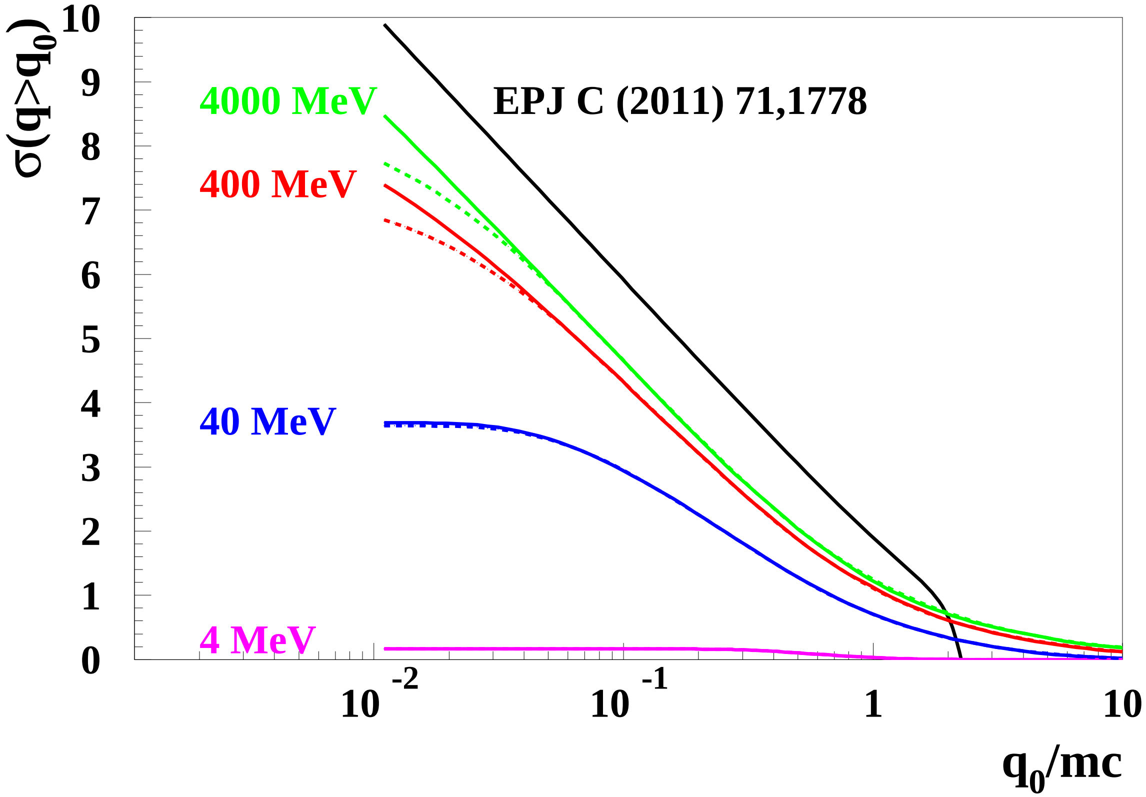
FIG. 7. The ratio R between the recoil distributions for pair production by electrons and by a very heavy nucleus. The curves a, b, c, and d refer to lab photon energies $\omega = 1000$, 100, 50, and 20 mc^2, respectively.

Without screening — with screening

"Pair Production by Photons on Electrons", K. J. Mork, Phys. Rev. 160, 1065 - 1071 (1967).

