MODHOC - Multiobjective Direct Hybrid Optimal Control

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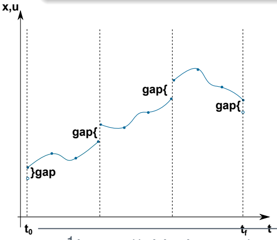
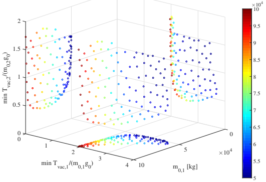
Content

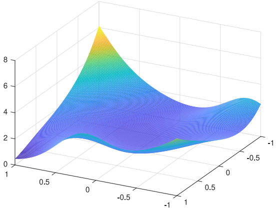
A quick description of MODHOC

What is MODHOC?

- MODHOC is an open source Matlab® framework[1] to solve general multi-phase multi-objective hybrid optimal control problems
- Perform trade-off studies for combined system and trajectory design
- Main Components:
  - DFET Direct Finite Elements Transcription
  - MACS Multi Agent Collaborative Search
  - NLP Nonlinear Programming solvers







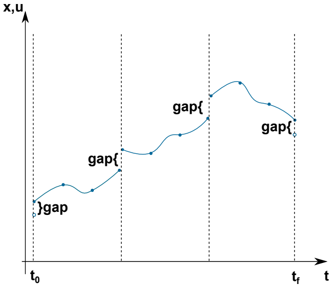
[1] https://github.com/strath-ace/smart-o2c

Direct Finite Elements Transcription[1]

A general, reliable and accurate transcription method

Features

- works for any dynamical system
- scheme is symplectic[2]: particularly suited for orbital mechanics
- scheme is equivalent to unconditionally stable[2], fully implicit RK methods[3]
- arbitrary order, h/p mesh refinement[1]
- can deal with DAEs, path constraints



[1] M. Vasile, Finite Elements in Time: A Direct Transcription Method for Optimal Control Problems, AIAA/AAS Astrodynamics Specialist Conference, Aug 2010

[2] M. Borri, C. Bottasso, A general framework for interpreting time finite element formulations, Computational Mechanics vol 13, Aug 1993

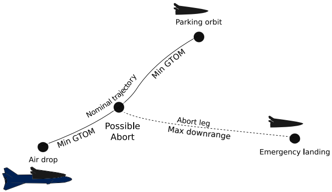
[3] C. Bottasso, A new look at finite elements in time: a variational interpretation of Runge-Kutta methods, Applied Numerical Mathematics vol 25, Dec 1997

Direct Finite Elements Transcription[1]

Deals with complex, multi-phase and multi-objective problems

Features

- multiple phases, each with its transcription settings
- phases can be connected in series and/or in parallel
- fixed number of phases, ordering defined by constraints
- each phase can have different and multiple objectives



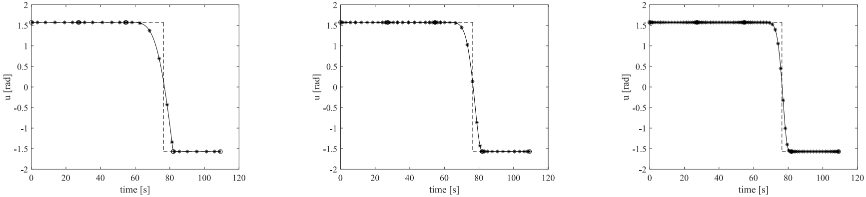
[1] Lorenzo A. Ricciardi, Christie A. Maddock and Massimiliano Vasile, Direct solution of multi-objective optimal control problems applied to spaceplane mission design, Journal of Gudiance, Dynamics and Control, accepted for publication

Direct Finite Elements Transcription[1]

A flexible and reliable scheme

Features

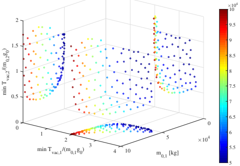
- independent approximation of states and controls: high flexibility, useful if control variable is discrete/constant/linear
- new approach with Bernstein basis: good for bang-bang controls, **theorem** of guaranteed satisfaction of inequality path constraints for all t



