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# Trajectory Design in High-Fidelity Models

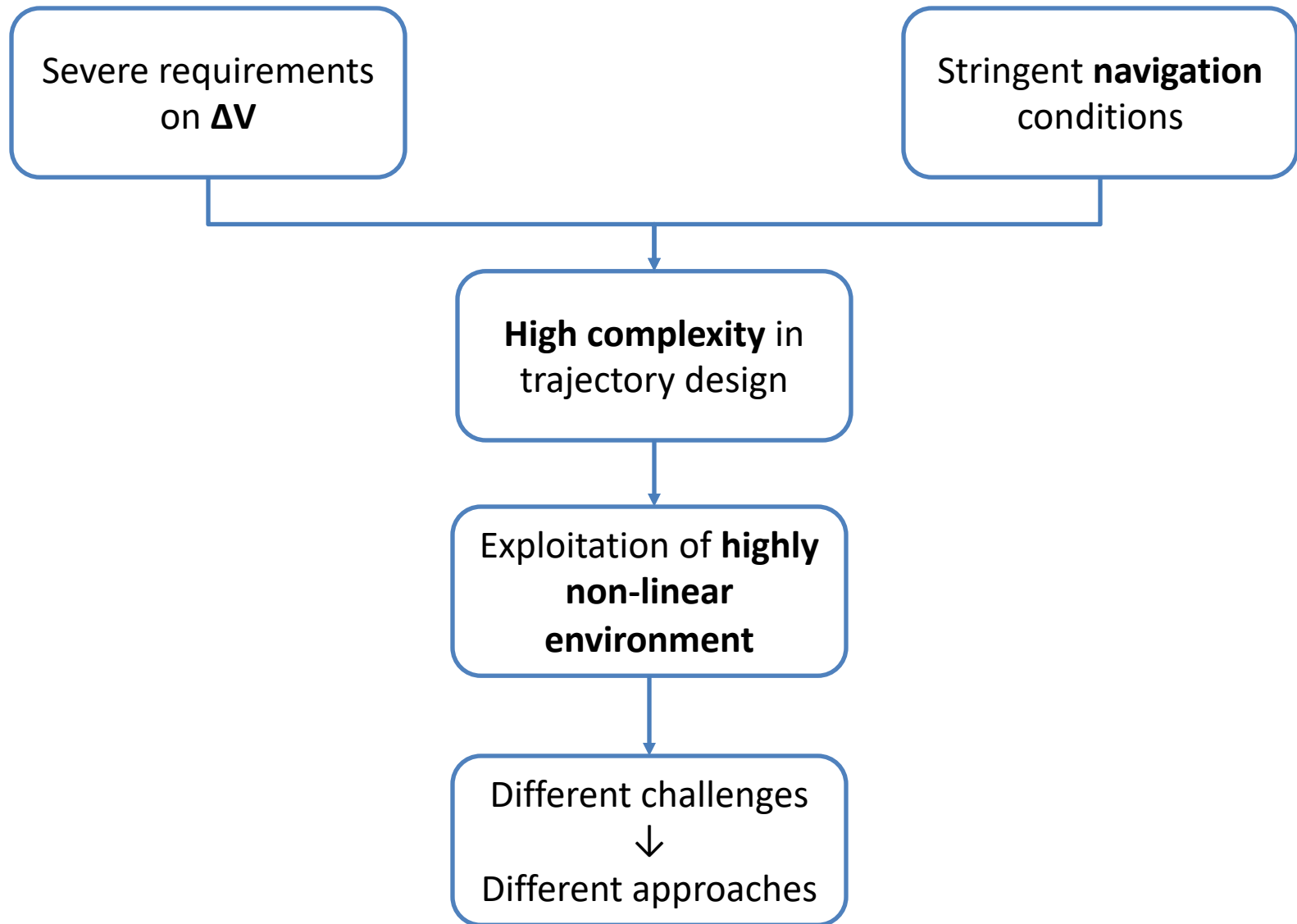
F. TOPPUTO, D.A. DEI TOS, K. V. MANI, S. CECCHERINI, **C. GIORDANO**, V. FRANZESE, Y. WANG

Department of Aerospace Science and Technology, Politecnico di Milano, Italia

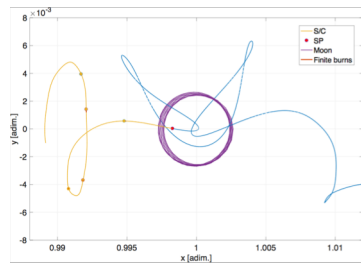
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Francesco Topputo

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[francesco.topputo@polimi.it](mailto:francesco.topputo@polimi.it)

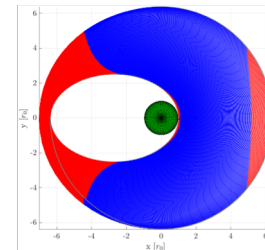
- Introduction
- General Overview
- PoliMi astrodynamics tools
  - ULTIMAT (Ultra Low-Thrust Interplanetary Mission Analysis Tool)
  - LT20 (Low-thrust Trajectory Optimization)
  - DIRETTO (DIREct collocation tool for Trajectory Optimization)
  - GRATIS (GRAvity TIdal Slide)
- Conclusions and future work



Limited-control  
authority spacecraft  
**ULTIMAT**

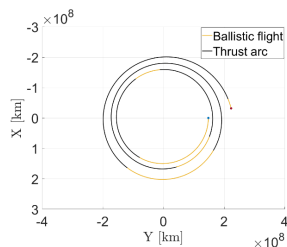


Low-thrust multispiral  
trajectories  
**LT20**

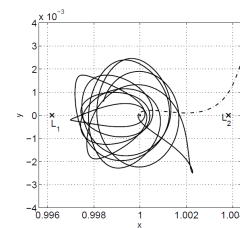


**CHALLENGES IN  
SPACE TRAJECTORY  
DESIGN**

Direct trajectory  
optimization  
**DIRETTO**



Highly non-linear  
dynamics exploitation  
**GRATIS**





## Ultra Low Thrust Interplanetary Mission Analysis Tool

### Why?

- Limited capability spacecrafts
- Impulsive, low thrust
- High navigation accuracy

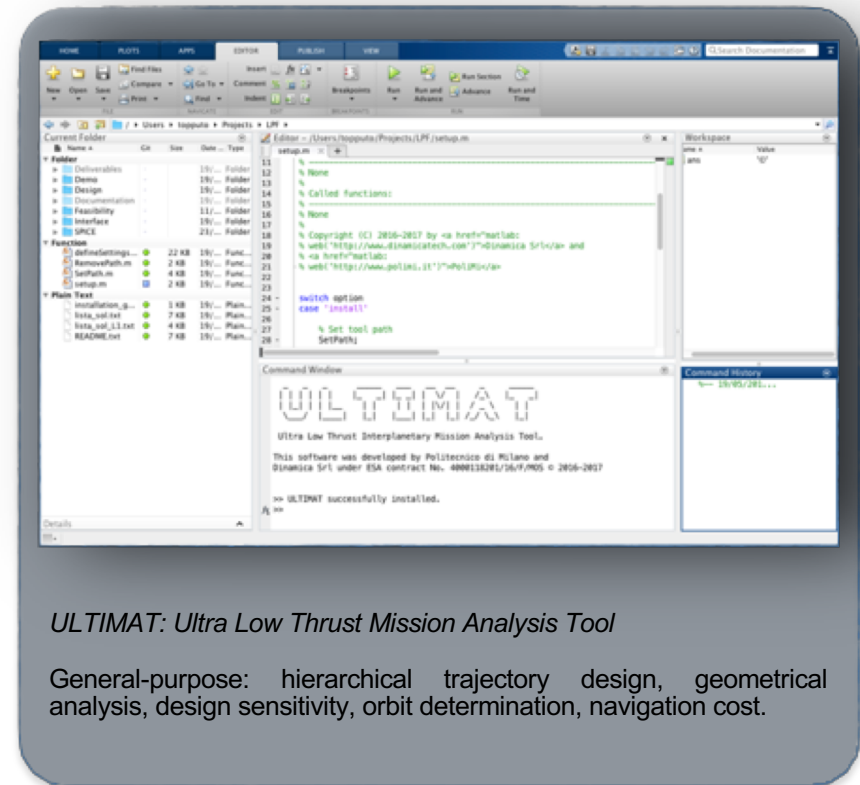
### Features

- Written in Matlab
- SPICE fully integrated
- Extensively tested
- Propagators validated (GMAT)