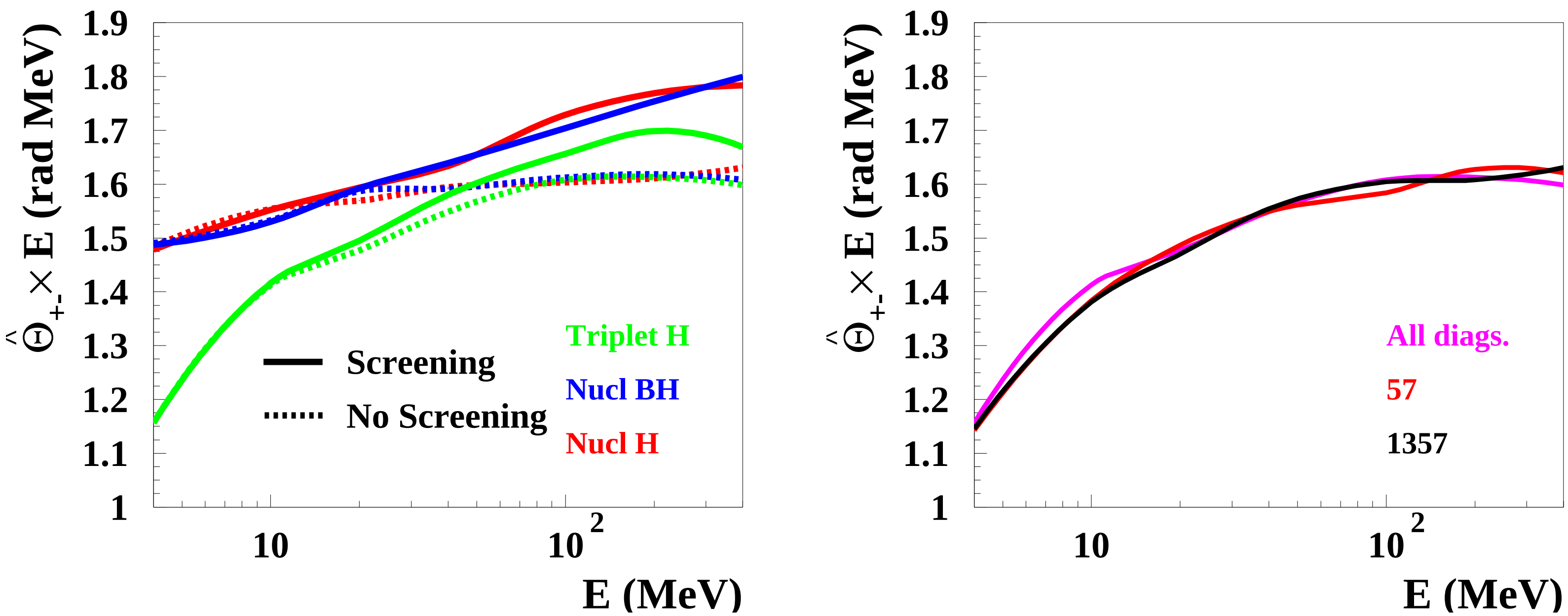
Benchmark: 2: triplet cross section above threshold q_0

- Triplet conversion: cross section for recoil electron momentum larger than q_0, $\sigma(q > q_0)$, as a function of q_0/mc, for various photon energies E; (Dashed is with form factor applied).
- High photon energy asymptotic expression by M. L. Iparraguirre and G. O. Depaola, Eur. Phys. J. C 71, 1778 (2011).



D.B. NIM A 729 (2013) 765

Benchmark: 3: Most probable opening angle $\hat{\theta}_{+-}$



	triplet
nuclear and triplet	with all diagrams
with/without screening	without exchange diagrams (1357)
for the H/BH pdf	with BH diagrams only (57)

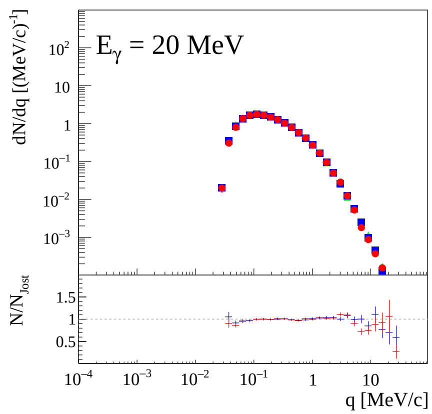
Olsen computes $\hat{\theta}_{+-} \approx \dfrac{1.6\,\mathrm{MeV}}{E}$

"Opening Angles of Electron-Positron Pairs", Haakon Olsen, Phys. Rev. 131, 406 - 415 (1963)