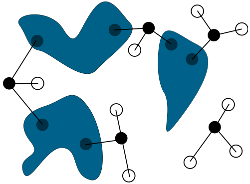
[1] Lorenzo A. Ricciardi and Massimiliano Vasile, Direct Transcription of Optimal Control Problems with Finite Elements on Bernstein Basis, Journal of Gudiance, Dynamics and Control, Oct 2018

Multi Agent Collaborative Search[1]

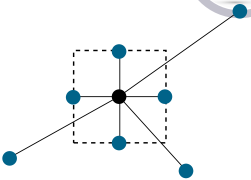
A powerful global multi-objective optimisation algorithm



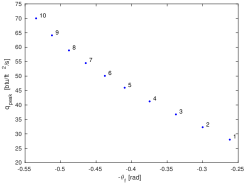
Multi/many objectives



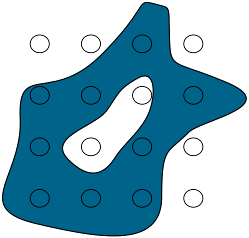
Parallel search



Global and local search



Well spread solutions



Mixed-integer nonlinear

Table 1: Solution Rankings for the Kessler Run (GTOC9)

Rank	Team Name	Missions	Removed	J in MEUR
1	Jet Propulsion Laboratory	10	123	731.2756
2	NUDT Team	12	123	786.21452
3	XSCC-ADL	12	123	821.37966
4	Tsinghua-LAD	12	123	829.57987
5	NPU	13	123	878.99821
6	Strathclyde++	14	123	918.9808
7	DLR	14	123	949.85389
8	Missions Learners	14	123	964.51134
9	The Aerospace Corporation	14	123	1004.4860
10	Team Jena	15	123	1022.9063
11	UT Austin	15	122	1044.1787
12	NJU Team	16	123	1047.9685
13	EFLMAN TEAM	14	119	1107.6936
14	CU Boulder	17	123	1150.8439
15	CAS-NUAA	14	123	1182.0632
16	MTU-UoM	16	122	1192.7433
17	NSSC-THU	16	122	1210.3333
18	Brute WORHP	18	123	1229.5475

Solved challenging problems

[1] L. A. Ricciardi and M. Vasile, Improved Archiving and Search Strategies for Multi Agent Collaborative Search, Advances in Evolutionary and Deterministic Methods for Design, Optimization and Control in Engineering and Sciences, Jan 2019

Multiobjective optimal control
Coupling MACS and DFET

The problem

- DFET generates NLP with many equality constraints
- MACS: global multiobjective solver, difficult to solve equality constraints to very strict tolerance
- NLP solvers can satisfy constraints to strict tolerance, but only locally optimal and single objective
- Need to find the right way to couple the solvers

The solution

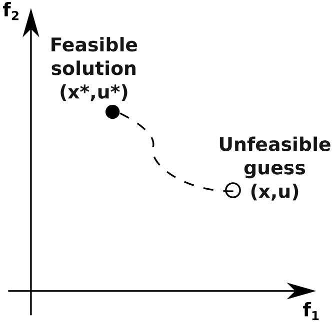
- Reformulate the problem to leverage the strengths of the tools
- Two different formulations employed: bi-level and single level

A bi-level formulation of MOOCP[1]

of multiobjective optimal control problems

Characteristics

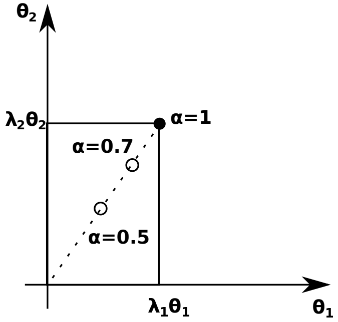
- outer level (MACS) generates guess
- inner level(NLP solver) satisfies constraints
- MACS evals objectives of feasible solution: dominance + Tchebychev scalarisation (non smooth)
- Solution of inner NLP very fast



