La force de l'innovation

OPEN SOURCE ORBIT DETERMINATION WITH SEMI-ANALYTICAL THEORY

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AGENDA

- ➔Context and target
- ➔DSST presentation
- ➔Mean Elements derivatives
- ➔State Transition Matrices computation
- ➔Short-periodic terms derivatives
- ➔Orbit Determination
- ➔Conclusion

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CONTEXT AND TARGET

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OREKIT HISTORY

Inception | 2002

Orekit instended as a basis for ground segments bids

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Collaborative tools
External commiters

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Collaborative tools
External commiters

Open governance | 2012

PMC representatives from agencies, academics, private companies

OREKIT FEATURES

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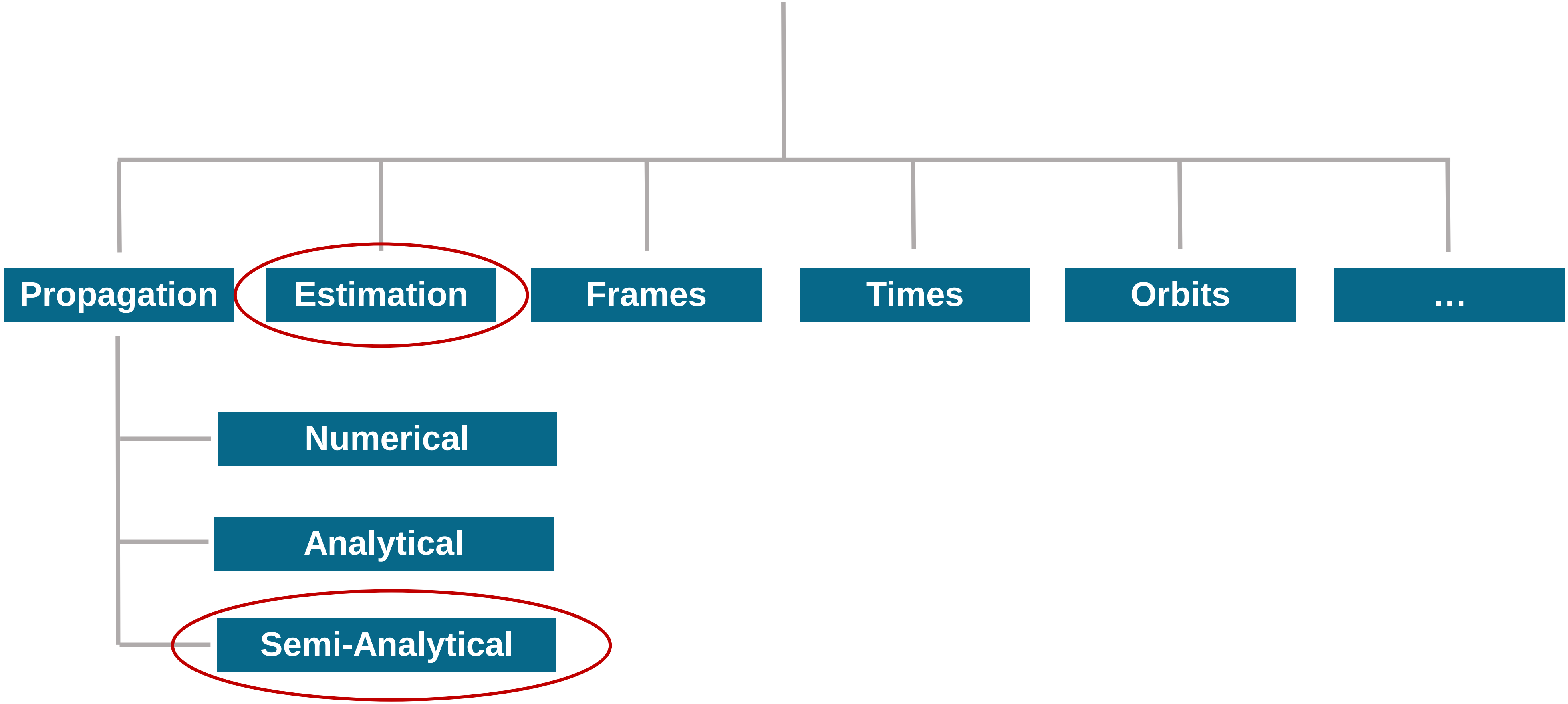
Orekit

- Propagation
  - Numerical
  - Analytical
  - Semi-Analytical
- Estimation
- Frames
- Times
- Orbits
- …

OREKIT FEATURES



TWO USE CASES

➔ **Fast Orbit Determination**

- **Several hundreds of thousands (Setty and al, 2016)**

Number of Orbit Determinations performed by the US Joint Space Operation Center per day to maintain their space objects catalog.

➔ **Need fast and accurate Orbit Determination**

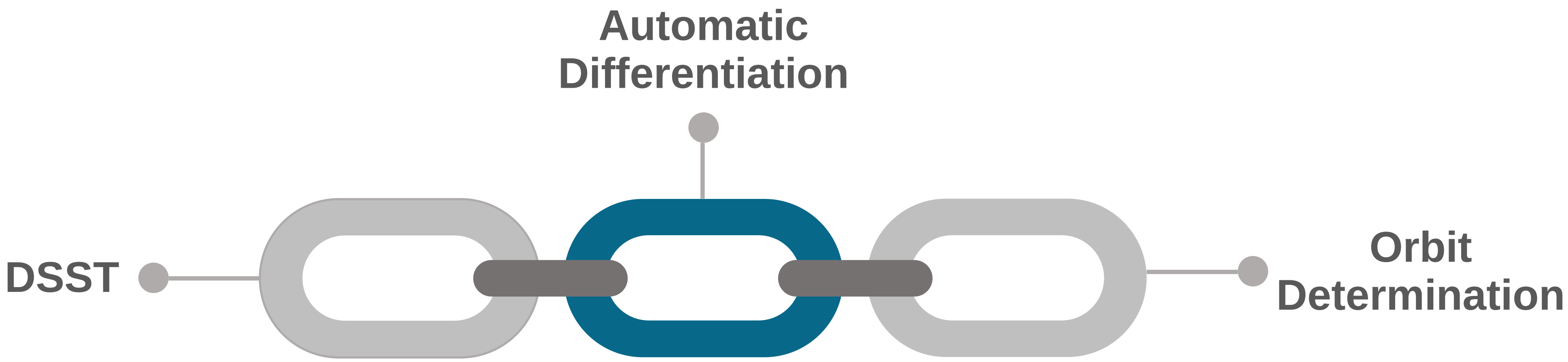
➔ **Mean Elements Orbit Determination**

➔ **Station keeping needs**

➔ Draper Semi-analytical Satellite Theory (DSST)

