Rate and collision probability of tethers and sails against debris or spacecraft

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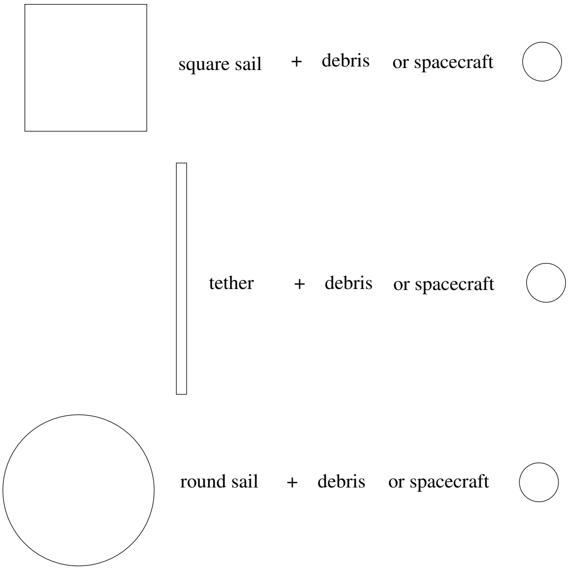
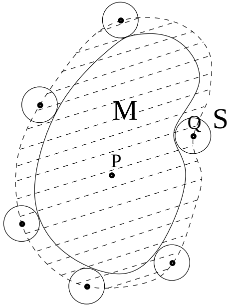
- Contrary to the sphere-sphere case, attitude matters.

Introduction



Introduction

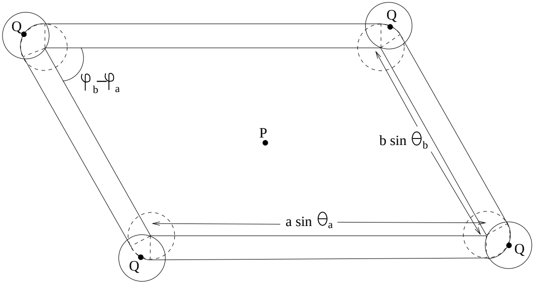
ρ is the projection on the b-plane of the pdf of the relative position.



probability of collision $= \int_{M\oplus(-S)} d^2r\ \rho(\vec{r})$

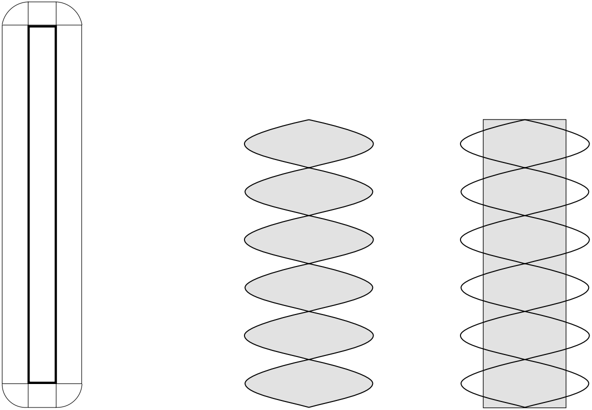
$$\text{rate of collision} = \frac{\bar{\rho}\ \mathrm{A}(M \oplus (-S))}{\text{time between encounters}} \to \phi \mathrm{A}(M \oplus (-S))$$

$$\text{prob. of } n \text{ collisions during a time } t = e^{-\phi \mathrm{A}(M\oplus(-S))t}\ \frac{(\phi \mathrm{A}(M \oplus (-S)))^n}{n!}$$



$$\mathrm{A}(M \oplus (-S)) = \mathrm{A}(M \oplus S) = \mathrm{A}(M) + \mathrm{per}(M)R + \pi R^2$$

Collision rate
Fixed attitude



Collision rate
Fixed attitude

$$\mathrm{A}(M \oplus (-S)) =$$

$$\mathrm{A}(M \oplus (-S)) =$$

- $$ab\sqrt{-\cos(\theta_a+\theta_b)\cos(\theta_a-\theta_b)}+2(a\sin\theta_a+b\sin\theta_b)R+\pi R^2$$

