Adaptive Pareto Front Sampling Based on Parametric Sensitivity Analysis in a Bi-Objective Setting

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Overview

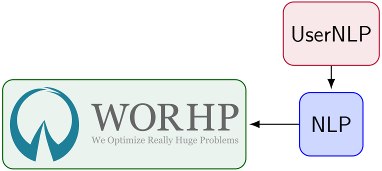
- New interface for WORHP
- Introduction to multiobjective optimization
- Adaptive Pareto front sampling

New WORHP Interface

WORHP

We Optimize Really Huge Problems

New WORHP Interface



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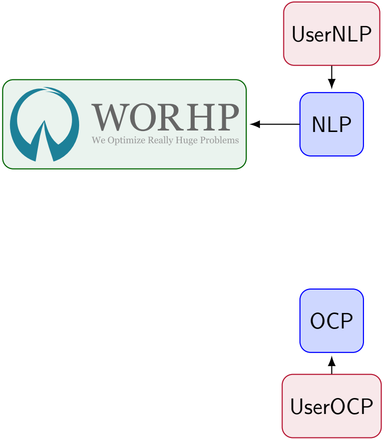

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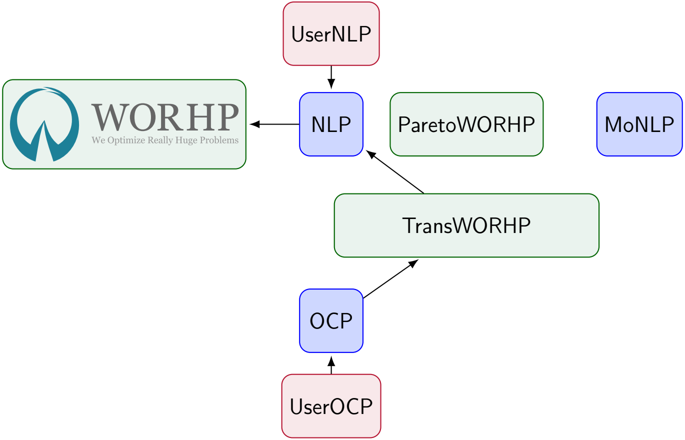

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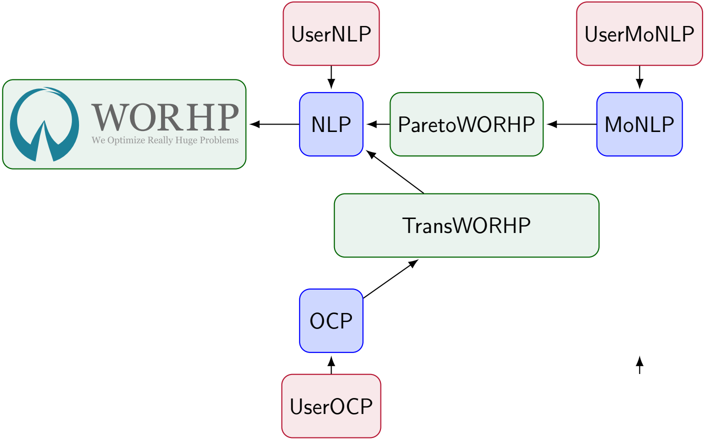

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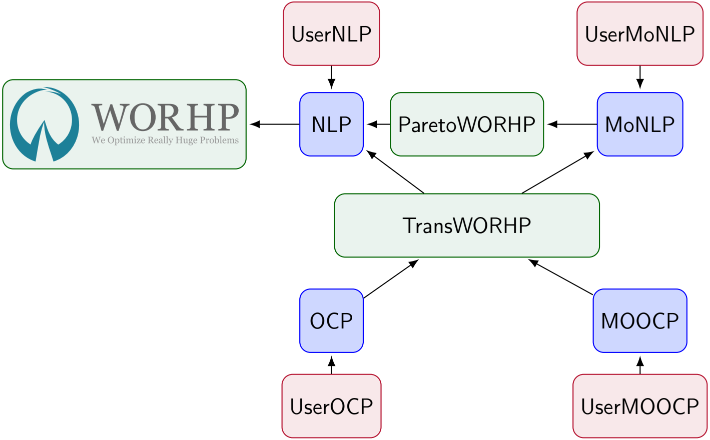
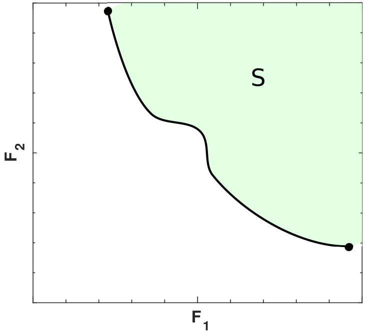