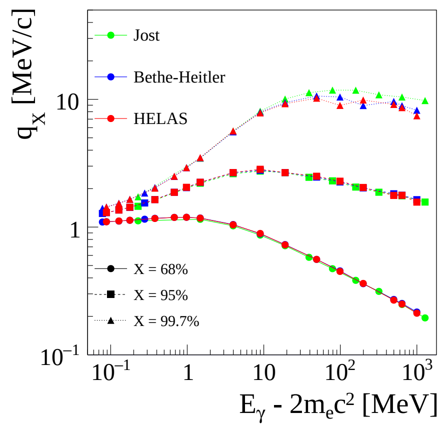
VEGAS-based generator: more on validations

- Extend to 1.1 MeV - 1 GeV

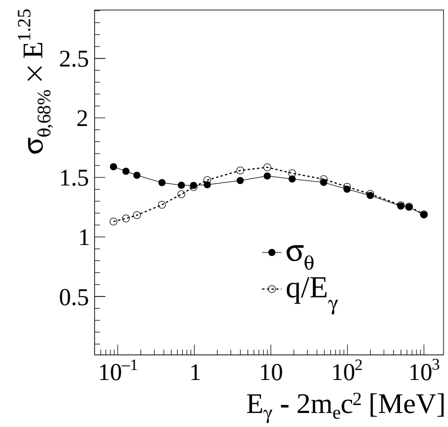
recoil momentum q



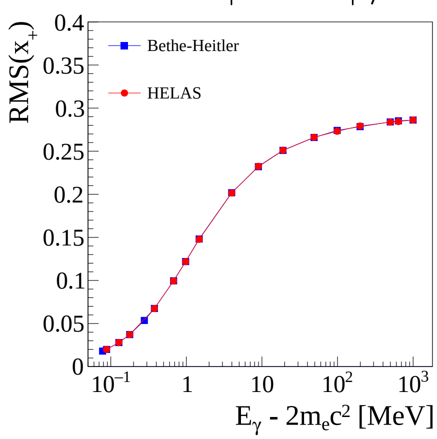
recoil momentum q containment



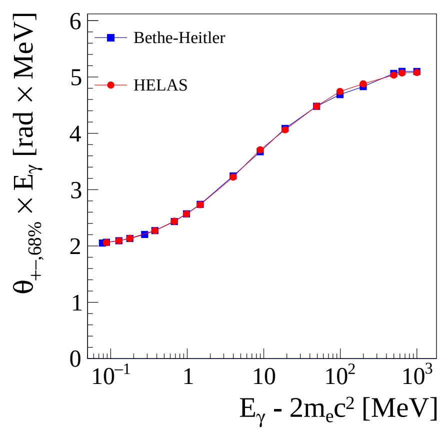
angular resolution containment



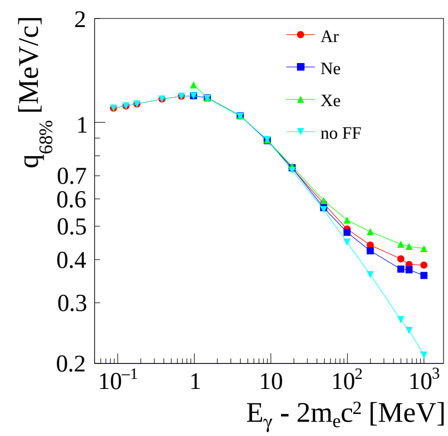
R.M.S. of $x_+ \equiv E_+/E$



opening angle containment



recoil momentum containment



R. Jost, "Distribution of Recoil Nucleus in Pair Production by Photons," Phys. Rev. 80, 189 (1950).

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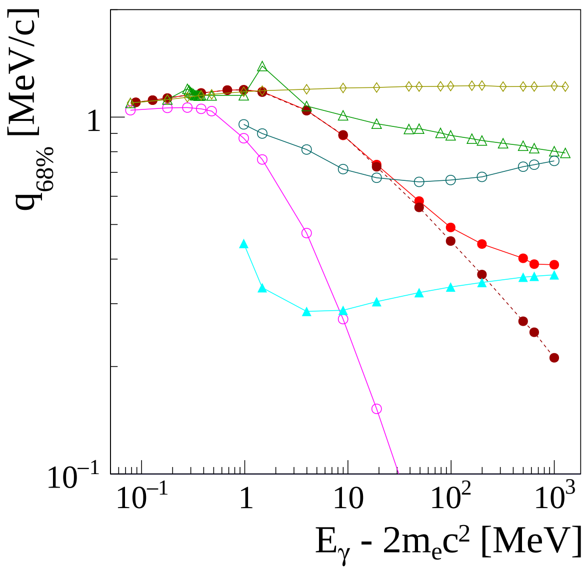
Conversion Event Generators: A Comparison

Name	Model	Generator
HELAS	HELAS Feynman amplitudes	BASES/SPRING
