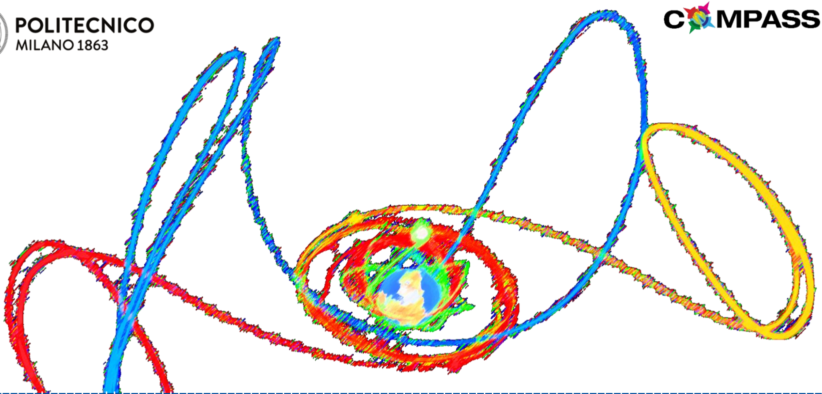

Semi-Analytical Framework for Precise Relative Motion in Low Earth Orbits

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7th ICATT 6-9 Nov 2018 DLR Oberpfaffenhofen, Germany

Foreword

- **Formation Flying** applications, multi-satellite missions for
  - sparse instruments
  - spacecraft rendezvous

- The **COMPASS project**
  - understanding and use of the orbital perturbations in several fields
  - here focus on the relative motion:
    - exploit the peculiarities of the orbital dynamics
    - to enhance current GNC (guidance navigation and control) algorithms
    - to improve the level of autonomy of such GNC systems

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Objectives and Motivations

- Development of a **framework for relative dynamics modelling**
  - semi-analytical: computational efficiency, smart state variables
  - precise: accuracy, long-scale scenarios
  - modular: included perturbations a/o accuracy to user's need
- Typical **applications**
  (special focus on the Low Earth Orbit LEO region)
  - optimal (long-time) relative guidance
  - relative navigation filters, initial relative orbit determination (computational load, convergence)

Contents

OE-based Parametrisation

- Relative dynamics: choice of the **state variables**
  - Orbital Elements (OEs) based set
  - seminal works from Schaub[1] and Alfriend[2]

- Main **advantages**
  - reducing the linearisation error in the initial conditions
  - simplifies the inclusion of orbital perturbations
  - celestial mechanics methods for efficient placement of orbit correction manoeuvres

[1] H. Schaub et Al., *Spacecraft formation flying control using mean orbit elements*, jAS 2000.

[2] D-W Gim, K.T. Alfriend, *State Transition Matrix of Relative Motion for the Perturbed Noncircular Reference Orbit*, jGCD, 2003.

Possible OE-based Parametrisations

- OE **differences** or **functions thereof**
  - possible singularities in their definition (classical, non-singular, equinoctial, Hoots)
  - canonical structure (Delaunay, Poincaré, Whittaker)

- Choice to be **driven by**
  - application domain (singularities)
  - conciseness/compactness of the related dynamical system
  - straightforward visualization of the relative orbits' geometry

Relative Orbital Elements

- Use of **Relative Orbital Elements** (ROEs)
  - suitable for LEO environment and further advantages

- **Definition** (d: deputy, c: chief)

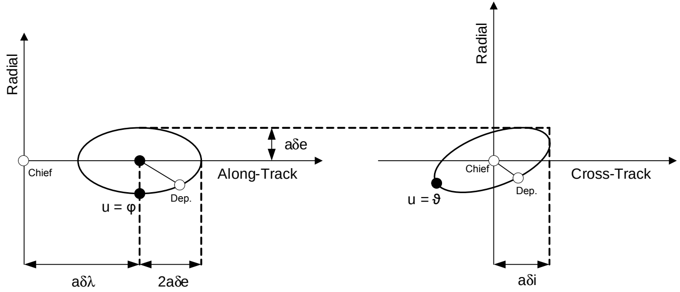
$\delta a =$	$(a_\mathrm{d} - a_\mathrm{c})/a_\mathrm{c}$	rel. semi-major axis
$\delta \lambda =$	$(u_\mathrm{d} - u_\mathrm{c}) + (\Omega_\mathrm{d} - \Omega_\mathrm{c}) \cos i_\mathrm{c}$	rel. mean longitude
$\delta e_x = e_{x,\mathrm{d}} - e_{x,\mathrm{c}}$	$\delta e_y = e_{y,\mathrm{d}} - e_{y,\mathrm{c}}$	rel. eccentricity vector
$\delta i_x = i_\mathrm{d} - i_\mathrm{c}$	$\delta i_y = (\Omega_\mathrm{d} - \Omega_\mathrm{c}) \sin i_\mathrm{c}$	rel. inclination vector

