OPTELEC: AN OPTIMISATION SOFTWARE FOR LOW-THRUST ORBIT TRANSFER INCLUDING SATELLITE AND OPERATION CONSTRAINTS

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ABSTRACT

Low-thrust orbit transfers to Geostationary Equatorial Orbit induce complex satellite platform and operational constraints. These constraints, which apply over many revolutions, need to be handled by dedicated transfer optimisation software. This paper presents the development, validation and the various usages and capabilities of Airbus Defence and Space's low-thrust transfer optimisation software, OptElec. The emphasis is on how the operations and how the specificities of the satellite impact the optimisation compared to an unconstrained transfer. Several benchmark scenarios are provided. A complex study case including multiple propulsion systems and taking into account operational and platform constraints is detailed. The effects of each of these aspects in terms of real time operations, satellite attitude profile, ΔV cost and mission duration are highlighted. The last section shows how the software can be updated to solve low-thrust Earth-Moon transfers.

***Index Terms*—** Low-thrust, Optimisation, Electric-Orbit-Raising, Constraints, Operations, Invariant Manifold

1. INTRODUCTION

Over the last few years, Airbus Defence and Space as satellite manufacturer, telecom operators and launch service providers have been looking into new solutions to reduce the cost of satellite payload mass into orbit. The use of low-thrust electric propulsion (EP) is a mean to achieve larger satellite payload mass into orbit for telecommunication satellites. Based on the current status of launcher injection performance and telecommunication satellite masses, low-thrust transfers of Geostationary Equatorial Orbit (GEO) satellites last several months and take up to hundreds of revolutions. In this context, low-thrust transfer optimisation technics were developed at Airbus Defence and Space to design the transfer optimisation software OptElec.

Traditionally, standard GTO to GEO transfers using chemical propulsion (CP) consist in optimising a limited number of high-thrust maneuvers. The compliance with satellite platform constraints and operational constraints is guaranteed by the launch window design. On the contrary, low-thrust transfers of GEO satellites require very long thrust phases. The complex satellite platform and operational constraints induced by the use of electric propulsion systems over these long thrust phases need to be handled by dedicated transfer optimisation software. The challenging problem of computing Electric Orbit Raising (EOR) transfers have been considered for many years [1-2] although constraints during the transfer are often ignored. Some researches introduce methods capable of dealing with a few constraints. Among them, Lyapunov feedback control methods such as the Q-Law [3-4] and the equinoctial Q-Law [5] are able to compute minimum-time and minimum-propellant transfers including J2 effects and coasting during eclipses; however, these type of methods which rely on heuristics to achieve a near optimum result show difficulties to reach precisely the desired targeted orbit. Direct methods have been applied too [6-8] including high fidelity environmental models with a mechanism for coast arcs during eclipses [9]. When using such methods the problem often falls into the category of large scale optimisation problems requiring a high number of optimisation variables. Indirect methods are also used [10] although they require a good initial guess to converge. A recent hybrid method combining heuristics and indirect methods has been proposed, solving the low-thrust transfer for minimum time and minimum propellant mass transfers with eclipses shut-off and slew rate limitations constraints [11].

This paper presents the development, validation and the capabilities of OptElec, a low-thrust transfer optimisation software. OptElec handles a wide variety of satellite platform design features and operational constraints, uses high-fidelity dynamics and satellite models and is able to achieve targets precisely: this makes OptElec adapted for use in an operational context.

2. SOFTWARE REQUIREMENTS

The design of OptElec was driven by three high-level objectives: use for a wide variety of low-thrust transfers, versatility with the possibility to mix multiple propulsion systems and integration to real-time operations i.e. spacecraft commanding.

2.1. Low-thrust transfer optimisation in Mission Analysis

The first purpose of OptElec is to be used for studies and mission analysis of Earth-bound low-thrust transfers in order to provide ΔV budget and transfer duration estimation, including computation of satellite attitude guidance. Eclipses length and occurrence, battery charge state, GEO ring crossing events and platform constraints related data are also provided. The family of injection orbit is not limited to GTO but can typically span from Low Earth Orbit (LEO) to highly eccentric Super Synchronous Transfer Orbit (SSTO). Minimum-time and minimum propellant mass low-thrust transfers can be designed. Also, for a given propulsion system, up to three levels of thrust are available: full thrust, reduced thrust (if available) and no thrust (coast arc). The software can optimise a selection of injection orbit parameters such as the right ascension of the ascending node, argument of perigee and anomaly. Finally, any combination of the following target orbit parameters can be used: classical keplerian parameters (a,e,i,Ω,ω,M), apogee and perigee altitude/radius, drift (in deg/day or deg/rev), equinoctial parameters (inclination vector, eccentricity vector), geographic or mean longitude. For these terminal constraints, fixed values but also lower bounds or upper bounds can be specified.

2.2. Adaptability

The software was developed with flexibility in mind, so that constraints, platform models and platform design features can be easily added. Furthermore, OptElec is to be used in various contexts: e.g. it can handle not only EOR optimisations but also transfers with medium or high thrust chemical propulsion only as well as hybrid transfers including any combination of low, medium or high thrust.

2.3. Operations: high fidelity dynamics and satellite modelling with platform and operational constraints

The main purpose of OptElec is to be used within the Spacecraft Operations Centre during the Launch and Early Operation Phase (LEOP) of geostationary satellites. Therefore, high fidelity environment and satellite modeling are needed: Earth potential perturbations (order and degree to be specified by the user), luni-solar third body perturbations and solar radiation pressure can be selected. The satellite platform and operational constraints are taken into account throughout the optimisation process. A list of constraints handled by OptElec is detailed in the following sub-sections.

2.3.1. Modeling the EP system

The EP basic model is defined with constant thrust and Isp values to be applied for the whole transfer. For operational purposes, the EP system can be further defined with multiple EP configurations, each of them defined with:

- Thrust and Isp (referred as “high level”)
- Reduced thrust and Isp (referred as “reduced level” and representing another PPS set-point)
- Piecewise time-dependent functions for the thrust and Isp efficiencies (to map with the predicted/observed thrust and Isp evolution with time)

2.3.2. Allowing multiple propulsion systems to be used for a given transfer

In addition to EP, other propulsion system can be used during the transfer such as high thrust propulsion e.g. Liquid Apogee Engine (LAE) or medium thrust propulsion e.g. Reaction Control Thrusters (RCT). The number of burns using high or medium thrust propulsion is not limited. The ΔV size or the mass used can be fixed or limited with an upper bound value. The position of the thrust direction of each propulsion system expressed in satellite body frame can be specified by the user.