Bethe-Heitler	Bethe-Heitler	BASES/SPRING
G4:emstandard	G4BetheHeitler	Geant4 10.02.01
G4:livermorepola	G4LivermorePolarizedGammaConversion	Geant4 10.02.01
G4:penelope	G4PenelopeGammaConversion	Geant4 10.02.01
EGS0	egs5, IPRDST$=0$	egs5 1.0.6
EGS2	egs5, IPRDST$=2$	egs5 1.0.6

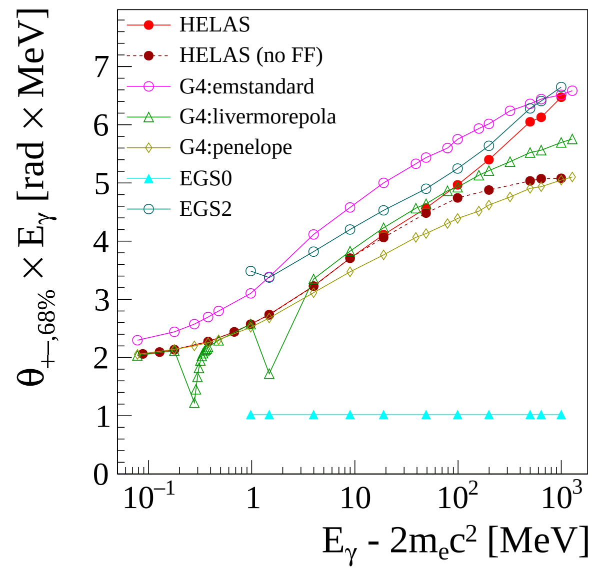
- *G4:livermore* has same kinematics as *G4:emstandard* : not shown here

Event Generator Comparison: Kinetic Variables

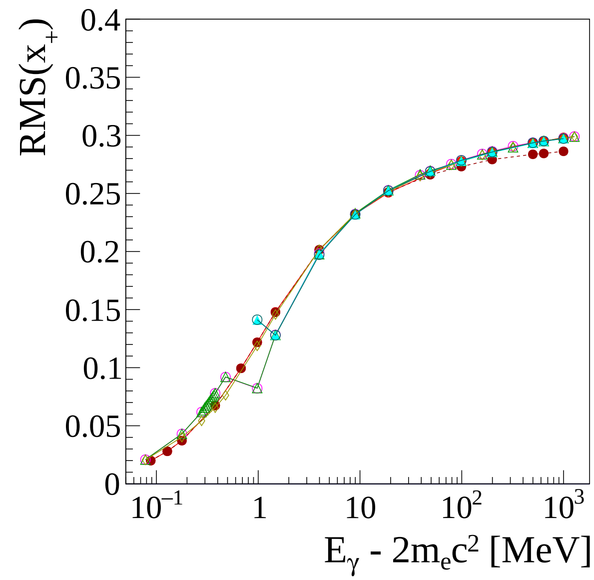
recoil momentum containment



opening angle containment



R.M.S. of $x_+ \equiv E_+/E$



- Recoil momentum incorrectly simulated by G4 and EGS5 physics models
- Inspection of code and/or documentation shows that they (most often) don't even conserve energy-momentum.