  - dimensionless state variable

$$\delta \boldsymbol{\alpha} = (\delta a, \delta \lambda, \delta e_x, \delta e_y, \delta i_x, \delta i_y)^\mathsf{T}$$

ROEs and Motion Visualisation

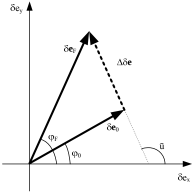
- ROEs **merge physical insight** of absolute and relative orbits
  - functions of the Hill-Clohessy-Wiltshire (HCW) integration constants[3]

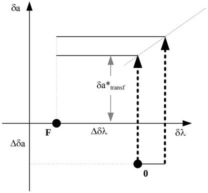


[3] S. D'Amico, *Relative Orbital Elements as Integration Constants of Hill's Equations*, DLR-GSOC TechNote 2005.

ROEs and Guidance

- **Interesting properties** deriving from Gauss' variational equations
  - relationship between delta-v optimal man. location and ROE changes[4]
  - length of ROE changes as metric of delta-v cost
  - analytical delta-v optimal manoeuvring scheme (3-T + 1-N)



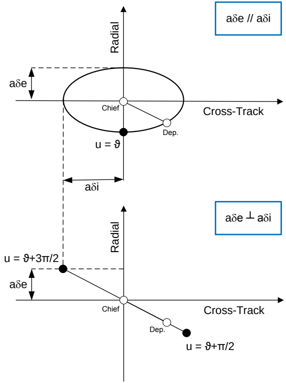


[4] G. Gaias, S. D'Amico, *Impulsive Maneuvers for Formation Reconfiguration using Relative Orbital Elements*, jGCD 2015.

ROEs and Formation Safety

- Straightforward **one-orbit minimum satellites' distance** normal to the flight direction

  - (almost-bounded) rel. eccentricity/inclination vectors phasing[5]
  - (drifting-orbits) available analytical expression accounting for δa[6]



[5] O. Montenbruck et Al., *E/I-Vector Separation for Safe Switching of the GRACE Formation*, jAST 2006.
[6] G. Gaias, J.-S. Ardaens, *Design challenges and safety concept for the AVANTI experiment*, ActaA 2016.

Flight Heritage of the ROE-based Approach

- **Spaceborne systems** of following DLR/GSOC experiments
  - GPS-based relative navigation (cooperative), formation-keeping
    - PRISMA mission:
      SAFE[7]- Spaceborne Autonomous Formation-Flying Experiment
    - TanDEM-X–TerraSAR-X mission:
      TAFF[8]- TanDEM-X Autonomous Formation Flying
  - Vision-based navigation (noncooperative), rendezvous
    - FireBIRD mission:
      AVANTI[9]- Autonomous Vision Approach Navigation and Target Identification

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[7] S. D'Amico et Al., *Spaceborne Autonomous Formation-Flying Experiment on the PRISMA Mission*, jGCD 2012.

[8] J.-S. Ardaens et Al. *Early Flight Results from the TanDEM-X Autonomous Formation Flying System*, 4th SFFMT 2011.

[9] G. Gaias, J. -S. Ardaens, *Flight Demonstration of Autonomous Noncooperative Rendezvous in Low Earth Orbit*, jGCD 2018.