- **Rapidity** of an analytical propagator
- **Accuracy** of a numerical propagator

➔ Target



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DSST PRESENTATION

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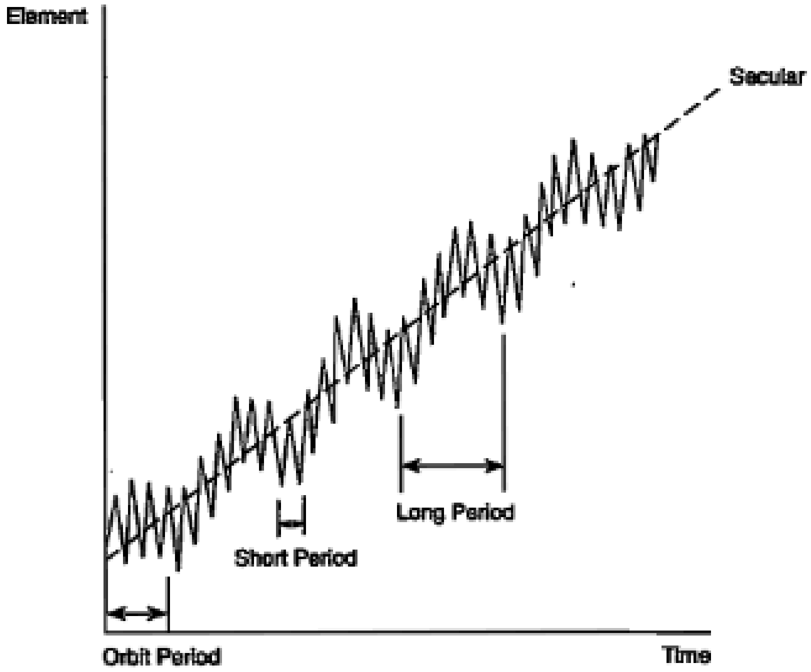
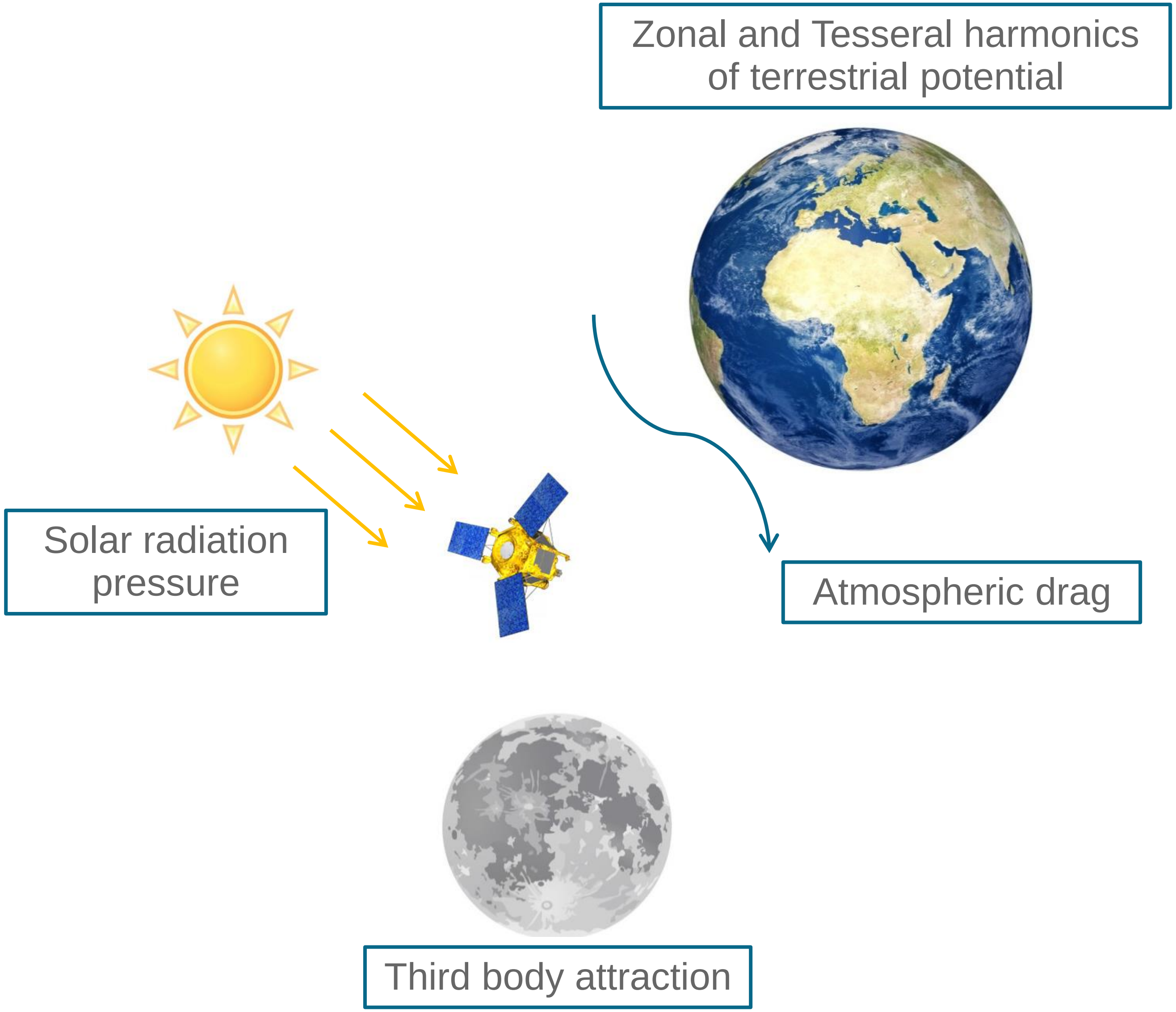
ORBITAL PERTURBATIONS





