EXPLORE DESIGN PERFECTION

Robust Design Optimisation of Space Missions

Mariapia Marchi (ESTECO SpA, Italy)

Outline

- Introduction: ITI project & partners
- Project goal
- Theoretical background
- Project scenarios & algorithms
- Software plug-in
- Plug-in & methods validation
  - CubeSat example
  - SSTL test case
  - ESA test case
- Conclusions & future developments

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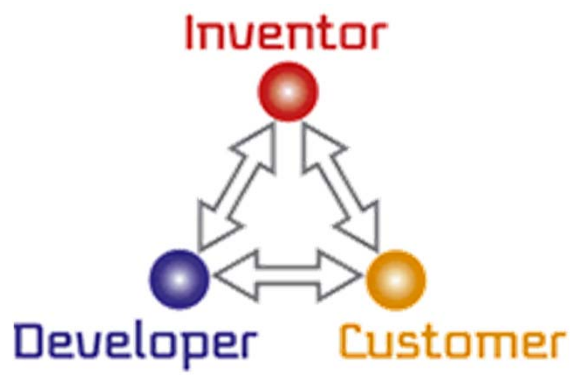
ITI Project & Partners

Development of Robust Design Optimisation of Space Missions (ITI project - type B)

University of Strathclyde



ESTECO SpA

SSTL

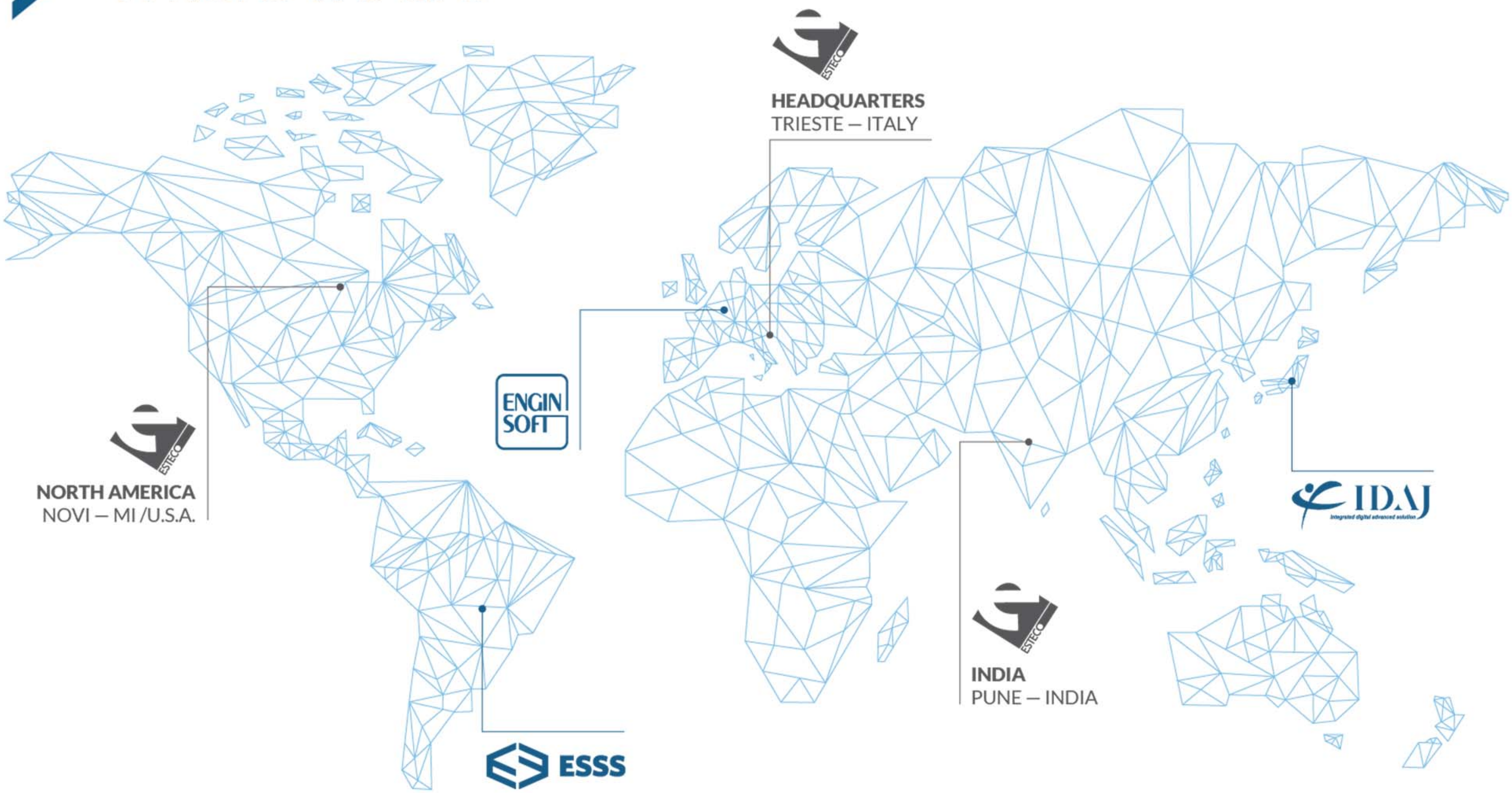
About ESTECO

ESTECO is an independent software provider, highly specialised in numerical optimisation and simulation data management with a sound scientific foundation and a flexible approach to customer needs. Our technology brings modularity, ease of use, standardisation, and innovation to the engineering design process.