Project Contributions

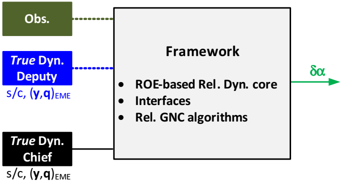
- **Further development** of the ROE-based approach to
  - improve the achievable precision (time-scale, consistency)
  - include more effects (general methodology, e.g. continuous control as special perturbation)

- **Specific contributions**
  - compact first-order dynamical system including the whole set of terms of the geopotential
  - closed-form State Transition Matrix (STM) for such first-order system
  - insight to efficiently model the effects of differential aerodynamic drag

Contents

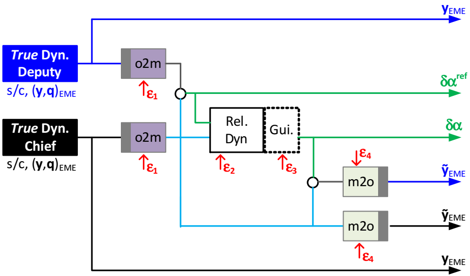
Framework Structure

- **Input**
  - chief s/c all infos
  - deputy: state or observations
- **Main elements**
  - core ROE-based relative dynamics
  - (case specific relative GNC algorithms)
- **Further characteristics**
  - mixed variables (Cartesian, OEs)
  - different reference systems



Propagation/Guidance Set Up

- **Goal**: relative trajectory close to the aimed reference $\delta\alpha^{\text{ref}}$,
  - simple propagation
  - control policy synthesis
- **minimising** the $\mathbf{y} - \tilde{\mathbf{y}}$ error in the deputy state
  - instruments operating in best conditions
  - minimum true position error



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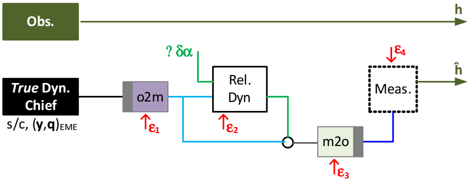
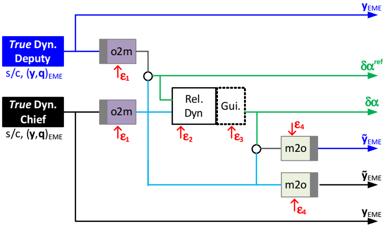
Navigation Set Up

- **Goal**: estimation of the relative state $\delta\boldsymbol{\alpha}$,
  - rel. navigation filter
  - initial rel. orbit determination

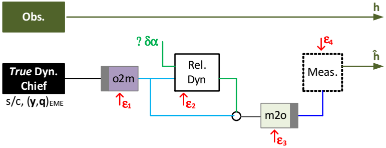
- **minimising** the $\mathbf{h} - \tilde{\mathbf{h}}$ observation residuals
  - accurate estimation
  - robustness, convergence



Accuracy and Consistency



Propagation/Guidance



Navigation

- **Simulation environment**: inaccuracies/inconsistencies cancel with each other (overestimation of the *true* precision of the framework)
- **True environment**: accuracy depending from whole chain of actions

Interfaces

- **Interfacing elements** to
  - convert Cartesian state into OEs
  - handle time synchronization and reference systems
  - two-way conversion of mean/osculating OEs
- Accurate **mean/osculating OEs** conversion
  - crucial step to achieve overall accuracy
  - *ad-hoc* algorithm, analytical, non-singular, computationally light
  - joint work with Dr. Lara (Univ. La Rioja), to be presented at ISSFD-2019

Contents

Earth Mass Distribution

- **Gravity field** expressed as geopotential function of spherical harmonics[10] order l, degree m

$$V_{lmpq} = -\frac{\mu}{a} J_{lm} \left(\frac{R_\oplus}{a} \right)^l F_{lmp}(i)\, G_{lpq}(e) \left\{ \begin{array}{c} \cos \\ \sin \end{array} \right\}_{(l-m)\ \text{odd}}^{(l-m)\ \text{even}} [\Psi_{lmpq}(\Omega, M, \omega, \theta)]$$

- **Mean OE set** out of interfacing blocks
  - mean: short- and long-periodic terms removed
  - secular terms: slow-varying variables, GNC insight
    - only $\dot{\Omega}$, $\dot{\omega}$, $\dot{M}$ function of $(a, e, i, J_2, J_2^2, J_4, J_6, ..., J_p)$, even zonal contributions
  - relative dynamics: relative secular terms

[10] W. M. Kaula, *Theory of Satellites Geodesy*, 1966.