Pascoletti-Serafini

- Objective function $F : \mathbb{R}^{N_x} \to \mathbb{R}^{N_F}$
- Inequality constraints $g : \mathbb{R}^{N_x} \to \mathbb{R}^{N_g}$
- Equality constraints $h : \mathbb{R}^{N_x} \to \mathbb{R}^{N_h}$
- Feasible Set $S \subset \mathbb{R}^{N_x}$

$$\text{"}\min_{x \in S}\text{"} \quad F(x) \in \mathbb{R}^{N_F}, \quad N_F \geq 2$$
$$\text{subject to} \quad g(x) \leq 0$$
$$h(x) = 0$$



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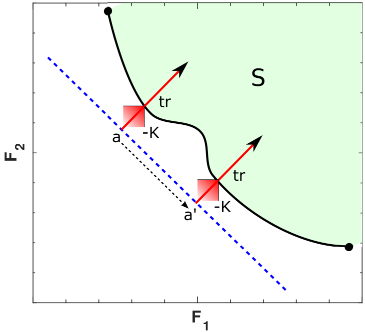
$$\min_{x,t} \quad t$$
$$\text{subject to} \quad F(x) \in a + tr - K, \quad g(x) \leq 0, \quad h(x) = 0,$$

- Point $a \in \mathbb{R}^{N_F}$
- Direction $r \in \mathbb{R}^{N_F}$
- Cone $K \in \mathbb{R}^{N_F}$



Parameter Perturbed NLP

$$\begin{aligned} \min_x \quad & f(x,p), \qquad f(x,p) \in \mathbb{R}, \quad p \in \mathbb{R}^{N_p} \\ \text{subject to} \quad & g(x,p) \leq 0, \\ & h(x,p) = 0 \end{aligned}$$

- Nominal parameter $p_0 \in \mathbb{R}^{N_p}$
- Sensitivity theorem: ‘In a neighborhood of p_0 the soltion x changes continously with the parameter’
- **Sensitivity differentials** $\frac{dx}{dp}(p_0)$, $\frac{df}{dp}(p_0)$, $\frac{dg}{dp}(p_0)$, $\frac{dh}{dp}(p_0)$ exist and can be explicitly stated

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Parameter Perturbed MONLP

$$
\begin{array}{ll}
\min\limits_{x,t} & t =: \hat{f}(x,t,p) \\
\text{subject to} & F(x) - (a + tr) \leq 0 \\
& g(x) \leq 0, \\
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\end{array}
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$$\nabla_p F = E_{N_F} - r_0 \left(\frac{d\hat{f}}{dp} \right)^T$$

[1] G. Eichfelder: Adaptive Scalarization Methods in Multiobjective Optimization

Multiobjective Optimal Control

$$
\begin{array}{llcll}
\min\limits_{u,x,t_f} & F(x(t)) & \in & \mathbb{R}^{N_F} & \\
\text{s.t.} & \dot{x}(t) & = & \xi(x(t), u(t), t), & t \in [0; t_f] \\
 & \omega(x(0), x(t_f)) & = & 0 & \\
 & C(x(t), u(t), t) & \leq & 0 & t \in [0; t_f],
\end{array}
$$

$$F_i = M_i(x(0), x(t_f)) + \int_0^{t_f} l_i(x(t, u(t), t))dt, \;\; i = 1, \ldots, N_F.$$

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Transcription

MONLP

Example - Definition

$$
\begin{array}{llcl}
\min\limits_{u,x,t_f} & F(x,u,t_f) & = & (-x_1(t_f), x_3(t_f)^2)^T \\
\text{s.t.} & \dot{x}_1(t) & = & x_2(t) \\
 & \dot{x}_2(t) & = & u(t) - \dot{\tau}(x(t)) \\
 & \dot{x}_3(t) & = & u(t)^2 \\
 & x_1(0) & = & 0 \\
 & t_f & = & 1 \\
 & u(t) & \geq & 0 \\
 & u(t) & \leq & 5
\end{array}
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TransWORHP