VOLTA is a *web platform* for multidisciplinary business process optimisation and the management of enterprise simulation data.

modeFRONTIER is a comprehensive solution for process automation and optimisation of the engineering design process.

Where we are



The company headquarters are located in Trieste, Italy with offices in North America and India

Our technology is distributed throughout the world by a network of qualified channel partners, who provide proficient local service and support to our customers.

modeFRONTIER Technology

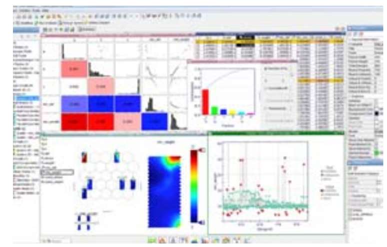
modeFRONTIER offers a **modular environment** giving access to different sets of functionalities

ACCESS THROUGH mF mP

WORKFLOW BUILDING

ACCESS THROUGH mF

PROCESS MONITORING



ACCESS THROUGH mF mS

ANALYSING RESULTS

Each is tailored
to achieve the ultimate goal
» **DESIGN OPTIMISATION**

Aerospace at the University of Strathclyde

- Long tradition of aerospace studies. In Prof Vasile's group:
- Several ESA studies (previous and current):
  - 4 Ariadna studies
  - ITI Development of a three-level bio inspired autonomous system
  - ITI Robust Mission Design Part A – Inventor
  - NPI Robust Mission Design
  - NPI Dynamics and control of formations at libration points
  - SYSNova – Contactless Asteroid Deflection
  - ITT Satellite Disposal from MEO
  - ITT Spacecraft Disposal from LPO and HEO
  - ITT Quantifying Uncertainty During Re-entry
  - NPI Multi-objective Hybrid Optimal Control

- Current collaboration with ESTECO under UTOPIAE - Uncertainty Treatment and Optimisation in Aerospace Engineering (EU-funded ITN project, until 2020)

Surrey Satellite Technology Limited (SSTL)

SURREY

- Pioneer of low cost satellites
- Focus applications, innovation and value
- Complete end to end solution
- Partnership approach to deliver low cost, high value capability

PRODUCTS

Operational space systems
Turn-key and bespoke space missions
Satellite platforms & Avionics
Training and Development

People Involved

ESTECO:

Carlo Poloni
Carlos Kavka
Paolo Vercesi
Mariapia Marchi

University of Strathclyde:

Massimiliano Vasile
Annalisa Riccardi
Carlos Ortega
Gianluca Filippi

SSTL:

Kathryn Graham
Kim Birkett
Jessica Page
Gemma Howard

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Project Goal

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Goal

Realisation of software prototype for Robust Design Optimisation of preliminary phases of space missions