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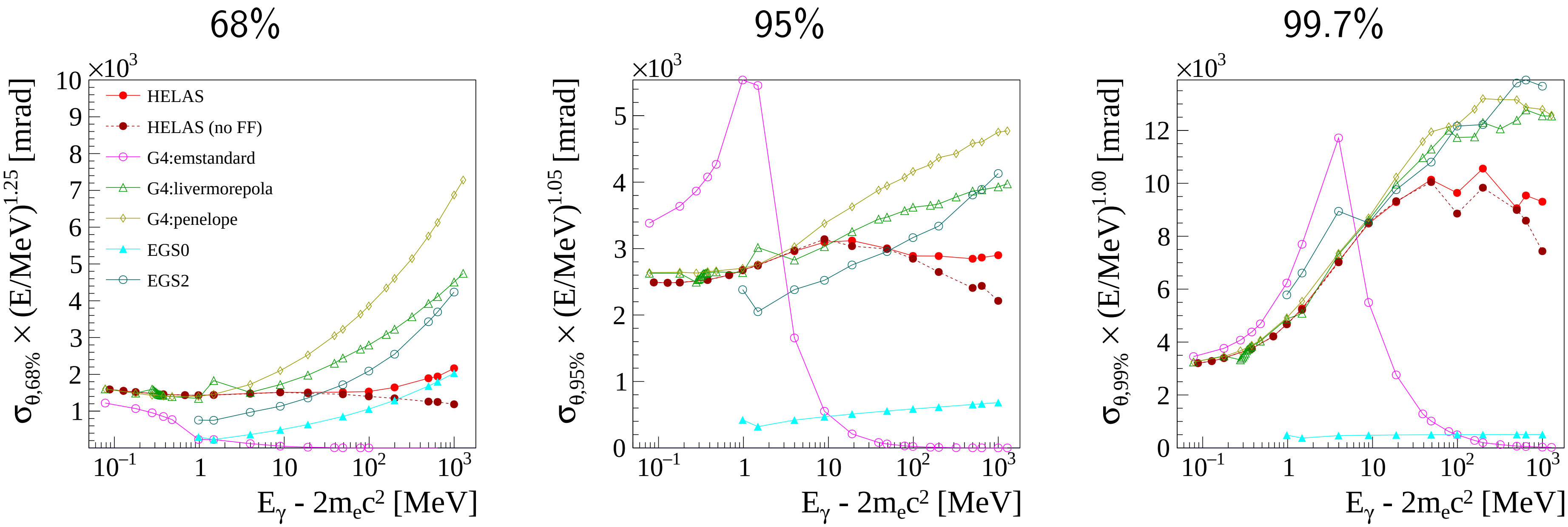
Event Generator Comparison: Angular Resolution



- HELAS: Form Factor (argon): an effect for $E > 100\mathrm{MeV}$
- HELAS: parametrization of σ_θ:

68%	$1.5\,\mathrm{rad}\,(E/\mathrm{MeV})^{-1.25}$
95%	$2.9\,\mathrm{rad}\,(E/\mathrm{MeV})^{-1.05}$
99.7%	$4-9\,\mathrm{rad}\,(E/\mathrm{MeV})^{-1.00}$

- G4 and EGS5 models have 68% angular resolution crazy.
  - Most often, e^+ and e^- generated back-to-back and polar angles generated independently : does not conserve energy-momentum.