Collision rate
Fixed attitude

$$\mathrm{A}(M \oplus (-S)) =$$

- $$ab\sqrt{-\cos(\theta_a+\theta_b)\cos(\theta_a-\theta_b)}+2(a\sin\theta_a+b\sin\theta_b)R+\pi R^2$$

- $$2(R+r)L\cos\alpha$$

$$\mathrm{A}(M \oplus (-S)) =$$

- $$ab\sqrt{-\cos(\theta_a + \theta_b)\cos(\theta_a - \theta_b)} + 2(a\sin\theta_a + b\sin\theta_b)R + \pi R^2$$

- $$2(R + r)L\cos\alpha$$

- $$2\left(\frac{w}{\pi} + R\right)L\cos\alpha$$

$$\mathrm{A}(M \oplus (-S)) =$$

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- $$2(R+r)L\cos\alpha$$

- $$2\left(\frac{w}{\pi}+R\right)L\cos\alpha$$

- $$\pi R_{sail}^2\cos\theta_M+4R_{sail}E(\sin^2\theta_M)R+\pi R^2$$

Collision rate
Random attitude

$$\overline{\mathrm{A}(M \oplus (-S))} =$$

- $$\overline{\mathrm{A}(M \oplus (-S))} =$$

$$\frac{ab}{2} + \frac{\pi(a+b)R}{2} + \pi R^2$$

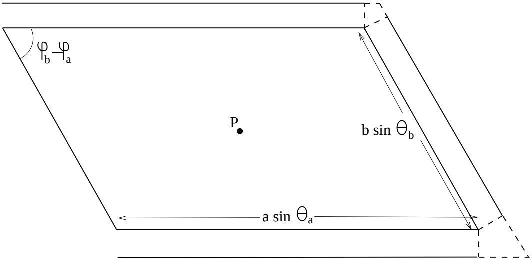
$$\overline{\mathrm{A}(M \oplus (-S))} =$$

- $$\frac{ab}{2} + \frac{\pi(a+b)R}{2} + \pi R^2$$

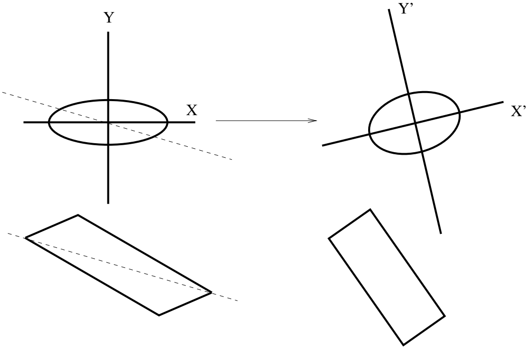
- $$\frac{\pi R_{sail}^2}{2} + \frac{\pi^2 R_{sail} R}{2} + \pi R^2$$

Collision probability
Rectangle

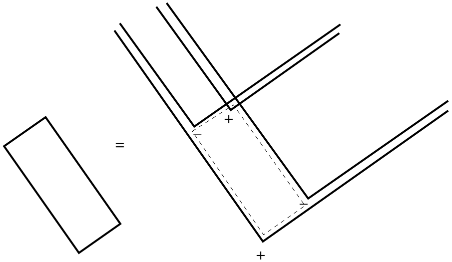
$$\int_{M\oplus(-S)} d^2r\ \rho(\vec{r}) \leq \int_{\text{parallelogram}} d^2r\ \rho(\vec{r})$$



The affine transformation of a parallelogram into a rectangle transforms the Gaussian into another Gaussian.

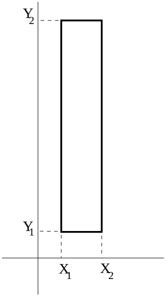


This allows the use of Genz's algorithm.

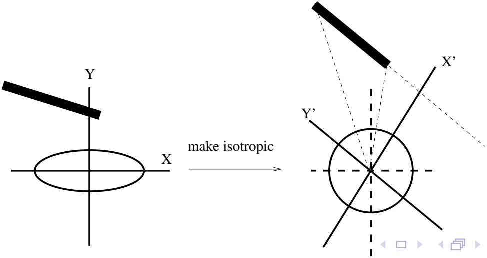


http://sdg.aero.upm.es/index.php/online-apps/gaussian-over-parallelogram

G gaussian over parallelogram G



Y2
Y1
X1
X2



Y
X
make isotropic
X'
Y'