2.3.3. Constraining the duration of a specific transfer phase

Any (sub) phase of the transfer (a burn when using CP, a low-thrust arc with same level definition or a coast arc when using EP) can be constrained in duration with an upper or lower limit. For instance, the user can constrain burns using LAE with a minimum or/and maximum duration or specify a minimum duration for coast arcs, when they exist, during the EOR phase.

2.3.4. Allowing event-type constraints

Event-type constraints can be included e.g. coast arcs needed for operations, date and orbit constraints for the beginning of a given phase of the transfer, etc…

2.3.5. Limitation of the slew rates

Slew rates (angular velocities of the spacecraft axes expressed in the spacecraft body frame) can be limited. Each axis can be constrained with a specific value. The value may differ whether the satellite is in a thrust or coast phase.

2.3.6. Ensuring the battery state level

A battery model is used to compute criteria representative of the battery charge level and the depth of discharge. The optimiser ensures that the battery charge capacity per orbit and depth of discharge do not cross some given threshold and can turn the thrust off or reduce the thrust level (if available) when needed. Alternatively, a simpler constraint formulation regarding the satellite battery is also available: the thrust is then turned off or reduced when the satellite is in eclipse. The satellite entry and exit into penumbra and umbra regions are precisely calculated. Only Sun by Earth eclipses are considered.

2.3.7. Maximising solar panels illumination

In order to guarantee maximum illumination of the solar panels, the Sun elevation, i.e. the Sun direction with respect to the X-Z satellite plane (the Y-axis defining the solar arrays axis) can be limited at all times with a maximum upper value. Alternatively the time-averaged Sun elevation angle can be constrained.

2.3.8. Ensuring celestial bodies out of any sensor's field of view

The Sun and/or Earth can be kept out of the field of view (FoV) of any given sensor, e.g. one can require the Sun and/or the Earth to be kept out of a Star Tracker's (STR) FoV. The sensor's FoV is modeled with a cone which is defined by its line of sight (LoS) expressed in the satellite body axis and its half-angle value.

2.3.9. Thermal constraints

Thermal constraints can be added by requiring a minimum or maximum Sun incidence with respect to any given direction expressed in spacecraft body frame.

2.3.9. GEO ring avoidance constraints

The GEO ring can be defined by the user by in terms of radial distance wrt the GEO distance (ΔR wrt 35786 km of altitude) and a normal distance wrt equatorial plane (ΔN). The optimiser will then modify the trajectory (provided by an initial guess, see 4.2) to guarantee that no crossing will occur during the transfer. Because the injection orbit and the target orbit may be within the GEO ring volume, transfer times during which the constraint actually applies may be defined.

2.3.10. Daily use during operations

Real time operations require fast-running software, without the need of an optimisation technics expertise. They also require re-optimisation at every transfer cycle – where a cycle typically corresponds to one week of operations.

After a cycle has been completed, the satellite is in a state that deviates from its nominal state, this being caused by propulsion under or over-performance, on-board guidance, triaxiality effects, attitude pointing errors, orbit determination errors, perturbations and operational events that are not modelled by the optimiser. Starting with the new state of the satellite, the software can automatically optimise the updated transfer strategy using the previous cycle optimisation solution.

3. TRANSFER OPTIMISATION PROBLEM

The general problem consists in minimising a performance index or cost function J defined by:

$$J = \phi\big(\boldsymbol{x}(t_0), t_0, \boldsymbol{x}(t_f), t_f; \boldsymbol{p}\big) + \int_{t_0}^{t_f} L(\boldsymbol{x}(t), \boldsymbol{u}(t), t; \boldsymbol{p})\, dt$$

where $\boldsymbol{x}$ is the state vector, $\boldsymbol{u}$ is the control vector, t_0 and t_f are the initial time and final time of the trajectory and $\boldsymbol{p}$ some parameters. In the context of GEO transfers, the objective is to maximise the beginning of life mass (or minimise the used propellant mass) or to minimise the transfer duration. Therefore, the cost function often takes the following simple forms:

$$J = t_f - t_0 \text{ or } J = m_p(t_f)$$

where $m_p(t_f)$ is the used propellant mass at the end of the transfer.

The dynamic of the system is defined by a set of differential equations:

$$\dot{\boldsymbol{x}} = f(\boldsymbol{x}(t), \boldsymbol{u}(t), t; \boldsymbol{p})$$

Initial conditions apply to the problem:

$$\psi_{min}^0 \le \psi(\boldsymbol{x}(t_0), t_0; \boldsymbol{p}) \le \psi_{max}^0$$

as well as final conditions or terminal constraints:

$$\psi_{min}^f \le \psi\big(\boldsymbol{x}(t_f), t_f; \boldsymbol{p}\big) \le \psi_{max}^f$$

Intermediate constraints or event-type constraints may eventually be added to the problem:

$$\psi_{min}^e \le \psi(\boldsymbol{x}(t_e), t_e; \boldsymbol{p}) \le \psi_{max}^e$$

where t_e is the time at which the event takes place and $t_0 < t_e < t_f$. The solution may further be subject to path constraints that takes the following form:

$$C_{min} \le C(\boldsymbol{x}(t), \boldsymbol{u}(t), t; \boldsymbol{p}) \le C_{max}$$

The general form of the path constraint C is a function of the state and control variable that is evaluated over a part of the trajectory.

In order to solve the orbital transfer optimisation problem, a direct approach is used in which the trajectory is decomposed into individual segments. The optimal control problem is then converted into a nonlinear programming problem (NLP). The general mathematical formulation of the NLP consists in determining the variables $\boldsymbol{z} \in \mathbb{R}^n$ that minimises the cost function J subjects to the equality and inequality constraints:

$$\text{Min } J(z) \text{, subject to}$$
$$c_{eq}(\boldsymbol{z}) = \boldsymbol{0}$$
$$c_{ineq}(\boldsymbol{z}) \leq \boldsymbol{0}$$

This NLP is then solved by using a first-order gradient-based optimisation method. This method requires the derivatives of all states, cost and constraints functions with respect to the optimisable variables. Given an estimate of the optimising value of $\boldsymbol{z}$ at the k^{th} iteration, $\boldsymbol{z}_k$, the solution is updated to:

$$\boldsymbol{z}_{k+1} = \boldsymbol{z}_k + \alpha_k \boldsymbol{s}_k$$

where $\boldsymbol{s}_k$ is the search direction that provides a direction in $\mathbb{R}^n$ along which $\boldsymbol{z}_k$ must be changed and α_k is some scalar ($\alpha_k \neq 0$) that determines the step length. This increment in the optimisable variables is calculated to minimise the cost function and to ensure the constraints are observed. This process is repeated until the cost function can no longer be reduced.