*25k lines, 270 files*

Updates managed with GIT

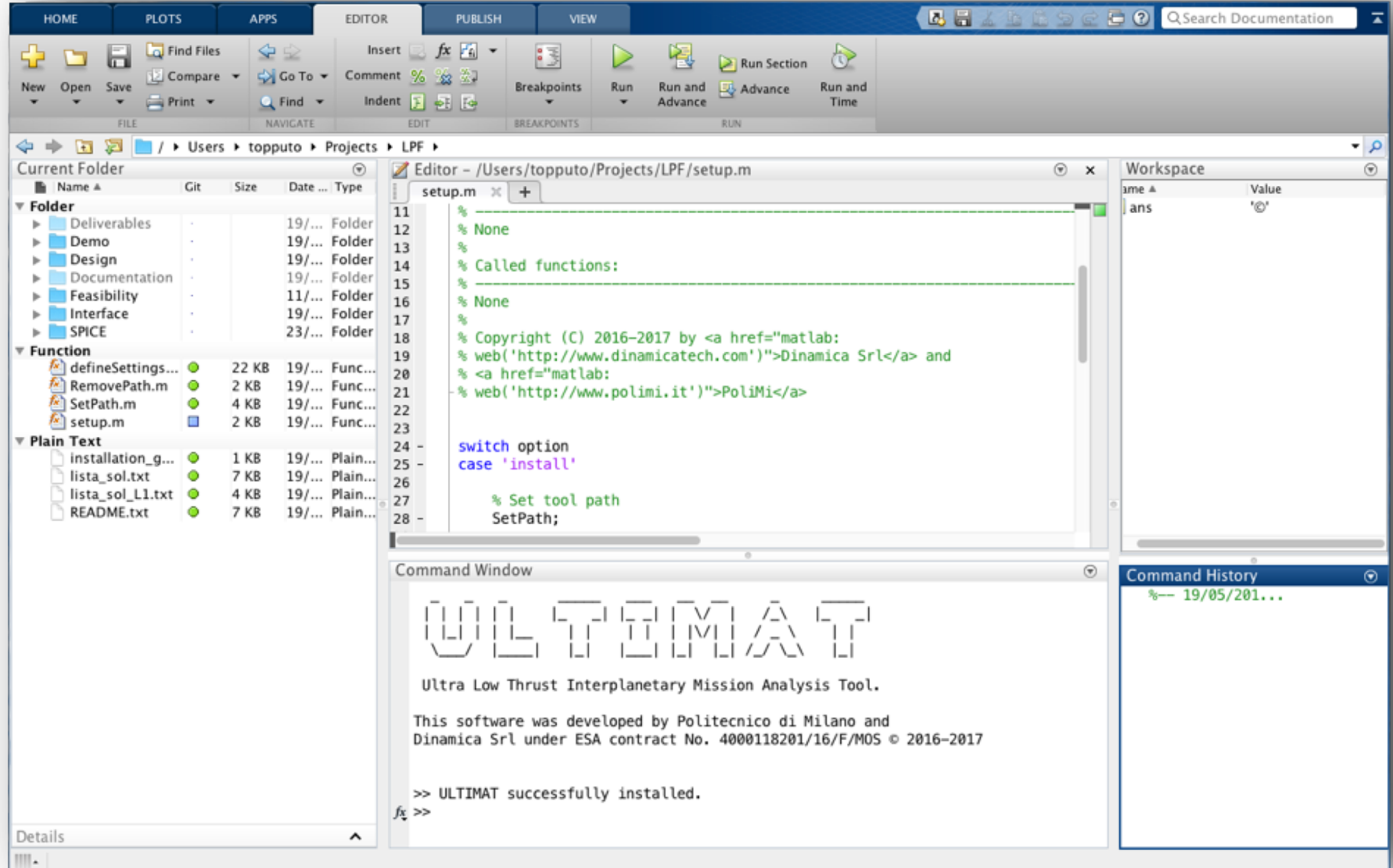
Completely **developed from scratch** (Oct 2016-Nov 2018)  
Kick off given by **ESA Contract No. 4000118201/16/F/MOS**

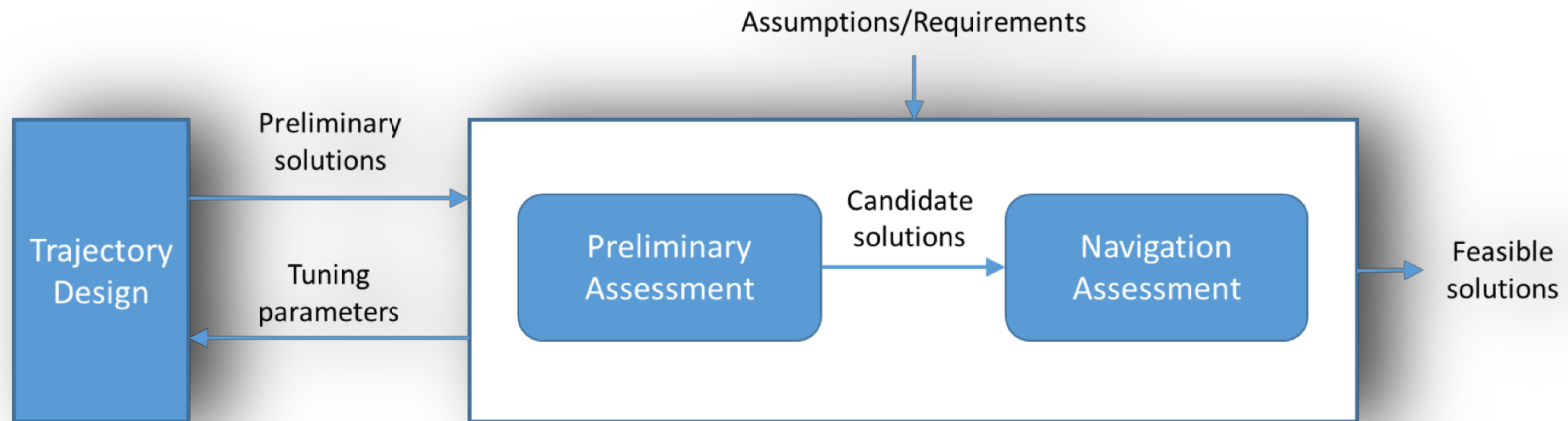


*ULTIMAT: Ultra Low Thrust Mission Analysis Tool*

General-purpose: hierarchical trajectory design, geometrical analysis, design sensitivity, orbit determination, navigation cost.