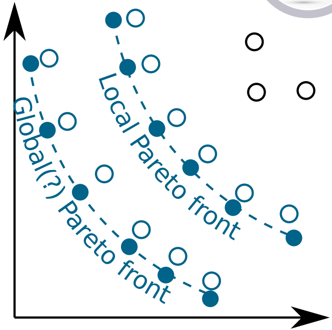
[1] Lorenzo A. Ricciardi, Christie A. Maddock and Massimiliano Vasile, Direct solution of multi-objective optimal control problems applied to spaceplane mission design, Journal of Gudiance, Dynamics and Control, accepted for publication

Characteristics

- Pascoletti-Serafini scalarisation (smooth)
- starts from previous feasible solution
- NLP solver optimises the scalarised problem



[1] Lorenzo A. Ricciardi, Christie A. Maddock and Massimiliano Vasile, Direct solution of multi-objective optimal control problems applied to spaceplane mission design, Journal of Gudiance, Dynamics and Control, accepted for publication



Synergy of two formulations

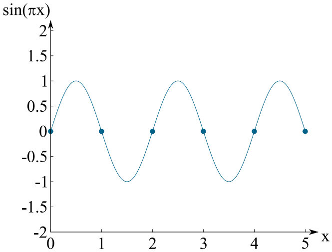
- bi-level formulation
  - global exploration, spreading
- single level formulation
  - guarantee of local optimality

- MACS uses Tchebychev scalarisation
- Single level uses Pascoletti-Serafini scalarisation
- The two scalarisations are equivalent
- Smooth transition between global exploration and local convergence

Treatment of discrete variables

A two pronged approach

- Outer level (MACS)
  - discrete variables remain discrete (modified heuristics)
- relaxation within NLP solver
  - a fully relaxed solution is first sought
  - a simple constraint is then added to impose integrality
- simultaneous treatment of discrete and continuous variables
- allows to treat nonlinear mixed-integer problems



Problem set-up

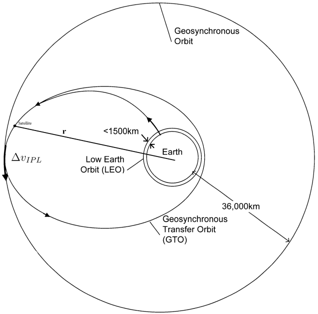
A streamlined process

- User writes one phase file per phase (template ready)
- Phase file contains transcription settings, reference to dynamical models, constraints, objectives
- Master phase file contains number of phases and aggregation function for objectives
- MACSoc settings in a setting file
- Transcription process is 1 click
- Running MACSoc is 1 click

Some Applications

Physical model

- modified equinoctial elements + mass
- 4 phases: 2 coast, 2 thrust
- limited thrust, finite pulse duration
- controls: direction of thrust
- optimisable phase durations
- 28deg change of inclination, circular to circular

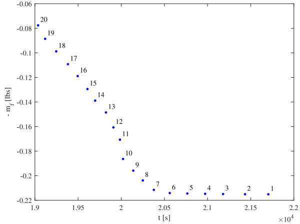


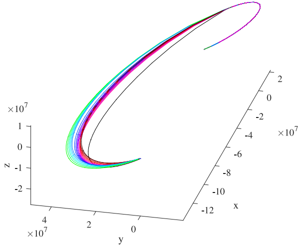
Objectives

- minimise mission time
- maximise final mass

Trade-off analysis

- Clear trade off between time and propellant mass
- Solution 1 matches with solution in literature[1]
- Solutions 1-7 (green-blue):
  - 1% difference of propellant
  - 10% difference of mission time
  - similar trajectories
- Solutions 8-20 (purple-black):
  - steeper increase of propellant mass
  - more pronounced difference of trajectories





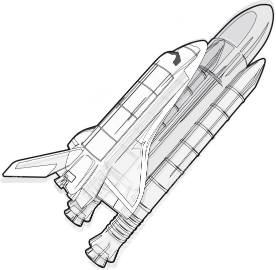
[1] J. Betts, Practical Methods for Optimal Control and Estimation Using Nonlinear Programming