4. METHODOLOGY

4.1. Multiple shooting approach

OptElec's optimisation technique is based on a direct optimisation method and uses multiple shooting (MS) to represent the trajectory. Such methods are capable of treating complex problems [12-13] including various path constraints, while offering high robustness. Although MS algorithms may be quite complex to initially apply, they offer many advantages for extended trajectory problems. This approach seems well adapted to treat the various considered scenarios while offering the robustness needed for operational software.

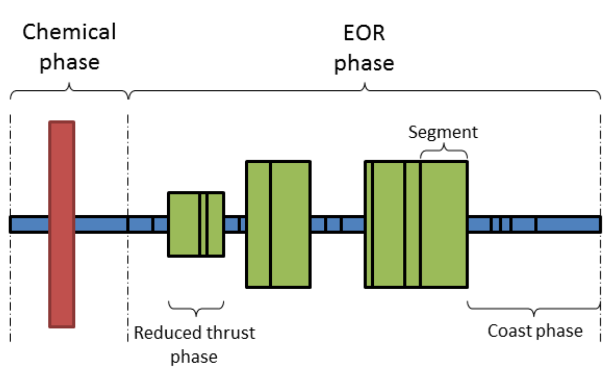

Fig. 1. Example of trajectory decomposition into segments.

The MS approach discretizes the orbit transfer into segments, each of them describing a portion of the trajectory with a given propulsion system and thrust level. A consecutive list of segments using the same propulsion system and thrust level is then called a sub-phase. A consecutive list of segments using the same propulsion system is called a phase.

For instance, Fig 1. describes a transfer with two phases: a chemical phase and an EOR phase. The chemical phase has 3 sub-phases: 2 coast phases (in blue) and one thrust phase (in red) made of one single thrust segment. The EOR phase is made of multiple sub-phases: coast phases, phases with maximum level of low-thrust (in green and higher size) and phases with reduced level of low-thrust (in green and lower size). Each of these sub-phases is composed of one or multiple segments.

4.2. Initial guess

The initial guess is provided by an in-house mission analysis software for chemical, hybrid or low-thrust transfers (GenetOp). These tools use analytical methods and heuristic methods in order to generate a variety of trajectories. Given an initial guess, OptElec performs an initial discretization of the transfer into multiple segments in order to generate a first initial solution.

Alternatively, a previous OptElec transfer solution can be used directly as initial guess ("warm start"). This previous solution may possibly be modified by the software to match with the orbit transfer problem to solve. Initial guess values for the optimisable parameters (referred as $\boldsymbol{z}_0$ for the NLP problem) can therefore obtained by two different means.

4.3. Adapting the software

The technics used to achieve the required flexibility in OptElec use cases (described in section 2.) are:

- adapting the initial discretization of the transfer (number of segment, segment length and/or duration, control variable definition) to the kind of transfer we are dealing with. In particular, non-linearities caused by high dynamics or highly non-linear path constraints can be reduced by refining the initial set of segments.
- selecting an appropriate scaling for the state vector, control and constraint vector and using fine-tuned optimisation parameters depending on the type of EOR problem to solve.

5. VALIDATION

A thorough validation of OptElec with respect to various cases found in the literature has been performed. Various kinds of orbits, minimum time transfers and minimum propellant mass transfers have been studied. Note that, unless stated otherwise, the reference cases use intra-orbit averaging which may induce some small differences when comparing results with OptElec.

5.1. Minimum time transfers

Different minimum time low-thrust scenarios have been tested. The reference cases are described in Table 1.

Table 1. Ref case definition.

Cases	Perturb	F [N]	Isp [s]	Mass [kg]	Sma [km]	Ecc [-]	Inc [deg]
1	-	0.35	2000	2000	24509	0.725	7
2	J2, thrust OFF in eclipse	1.86948	1800	5500	6780	0	5.2
3		0.40171	3300	1200	6927	0	28.5
4		0.20085	3300	450	24364	0.731	27

The first case is a GTO to GEO case without perturbations. J2 perturbation is included and the thrust is turned off during eclipses for cases 2 to 4. Cases 2 and 3 are LEO to GEO cases and case 4 is a GTO to GEO case. The results obtained with OptElec and the reference results are summarized in Table 2.

Table 2. Minimum-time EOR validation.

Cases	Mission	References	Transfer duration (days) Ref	Transfer duration (days) OptElec
1	GTO-GEO	MIPELEC [9]	137.5	137.4
2	LEO-GEO	Ref [6]	167.8	167.7
3	LEO-GEO		198.8	199.2
4	GTO-GEO		66.6	66.7

Table 2 shows that the minimum time cases results obtained with OptElec match very well with the reference ones.

5.2. Minimum propellant mass transfers

For minimum propellant mass cases, a maximum or fixed transfer duration is given; the thrust is then reduced or turned off when the efficiency of thrusting is low. The reference test cases are all taken from [11].
The presented case is a typical GTO ($h_a = 35786$ km, $h_p = 300$ km, $i_0 = 6$ deg, $\Omega_0 = 270$ deg, $\omega_0 = 180$ deg) to GEO transfer, with Keplerian orbit propagation, without constraint and with F= 0.8N, I_{sp}= 1800 s and m_0= 5000 kg. OptElec's result for the minimum time transfer is 149.3 day transfer with a ΔV of 2195.1 m/s. The minimum propellant mass cases are for transfers of 150, 153, 160 and 170-days. For those cases, the thrust level is either "ON" or "OFF". The results given by OptElec are provided in Table 3 and matches very well with those obtained with LOTTO [11].

Table 3. Minimum propellant mass transfer validation.

Cases	Mission	References	ΔV [m/s] Ref	ΔV [m/s] OptElec
5	Min-time	LOTTO [11]	2196.9	2195.1
6	150 days		2166.8	2162.0
7	153 days		2091.8	2090.2
8	160 days		1993.6	1992.3
9	170 days		1900.1	1898.7

Figure 2 shows the true anomaly of the coast arcs locations (in red) for the 160-day transfer; single coast arcs initially appear around Day 24 and are located around the perigee. Around Day 102, the single coast arcs are broken into two coast arcs. The last coast arcs appear around Day 140 where they are located near the orbit's nodes.