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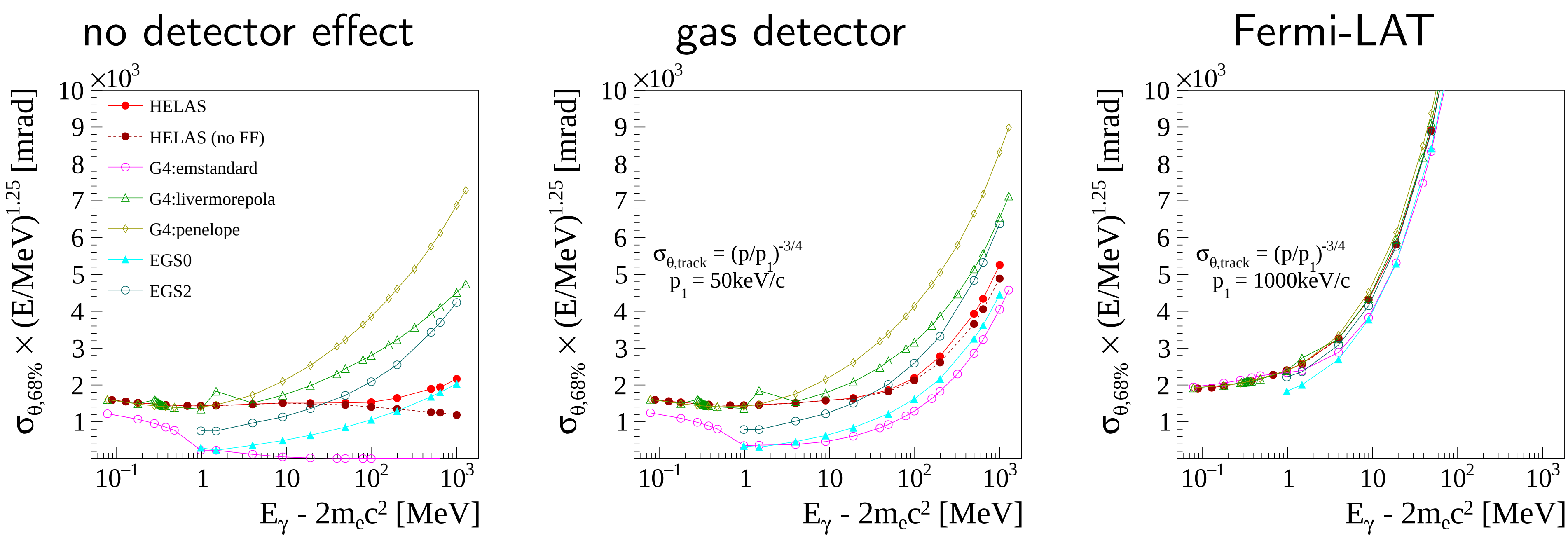
Event Generators and Telescopes



- For dense, high-Z telescopes, the multiple scattering washes out track correlations and therefore the event generator differences