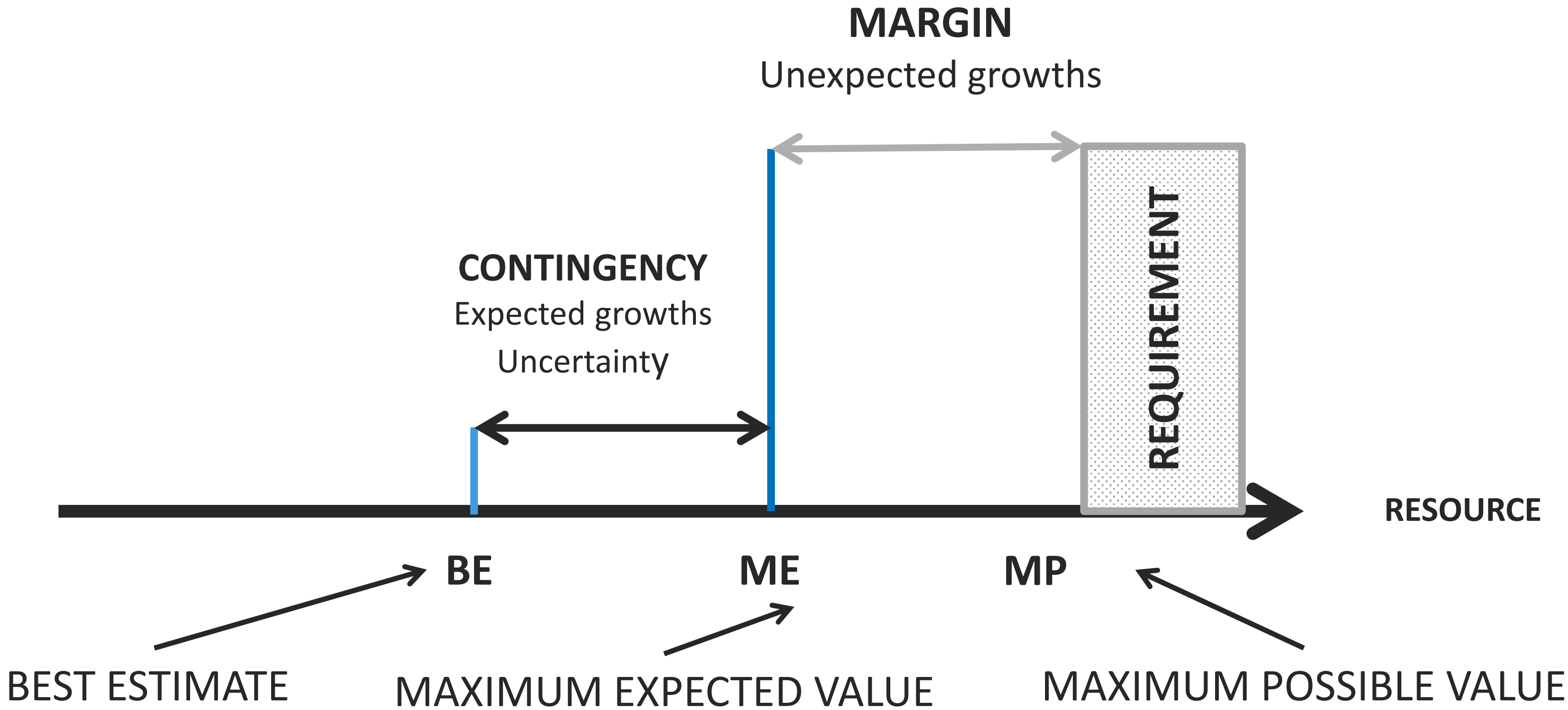
- Efficient estimate of propagation of epistemic uncertainty in space models
- Exact quantification of uncertainty on system budgets
- Minimisation of uncertainty impact budgets

Why?

Margin Approach	Robust Optimisation
• Computation of a design solution • Add safety margin to every subsystem or components to compensate for uncertainties • Add margin to final overall system solution • Does not account for optimality & uncertainty propagation	• Budget optimisation & Uncertainty Quantification (UQ) • Several approaches depending on uncertainty – (Deterministic optimisation + UQ) – Robust optimal solutions – Reliable optimal solutions – Computation of optimal worst case scenario • Accounts for optimality & uncertainty propagation