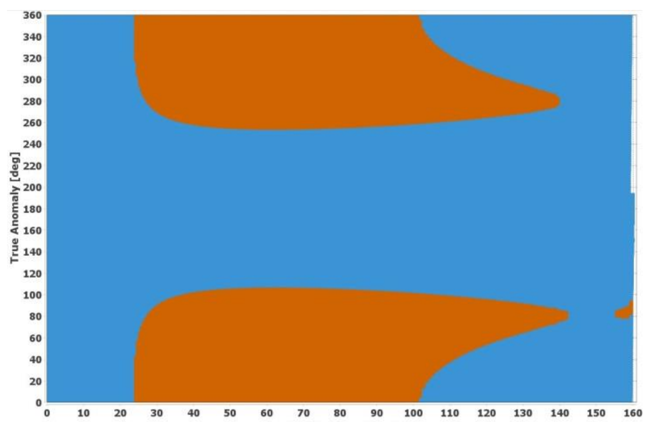

Fig. 2. Case 8 (160 day transfer): true anomaly of the coast arc location.

6. A STUDY CASE: LOW-THRUST TRANSFERS WITH MIXED PROPULSION SYSTEMS AND CONSTRAINTS

We now introduce a study case illustrating a realistic low-thrust transfer with operational constraints. The case is a GTO to GEO transfer with initial high inclination ($h_a = 35786$ km, $h_p = 300$ km, $i_0 = 20$ deg, $\Omega_0 = 90$ deg, $\omega_0 = 180$ deg) and an initial mass m_0= 3500 kg. The injection date is Jan. 01st 2017, 00h00m. The target orbit has a drift rate of 0.5 deg/day, i = 0 deg, e = 0 and a geographic longitude of 100 deg East. For the EP system, the simplified model is used: F = 1.0 N and I_{sp}= 2000 s and constant throughout the transfer. The satellite is equipped with 2 RCTs providing a thrust F= 10 N each and I_{sp}= 290 s. Earth potential (J_2 and $J_{2,2}$) and third body gravitational perturbations by Sun and Moon are considered. The thrust direction is aligned with the satellite Z-axis. The satellite is equipped with one STR which orientation is [-0.6, 0.8, 0] in the satellite body frame.

6.1. Minimum time transfer

We first consider the minimum-time problem with two distinct phases: the transfer starts with a chemical phase followed by an EOR phase. We further assume that the chemical phase uses RCT burns only.

We start by only including some operational constraints. Then, we consider thrust reduction during eclipses. Finally, we take into account platform constraints and additional operational constraints to this transfer.

6.1.1. Case-MT0: Minimum-Time problem with mixed propulsion systems and some operational constraints

This case considers the following constraints:

- The transfer starts with a first chemical phase using 1 or 2 burns using RCTs.
- The 1st RCT burn must occur after a minimum of 2 full revolutions after injection.
- The maximum allocated propellant mass for this chemical phase is 40 kg.
- The EOR phase must start on the 5th of January 2017 00h00m (e.g. for operational reasons such as ground station visibility or operation schedule, etc …)
- Terminal constraints on the target orbit.

Two RCT burns are proposed in the initial guess solution. Therefore, the initial discretization of the segments contains two RCT burns, with coast arcs in between. The RCT burns are not limited by a minimum duration so the optimiser could theoretically suppress one burn by decreasing its duration to zero. However in practice the optimiser keeps them both in order to reduce the ΔV loss induced by manoeuvre spreading. The other constraints are event-type constraints: e.g. an equality constraint is applied in order to guarantee that the EOR-phase starts on the 5th of January 2017 00h00m.

The result is a transfer in 94.3 days (90.3 days for the EOR-phase) and a total ΔV of 2428.2 m/s (including the RCT burns). The optimiser uses the 40 kg of chemical propellant available to perform 2 RCT burns located around the apogee. They raise the perigee by roughly 224 km and decrease the inclination by ~0.8 deg before EOR starts. All other orbital elements remain unchanged by this first chemical phase.

6.1.2. Case-MT1: MT0 + thrust reduction during eclipses

We now use the same assumptions than case MT0 and include power-related constraints. Since the full expression of the constraint (battery charge and depth of discharge) is rather complex and platform-dependent, we will consider that the constraint applies when the spacecraft is in eclipse. When it happens, the thrust level must be reduced by half (0.5 N available with unchanged Isp).

With this additional constraint, we now have four different types of sub-phases. There are sub-phases containing coast segments, other sub-phases in between containing one RCT burn segment each, and multiple sub-phases contain for each of them several low-thrust segments with full or reduced thrust.

The result of this case, illustrated in Fig. 3, is a transfer of 95.7 days (91.7 days for the EOR phase) and a total ΔV of 2417.6 m/s.

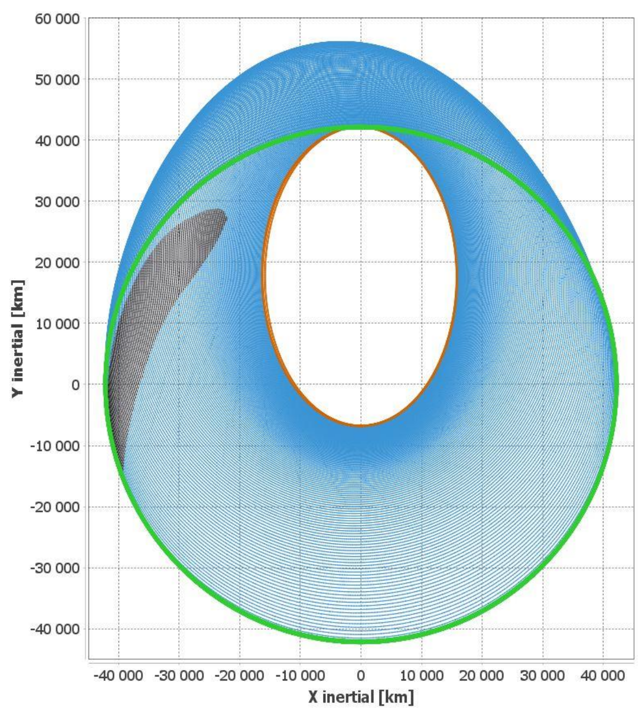

Fig. 3. Case MT1: trajectory in equatorial plane. Coast arcs are in red, full thrust arcs in blue and reduced thrust arcs in black. The GEO orbit is in green.