$$c_i(t) = \overline{c_i(t)} + \sum_{j=1}^{N} k_{ij}\eta_{ij}(t), \qquad i = 1, 2, 3, 4, 5, 6$$

MATHEMATICAL MODEL OF DSST

$$c_i(t) = \boxed{\overline{c_i(t)}} + \sum_{j=1}^{N} k_{ij}\eta_{ij}(t), \qquad i = 1, 2, 3, 4, 5, 6$$

Mean Elements

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Mean Elements

Short-periodic terms

DEVELOPMENT STEPS

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Force models configuration

Computation of the mean elements derivatives and the short-periodic terms derivatives by automatic differentiation.

State Transition Matrices

Computation of the State Transition Matrices thanks to the variational equations.

Orbit Determination

Perform the DSST-OD with the Orekit's Batch Least Squares algorithm and the Kalman Filter.

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MEAN ELEMENTS DERIVATIVES

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➔GOAL

$$\begin{bmatrix} Y_i & \frac{\partial Y_i}{\partial Y_1} & \frac{\partial Y_i}{\partial Y_2} & \cdots & \frac{\partial Y_i}{\partial Y_6} & \frac{\partial Y_i}{\partial P_1} & \cdots & \frac{\partial Y_i}{\partial P_N} \end{bmatrix}$$

- **Y_i:** Orbital element
- **P_k:** Force model parameter
- **N:** The number of force model parameters taken into account for the Orbit Determination

➔GAIN

- Safer implementation
- Simpler validation

MEAN ELEMENTS DERIVATIVES 1/2

- Each DSST-specific force model on Orekit has a method allowing the computation of the mean elements rates.

$$\mathbf{Y} = [\mathbf{a}\ \mathbf{h}\ \mathbf{k}\ \mathbf{p}\ \mathbf{q}\ \boldsymbol{\lambda}] \longrightarrow \dot{\mathbf{Y}} = \begin{bmatrix} \dot{\mathbf{a}} \\ \dot{\mathbf{h}} \\ \dot{\mathbf{k}} \\ \dot{\mathbf{p}} \\ \dot{\mathbf{q}} \\ \dot{\boldsymbol{\lambda}} \end{bmatrix}$$

- Method implemented for the states based on the real numbers ✔

- Need to be implemented to provide the Jacobians of the mean elements rates by automatic differentiation.

MEAN ELEMENTS DERIVATIVES 2/2

$$\mathbf{Y} = [\mathbf{a}\ \mathbf{h}\ \mathbf{k}\ \mathbf{p}\ \mathbf{q}\ \boldsymbol{\lambda}]$$

MEAN ELEMENTS DERIVATIVES 2/2

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Mean Elements Rates

$$\dot{\mathbf{Y}} = \begin{bmatrix} \dot{\mathbf{a}} \\ \dot{\mathbf{h}} \\ \dot{\mathbf{k}} \\ \dot{\mathbf{p}} \\ \dot{\mathbf{q}} \\ \dot{\boldsymbol{\lambda}} \end{bmatrix}$$

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Mean Elements Rates

$$\dot{\mathbf{Y}} = \begin{bmatrix} \dot{a} \\ \dot{h} \\ \dot{k} \\ \dot{p} \\ \dot{q} \\ \dot{\lambda} \end{bmatrix}$$

Automatic Differentiation

$$\dot{\mathbf{Y}}' = \begin{bmatrix} \dot{a} & \partial_a \dot{a} & \partial_h \dot{a} & \dots & \partial_\lambda \dot{a} & \partial_{P_1} \dot{a} & \dots & \partial_{P_N} \dot{a} \\ \dot{h} & \partial_a \dot{h} & \partial_h \dot{h} & \dots & \partial_\lambda \dot{h} & \partial_{P_1} \dot{h} & \dots & \partial_{P_N} \dot{h} \\ \vdots & \vdots & \vdots & & \vdots & \vdots & & \vdots \\ \dot{\lambda} & \partial_a \dot{\lambda} & \partial_h \dot{\lambda} & \dots & \partial_\lambda \dot{\lambda} & \partial_{P_1} \dot{\lambda} & \dots & \partial_{P_N} \dot{\lambda} \end{bmatrix}$$

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STATE TRANSITION MATRICES COMPUTATION

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JACOBIANS OF MEAN ELEMENTS RATES

$$\dot{\mathbf{Y}}' = \begin{bmatrix} \dot{\mathbf{a}} & \partial_{\mathbf{a}}\dot{\mathbf{a}} & \partial_{\mathbf{h}}\dot{\mathbf{a}} & \cdots & \partial_{\lambda}\dot{\mathbf{a}} & \partial_{\mathbf{P_1}}\dot{\mathbf{a}} & \cdots & \partial_{\mathbf{P_N}}\dot{\mathbf{a}} \\ \dot{\mathbf{h}} & \partial_{\mathbf{a}}\dot{\mathbf{h}} & \partial_{\mathbf{h}}\dot{\mathbf{h}} & \cdots & \partial_{\lambda}\dot{\mathbf{h}} & \partial_{\mathbf{P_1}}\dot{\mathbf{h}} & \cdots & \partial_{\mathbf{P_N}}\dot{\mathbf{h}} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \dot{\lambda} & \partial_{\mathbf{a}}\dot{\lambda} & \partial_{\mathbf{h}}\dot{\lambda} & \cdots & \partial_{\lambda}\dot{\lambda} & \partial_{\mathbf{P_1}}\dot{\lambda} & \cdots & \partial_{\mathbf{P_N}}\dot{\lambda} \end{bmatrix}$$