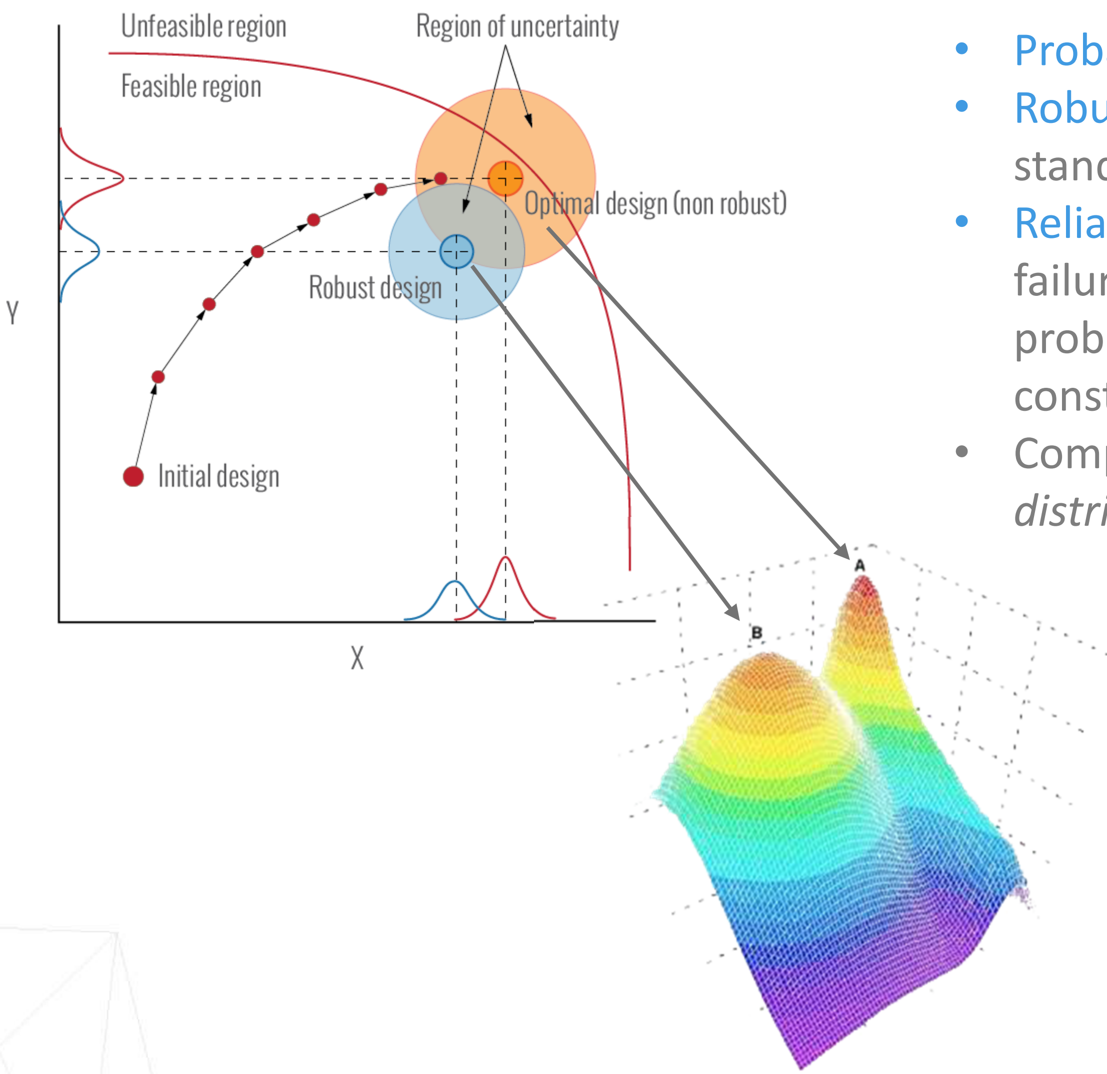
Margin & Contingency

- AIAA Standard S-120-2006e, Mass Properties Control for Space Systems



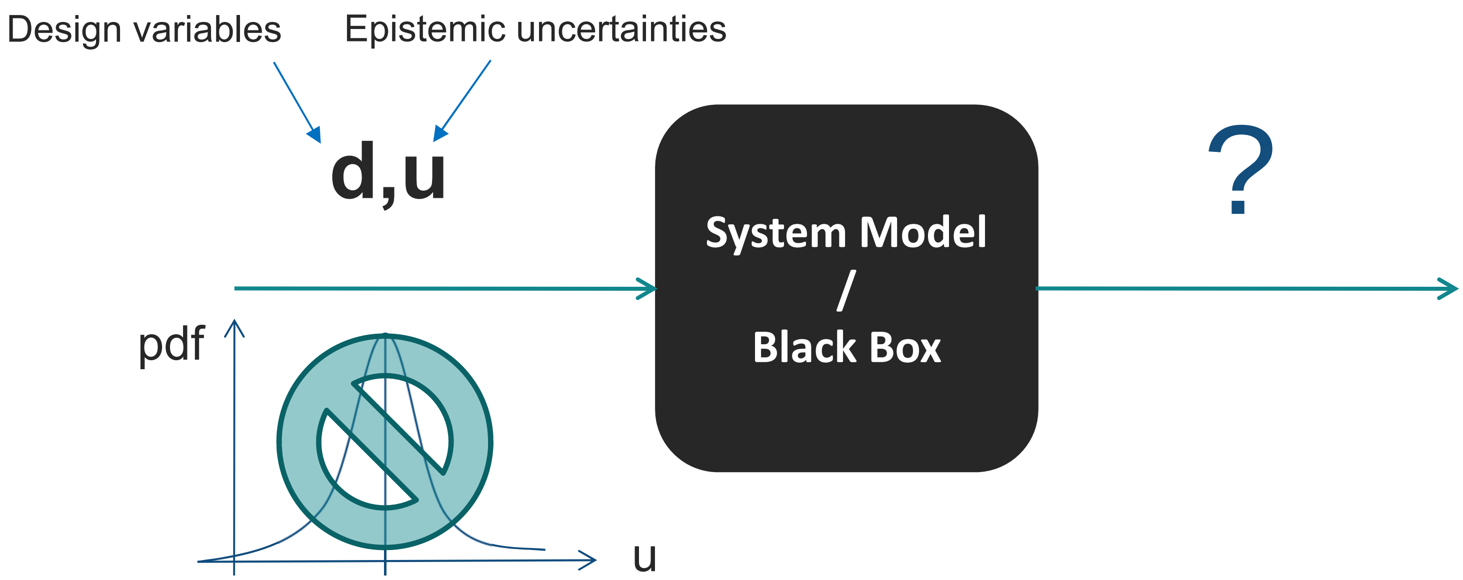
“Classic” Optimisation under Uncertainty

What is the impact of input uncertainty on system responses?