The total time spent in eclipse is about 89 hours and since the thrust is reduced by half during eclipses the transfer is longer than case MT0 by 1.4 day. The eclipses occur after 22 days of transfer and last until the end of the transfer. The overall ΔV cost is slightly reduced compared to case MT0. It seems to indicate that the eclipses occur at portions of orbit where thrusting at full capacity is not so efficient. Again, the optimiser uses the 40 kg of chemical propellant available to perform two RCT burns located around the apogee. However, the presence of the thrust reduction during eclipses constraint has changed the optimal RCT burns: they now raise the perigee by ~205 km and decrease the inclination by ~0.9 deg before EOR starts which slightly differs from case MT0.

6.1.3. Case-MT2:MT1 + operation and platform constraints

We now present a realistic operational scenario with additional constraints used on top of those proposed for the cases MT0 and MT1:

- The minimum apogee radius after the chemical phase must be 42365 km (200 km above the GEO radius, so that the risk of collisions with GEO satellites is reduced). A third RCT burn is allowed. The maximum allocated chemical propellant mass remains 40 kg.
- The maximum attitude slew rates are: 100 deg/h, 200 deg/h, 100 deg/h respectively around X, Y and Z satellite axes. Values apply for both coast and thrust arcs.
- The Sun elevation with respect to the satellite X-Z plan must be contained within 30 deg at all times.
- The STR must not be blinded at any time by the Earth or by the Sun. The exclusion half-cone angle is set to 21 deg for the Earth and 26 deg for the Sun.
- Any EOR sub-phase (phase with reduced or full low-thrust) must last at least 30 minutes.

We have applied an additional event-type constraint (minimum apogee radius) at the end of the chemical phase. The additional path constraints apply on the EOR-phase and are expected to modify the attitude profile with respect to case MT1.

The result is a transfer of 95.8 days (91.8 days for the EOR phase) and a total ΔV of 2418.4 m/s. Despite the addition of constraints, the total transfer duration and ΔV cost has barely increased compared to case MT1 (+0.07%).

This time, three RCT burns are used by the optimiser. The first RCT burn (3.5 m/s) is performed around the perigee to increase the apogee by ~200 km and guarantee that the EOR starts with an altitude 200 km above the GEO radius.

The remaining propellant is used with 2 burns located around the apogee to raise the perigee and decrease the inclination before EOR starts in a similar manner than case MT1.

Figure 4 shows the Sun azimuth (Sun projection on X-Z satellite plane, counted positively from +Zsat to +Xsat) and Sun elevation (counted positively towards +Ysat) angles for cases A1 and A2 during the entire EOR-phase.

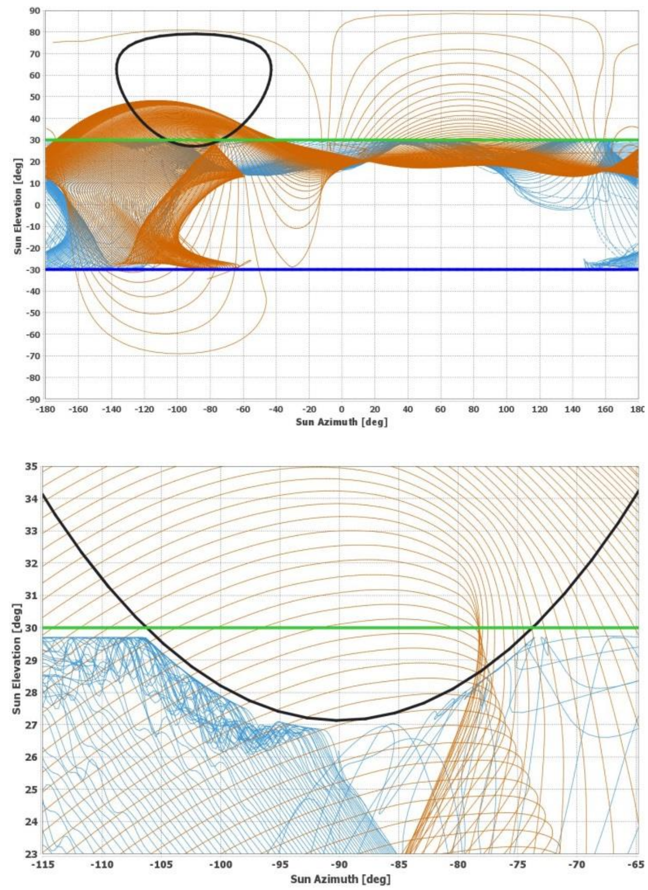

Fig. 4. (Top) Sun elevation vs sun azimuth during the EOR-phase for the unconstrained (case MT1, in red) and constrained case (MT2, in blue). The horizontal lines show the limits for the sun elevation constraint. The black line represents the Sun STR blinding constraint. The bottom figure is a close-up near the Sun STR blinding constraint.

As expected, the unconstrained case (in red) violates the Sun elevation and Sun STR blinding constraints. On the other hand, the Sun elevation constraint is observed (blue lines within the limits) for case MT2. The bottom figure is a close-up around the Sun STR blinding constraint. One can see that the blue lines never cross the constraint threshold. Clearly the spacecraft attitude has been modified to observe the constraints. Note that the visible margin with respect to the thresholds actually corresponds to the apparent radius of the Sun.

Contrary to the Sun, the Earth apparent diameter as seen from the spacecraft strongly varies along the orbit and during the transfer. At beginning of EOR phase, the Earth apparent diameter varies between ~18 and 140 deg over one revolution.

Figure 5 shows the Earth and Sun STR blinding margin angle (closest angle between the STR's FoV and the apparent Earth/Sun) during the first day of the EOR-phase, for the unconstrained and constrained solutions.