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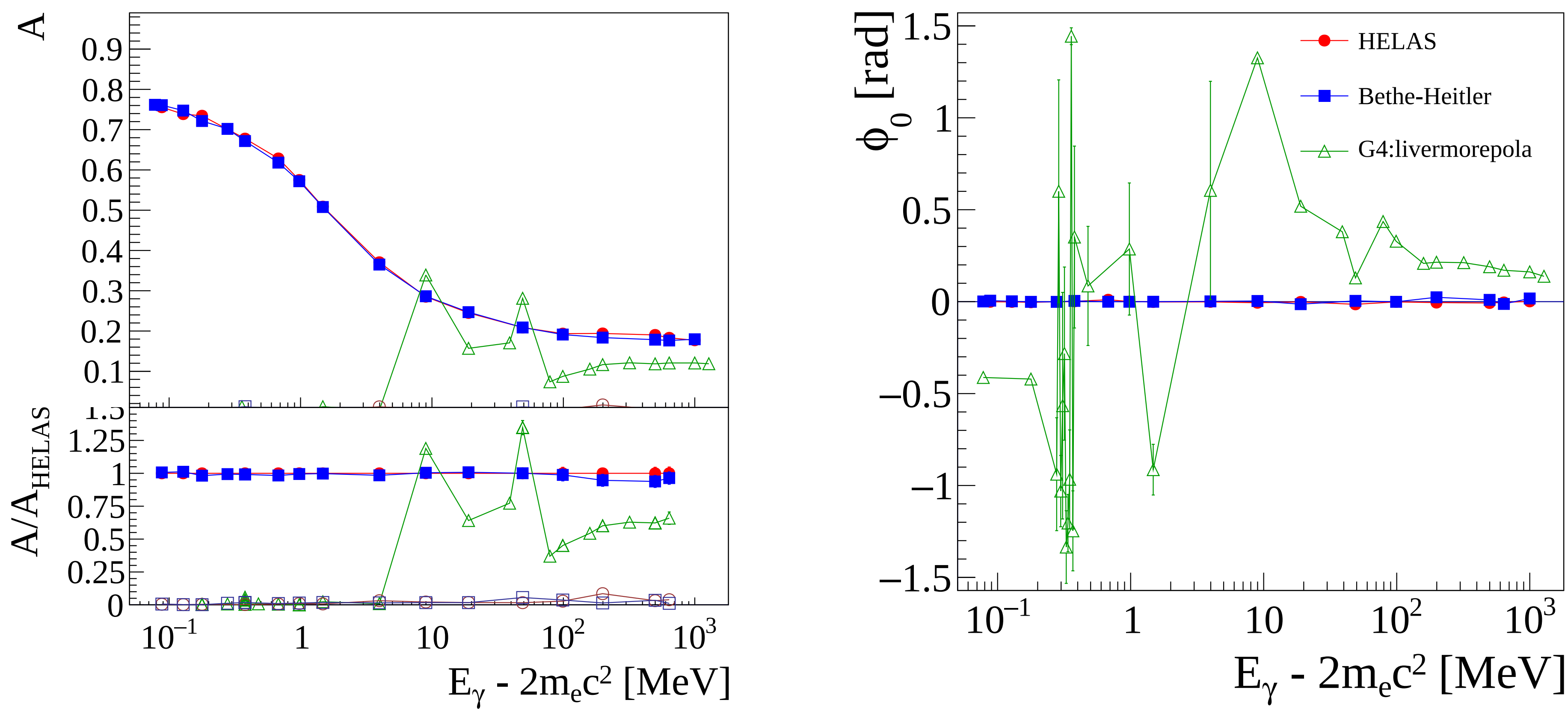
Event Generators and Photon Linear Polarization



- Simulation of polarized γ conversion to pairs by G4:livermorepola surprising (polarisation asymmetry and polarisation angle)

@ 100 MeV, $$\frac{\mathcal{A}_{(\text{HELAS or BH})}}{\mathcal{A}_{\text{G4:livermorepola}}} = \frac{(19.1 \pm 0.4)\%}{(8.7 \pm 0.6)\%} \approx 2.2$$

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Multi-dimensional integration and event generation with package BASES/SPRING

- Method: The VEGAS algorithm, due to G. P. Lepage (J. Comput. Phys. 27 (1978) 192)
- Integration:
  - n-D hyper-volume first segmented in identical cells (n-dimensional bins);
  - pdf and its variance evaluated in each cell, by shooting a number of points;
  - hyper-volume segmentation then tuned iteratively, so as to minimize the variance;
  - upon convergence, an optimal tabulation of the pdf is obtained.
- Event generation:
  - an "exact" generation from $f(\lambda)$ is then obtained easily from the tabulated $f_0(\lambda)$ the acceptance-rejection method
- The pdf depends on photon energy E (and target nature through $F(q^2)$)
  - the integration step is time consuming (few seconds)
  - generation of batches of zillions events with same energy easy.
- Presently moving to a parametrized(E) version for each variable.

No sign of (Bethe-Heitler)-HELAS difference: **HELAS dropped**

Perspectives for the future: a Geant4 Physics Model

- Fortran $\rightarrow$ C++ $\rightarrow$ private G4EmModel
  - G4 experts expertise will be welcome.

- Donation to the Geant4 Collaboration if agreeable by the G4 Coll.