- Probabilistic uncertainties
- Robustness measures: mean, standard deviation,…
- Reliability measures: minimize failure probabilities, maximize probability of respecting constraints…
- Computed exploiting *probability distribution information*

What is Expected Budget Value if Uncertainty is not Aleatory?



Theoretical Background

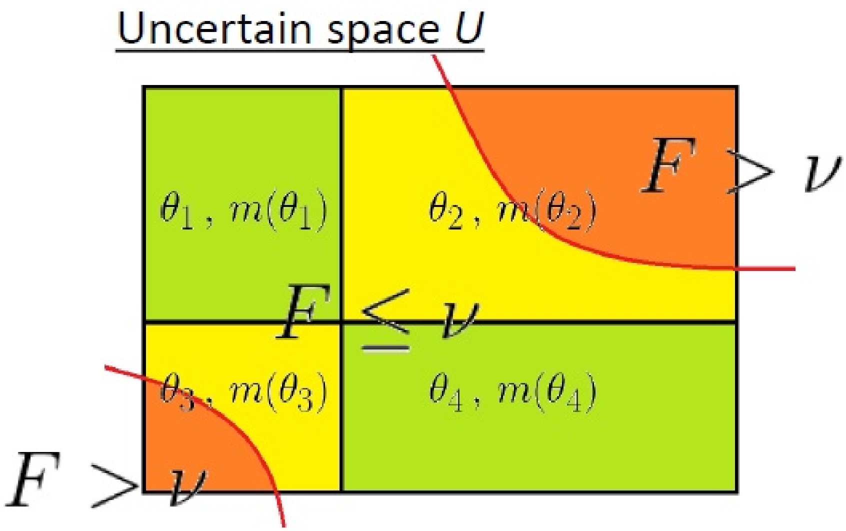
Epistemic Uncertainty

- Due to a lack of knowledge
- Reducible as knowledge is gained
- Cannot be described with probability distributions
- Characterised by set of intervals with associated confidence level (belief that uncertain variable is inside interval) (Basic Probability Assignment - BPA)
- BPAs defined by experts
- Focal elements
- Independent uncertainties: BPA space is Cartesian product of intervals
- Evidence theory as mathematical framework to treat epistemic variables and make inferences

Belief and Plausibility

- Belief: collects all probability masses *fully supporting* a hypothesis
- Plausibility: collects all probability masses *not contradicting* it
- Belief and Plausibility as lower and upper bounds of unknown probability distribution

$F \leq \nu$	
Belief (*Bel*)	$m(\theta_1) + m(\theta_4)$
Plausibility (*Pl*)	$\sum_{i=1}^{4} m(\theta_i)$
Probability (*P*)	unknown $Bel < P < Pl$