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TransWORHP

Pascoletti-Serafini

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TransWORHP

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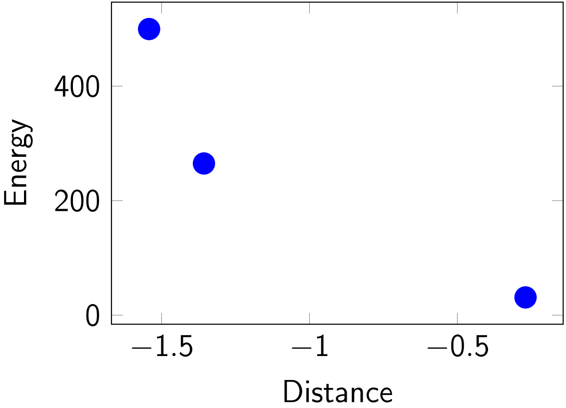
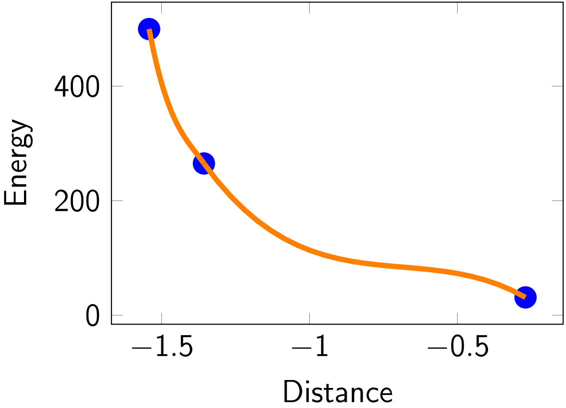
Example - Pareto Front Sampling (1)

- Find samples of pareto front



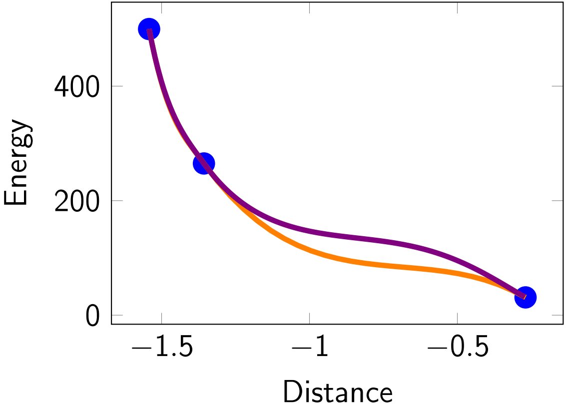
Example - Pareto Front Sampling (1)

- Find samples of pareto front
- Interpolate Pareto front in objective space based on sensitivities



Example - Pareto Front Sampling (1)

- Find samples of pareto front
- Interpolate Pareto front in objective space based on sensitivities
- Interpolate Pareto front in parameter space and apply objectives



Example - Pareto Front Sampling (1)

- Find samples of pareto front
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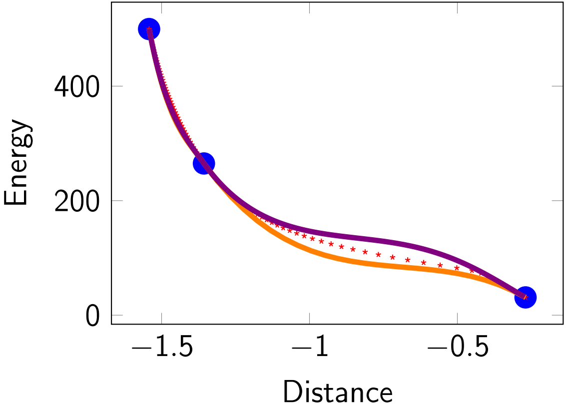
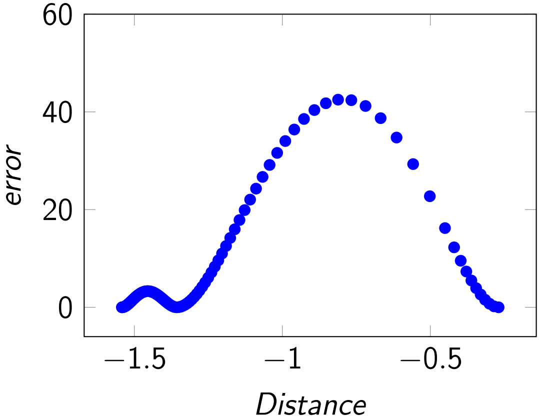
- Compare to true Pareto front