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$\dot{\mathbf{Y}}$

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$$\dot{\mathbf{Y}}$$

$$\frac{\partial \dot{\mathbf{Y}}}{\partial \mathbf{Y}}$$

JACOBIANS OF MEAN ELEMENTS RATES

$$
\dot{\mathbf{Y}}' = \left[\begin{array}{c|cccc|ccc}
\dot{a} & \partial_a \dot{a} & \partial_h \dot{a} & \dots & \partial_\lambda \dot{a} & \partial_{P_1} \dot{a} & \dots & \partial_{P_N} \dot{a} \\
\dot{h} & \partial_a \dot{h} & \partial_h \dot{h} & \dots & \partial_\lambda \dot{h} & \partial_{P_1} \dot{h} & \dots & \partial_{P_N} \dot{h} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
\dot{\lambda} & \partial_a \dot{\lambda} & \partial_h \dot{\lambda} & \dots & \partial_\lambda \dot{\lambda} & \partial_{P_1} \dot{\lambda} & \dots & \partial_{P_N} \dot{\lambda}
\end{array}\right]
$$

$\dot{\mathbf{Y}}$

$\frac{\partial \dot{\mathbf{Y}}}{\partial \mathbf{Y}}$

$\frac{\partial \dot{\mathbf{Y}}}{\partial \mathbf{P}}$

APPLICATION OF THE VARIATIONAL EQUATIONS

$$\frac{\mathbf{d}\left(\frac{\partial \mathbf{Y}}{\partial \mathbf{Y_0}}\right)}{\mathbf{dt}} = \frac{\partial \dot{\mathbf{Y}}}{\partial \mathbf{Y}} \times \frac{\partial \mathbf{Y}}{\partial \mathbf{Y_0}}$$

$$\frac{\mathbf{d}\left(\frac{\partial \mathbf{Y}}{\partial \mathbf{P}}\right)}{\mathbf{dt}} = \frac{\partial \dot{\mathbf{Y}}}{\partial \mathbf{Y}} \times \frac{\partial \mathbf{Y}}{\partial \mathbf{P}} + \frac{\partial \dot{\mathbf{Y}}}{\partial \mathbf{P}}$$

VALIDATION

- Computation of $\frac{\partial Y}{\partial Y_0}$ and $\frac{\partial Y}{\partial P}$ matrices by finite differences and comparison to those previously obtained.

- Computation of $\frac{\partial \mathbf{Y}}{\partial \mathbf{Y_0}}$ and $\frac{\partial \mathbf{Y}}{\partial \mathbf{P}}$ matrices by finite differences and comparison to those previously obtained.

Problem : Different matrices !

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- Newtonian Attraction derivatives were not taken into account in the computation of the state transition matrices.
- Some dependencies to the central attraction coefficient were implicit and therefore not differentiated.

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- Newtonian Attraction derivatives were not taken into account in the computation of the state transition matrices.
- Some dependencies to the central attraction coefficient were implicit and therefore not differentiated.

Problem solved ✔

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SHORT-PERIODIC TERMS DERIVATIVES

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SHORT-PERIODIC TERMS DERIVATIVES

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Automatic Differentiation

Compute the Jacobians of the short-periodic terms into the DSST-specific force models

Addition of the contribution

Add the contribution of the short-periodic derivatives after the numerical integration of the mean elements rates

Validation

Compute the state transition matrices (with the short-periodic derivatives) by finite differences

SHORT-PERIODIC TERMS DERIVATIVES

Automatic Differentiation

Compute the Jacobians of the short-periodic terms into the DSST-specific force models

Validation

Compute the state transition matrices (with the short-periodic derivatives) by finite differences

Addition of the contribution

Add the contribution of the short-periodic derivatives after the numerical integration of the mean elements rates

The user has the choice to use only the mean elements derivatives or adding the short-periodic terms derivatives