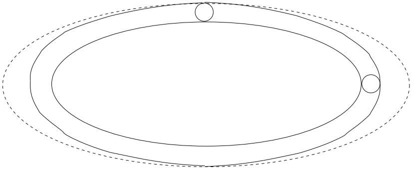
$$\frac{1}{4}\left\{\mathrm{erf}\left[\frac{D^2L/2 + x\sigma_y^2\cos\Lambda + y\sigma_x^2\sin\Lambda}{\sqrt{2}D\sigma_x\sigma_y}\right] + \mathrm{erf}\left[\frac{D^2L/2 - x\sigma_y^2\cos\Lambda - y\sigma_x^2\sin\Lambda}{\sqrt{2}D\sigma_x\sigma_y}\right]\right\}$$

$$\left\{\mathrm{erf}\left[\frac{\Delta/2 + |y\cos\Lambda - x\sin\Lambda|}{\sqrt{2}D}\right] + \mathrm{erf}\left[\frac{\Delta/2 - |y\cos\Lambda - x\sin\Lambda|}{\sqrt{2}D}\right]\right\},$$

where:

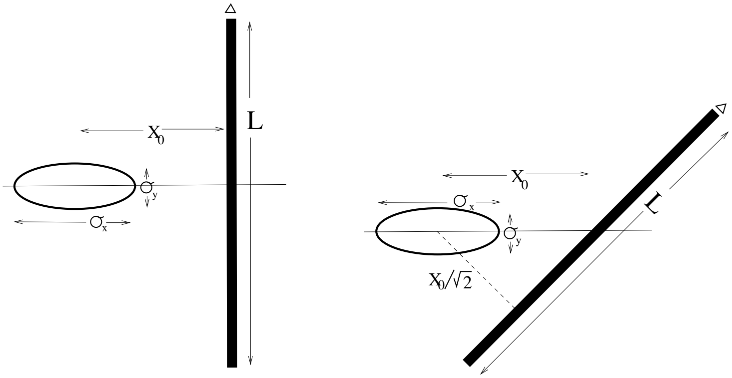
$$D \equiv \sqrt{\sigma_x^2\sin^2\Lambda + \sigma_y^2\cos^2\Lambda}.$$

Collision probability
Round sail



$\Lambda = 45^o$ and 90^o; $L = 2000$ m; $\Delta = 7$ cm, 30 cm, 2 m, and 8 m.

x, y, σ_x, σ_y.



$\Lambda = \pi/2, \ \Delta = 0.07$ m

$\sigma_x \ \backslash p$	10^{-3}	10^{-4}	10^{-5}
2	4.6	6.3	7.6
8	12.6	21.3	27.4
32	0	66.6	95.7
128	0	160	318
512	0	0	943
2048	0	0	1613

$\Lambda = \pi/2, \ \Delta = 8$ m

$\sigma_x \ \backslash p$	10^{-3}	10^{-4}	10^{-5}
2	10.2	11.4	12.6
8	29	34	38.1
32	97	119	138
128	325	425	506
512	482	1200	1837
2048	1929	4800	6508

The contribution to the overall probability of collision of the end masses is almost always negligible compared to the contribution of the tether. The reason is that the perimeter of the tether, p_t, is about 1,000 longer than the perimeter of any of the end masses, p_m.

Conclusions

- For a tether of length $2R$ and diameter d the area of its average projection is $\pi dR/2$, while the area of the projection of the circumscribing sphere is πR^2. For a disk of radius R and its circumscribing sphere the average projected areas are, respectively, $\pi R^2/2$ and πR^2.

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- Thus for these two cases the ratios of the area of the average projection to the area of the projection of the circumscribing sphere are $d/(2R)$ (in reality less than 0.01) and $1/2$, respectively.

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- Thus for these two cases the ratios of the area of the average projection to the area of the projection of the circumscribing sphere are $d/(2R)$ (in reality less than 0.01) and $1/2$, respectively.
- **No additional computational cost, since the formulae are all analytical except for the case of the collision probability of a rectangle, for which a very fast algorithm, available as an app, is provided.**

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In general one may decompose a complex aircraft in various pieces, and separately compute the probability of collision for each. When one of the pieces is a solar panel then the formulae for the rectangle can be used.