Linearised Relative Dynamics

- **First-order relative dynamics** in ROEs

$$\delta\dot{\boldsymbol{\alpha}} = A(\boldsymbol{\alpha}_{\mathrm{c}})\,\delta\boldsymbol{\alpha} \qquad \text{with} \qquad \boldsymbol{\alpha}_{\mathrm{c}} = (a, e, i, \Omega, \omega, u)^{\mathsf{T}}$$

  - approach as in [11]

$$\frac{d}{dt}(\delta\alpha_i) = \frac{d}{dt}\left(f_i(\boldsymbol{\alpha}_{\mathrm{d}}) - f_i(\boldsymbol{\alpha}_{\mathrm{c}})\right) \approx \sum_j \left.\frac{\partial g_i}{\partial \alpha_j}\right|_{\mathrm{c}} \Delta\alpha_j$$

  - linearised relations between $\delta\boldsymbol{\alpha}$ and $\Delta\boldsymbol{\alpha}$
    - only partials w.r.t. a, e, i
    - recurring structure and dependence on only (a, e, i) and J_{even}

[11] G. Gaias et Al., *Model of J_2 Perturbed Satellite Relative Motion with Time- Varying Differential Drag*, CelMechDA 2015.

System Plant Matrix

- **Plant matrix** with structure

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ \bullet & 0 & \bullet & \bullet & \bullet & 0 \\ \bullet & 0 & \bullet & \bullet & \bullet & 0 \\ \bullet & 0 & \bullet & \bullet & \bullet & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \bullet & 0 & \bullet & \bullet & \bullet & 0 \end{bmatrix}$$

  - same structure of J_2 only case
  - now $\bullet$ account for whole J_p terms
  - linear time variant (LTV) due to $\omega(t)$ in rel. eccentricity vector (i.e., third and fourth rows)

First-Order State Transition Matrix

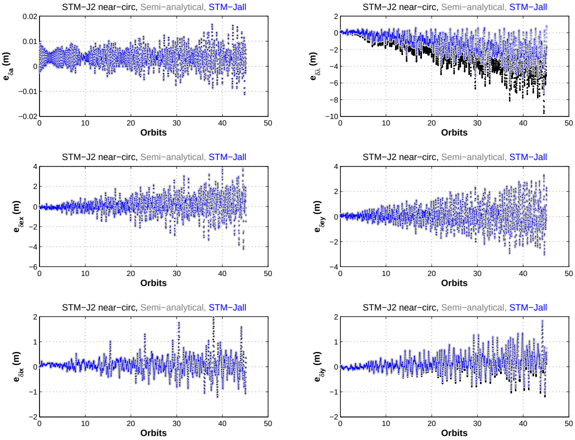
- **Linear time invariant** (LTI) system using the approach of[12]
  - change of variables $T : \delta\boldsymbol{\alpha} \mapsto \delta\boldsymbol{\alpha}'$ $\qquad \delta\mathbf{e}' = R(\omega)\delta\mathbf{e}$
  - resulting $\tilde{A}$ is nilpotent
  - original STM from: $\Phi_{\text{Jall}} = T^{-1}(\boldsymbol{\alpha}_{\text{c}}(t_{\text{f}}))\left(I + \tilde{A}(\boldsymbol{\alpha}_{\text{c}})\right) T(\boldsymbol{\alpha}_{\text{c}}(t_0))$

$$\tilde{A} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ \bullet & 0 & \bullet & 0 & \bullet & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \bullet & 0 & \bullet & 0 & \bullet & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \bullet & 0 & \bullet & 0 & \bullet & 0 \end{bmatrix} \qquad \Phi_{\text{Jall}} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ \bullet & 1 & \bullet & \bullet & \bullet & 0 \\ \bullet & 0 & \bullet & \bullet & \bullet & 0 \\ \bullet & 0 & \bullet & \bullet & \bullet & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ \bullet & 0 & \bullet & \bullet & \bullet & 1 \end{bmatrix}$$

[12] A. W. Koenig Al., *New State Transition Matrices for Spacecraft Relative Motion in Perturbed Orbits*, jGCD 2017.