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ORBIT DETERMINATION

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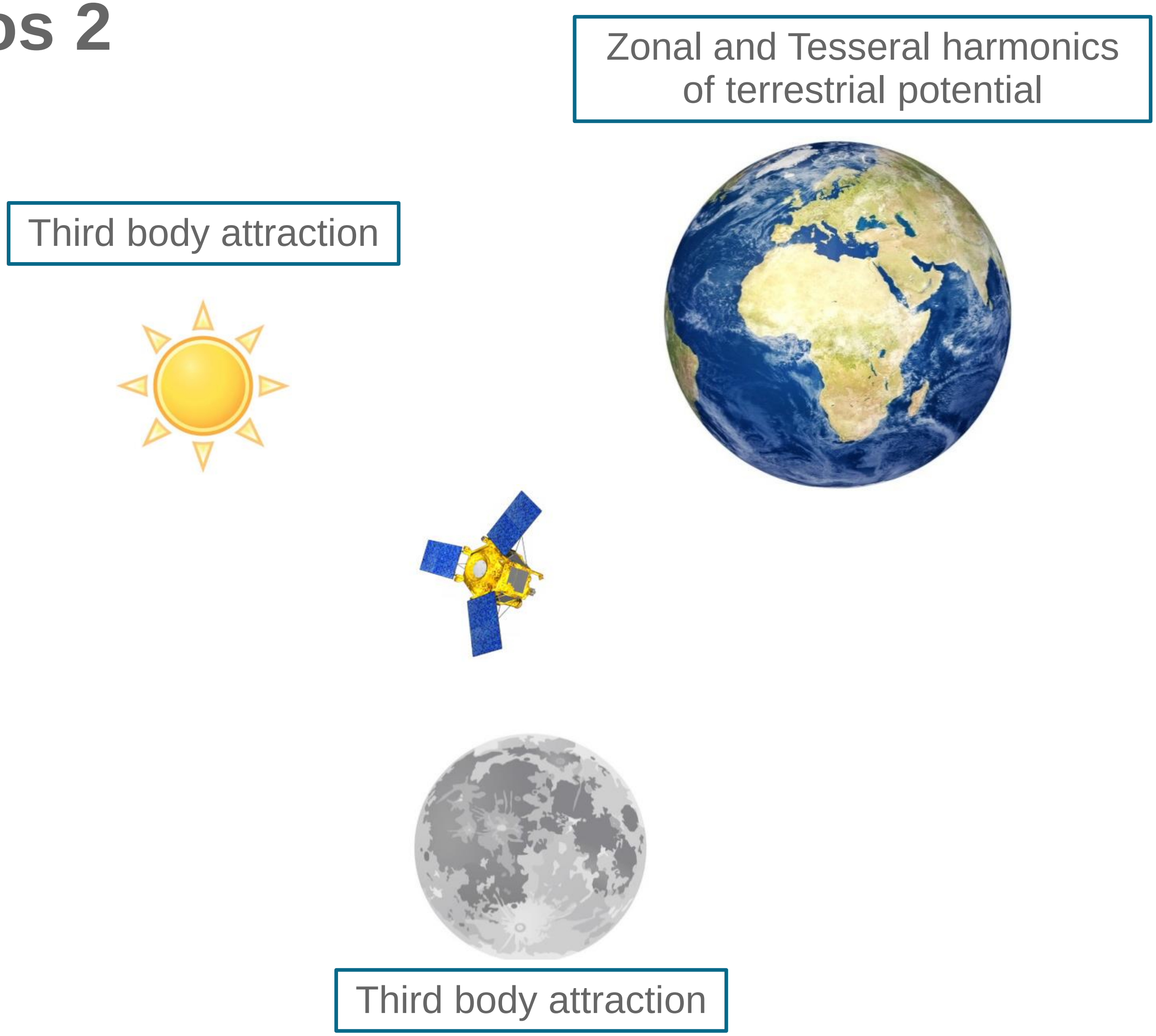
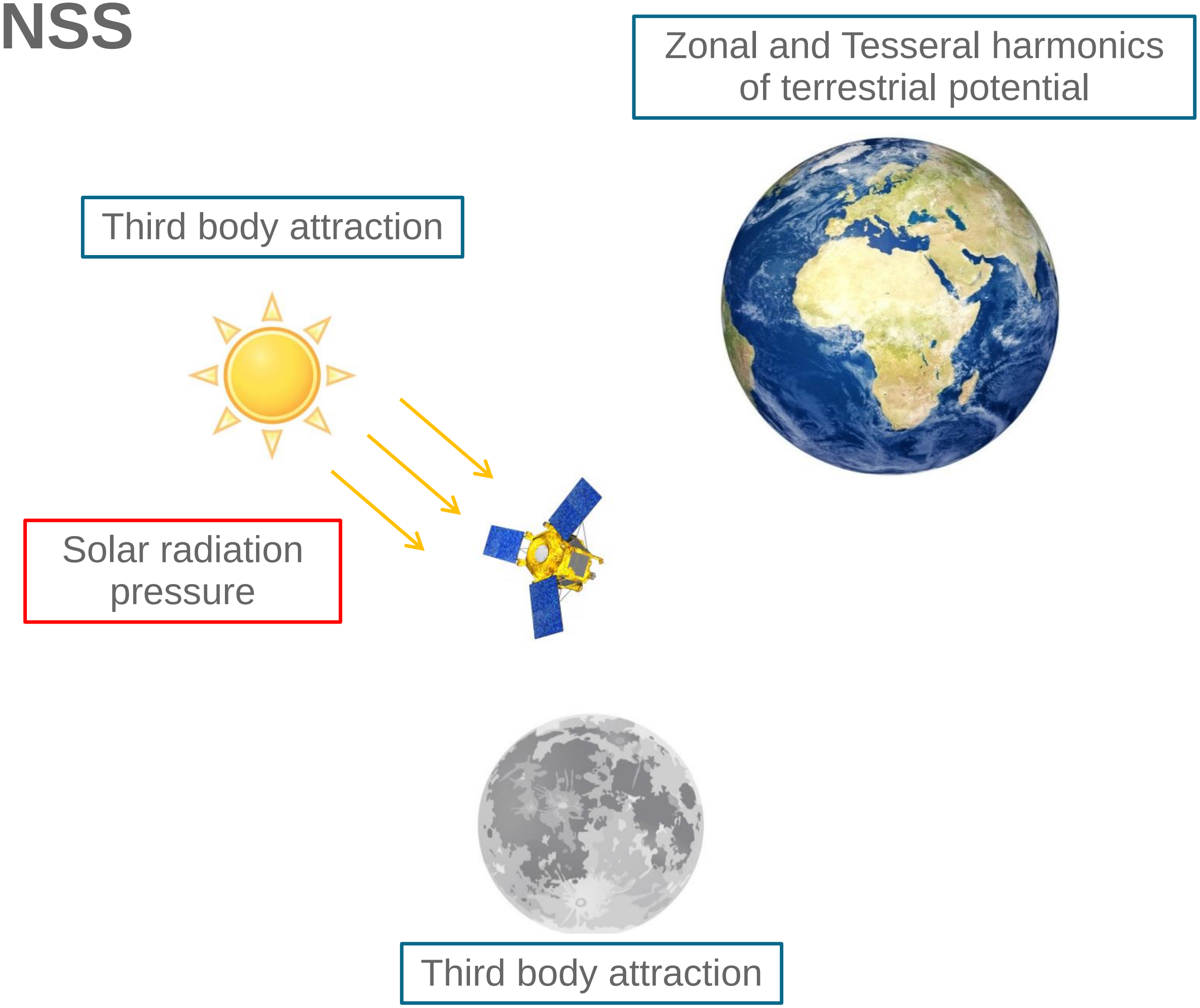
FORCE MODELS USED: LAGEOS 2

Lageos 2



FORCE MODELS USED: GNSS

GNSS



TEST CASES: INTEGRATION STEP

DSST

Minimum step (s)	6000
Maximum step (s)	86400
Tolerance (m)	10

Numerical

Minimum step (s)	0,001
Maximum step (s)	300
Tolerance (m)	10

➔ The DSST has **significant advantage** compared to the numerical propagator for the integration step. This because the elements computed numerically by the DSST are the mean elements.