A three objective vehicle optimisation

Problem description

Physical model

- 3DOF model, ECEF frame
- two stage vehicle, air dropped
- controls: α, β, throttle
- optimisable T_{vac}, m_{prop} of each stage
- (proprietary) mass models
- simplified aerodynamics



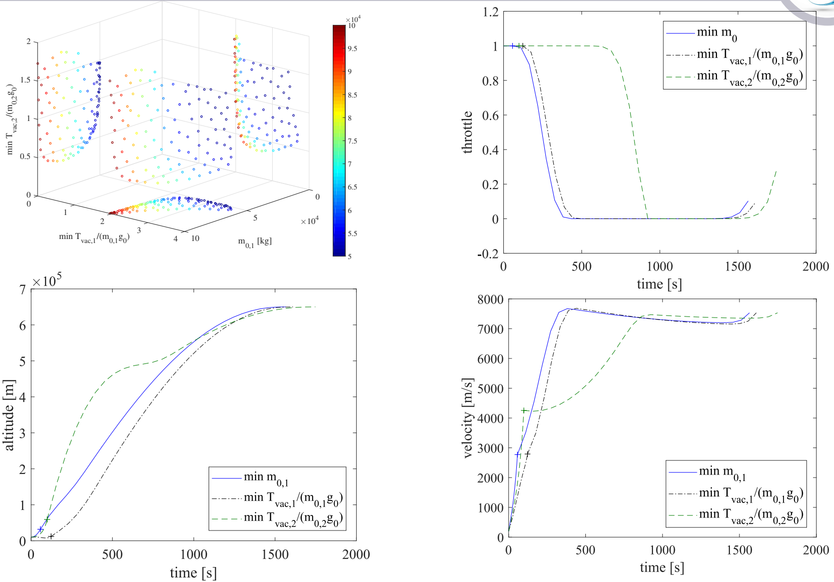
Objectives

- minimise GTOM
- minimise $T_{vac,1}/GTOM_1$
- minimise $T_{vac,2}/GTOM_2$

Constraints

- 650km altitude circular orbit, equatorial
- 500kg payload delivered
- GTOM<=100 ton
- $T_{vac,1} \leq$2MN, $T_{vac,2} \leq$200kN

A three objective vehicle optimisation
Some results



A three objective vehicle optimisation

Some interesting design considerations for the three extreme cases

Solution	Stage	Vacuum thrust [kN]	Thrust weight ratio	Δv [$\mathrm{km\,s^{-1}}$]
$\min(m_{0,1})$	1	1682.611	3.432	2.836
	2	126.100	1.467	5.664
$\min(T_{vac,1}/m_{0,1}g_0)$	1	1930.137	1.968	4.115
	2	200.000	1.730	6.003
$\min(T_{vac,2}/m_{0,2}g_0)$	1	2000.000	2.571	4.565
	2	15.124	0.351	4.606

Table: Engine sizing, T/W and ΔV

Solution	Stage	Initial mass [t]	Propellant mass [t]	Dry mass [t]
$\min(m_{0,1})$	1	49.995	29.632 (59.27%)	20.363 (40.73%)
	2	8.765	7.063 (80.59%)	1.699 (19.38%)
$\min(T_{vac,1}/m_{0,1}g_0)$	1	100.000	72.830 (72.83%)	27.170 (27.17%)
	2	11.789	9.717 (82.42%)	2.071 (17.57%)
$\min(T_{vac,2}/m_{0,2}g_0)$	1	79.318	60.629 (76.44%)	18.689 (23.56%)
	2	4.390	3.234 (73.66%)	1.156 (26.34%)