Achievable Accuracy

$$a\delta\boldsymbol{\alpha}_0 = (-200.0, 4500.0, 0.0, 250.0, 0.0, 300.0)^T \text{ meters; } 6 \times 6 \text{ field}$$



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Summary Geopotential Effect

- Inclusion of the **Earth mass distribution perturbation**
  - linearised A valid for whatever order, $l > 6$ negligible contributions
  - if small e approx. acceptable $\Rightarrow$ consistent to neglect $l > 2$ terms
  - errors in the initialization (i.e., $\delta\boldsymbol{\alpha}_0$) nullify the benefit of including the effects of higher gravitational orders

- $\Phi_{\mathbf{Jall}}$ very practical for LEO subject to negligible drag (i.e., > 700 km)
  - linearisation in OEs $\Rightarrow$ whole typical formation-flying domain
  - Φ_{Jall} valid also for the eccentric case

Contents

Aerodynamic Drag

- **General difficulties** in modelling this perturbation
  - inaccuracy/complexity of upper-atmosphere density models
  - ballistic coefficient $B = C_\mathrm{D} S/m$ depending on attitude
  - unknown drag coefficient C_D, function of attitude

- Possible OEs-based **approaches**
  - physical: expanding absolute mean OEs time variation
  - engineering: introducing empirical rel. acceleration

- Both require **additional parameters** to be estimated

Physical Approach

- **Methodology** proposed in[13]
  - one-orbit averaged Gauss variational eqs. subject to drag
  - exponential model of density, only tangential acc., no $\mathbf{v}_{\mathrm{atm}}$[14]
  - expansion of $\dot{\bar{a}}$ and $\dot{\bar{e}}$ w.r.t. a and e

- **Remarks**
  - approach not portable to ROEs as done for the geopotential
  - linearised equations require numerical integration
  - at least 2 additional parameters (i.e., *true* ρ_{p} and mean ΔB)
  - little insight in the relative acceleration in local orbital frame

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[13] D. Mishne, *Formation Control of Satellites Subject to Drag Variations and J_2 Perturbations*, jGCD 2004.
[14] D. King-Hele, *Theory of satellite orbits in an atmosphere*, 1964.

Engineering Approach

- **Methodology** proposed in[11]
  - non-conservative acc. more tractable in the local Cartesian frame RTN
  - equivalence between the linearised dynamics in RTN and OEs[15]
  - HCW eqs. two sharp pass-bands filter - centred on 0, $1/P$ freq.[16]

- **Remarks**
  - 3 additional parameters, coeff. of the empirical acceleration
  - they correspond to mean $\delta\dot{a}$, $\delta\dot{e}_x$, $\delta\dot{e}_y$ due to Δdrag
  - availability of closed-form STM for the state $(\delta\alpha, \delta\dot{a}, \delta\dot{e}_x, \delta\dot{e}_y)^T$

[11] G. Gaias et Al., *Model of J_2 Perturbed Satellite Relative Motion with Time- Varying Differential Drag*, CelMechDA 2015.
[15] A. J. Sinclair et Al., *Calibration of Linearized Solutions for Satellite Relative Motion*, jGCD 2014.
[16] O. L. Colombo, *The dynamics of global position system orbits and the determination of precise ephemerides*, jGR 1989.

Engineering Approach Reworked

- **HCW** equations further **simplified** using Leonard's change of variables[17]

$$\mathbf{x} = (x, \dot{x}, y, \dot{y})^{\mathsf{T}} \mapsto \boldsymbol{\kappa} = (\bar{x}, \bar{y}, \gamma, \beta)^{\mathsf{T}} \quad \begin{aligned} \gamma &= x - \bar{x}, \qquad & \bar{x} &= 4x + 2\dot{y}/n \\ \beta &= y - \bar{y}, \qquad & \bar{y} &= y - 2\dot{x}/n \end{aligned}$$

  - HCW $\mapsto$ decoupled double-integrator in $\bar{y}$ and harmonic oscillator in β

- In-plane **ROEs related to** $\boldsymbol{\kappa}$

$$a(\delta a_0, \delta\lambda_0, \delta e_{x0}, \delta e_{y0})^{\mathsf{T}} = (\bar{x}_0, \bar{y}_0, -\gamma_0, -\beta_0/2)^{\mathsf{T}}$$