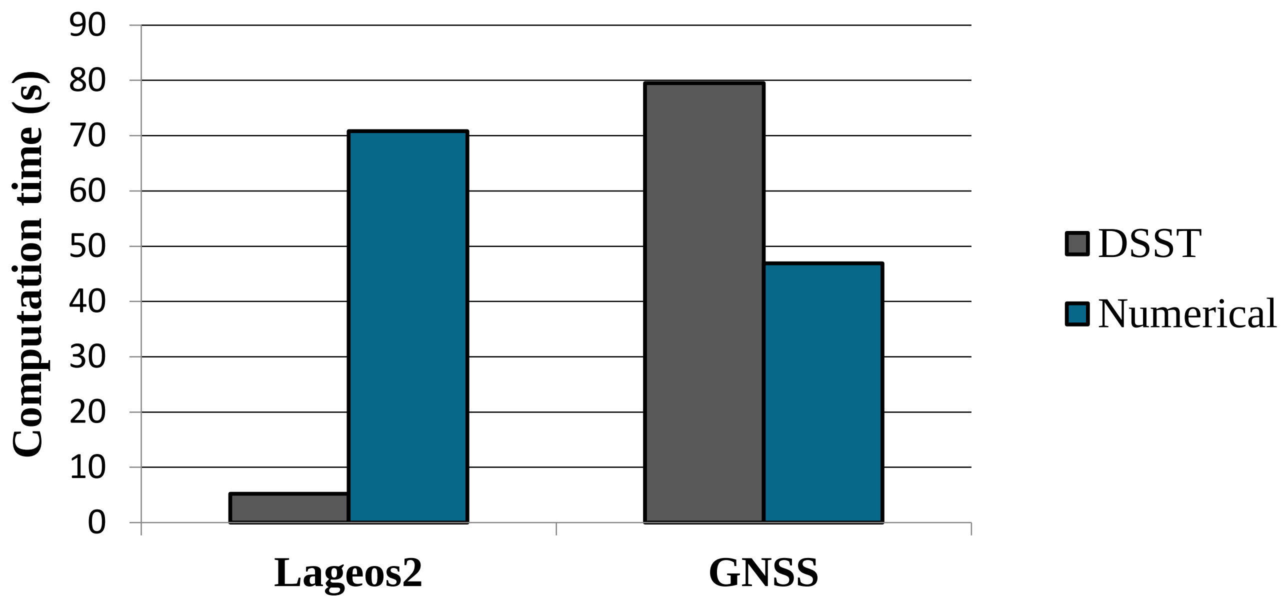
TEST CASES: SHORT-PERIODIC TERMS DERIVATIVES

Case	Zonal	Tesseral	Third Body
1	×	×	×
2	✓	×	×
3	✓	✓	×
4	✓	✓	✓

➔ Gradual addition of the short-periodic terms derivatives to highlight the main contributions .

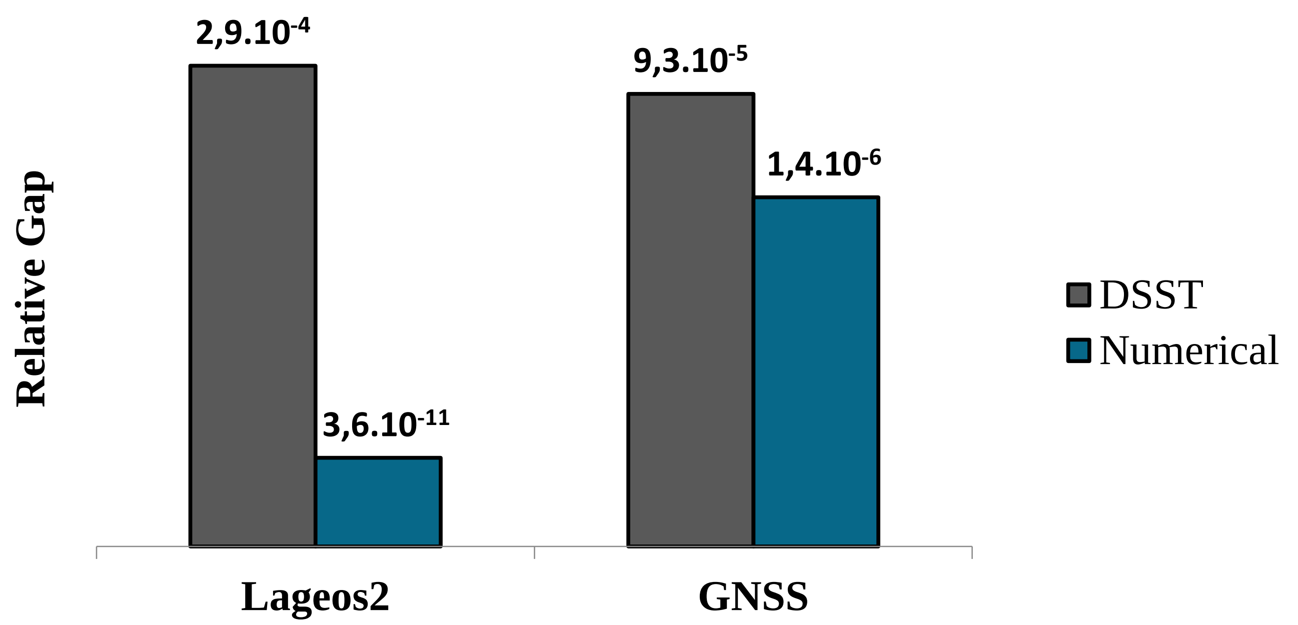
➔ Performed tests for Lageos2 Orbit Determination.