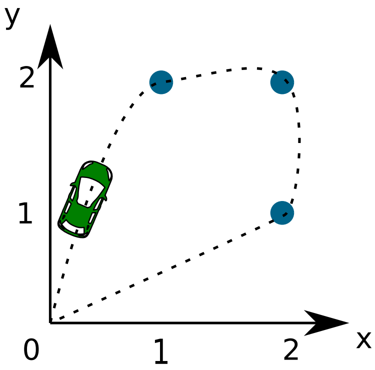
Table: Mass breakdown

Physical model

- simple 2D vehicle model
- limited acceleration and steering rate
- 3 target locations
- order not specified
- no constraints on velocity at target locations

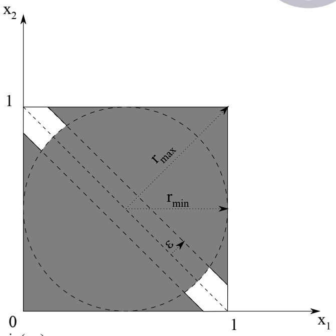
Objectives

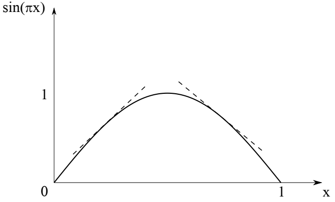
- minimise time
- minimise energy consumption



Proposed formulation

- 1 binary variable per target per phase
- arranged as a 3x3 matrix $a_{i,j}$
- relax $a_{i,j}$
- Write geometrical inequality constraints
- Relaxed solution almost close to fully feasible
- Integrality constraints easy to enforce
- Benefits from global approach





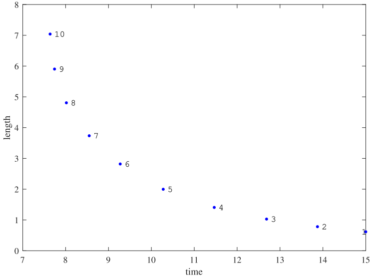
Important remarks

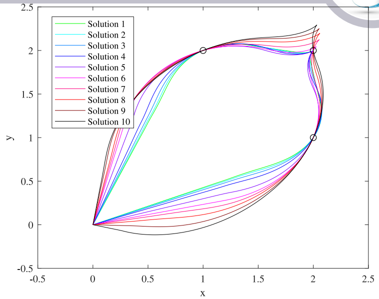
- Von Styrk and Glocker[1] studied Min time problem (obj 1)
- Von Styrk and Glocker[1] reported several different locally optimal solutions for same order of targets
- Von Styrk and Glocker[1] stated importance of initial guess for continuous variables
- MODHOC found the same solution as the best solution found in literature in a single run
- Problem has global nature
- Simultaneous treatment of discrete and continuous variables helpful

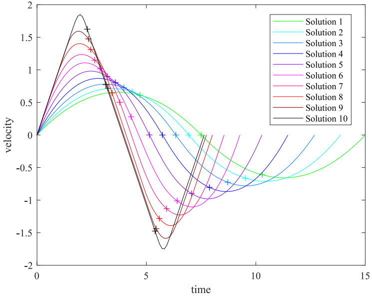
[1]O. Von Stryk and M. Glocker, Numerical mixed-integer optimal control and motorized traveling salesmen problems, Journal Européen des Systèmes Automatisés, vol 35, 2001

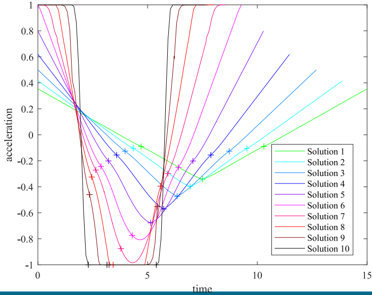
A dynamic Travelling Salesman Problem

Some results









Conclusions and final remarks

Conclusions

The present

- MODHOC can handle complex multi-objective hybrid optimal control problems
- MODHOC does not need an initial guess, performs global search but ensures local optimality
- Already being used in preliminary design phases of new launch vehicle (Orbital Access Ltd, BAE, Reaction Engines)