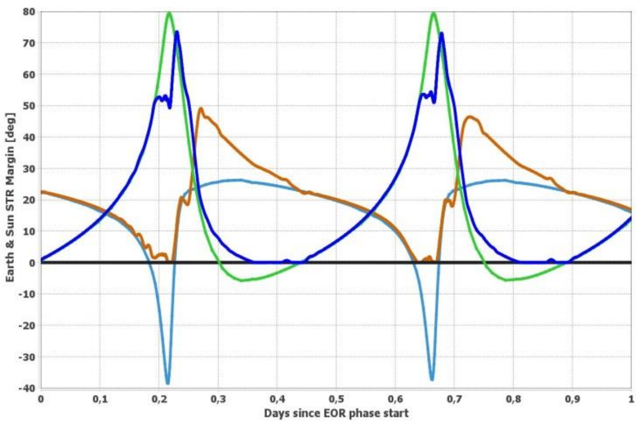

Fig. 5. Case MT2: Earth and Sun STR blinding margins during the first day of the EOR phase. Values below zero mean the Earth (resp. Sun) enters the STR's FoV. The light blue (case MT1) and the red (case MT2) lines are for the Earth blindings. The green (case MT1) and the dark blue (case MT2) lines are for the Sun blindings.

As expected, the margin angle of the unconstrained case has negative values, meaning that the Sun and the Earth enter the FoV of the STR. On the other hand, for the case MT2, the margin angle never crosses the threshold. This is also illustrated in Fig. 6 [15] where the Earth, as seen from the spacecraft (volume in green), osculates the STR's FoV (volume in red). To observe the constraints, the optimiser uses the available degree of freedom: the rotation angle around the thrust direction. When necessary, the thrust steering angles are also modified, inducing an over-cost.

Finally, the slew rate constraints and the phases with minimum duration constraint (forcing the reduced thrust phase to last at least 30 minutes) are observed without significant over-cost.

6.2. Minimum propellant mass transfer

We now treat the minimum propellant mass transfer case: the total transfer duration, including the chemical phase, is fixed to 100 days. In general, three levels of thrust can be available during the EOR phase: full thrust, reduced thrust and coast arcs. Reduced thrust arcs and coast arcs can be used during eclipses to cope with the battery charge constraint. They can also be used during non-efficient orbit arcs for reducing the propellant consumption. The user defines what they can be used for. In our case, coast arcs are now authorized during EOR to minimise the propellant usage and the reduced thrust shall be used when thrusting in eclipse. Cases MP1 (resp. MP2) is similar to MT1 (resp. MT2) in terms of operational and platform constraints.

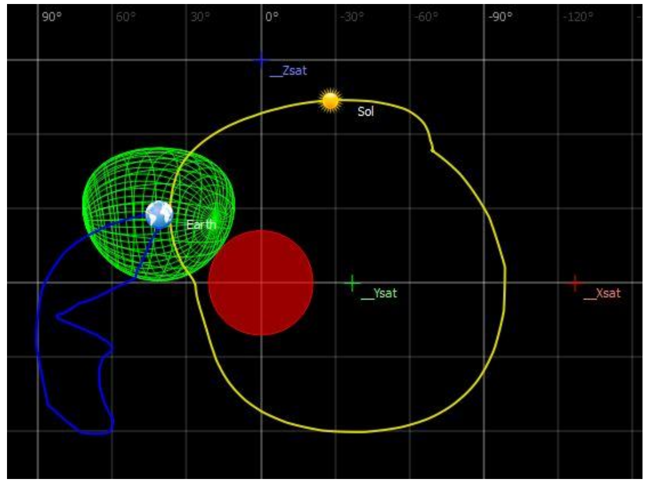

Fig. 6. Case MT2 – 0.642 days after beginning of the EOR phase: Earth and Sun path (resp. in blue and yellow) over one revolution as seen from the STR body frame. The green volume around the Earth represents the Earth as seen from the STR. The STR's FoV is in red. One can see that this volume does not cross the FoV limits, hence validating the constraint.

The results for cases MP1 and MP2 are summarized in Table 4. When comparing minimum time and minimum propellant mass cases, one can see that increasing the transfer duration by slightly more than 4 days has allowed a significant reduction in total ΔV (a bit more than 150 m/s, i.e. -6.3%). When comparing the unconstrained and constrained minimum propellant cases, we see that including the constraints have increased the total ΔV by ~0.16%.

Table 4. Summary of the study case results.

Cases	Transfer Type [time/mass]	Transfer duration [days]	ΔV [m/s]	Constraints Over-cost [%]
MT0	Time	94.30	2428.2	N/A
MT1	Time	95.71	2417.6	0.07
MT2	Time	95.78	2418.4	
MP1	Mass	100	2263.1	0.16
MP2	Mass	100	2266.7	

Figure 7 shows the MP2 case trajectory, including the coast arcs and reduced thrust phases. The 3 RCT burns are too small to be seen (hidden by the coast arcs between each RCT burns at the beginning of the transfer). The utilization of the RCT burns by the optimiser is very similar to what was done for case MT2. Also, for our particular case, the coast arcs of the EOR-phase are separated from the reduced thrust arcs (eclipses).

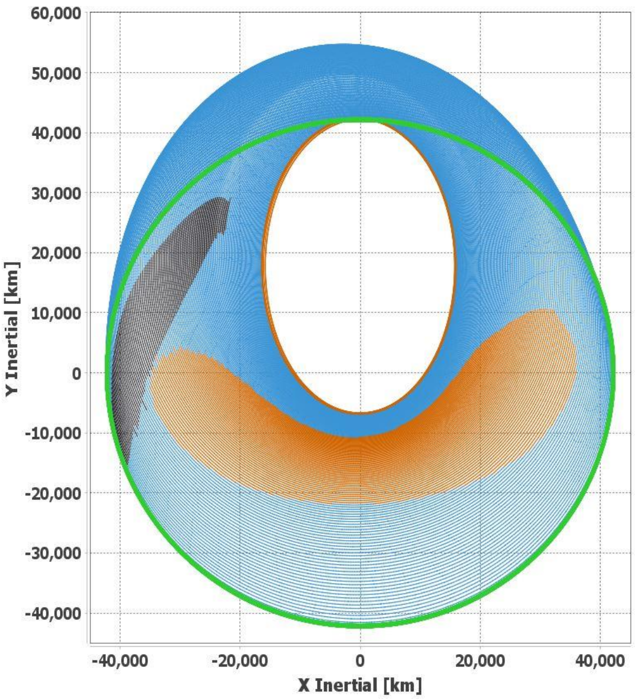

Fig. 7. Case B2: trajectory in equatorial plane. Coast arcs are in red, full thrust arc in blue and reduced thrust arcs in black. The GEO orbit is in green.

The slew rate constraints are observed whether the satellite is in a thrust phase or in a coast phase. The phase with minimum duration constraint (forcing the reduced thrust phase and the coast phases to last at least 30 minutes) has now a minor impact (~1 m/s over-cost) since quite a few coast arcs are forced to last 30 minutes while they would be shorter if unconstrained.