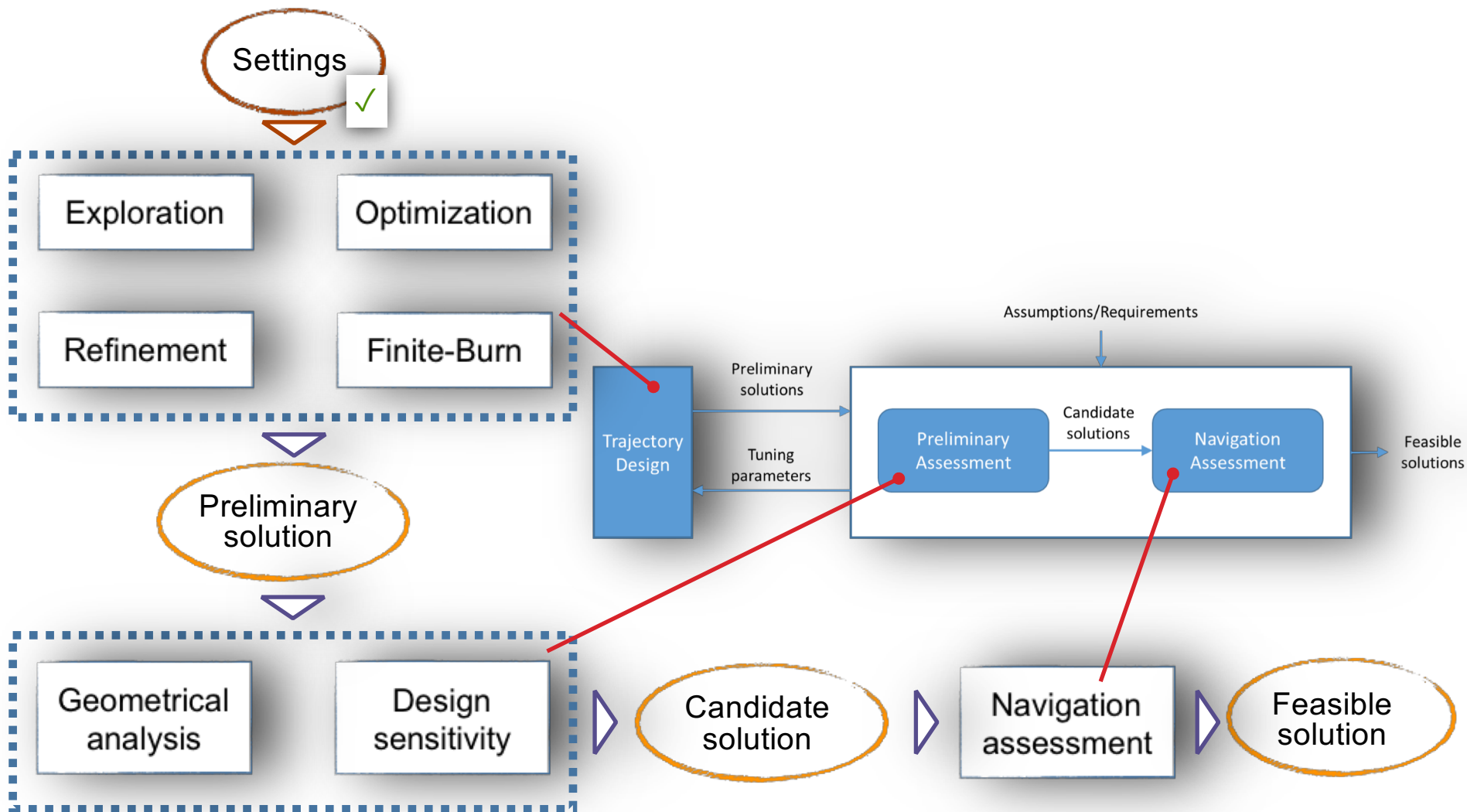
# Look and feel



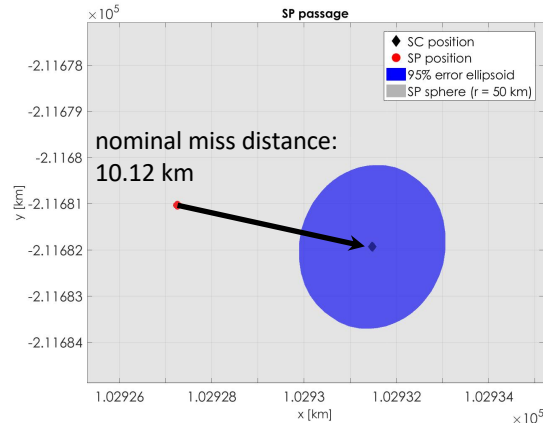
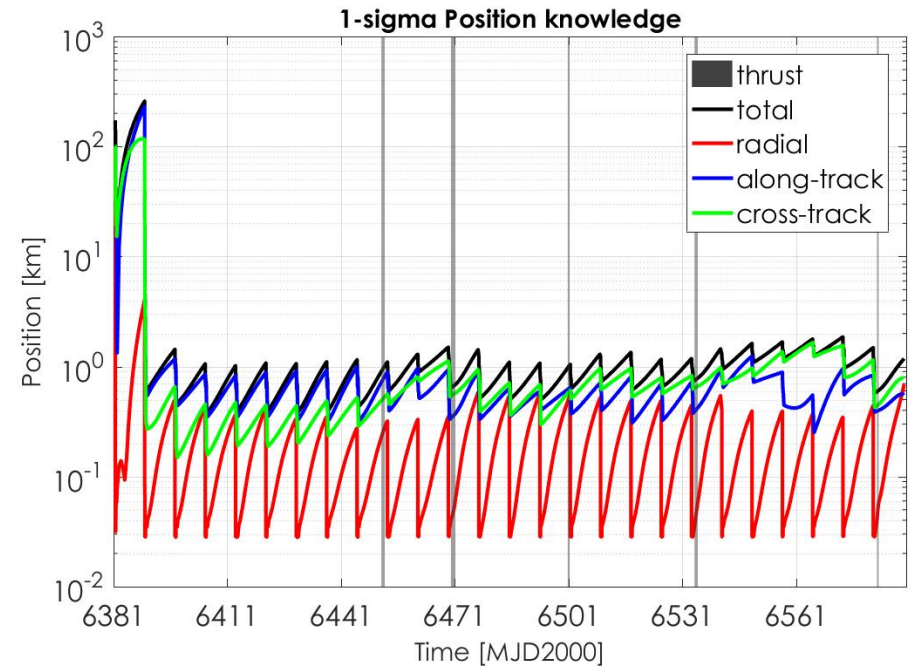
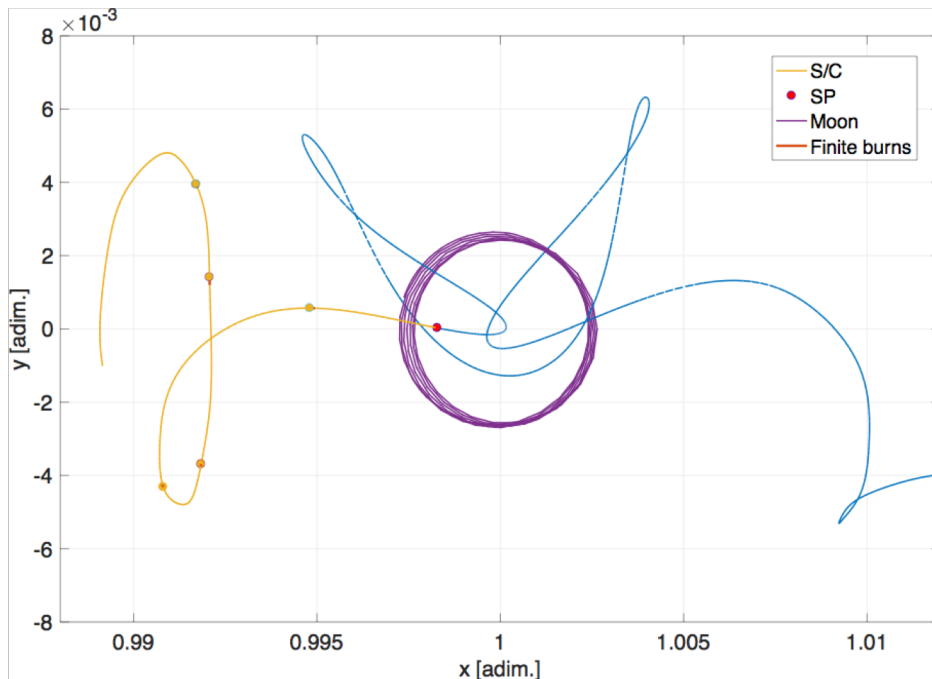


- **Design.** Developing *methods* and *tools* to derive orbits
  - Defined in highly nonlinear vector fields
  - Having very limited control authority
- **Navigation.** Developing *methods* and *tools* to assess feasibility of highly nonlinear, limited control orbits
  - High fidelity models for ground and space systems
  - Requirements needed to fly such orbits

# Workflow



# LPF-to-SP mission extension example



- Original, state-of-the-art trajectory design methodologies formulated and implemented
- Hundreds of potentially applicable solutions derived
- Parametric analyses for feasibility assessment performed
- General purpose tool developed, can be used for future similar applications

# Project repository > Structure

Project **Repository** Registry Issues 0 Merge Requests 0 Pipelines Wiki Settings

**Files** Commits Branches Tags Contributors Graph Compare Charts Locked Files

master

LPF /



Find file



Name	Last commit > <a href="#">d6c96b36</a> about 3 hours ago - small mods <a href="#">History</a>	Last Update
Deliverables	ISTS class mods	6 days ago
Demo	minor changes in main_demo	6 days ago
Design	small mods	about 3 hours ago
Documentation	Added setup function to install and uninstall the tool and updated documentati...	3 days ago
Feasibility	Improved organization of solutions in Interface subfolders	5 days ago
Interface	LPF refined solutions	2 days ago
SPICE	small mods to DEMO	a week ago
.gitignore	non ignore separate sols	a week ago
README.txt	Added setup function to install and uninstall the tool and updated documentati...	3 days ago
RemovePath.m	Added setup function to install and uninstall the tool and updated documentati...	3 days ago
SetPath.m	Added Documentation folder to the list of folders to be ignored in SetPath.m	6 days ago
defineSettings.m	small mods	about 3 hours ago
installation_guide.txt	Added setup function to install and uninstall the tool and updated documentati...	3 days ago

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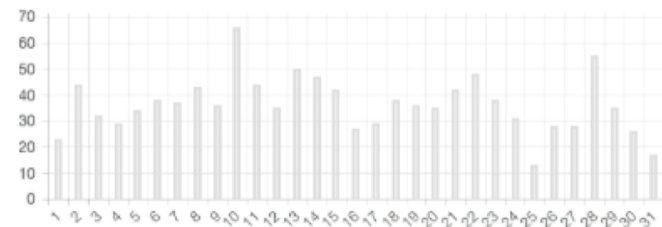
## Commit statistics for master Aug 30 - Oct 17

- Total: **1126 commits**
- Average per day: **1.4 commits**
- Authors: **14**

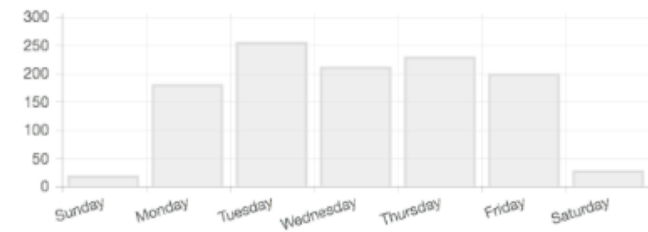
master

LPF

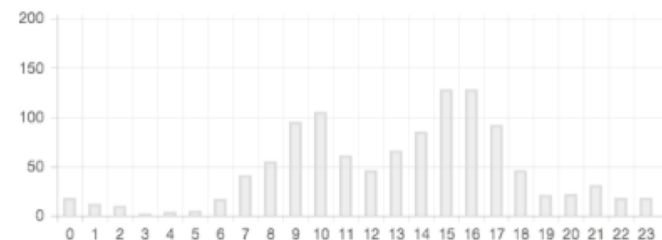
Commits per day of month



Commits per weekday

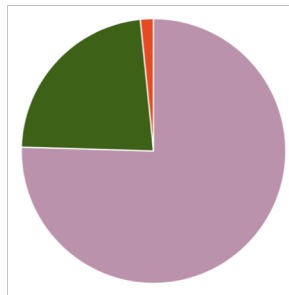


Commits per day hour (UTC)