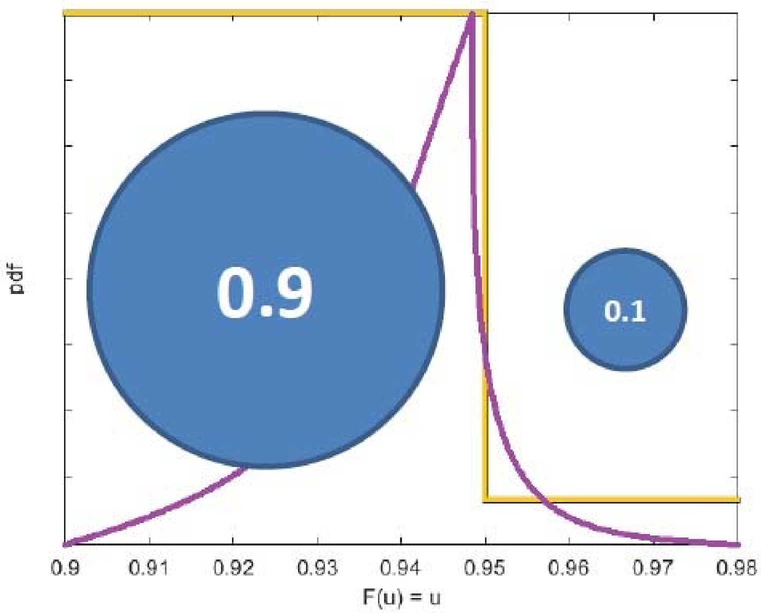
BPAs are not Probability Distributions

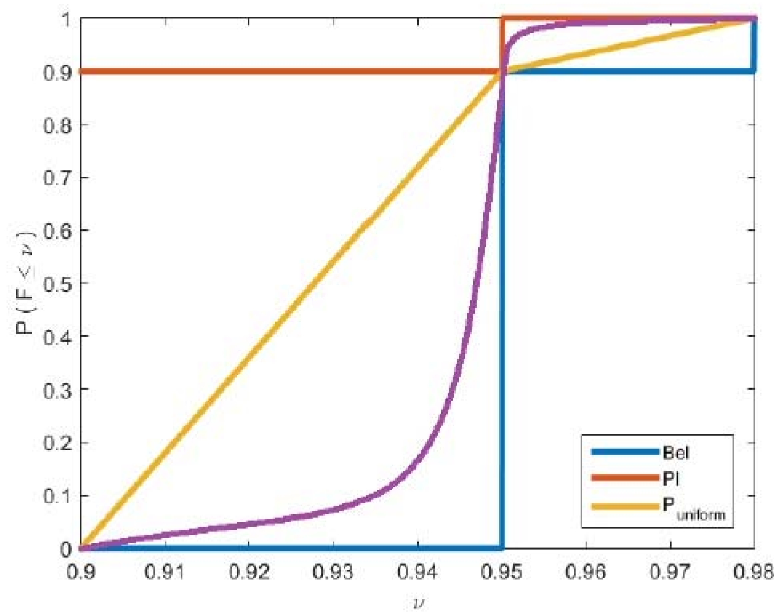
- System budget F:

$$F(d,u) = d + u$$
$$d \in [0,1],\ u \in [0.90, 0.98]$$

θ	u	BPA
θ_1	[0.90, 0.95]	0.9
θ_2	[0.95, 0.98]	0.1

- Optimal set: d*=0, thus budget F(d*,u)=u

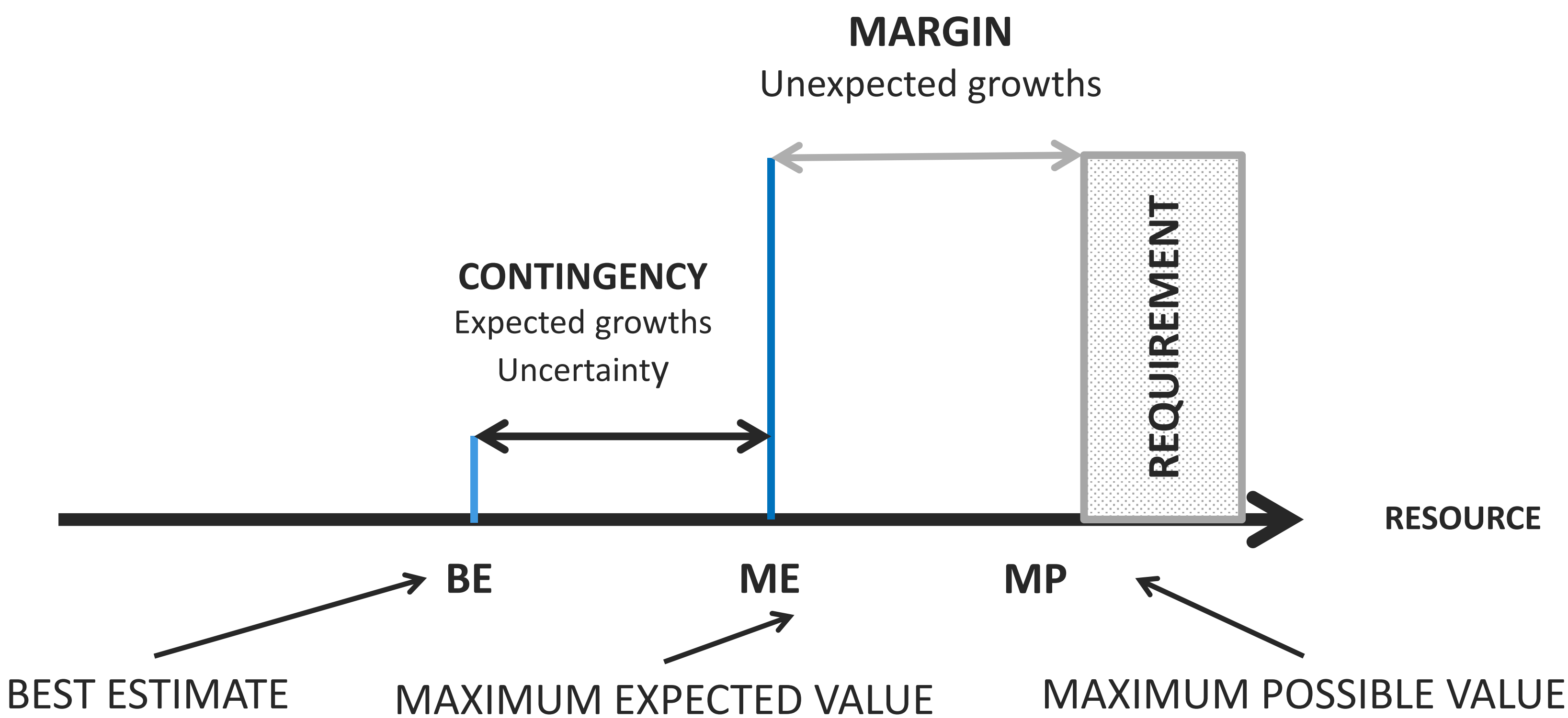




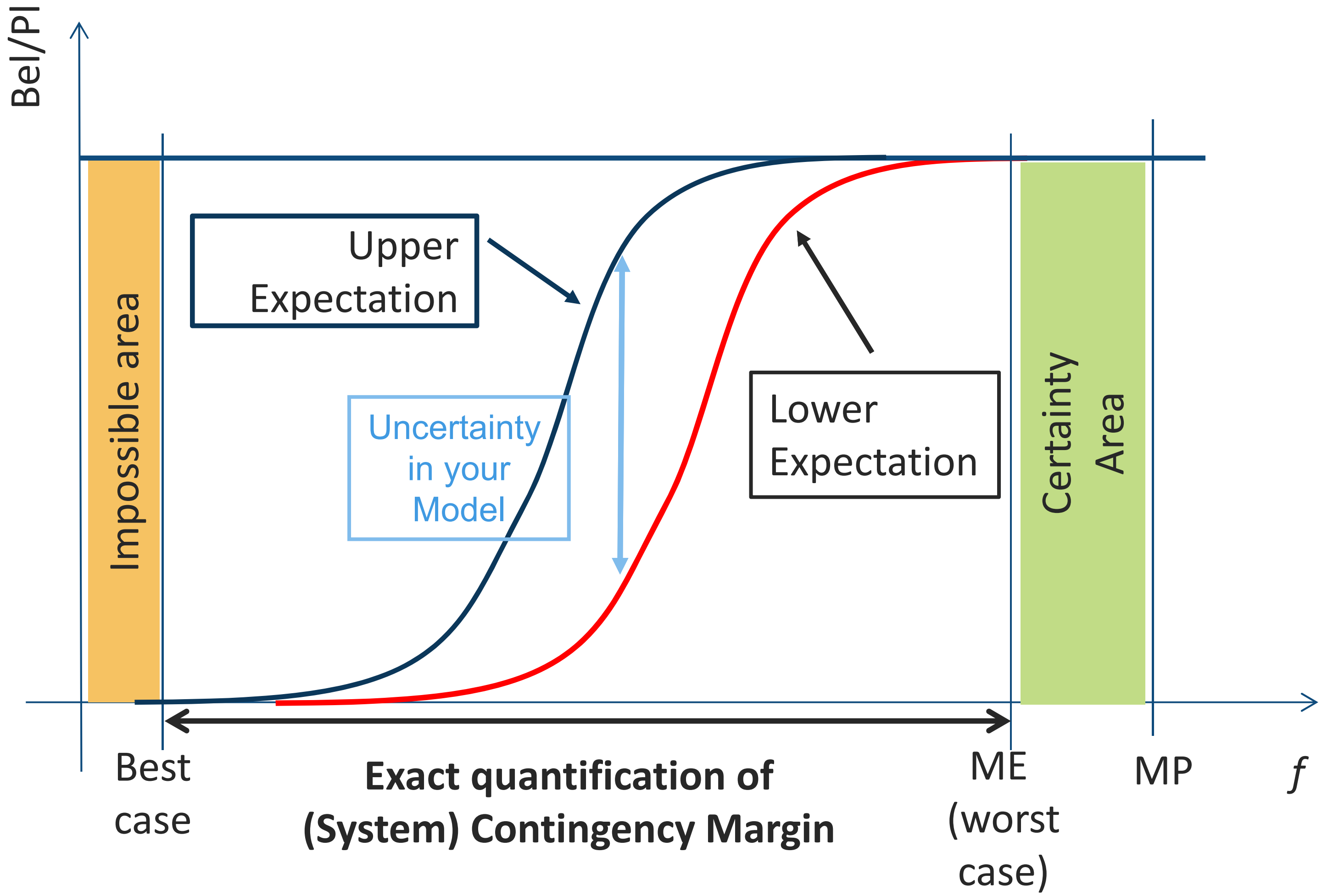
Miscellanea

- If variables are not independent: need more general BPA structures than Cartesian product of intervals
- If BPAs do not sum to 1 → ignorance in the system: if uncertain variable space is the whole universe, then ignorance add to worst case Belief
- If conflicting BPA information from different experts, need rules for aggregating information