6.3. Interpretation

In order to observe all the attitude and slew rate constraints, the optimiser uses in priority the remaining degree of freedom, i.e. the rotation around the Zsat axis (here equal to the thrust direction) but it may also be forced to change the thrust steering angles in order to observe all the constraints. The resulting effect is an increase in the cost function as the thrust orientation is not optimal anymore.

Also it is often found that, taken individually, some constraints might be easy to observe. It is the combination of multiple constraints that makes the optimiser tweaks the unconstrained thrust profile which induces an increase of the cost function. This is illustrated in Fig 8. where the out-of-plane elevation angle and yaw angle (rotation around thrust axis) are modified with respect to the optimum unconstrained angles: the elevation angle is tweaked and the yaw angles takes values different from 0 deg (unconstrained case) in order to satisfy the constraints. As seen in the results of our study case, the over-cost is here acceptable.

However, in practice, the over-cost depends on various aspects of the mission (initial orbit inclination and eccentricity, initial Sun position with respect to the injection orbit, etc.) and on the constraints (their number, how strict they are, etc.). This needs to be evaluated during the mission analysis. If needed, the over-cost can be compensated by slightly increasing the allowed maximum transfer duration since great propellant mass savings can be made within a fraction of day or a few days.

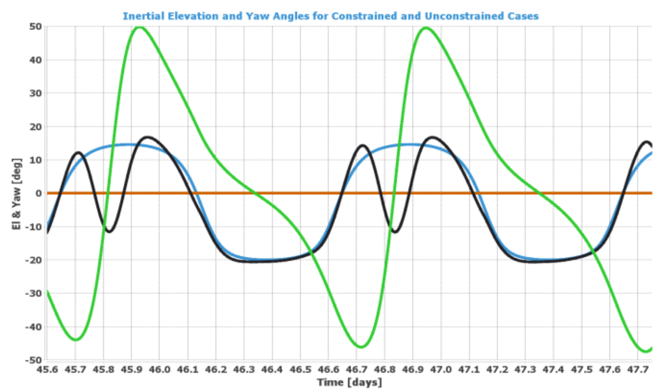

Fig. 8. Thrust elevation and yaw angles taken from an operational EOR transfer – in-flight data (slew rates, sun incidence and thermal constrains apply): the unconstrained optimal case angles are shown in blue (el) and red (yaw) while the operational constrained case angles are shown in black (el) and green (yaw).

7. CHEMICAL TRANSFER

The purpose of this section is to show that the approach taken for designing OptElec allows its use in wider contexts such as a chemical transfer to GEO. The initial orbit is a sub-synchronous orbit with $h_a = 28900\ km,\ h_p = 290\ km, i_0 = 28\ deg,\ \Omega_0 = 170$ deg, $\omega_0 = 180\ deg, \theta_0 = 20\ deg$. The initial mass is m_0= 4500 kg. For the chemical propulsion system, F= 450N, I_{sp}= 320 s. Earth potential (zonal and tesseral harmonics up to 10), third body gravitational perturbations by Sun and Moon and SRP are considered. The following constraints apply to the transfer:

7.1. Mission analysis constraints

For this injection orbit it is clear that two types of maneuvers must be applied: first, maneuvers at perigee (PVA) followed by maneuvers at apogee (AEF). In order to achieve the minimum ΔV cost (the optimiser's objective), maneuvers at perigee must be performed first. We will assume that the following mission analysis ascent profile fulfils the operational constraints and requires that:

- 2 PVA must be performed, followed by 3 AEF.
- The 1st PVA occurs 5 revs after separation. The 2nd PVA occurs 3 revs after PVA1.
- The first AEF occurs 2.5 revs after the last PVA. The 2nd and 3rd AEF occurs 2 revs after the AEF before them.
- The last burn (AEF) must not exceed 350 m/s.

7.2. Platform constraints

- The maximum burn duration is 2 hours.
- The minimum burn durations is 120 s.
- AEFs must be fixed in inertial frame while PVA may have time-varying thrust direction.

7.3. Targets

The following targets are considered:

- Drift = 1 deg/day
- Maximum apogee radius: 42125 km (to avoid Geo ring crossings before the final drift phase)
- $i = 0.1$ deg, $\Omega = 270$ deg.
- $l_{geo} = 0$ deg.

7.4. Results

For this case, OptElec uses one single segment for each maneuver. Furthermore, one single segment is used to model the coast phases between thrust segments. Both equality and inequality constraints are used.

The optimisation result is provided in Table 5 (case C1). The optimal ascent profile requires that the last burn ΔV is exactly 350 m/s, which indicates that the total ΔV would be reduced if a larger ΔV size were allowed for the last burn. The same applies for the maximum apogee radius constraint as the achieved apogee radius is exactly 42125 km. If the maximum ΔV size for the last burn constraint is now removed (case C2), the total ΔV is as expected reduced to 1972.3 m/s and the optimum ΔV size for the last burn is 468.9 m/s. The total ΔV has been reduced because the overall Robbins penalty has decreased due to more “well balanced” AEFs. If we now let OptElec optimise the AoP at injection (case C3), the total ΔV is lowered to 1959.2 m/s with an optimum AoP at injection of 176.18 deg. In this case, optimising the AoP at injection enables significant ΔV gains.

Table 5. Comparison of the optimised chemical transfers.

Cases	AoP at injection		Last ΔV		Total ΔV [m/s]
	Status	AoP [deg]	Status	ΔV [m/s]	
C1	Fixed	180	Max	350	1974.2
C2	Fixed	180	Free	468.9	1972.3
C3	Free	176.18	Max	350	1959.2

8. TO THE MOON

In this section, we show how OptElec can be used to solve other low-thrust transfer optimisation problems, such as Earth to Halo transfers.

To do so, the software have been upgraded so that the central gravity field can be chosen (Earth, Moon, other planets) and the initial and target orbit can be expressed in the reference frame specified by the user (e.g. initial orbit is expressed in an Earth centred inertial reference frame and the target orbit is expressed in a Moon centred inertial reference frame).