### Programming languages used in this repository

Matlab	75.48 %
TeX	22.92 %
HTML	1.61 %
Objective-C	0.0 %





## Extensive documentation in html

### Matlab Directories

- 📁 ULTIMAT
- 📁 ULTIMAT/Design/Common
- 📁 ULTIMAT/Design/Exploration/EvolutionaryAlgorithm/EA\_R
- 📁 ULTIMAT/Design/Exploration/GridSearch/GS\_RFBP
- 📁 ULTIMAT/Design/Exploration/GridSearch/GS\_RNBP-RPF
- 📁 ULTIMAT/Design/Exploration/GridSearch/GS\_RTBP
- 📁 ULTIMAT/Design/Exploration/LPF
- 📁 ULTIMAT/Design/FiniteBurn/FBM\_RFBP
- 📁 ULTIMAT/Design/FiniteBurn/FBM\_RFBP/Integration
- 📁 ULTIMAT/Design/FiniteBurn/FBM\_RNBP-RPF
- 📁 ULTIMAT/Design/Models/EME2000
- 📁 ULTIMAT/Design/Models/RFBP/Integration
- 📁 ULTIMAT/Design/Models/RFBP/Transformations
- 📁 ULTIMAT/Design/Models/RNBP-RPF/Integration
- 📁 ULTIMAT/Design/Models/RNBP-RPF/Transformations
- 📁 ULTIMAT/Design/Models/RNBP-RPF/testing\_RNBP-RPF
- 📁 ULTIMAT/Design/Models/RTBP/Basics
- 📁 ULTIMAT/Design/Models/RTBP/Integration
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- 📁 ULTIMAT/Design/Models/RTBP/Orbits
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- 📁 ULTIMAT/Design/SaddlePoint/SP\_RTBP
- 📁 ULTIMAT/Feasibility/General
- 📁 ULTIMAT/Feasibility/Navigation\_assessment
- 📁 ULTIMAT/Feasibility/Preliminary\_assessment
- 📁 ULTIMAT/Interface/Relevant\_ephemerides/Auxiliary\_files

### Matlab Index

plotThrustProfile	setFolders	
plotTrajectory	setObjectParam	
plot_multiGArunResult	setStationParam	
prep_OBJECT	setSubField	
printFigure	setsolName	StopFcn_msrfbp
printMessage	settings2p	StopFcn_msrfp
processNoise	settings2p	StopFcn_msrfp_LPF
progress	settings2p	StopFcn_refine
propagateSolution	settings2p	accLT
pseudoMeasurements	setup	accSRP
qrgivens	setupEstim	act
r3bpFlow3D	sing2sub	act_fg
r3bpOrbit3D	singleSol4	addFolder
r3bpOrbit3D_1dV	sph2cart_	angle2dir
r3bpOrbit3D_eval	ssb2syn	anglesyn2ssb
r3bpSymmetry3D	start	assign_data
r3bpVectorField3D	syn2ssb	automatise_ms
r4bpFlow	synodic2P	bracket
r4bpFlow_compact	synodic2P	build_nbstate
r4bpFlow_thrust	test_nbpt	build_state
r4bpFlow_thrust_compact	test_rnbpt	checkBodyDistance
r4bpOrbit	testinterp3	checkCoverage
r4bpOrbit_NdV	thrust_hist	checkEclipses
r4bpOrbit_compact	trial_and_	checkGA
r4bpOrbit_eval	turn_thrus	checkMCResults
r4bpOrbit_minSP	unix2dos	checkNavResults
r4bpOrbit_thrust	validateSc	checkSP
r4bpOrbit_thrust_compact	writelnInput	check_mexfiles
r4bpVectorField		const_func
r4bpvarFlow		constraints_
r4bpvarFlow_compact		crash_on_EarthMoon
r4bpvarOrbit		createIntSPK
		createKernel
		createObject
		createStation
		cspice_spkzr
		generate_fg
		geometricAnalysis
		getNoisyProfiles
		getSol
		grad_rpf
		gradvar_rpf
		indirect_lt_traj
		interpMatrix
		isTooClose
		linspaceNdim
		linspaceMeas
		loadKernel
		loadMAT
		loadObject
		loadSpiceKernels
		loadStation
		loadedKernels
		lookatsols
		lt_traj
		mainNavigation
		mainPreliminary
		main_EXPLORATION_RPF
		main_FBM_RPF
		main_LPFEXPLORATION
		main_MSFRFBP
		main_MS_RPF
		main_REFINE_EME2000
		main_ga
		main_gamultiobj
		main_missDis_1dV
		main_missDis_DNS
		main_missDis_mani

Developed by Politecnico di Milano and Dinamica Srl under ESA contract No. 4000118201/16/F/MOS © 2016-2017



# Low-Thrust Trajectory Optimization

Last trend in space engineering is the exploitation of **electric propulsion**

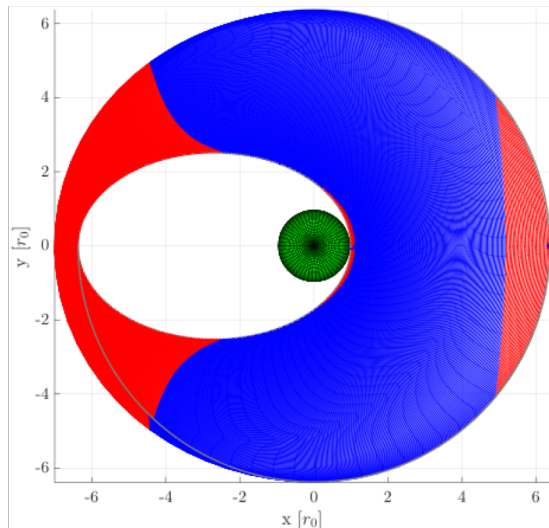
$I_{sp}$   $T$

Managing path propelled by EP becomes crucial in designing feasible missions

High time-of-flight

Multi-revolution transfers

Low-control authority



GTO to GEO fuel-optimal transfer,  $T/m = 2.5 \times 10^{-4} \text{ m/s}^2$ ,  
 $I_{sp} = 3000 \text{ s}$  and transfer time  $\approx 150$  days.

New challenges in trajectory optimization  
answered by **LT2O**

## Different Dynamics model and Coordinates Systems

- A. Restricted Two-Body problem
  - i) Cartesian Coordinates and Earth's oblateness
  - ii) Modified Equinoctial Elements
- B. Restricted three-body problem, Cartesian Coordinates
- C. Restricted four-body problem, Cartesian Coordinates

## Indirect Methods Optimization for TPBVP

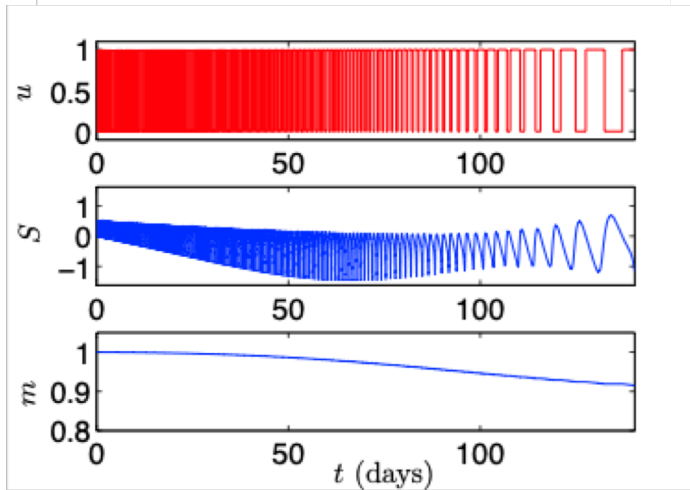
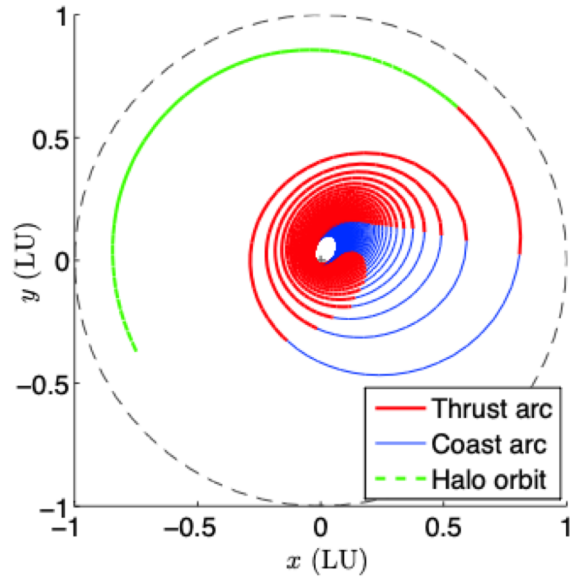
- A. Time-optimal problems  $J = \int 1 dt$
- B. Energy-optimal problems  $J = \int u - u\varepsilon(1 - u) dt$
- C. Fuel-optimal problems  $J = \int u dt$

### End-to-end optimization needs

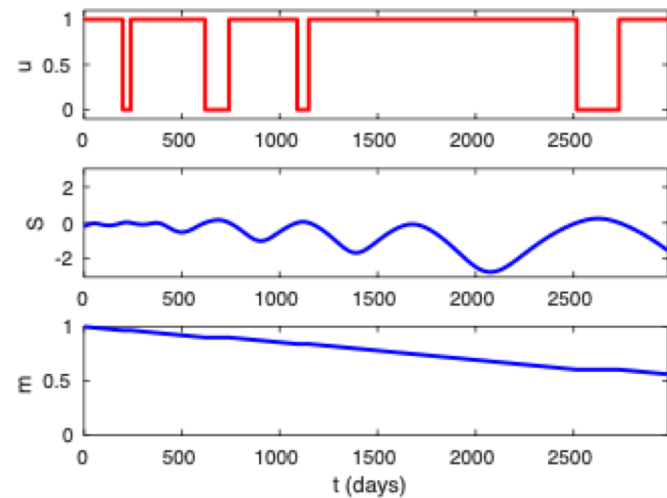
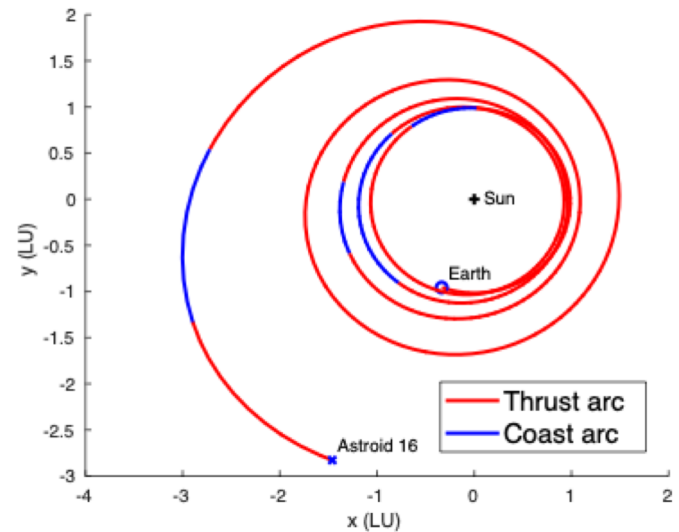
- Shooting Methods for initial costate
- Homotopy procedures on
  - Thrust level
  - Epsilon value
  - Orbital parameters
- Switching Technique and STM propagation for bang-bang control