BATCH LEAST SQUARES / MEAN ELEMENTS

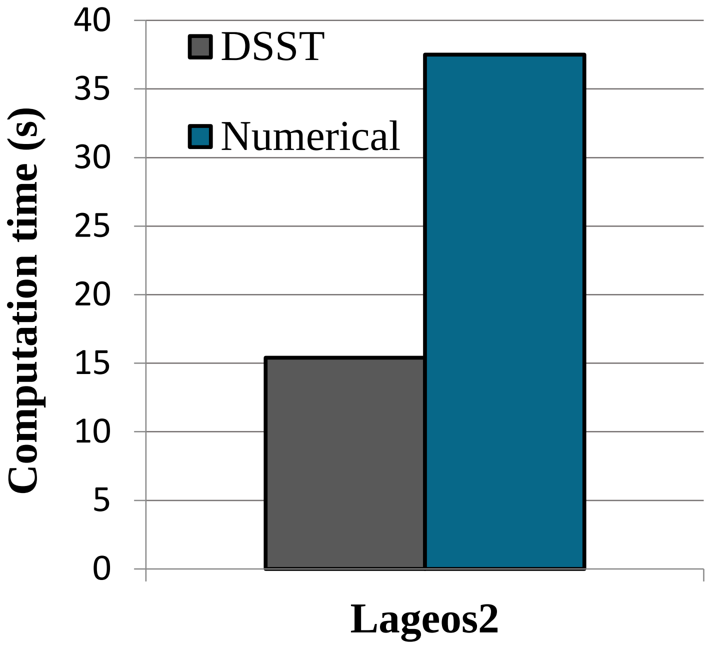


RAPIDITY

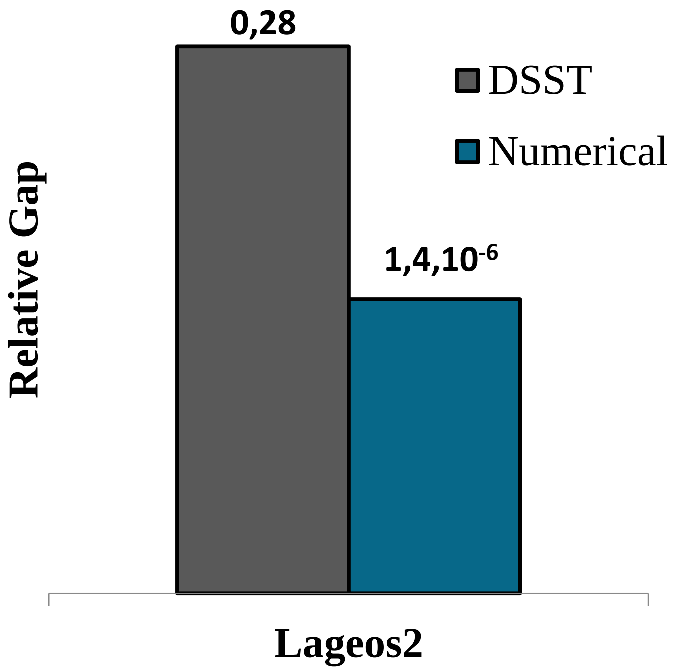
ACCURACY



KALMAN FILTER / MEAN ELEMENTS

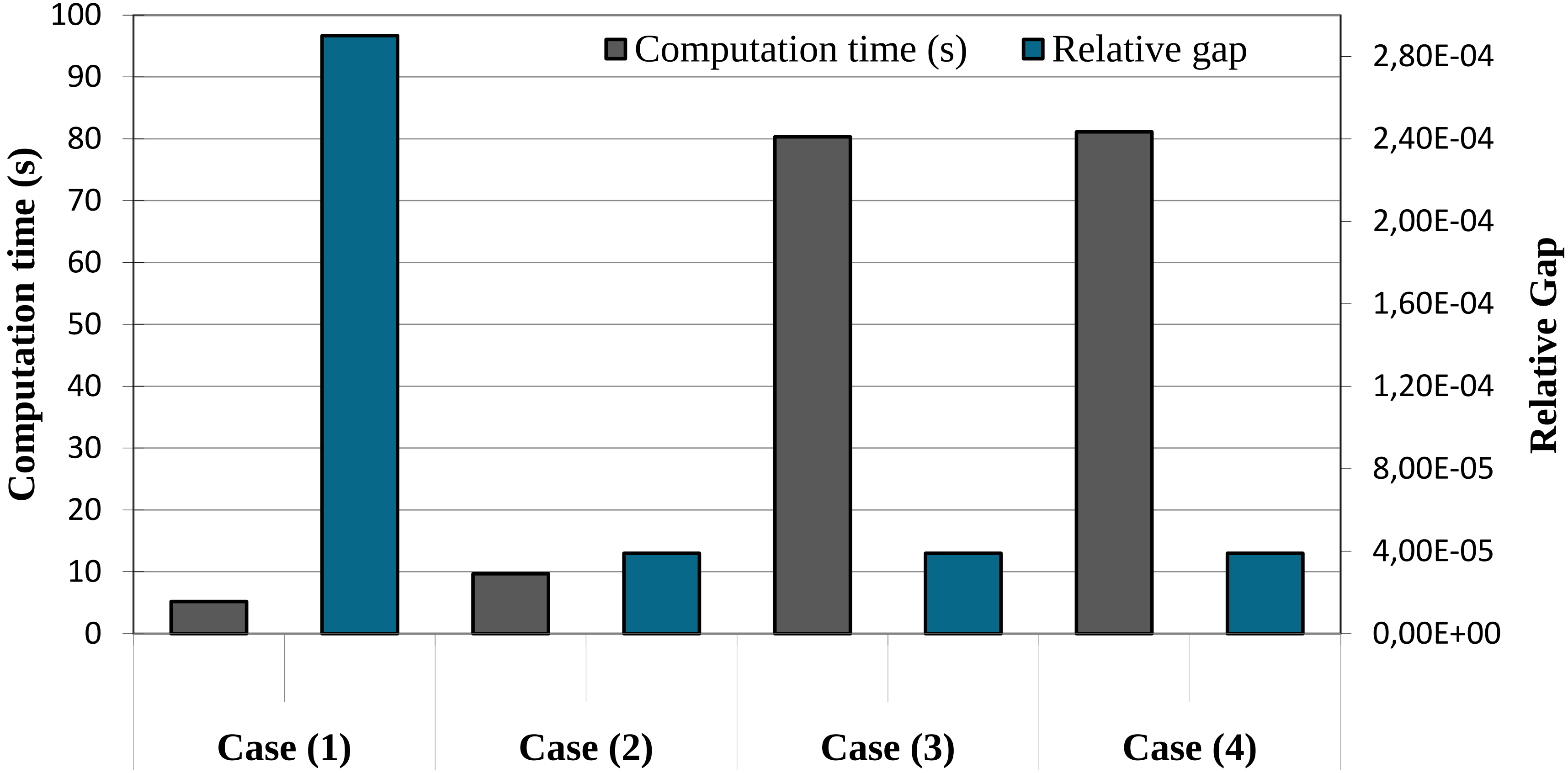


RAPIDITY

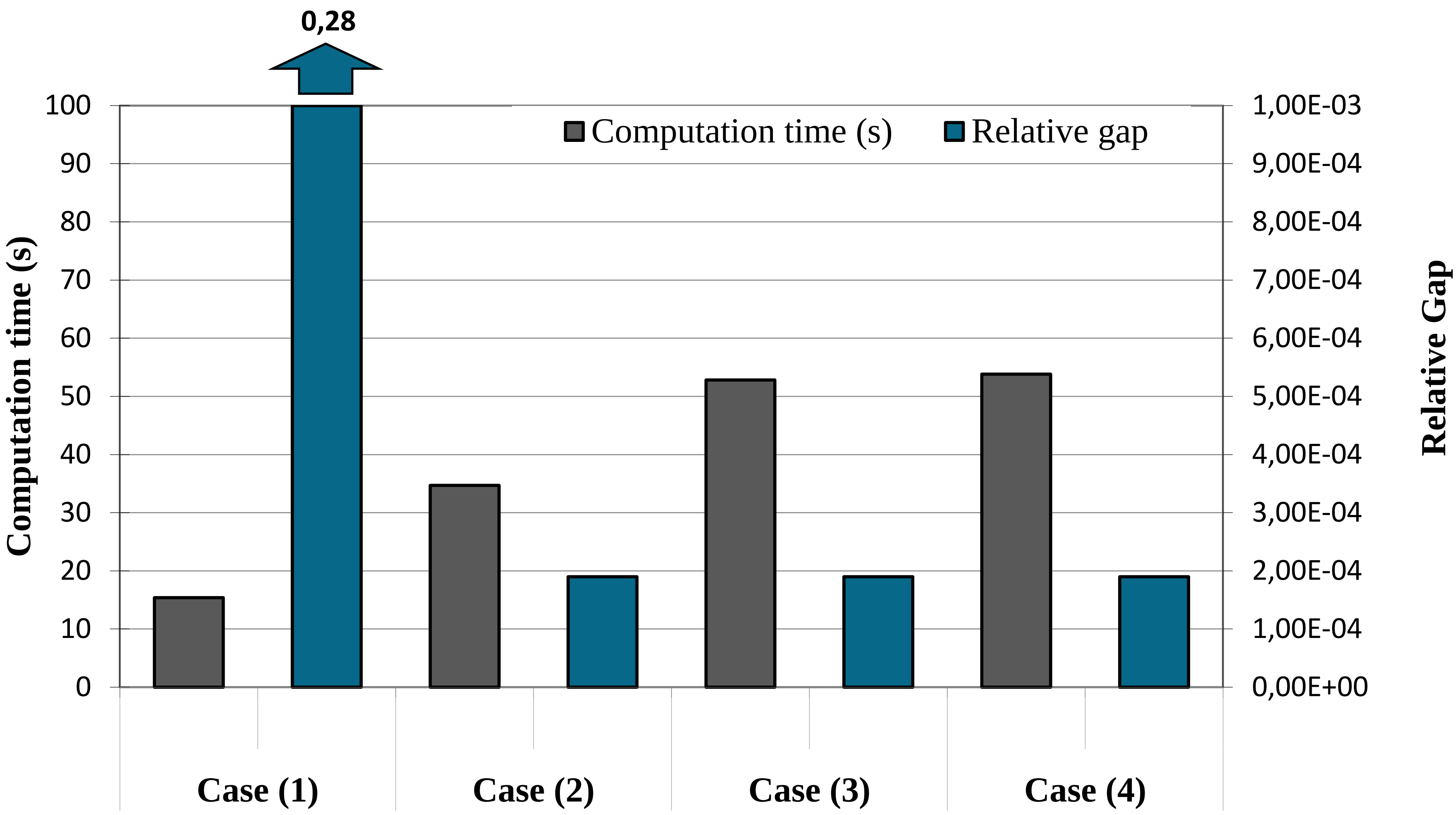


ACCURACY

BATCH LEAST SQUARES / LAGEOS2 SP DERIVATIVES



KALMAN FILTER / LAGEOS2 SP DERIVATIVES



CONCLUSION

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DSST

OUTLOOKS

Need acces to « optimal » values for force models initialization

DSST

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Need acces to « optimal » values for force models initialization

Need to optimize critical computation steps.

DSST

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DSST

Improve the Kalman Filter Orbit Determination with the DSST.

Thank you for your attention