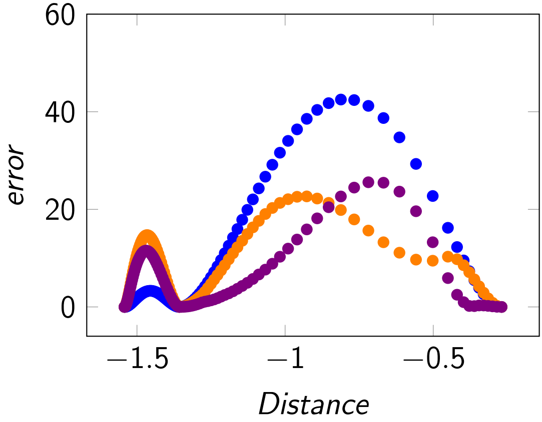
Example - Approximation errors (1)

- Calculate distance between both approximations



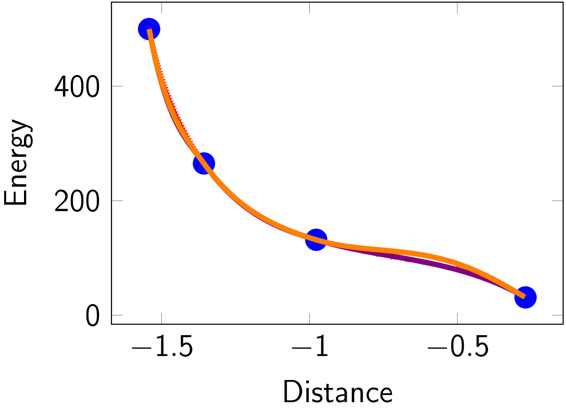
Example - Approximation errors (1)

- Calculate distance between both approximations
- Compare with approximation error made in objective space and parameter space



Example - Pareto Front Sampling (2)

- Find samples of pareto front
- Interpolate Pareto front in objective space based on sensitivities
- Interpolate Pareto front in parameter space and apply objectives
- Compare to true Pareto front



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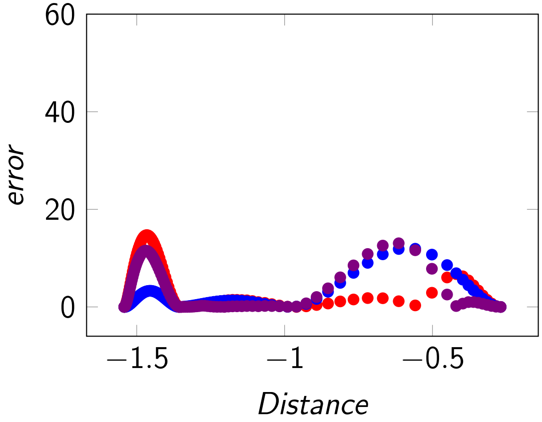
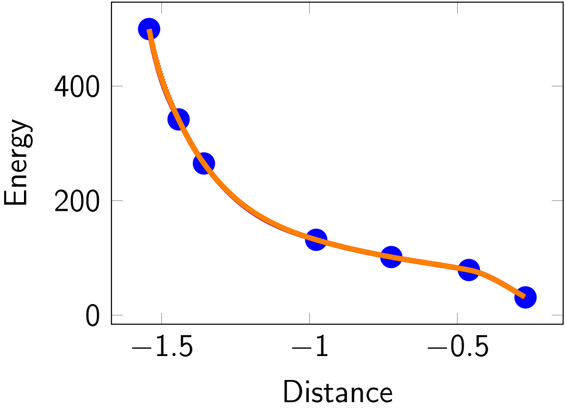


Example - Pareto Front Sampling (3)

- Find samples of pareto front
- Interpolate Pareto front in objective space based on sensitivities
- Interpolate Pareto front in parameter space and apply objectives
- Compare to true Pareto front



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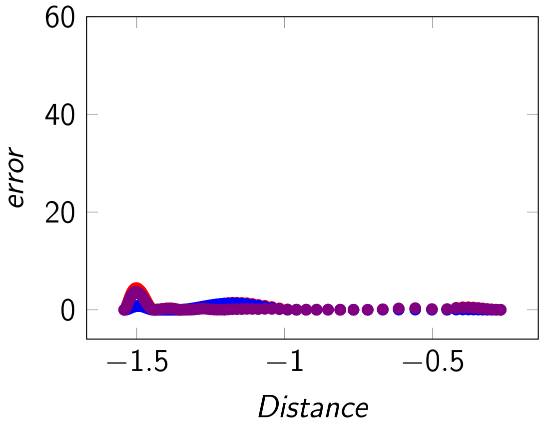

Conclusion

- Polynomial interpolation of 3rd grade for Pareto front in objective and parameter space
- Distance between approximations can be used as an indicator for approximation quality
- Adaptive stepsize refinement based on this indicator is possible
- Further research needed for "$\Leftrightarrow$"