# Low-Thrust Trajectory Optimizer — Case study (1)

GTO-L1 EM fuel-optimal transfer with 150 switches  
 $T/m_0 = 4 \times 10^{-4} \text{ m/s}^2$ ,  $I_{sp} = 3000 \text{ s}$ ; cartesian coordinates

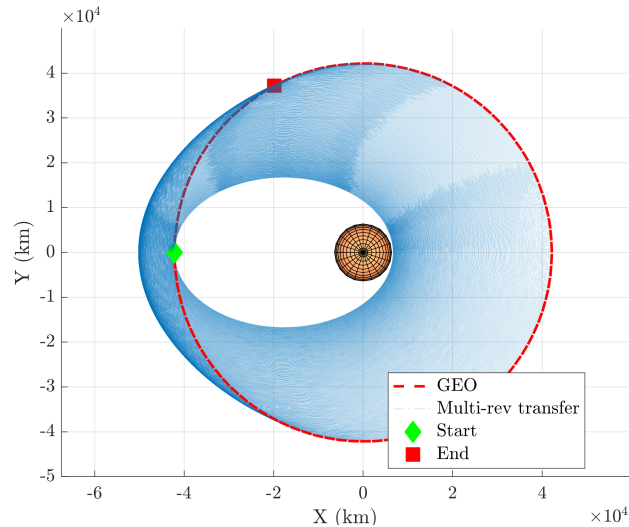
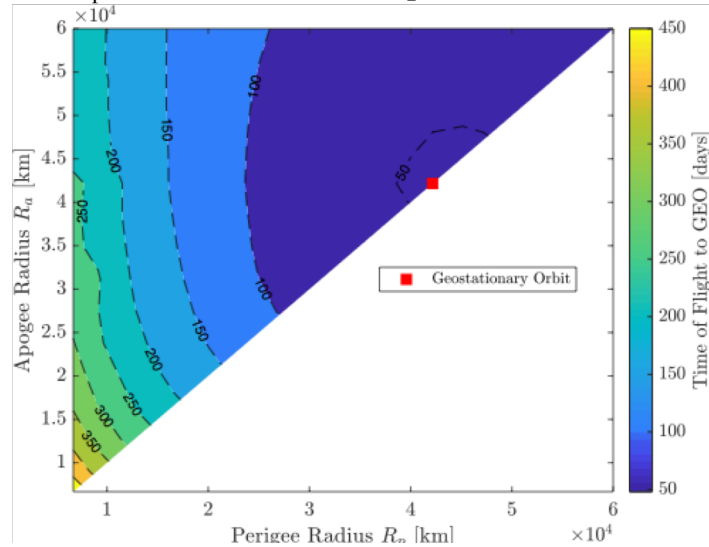


Fuel-optimal transfer from the Earth to the Asteroid 16.  
 $T/m_0 = 8.7 \times 10^{-3} \text{ m/s}^2$ ,  $I_{sp} = 2640 \text{ s}$ , cartesian coordinates.

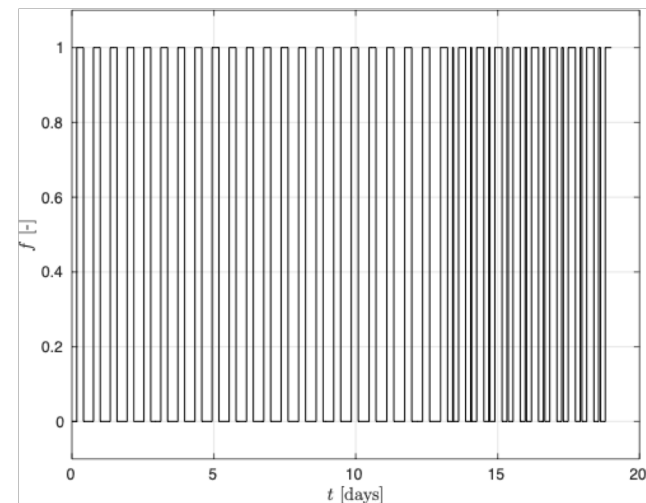
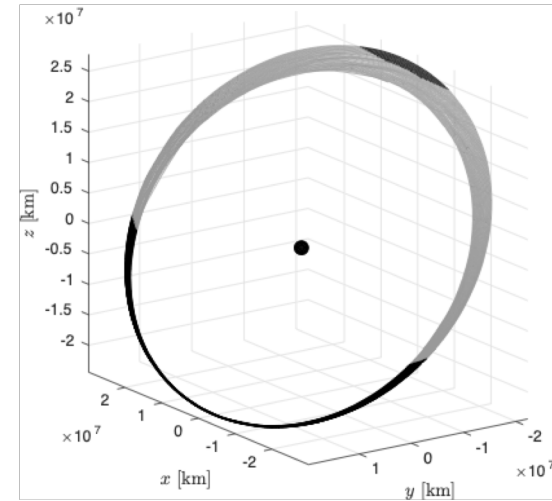


# Low-Thrust Trajectory Optimizer — Case study (2)

Time-optimal transfers to GEO,  $T/m_0 = 1 \times 10^{-4} \text{ m/s}^2$  and  $I_{sp} = 2000 \text{ s}$ ; modified equinoctial elements



End-of-Life disposal of a Galileo satellite via a fuel-optimal strategy,  $T/m_0 = 2.2 \times 10^{-4} \text{ m/s}^2$ , specific impulse of 4000 s, and transfer time  $\approx 19$  days; modified equinoctial elements



# DIREct collocation Tool for Trajectory Optimization

- Direct transcription to solve optimal control problems
- State and control variables are discretized and non-linear optimal control problem (NLOC) is converted to a non-linear programming (NLP) problem

Why?

- NLOC
  - Non-linear dynamics of states and costates
  - Two point BVP needs to be solved
  - Accurate but complex and time consuming
- NLP
  - A decisional problem with a scalar objective function and a vector of constraints
  - Does not involve dynamics of costates
  - Less accurate than NLOC but fast

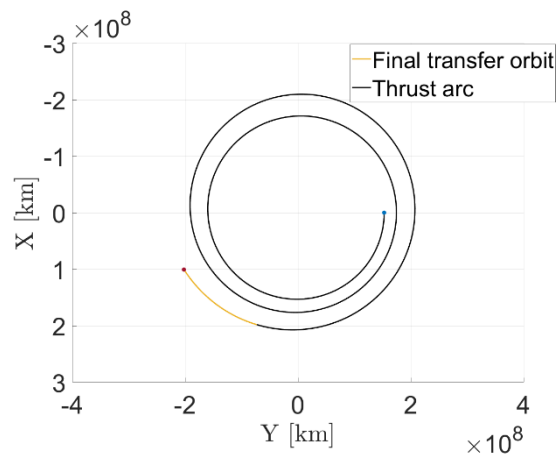
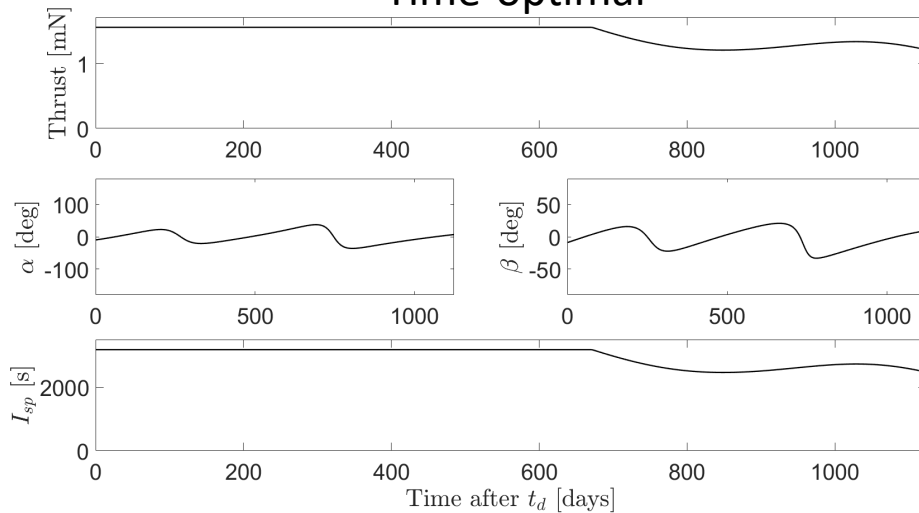
# Features