Conclusion

- Presently available physical models on the market inapropriate:
  - for simulation of high-resolution γ telescopes with pairs
  - for simulation of γ polarimeters with pairs
- Electron/ positron energy share distribution found OK.
- VEGAS-based exact, 5D, polarized event photon conversion event generator built, validated, published.
- De-VEGAS-ification in progress $\rightarrow$ (several)-$\mu\mathrm{s}$ single-photon generation

  validation already extended to $E = 1\,\mathrm{PeV}$
- Geant4 Physics Model later this year.

Back-up slides

Form Factors

- Form factors must be taken into account as the fraction of the total cross section lost increase with energy :

$$\frac{\sigma_{\mathrm{noFF}}}{\sigma_{\mathrm{FF}}} = \frac{41 \log\left(2E/m\right) - 109}{41 \log\left(183 Z^{-1/3}\right) - 1} \tag{1}$$

- for 2 elements Z_1 and Z_2;

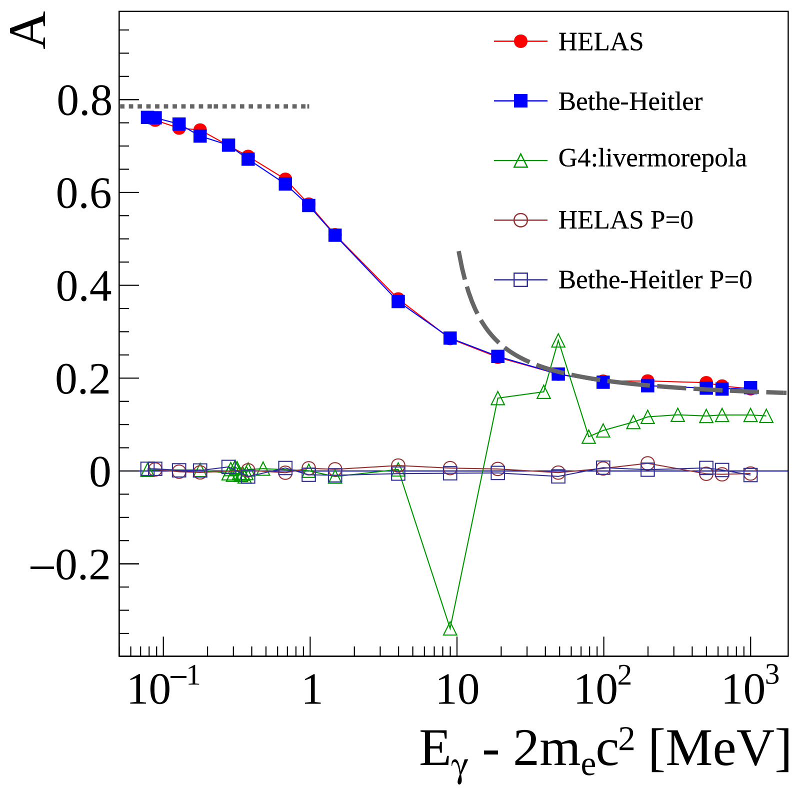
  noting that $(1 - F(q^2, Z_1)) < (1 - F(q^2, Z_2))$ for $Z_1 > Z_2$,

  - use Z_2 mock-up to shoot Z_1
  - The efficiency loss would be asymptotically

$$\frac{\sigma_{\mathrm{FF},Z_1}}{\sigma_{\mathrm{FF},Z_2}} = \frac{41 \log\left(183 Z_1^{-1/3}\right) - 1}{41 \log\left(183 Z_2^{-1/3}\right) - 1}, \tag{2}$$

For example, $\frac{\sigma_{\mathrm{FF},Ar}}{\sigma_{\mathrm{FF},Xe}} \approx 1.095.$ (after Z^2 dependence rescaled).

Polarisation asymmetry : asymptotic expressions



- Low energy

$$A = \frac{\pi}{4}.$$

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- High energy

$$A \approx \frac{\frac{4}{9}\ln 2E - \frac{20}{28}}{\frac{28}{9}\ln 2E - \frac{218}{27}}$$

V. F. Boldyshev et al., Yad. Fiz. 14 (1971) 1027, (Sov.J.Nucl.Phys. 14 (1972) 576).