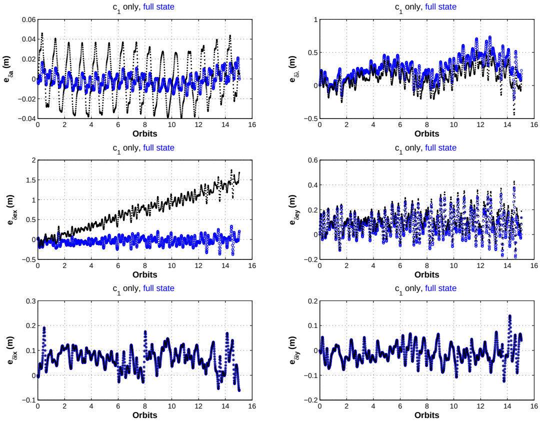
$$a\delta\boldsymbol{\alpha}^{\text{ip}}(t) = M\boldsymbol{\kappa}(t) \qquad M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -\cos(nt) & +\sin(nt)/2 \\ 0 & 0 & -\sin(nt) & -\cos(nt)/2 \end{bmatrix}$$

- More **compact** development, **further insight**

[17] C. L. Leonard et Al., *Orbital Formation keeping with Differential Drag*, jGCD 1989.

Achievable Accuracy

$a\delta\boldsymbol{\alpha}_0 = (0.0, 4500.0, 0.0, 250.0, 0.0, 300.0)^T$ meters; 6×6 field and drag



AVANTI scenario, but varying attitude not considered

Meaning of the Empirical Acceleration

- **True Δdrag acceleration** in local RTN frame

$$\mathbf{a}_{\Delta \mathrm{D}}^{(\mathrm{RTN,c})} = R_{\mathrm{RTN,d}}^{\mathrm{RTN,c}} \, R_{\mathrm{TOD}}^{\mathrm{RTN,d}} \, \mathbf{a}_{\mathrm{D,d}}^{(\mathrm{TOD})} - R_{\mathrm{TOD}}^{\mathrm{RTN,c}} \, \mathbf{a}_{\mathrm{D,c}}^{(\mathrm{TOD})}$$

$$\mathbf{a}_{\mathrm{D}}^{(\mathrm{TOD})} = -\frac{1}{2}\rho B \left\| \mathbf{v} - \mathbf{v}_{\mathrm{atm}} \right\| \left(\mathbf{v} - \mathbf{v}_{\mathrm{atm}} \right)$$

- Possible **simplifications** ($\mathbf{v}_{\mathrm{atm}}$, frames)

$$a_{\Delta \mathrm{D}}^{(\mathrm{T})} = -(1/2)\rho_{\mathrm{d}} B_{\mathrm{d}} v_{\mathrm{d}}^2 + (1/2)\rho_{\mathrm{c}} B_{\mathrm{c}} v_{\mathrm{c}}^2$$

$$a_{\Delta \mathrm{D}}^{(\mathrm{T})} = -(1/2)\rho_{\mathrm{c}} v_{\mathrm{c}}^2 (\bar{B}_{\mathrm{d}} - \bar{B}_{\mathrm{c}}(1+b))$$

- **Empirical acceleration** (trigonometric approximation)

$$a_{\Delta \mathrm{D}}^{(\mathrm{T})} = c_1 + c_2 \sin\left(\frac{2\pi}{P} t\right) + c_3 \cos\left(\frac{2\pi}{P} t\right)$$

$$c_1 = \frac{n}{2} a \delta \dot{a} \qquad c_2 = n a \delta \dot{e}_x \qquad c_3 = n a \delta \dot{e}_y$$

Summary Drag Effect

- Inclusion of the **differential drag perturbation**
  - closed-form STM for the linearised dynamics s.t. time-varying Δdrag
  - the small e approximation is used to derive such STM
  - at most 3 additional parameters need to be estimated

- These **parameters** (i.e., $\delta\dot{a}$, $\delta\dot{e}_x$, and $\delta\dot{e}_y$)
  - represent the mean elements variation to the net of the geopotential effects (known very precisely)
  - density-model-free modelling/estimation
  - $\delta\dot{a}$ catches the mean effect due to $\rho v^2 \Delta B$
  - $\delta\dot{e}_x$, and $\delta\dot{e}_y$ catch the time-varying effects (e.g., $\rho(t)$, $\Delta B(t)$)

Contents

Conclusions

- Description of semi-analytical framework for formation flying in LEO
- Elements, connections, and dynamical model to achieve high accuracy
- Linearised models taking into account both main perturbations
- Closed-form state transition matrices suitable for onboard applications

This project has received funding from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme (grant agreement No 679086 – COMPASS)

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