- **Equations of motion** transcribed into a set of **equality constraints**
- **Time domain discretized** as nodes (uniform and non-uniform)
- **Discretized states and control variables** treated as NLP variables
- **Differential equations** replaced by **defect constraints**
- Defect constraints derived from the **collocation method**
- Collocation methods
  - Hermite-Simpson (low-order)
  - Gauss-Lobatto (variable and high-order)
- Gradients of objective function and constraints assembled and supplied to NLP solver

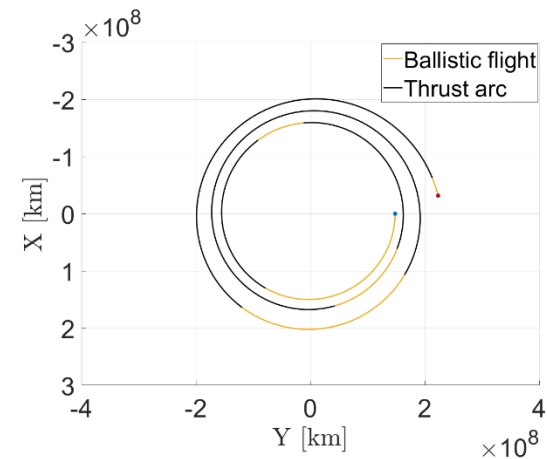
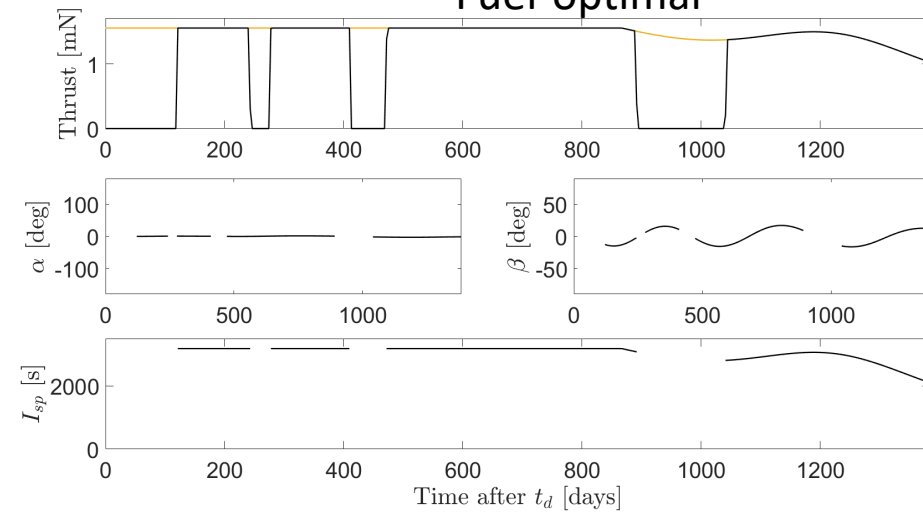
- Objective functions for minimization
  - Time
  - Fuel
  - Energy
- Implemented dynamics
  - 2-body problem with spherical dynamics
  - Circular restricted 3-body problem
  - Real solar system dynamics (incl. ephemerides obtained from SPICE)
- Low-thrust options
  - Engine model with fixed and variable Thrust & Specific Impulse
- NLP Solver
  - Fmincon
  - IPOPT

# Example: Low-thrust trajectory optimization of a stand-alone 16U CubeSat to Mars

Time-optimal



Fuel-optimal





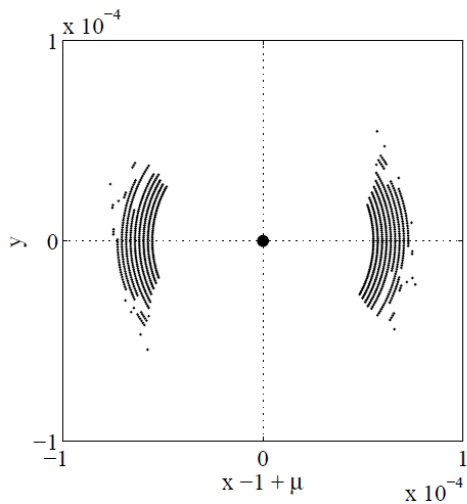
# GRAvity Tidal Slide

## Why?

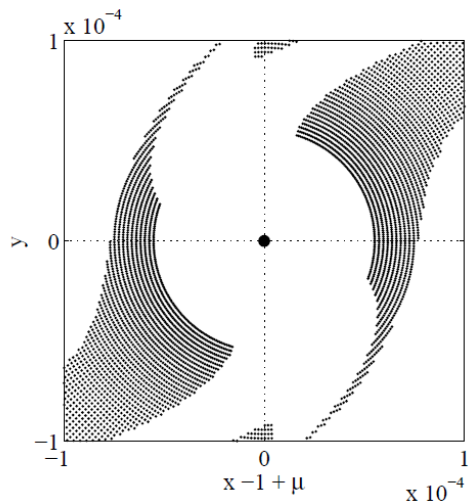
- Find a way to exploit nonlinear gravity fields unique features
- Allow small satellites to have *free transfers*
- Overcome single-point failures

## Aim

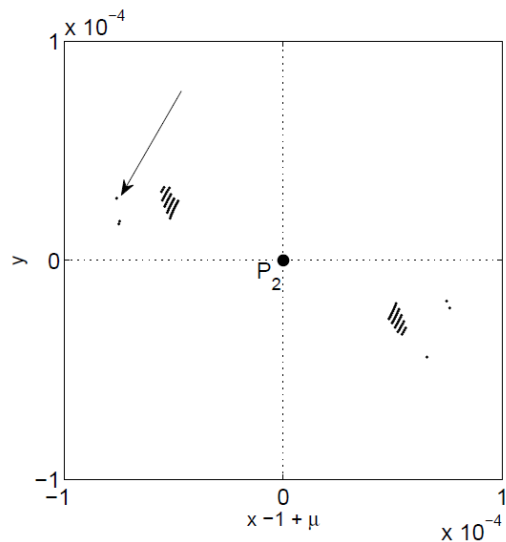
- **Compute and classify** stable and unstable sets associate to ballistic capture by
  - Sampling
  - Integration
- Different models implemented
  - CR3BP
  - ER3BP
  - RFBP
  - Real solar-system model (RnBP) with SRP and non-spherical gravity



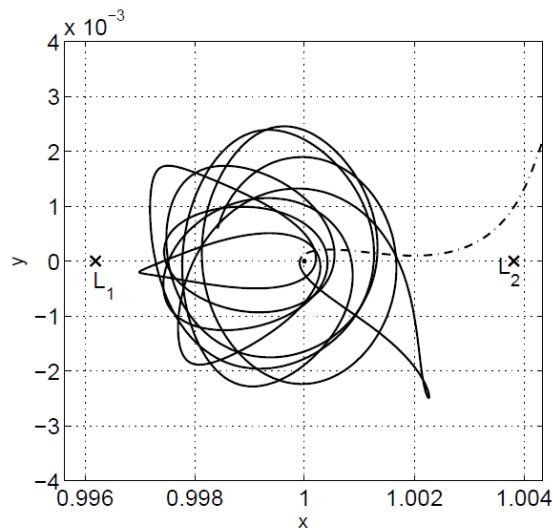
(a)  $\mathcal{W}_9(0, 0.95)$



(b)  $\mathcal{W}_{-1}(0, 0.95)$



(a)  $\mathcal{C}_{-1}^9(0, 0.95)$



(b) A sample orbit in  $\mathcal{C}_{-1}^9(0, 0.95)$

Sample of Mercury  
ballistic captures with 9  
stable revolutions

# Conclusions

- Different astrodynamics tools are able to cope with diverse mission specs
- Dedicated solutions with general software can be generated
- Tools can design current and future trajectories for space missions

## Future work

### *ULTIMAT*

- Implement different navigation and station keeping techniques
- Stochastic mission analysis

### *LT20*

- Add perturbations and eclipses
- Add radiation optimal trajectory
- Include real-engine models

### *DIRETTO*

- Pseudospectral methods

## ULTIMAT

- Topputo, F., Dei Tos, D. A., Rasotto, M., & Renk, F. (2018). Design and Feasibility Assessment of Ultra Low Thrust Trajectories to the Sun-Earth Saddle Point. In *2018 Space Flight Mechanics Meeting* (p. 1691)
- Topputo, F., Dei Tos, D. A., & Rasotto, M. (2017). Feasibility of ultra-low thrust transfers in L1/L2 Sun–Earth–Moon systems. *Final Report, ESA Contract No. 4000118201/16/F/MOS*
- Cipriano, A. M., Dei Tos, D. A., & Topputo, F. (2018). Orbit design for LUMIO: The lunar meteoroid impacts observer. *Frontiers in Astronomy and Space Sciences*, 5, 29.