The future

- Expand user base
- Develop GUI (prototype ready)
- Introduce Uncertainty Quantification
- High performance (re)implementation
- Several ideas to improve algorithms

Thanks

Recast ODEs into weak form

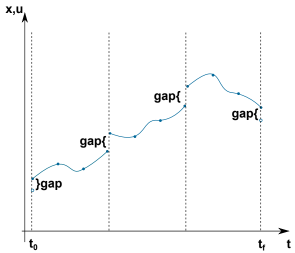
$$\int_{t_0}^{t_f} \dot{\mathbf{w}}^T\mathbf{x}+\mathbf{w}^T\mathbf{F}(\mathbf{x},\mathbf{u},t)dt-\mathbf{w}_f^T\mathbf{x}_f^b+\mathbf{w}_0^T\mathbf{x}_0^b=0$$

Slice time into finite elements

$$D=\bigcup_{j=1}^{N} D_j(t_{j-1},t_j)$$

Express vars on spectral basis

$$\mathbf{x}(t)=\overset{N}{\underset{j=1}{\mho}}\mathbf{X}_j=\overset{N}{\underset{j=1}{\mho}}\sum_{s=0}^{l} f_{sj}(t)\,\mathbf{x}_{sj}$$

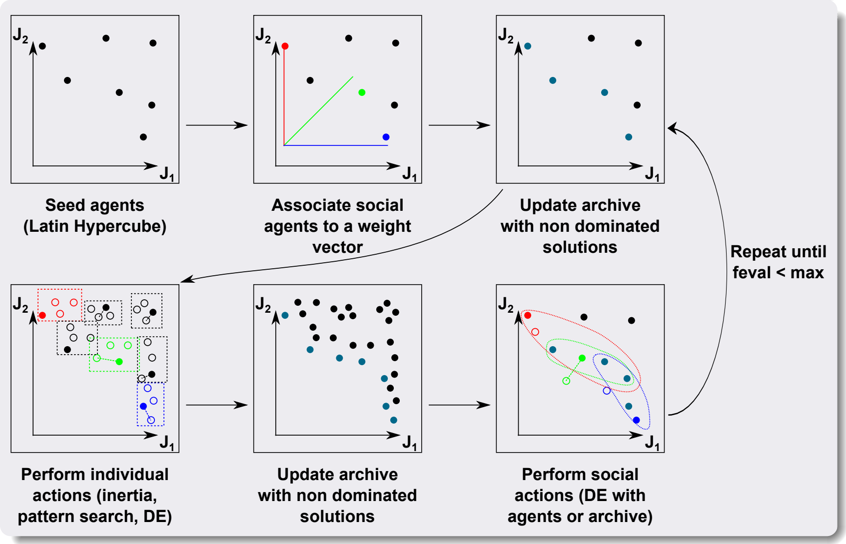


Numerically integrate to get nonlinear system

$$\sum_{k=1}^{l+1}\sigma_k\left[\dot{\mathbf{W}}_j(\tau_k)^T\mathbf{X}_j(\tau_k)+\mathbf{W}_j(\tau_k)^T\mathbf{F}_j(\tau_k)\frac{\Delta t}{2}\right]-\mathbf{W}_{p+1}^T\mathbf{X}_j^b+\mathbf{W}_1^T\mathbf{X}_j^b=0$$

MACS

A pictorial representation of the algorithm

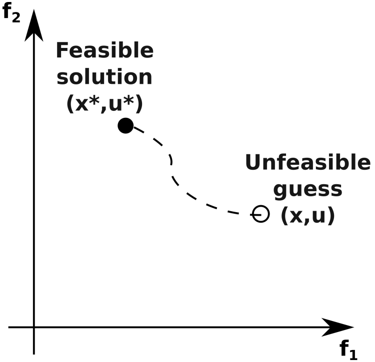


Original problem

$$\min \mathbf{J}(\mathbf{x},\mathbf{u}) \; s.t \quad \begin{array}{l} \dot{\mathbf{x}} = \mathbf{F}(\mathbf{x},\mathbf{u},t) \\ \mathbf{g}(\mathbf{x},\mathbf{u},t) \geq 0 \\ \psi(\mathbf{x}_0,\mathbf{x}_f,t_0,t_f) \geq 0 \\ t \in [t_0,t_f] \end{array}$$