In our example, the first part of the transfer consists in a minimum time low-thrust transfer up to an entry point of a stable manifold associated to a EML2 Halo orbit; the second part of the transfer consists in a coast phase on the stable manifold towards the EML2 Halo orbit. The amplitude Az, the initial position on the Halo orbit and the transfer duration along the stable manifold are fixed. In the literature, the latter two parameters are sometimes optimised while the low-thrust phase consists in a tangential spiraling [16]. This example focuses on the low-thrust part of the transfer: to do so, a stable manifold associated to the Halo orbit has been chosen and backward propagated and the low-thrust phase is optimised under operational constraints. The initial orbit is a sub-GTO orbit (Ha = 20000 km, Hp = 200 km and Inc = 28 deg).

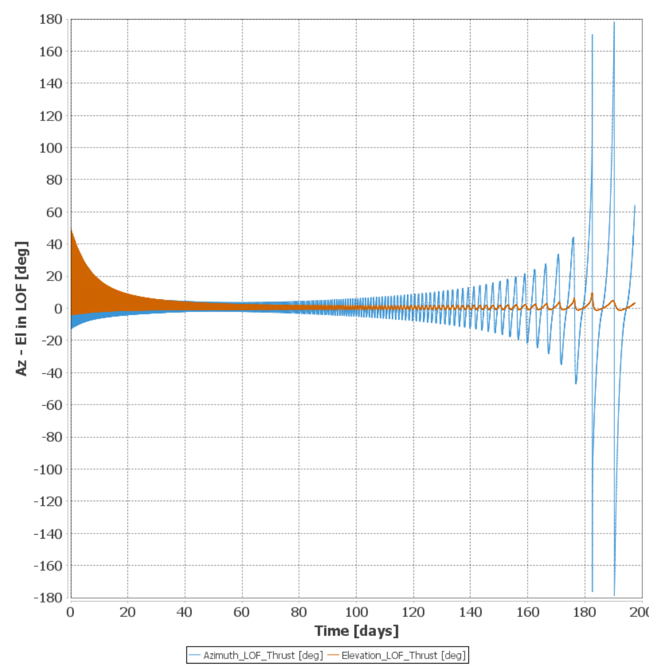

Fig. 9. Optimised in plane (az, in blue) and out-of-plane (el, in red) angle of the thrust vector. The in-plane angle shows some deviations wrt the velocity direction (az = 0 deg). The out-of-plane angle is different from 0 deg to correct for the orbit plane.

A first solution for the low-thrust transfer phase that targets a point close to the end point of the stable manifold is generated and serves as initial guess to OptElec. The software optimises the low-thrust phase in a perturbed environment (Earth harmonics, Sun and Moon gravity field and SRP) and considers some platform constraints (thrust off during eclipses, maximum slew rates set to 100-200-100 deg/h, maximum Sun incidence set to 20 deg). The solution that is found corrects for the defects between the end of the thrust phase and the beginning of the stable manifold phase, observes the constraints and optimises the transfer duration. Fig. 9 shows the thrust profile evolution and Fig 10 shows the transfer trajectory.

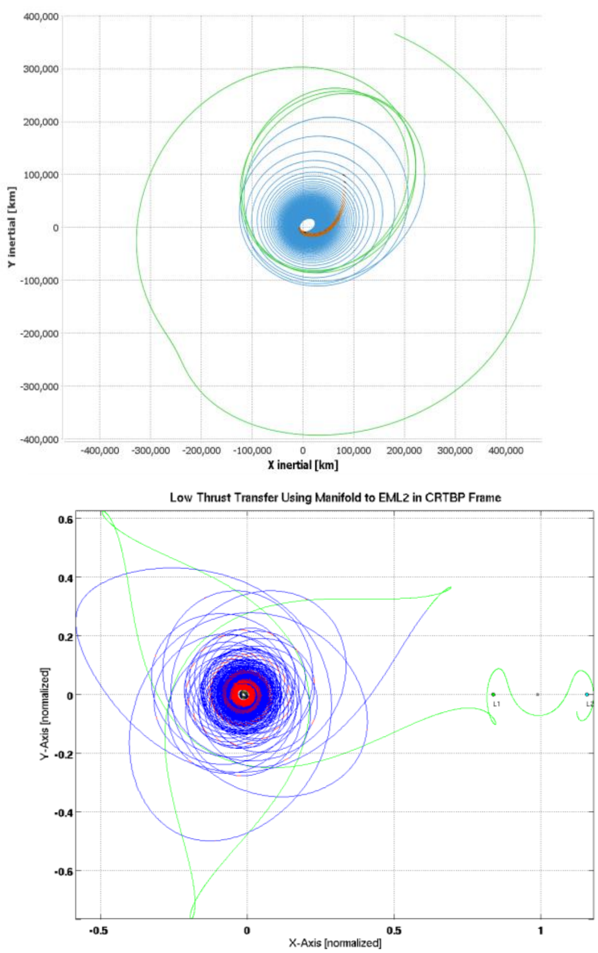

Fig. 10. Transfer trajectory from the Earth injection orbit to a L2 halo orbit. Earth-centered inertial frame (top), Earth–Moon rotating frame (bottom). The thrust arcs are in blue, the coast arcs for eclipses are in red and the stable manifold arc in green.

A step further would consists in a more global optimisation process: for a given Az amplitude of the Halo orbit, the optimisable parameters would be the position on the Halo orbit and the backward propagation duration on the manifold but also the entire low-thrust transfer phase (steering angles, thrust module history). Allowing for an optimised low-thrust phase (not only-tangential steering, coast arcs or reduced thrust phase allowed) would enable to search for other solutions to the minimum time or minimum propellant Earth to Halo transfer problem. In addition, in a two-step process, starting from the optimised solution of an unconstrained scenario, OptElec could then include platform and operational constraints for a more realistic transfer scenario.

9. CONCLUSIONS

The Airbus Defence and Space OptElec software has been developed to optimise mixed propulsion transfers and has been successfully validated. OptElec handles a wide variety of satellite platform design features and operational constraints and uses high-fidelity dynamics and satellite models which make it well suited for operational use. OptElec has already been used for GEO chemical transfers and for Electric Orbit Raising during LEOP with mixed propulsion system. It is regularly used at mission analysis level for all kinds of missions (LEO, MEO, GEO) and is currently being upgraded to solve low-thrust Earth-Moon transfers, electric orbit raising and insertion around other planets.

10. ACKNOWLEDGEMENT

The author is particularly thankful to Philippe Bertrand, Sylvain Rouard, Steve Kemble, Mark Watt and Kevin McManamon for their contribution to the development of OptElec. The author would also like to thank Georges Krier for providing additional results to his previous paper [11].

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