## LT2O

- Zhang, C., Topputo, F., Bernelli-Zazzera, F., & Zhao, Y. S. (2015). Low-thrust minimum-fuel optimization in the circular restricted three-body problem. *Journal of Guidance, Control, and Dynamics*, 38(8), 1501-1510

## DIRETTO

- Zhang, C., Topputo, F., Bernelli-Zazzera, F., & Zhao, Y. S. (2013). An Exploration of Numerical Methods for Low-Thrust Trajectory Optimization in n-Body Models. In *64th International Astronautical Congress (IAC)*

## GRATIS

- Hyeraci, N., & Topputo, F. (2010). Method to design ballistic capture in the elliptic restricted three-body problem. *Journal of guidance, control, and dynamics*, 33(6), 1814-1823
- Luo, Z. F., Topputo, F., Bernelli-Zazzera, F., & Tang, G. J. (2014). Constructing ballistic capture orbits in the real solar system model. *Celestial Mechanics and Dynamical Astronomy*, 120(4), 433-450
- Topputo, F., & Belbruno, E. (2009). Computation of weak stability boundaries: Sun–Jupiter system. *Celestial Mechanics and Dynamical Astronomy*, 105(1-3), 3



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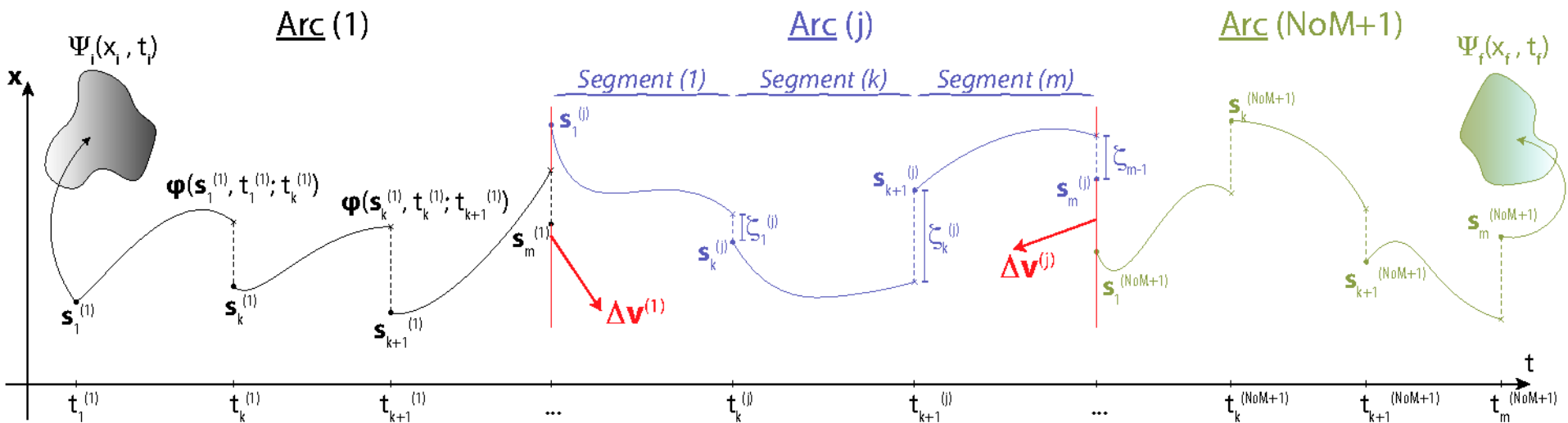
# Trajectory Design in High-Fidelity Models

## THANK YOU FOR THE ATTENTION

## QUESTIONS?

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Department of Aerospace Science and Technology, Politecnico di Milano, Italia



# maneuvers (NoM)

# segments (m-1)



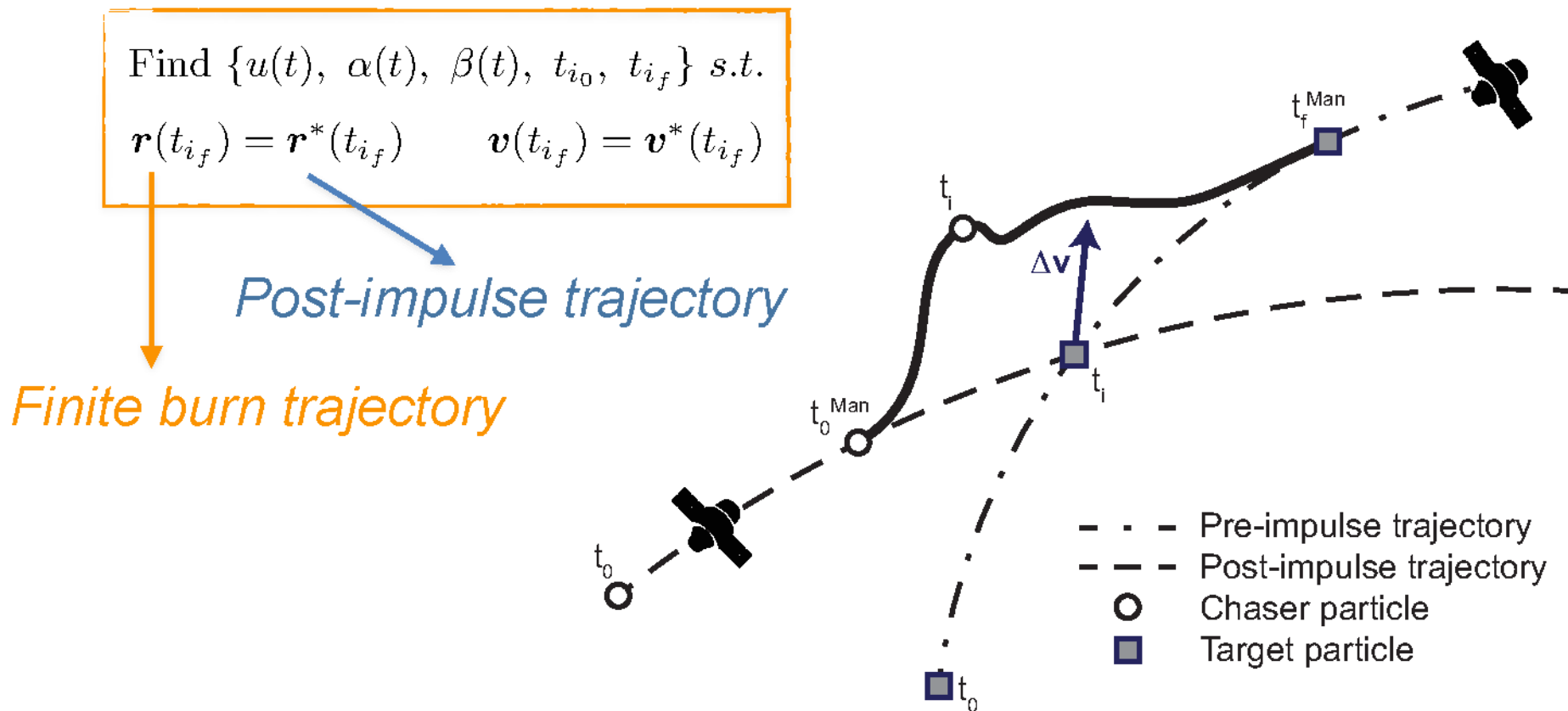
fixed parameters

Segment

Multiple shooting discretization domain (m-1) →  $t_1^{(j)} < t_k^{(j)} < \dots < t_m^{(j)}$