Bi level reformulation

$$\min \tilde{\mathbf{J}}(\mathbf{x}^*,\mathbf{u}^*) \; s.t.$$
$$(\mathbf{x}^*,\mathbf{u}^*) = \mathrm{argmin}\{f(\mathbf{x},\mathbf{u}) | \tilde{\mathbf{c}}(\mathbf{x},\mathbf{u}) \geq 0\}$$
$$f(\mathbf{x},\mathbf{u}) = const$$



- MACS generates trial solution, NLP makes it feasible
- Solution of inner NLP very fast

A single level refinement strategy

The Pascoletti-Serafini scalarisation, allows gradient based approaches

Original problem

$$\min \mathbf{J}(\mathbf{x},\mathbf{u}) \; s.t \quad \begin{array}{l} \dot{\mathbf{x}} = \mathbf{F}(\mathbf{x},\mathbf{u},t) \\ \mathbf{g}(\mathbf{x},\mathbf{u},t) \geq 0 \\ \psi(\mathbf{x}_0,\mathbf{x}_f,t_0,t_f) \geq 0 \\ t \in [t_0, t_f] \end{array}$$

Single level reformulation

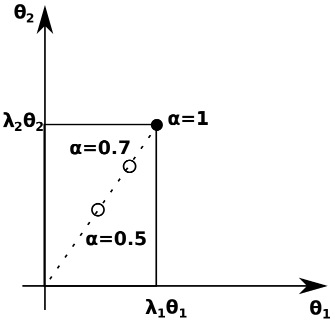
$$\min_{\alpha>0} \; \alpha$$

s.t.

$$\lambda_i \vartheta_i(\overline{\mathbf{x}},\overline{\mathbf{p}}) \leq \alpha \qquad i = 1,..,m$$

$$\mathbf{c}(\overline{\mathbf{x}},\overline{\mathbf{p}}) \geq 0$$

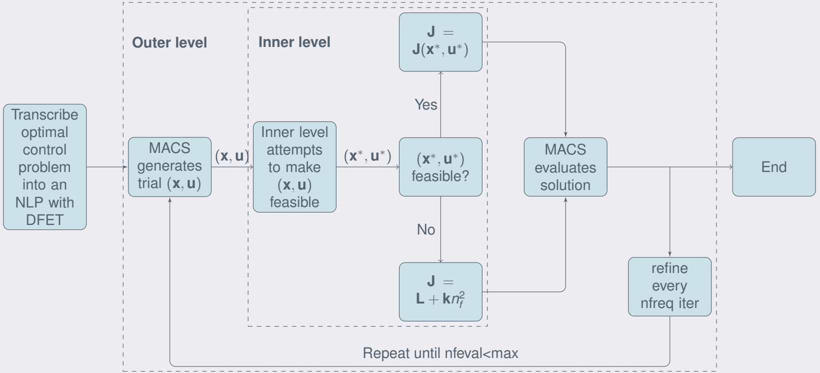
(A.K.A. goal-attainemnt method)



Starting guess generated by bi-level, NLP makes it locally optimal

Integration of MACS and DFET

A Bi-level algorithm with single level refinement



A dynamic Travelling Salesman Problem
Treatment of combinatorial part

Literature constraint formulation[1]

- 1 binary variable per target per phase
- arranged as a 3x3 matrix $a_{i,j}$
- we want only 1 element per row and column equal to 1
- relax $a_{i,j}$, impose sum rows and column =1

Problems

- All elements equal to 1/3 is a legit solution
- More equality constraints than unknowns!
- NLP solver fails

[1]O. Von Stryk and M. Glocker, Numerical mixed-integer optimal control and motorized traveling salesmen problems, Journal Européen des Systèmes Automatisés, vol 35, 2001