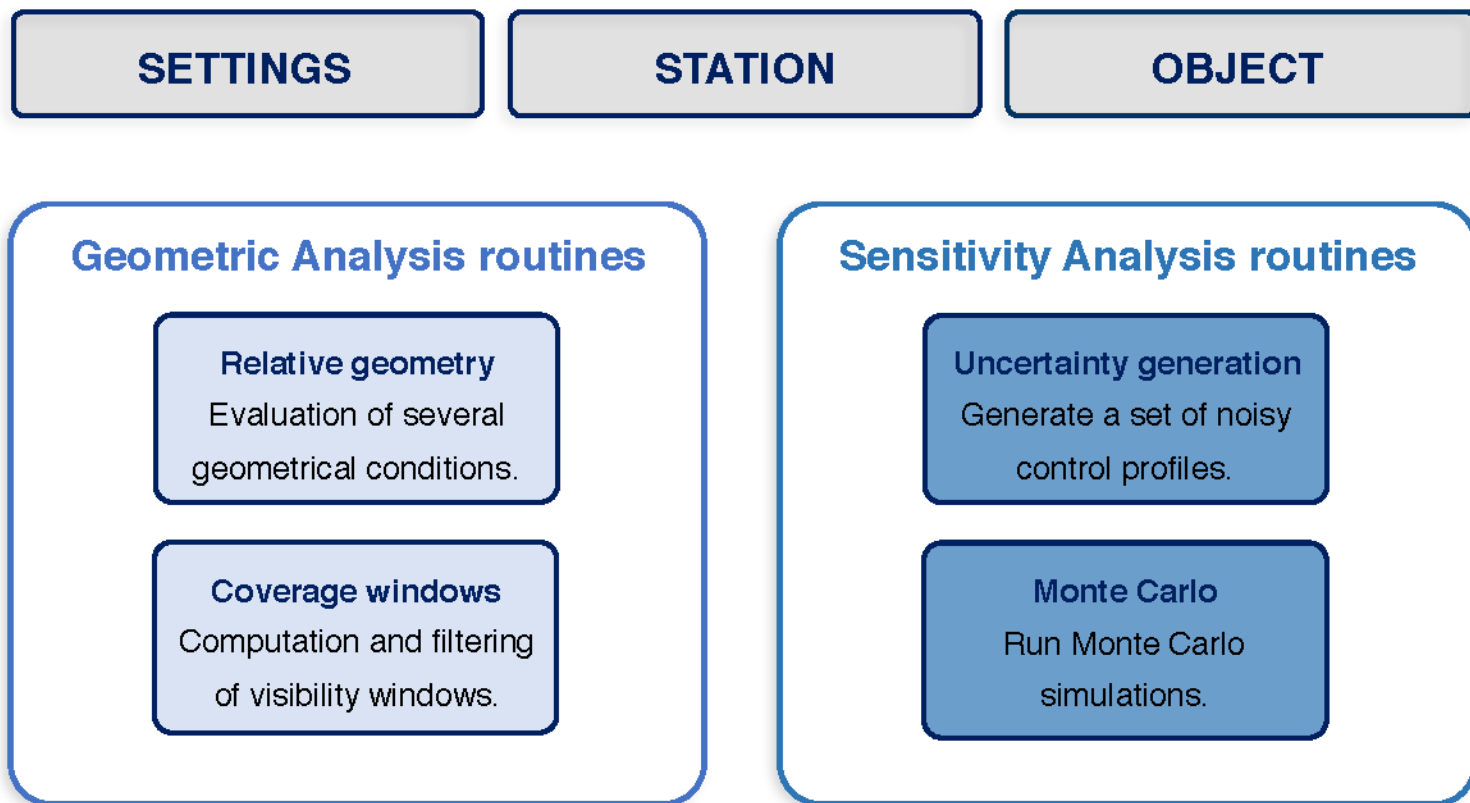
Arc

Ballistic arc separated by maneuvers (NoM+1) → Standard MS



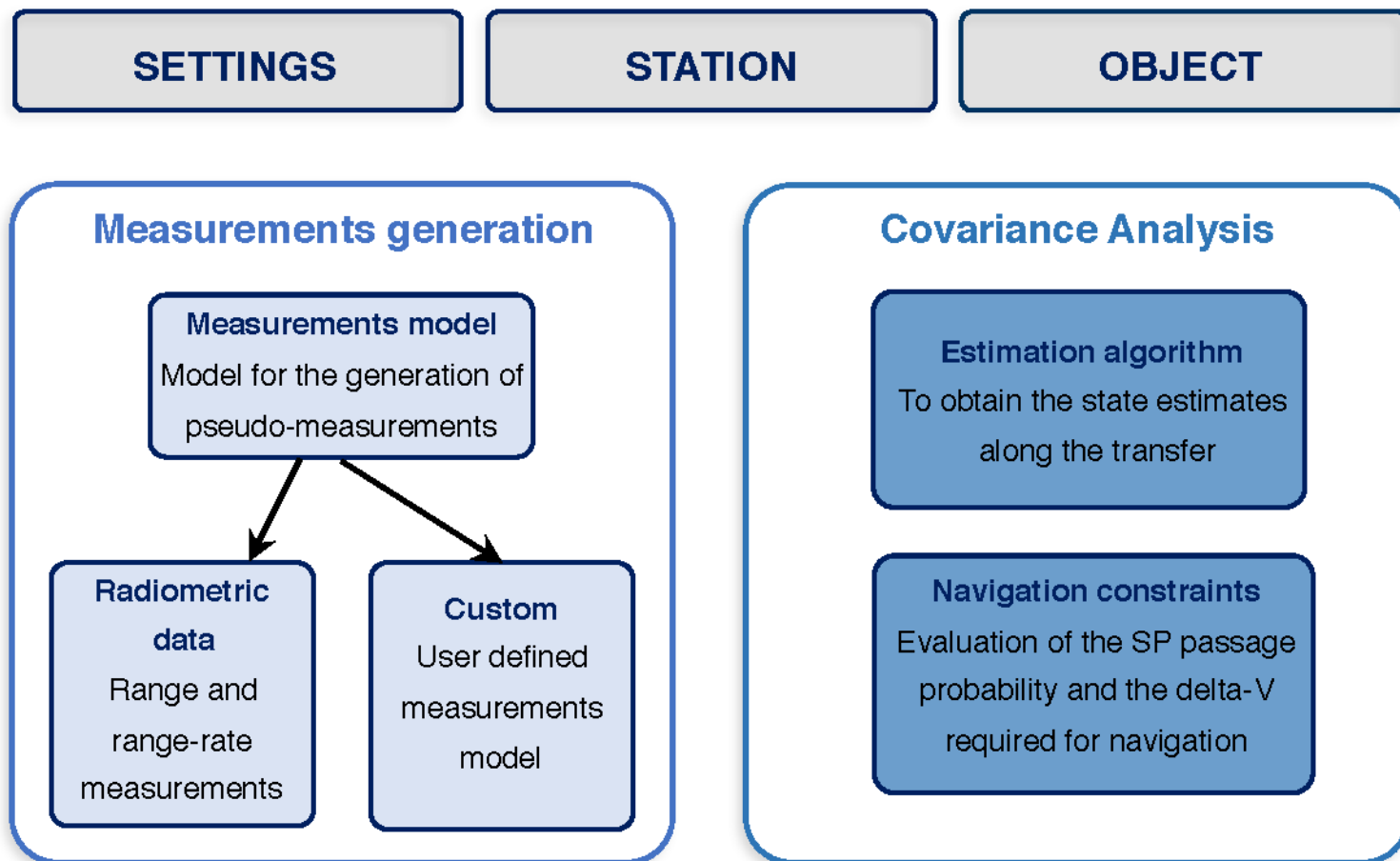
→ The **finite burn** optimization problem for a single impulse is treated as a **pseudo-rendezvous** problem where the target particle flies along the post-impulse trajectory

First exploration of the computed solutions and initial pruning of those not compliant with geometric and/or sensitivity constraints.

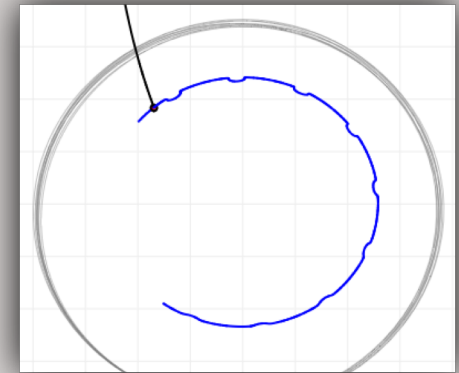
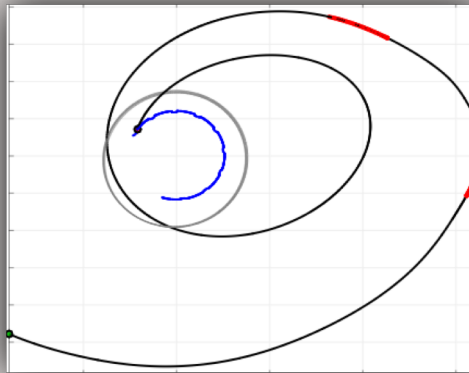
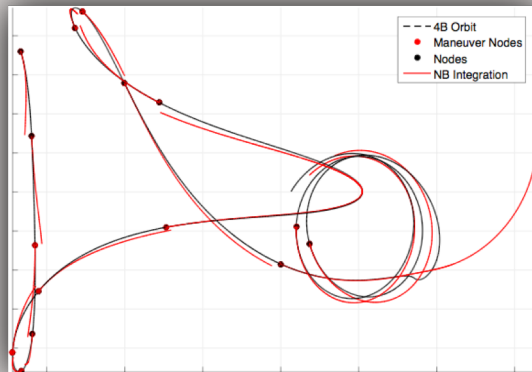




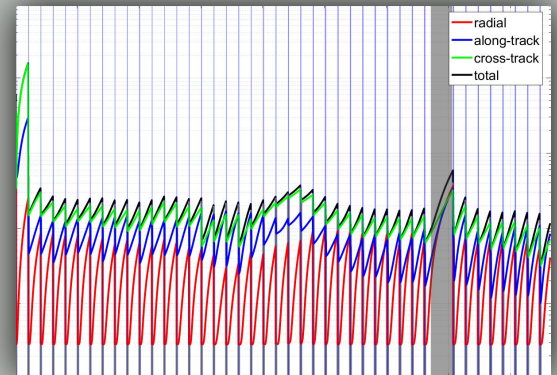
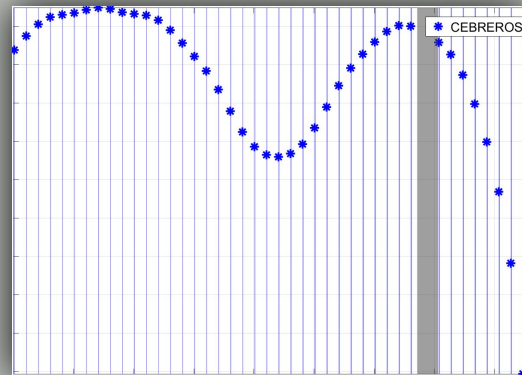
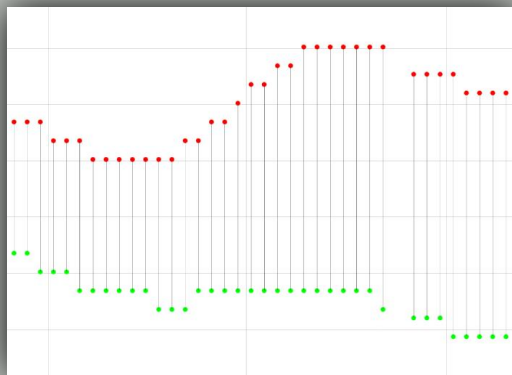
Detailed feasibility assessment through the simulation of radiometric data and computation of the achievable position and velocity knowledge during the entire transfer trajectory.



Developed methods implementing *advanced techniques* in trajectory optimization in *sophisticated models*



Developed methods to assess *solution feasibility* (OD, navigation, sensitivity, geometrical analyses)



Applied methods to *LPF*, derived top-level *requirements*, gained *knowledge* to treat similar problems