Contingency & Evidence



Evidence-Based Design: System Characterisation

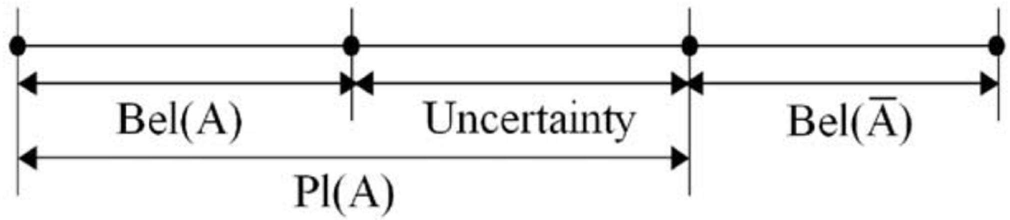


Belief & Plausibility (II)

- All BPAs sum to 1:

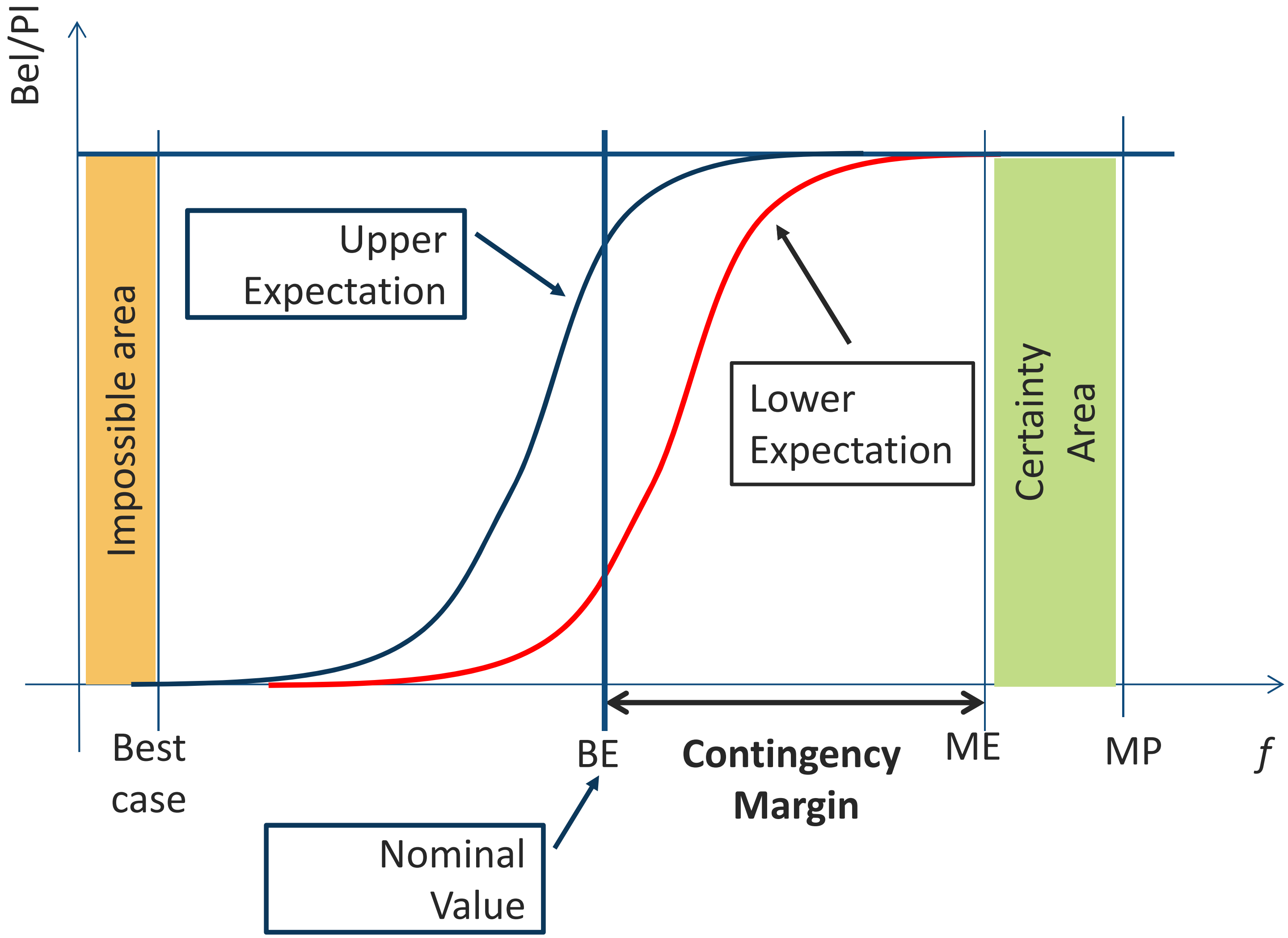
$$Pl(\mathbf{A}) + Bel(\bar{\mathbf{A}}) = 1$$

- Plausibility includes uncertainty:



- Uncertainty = Plausibility - Belief

Evidence-Based Design: Design Characterisation



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Project Scenarios & Algorithms

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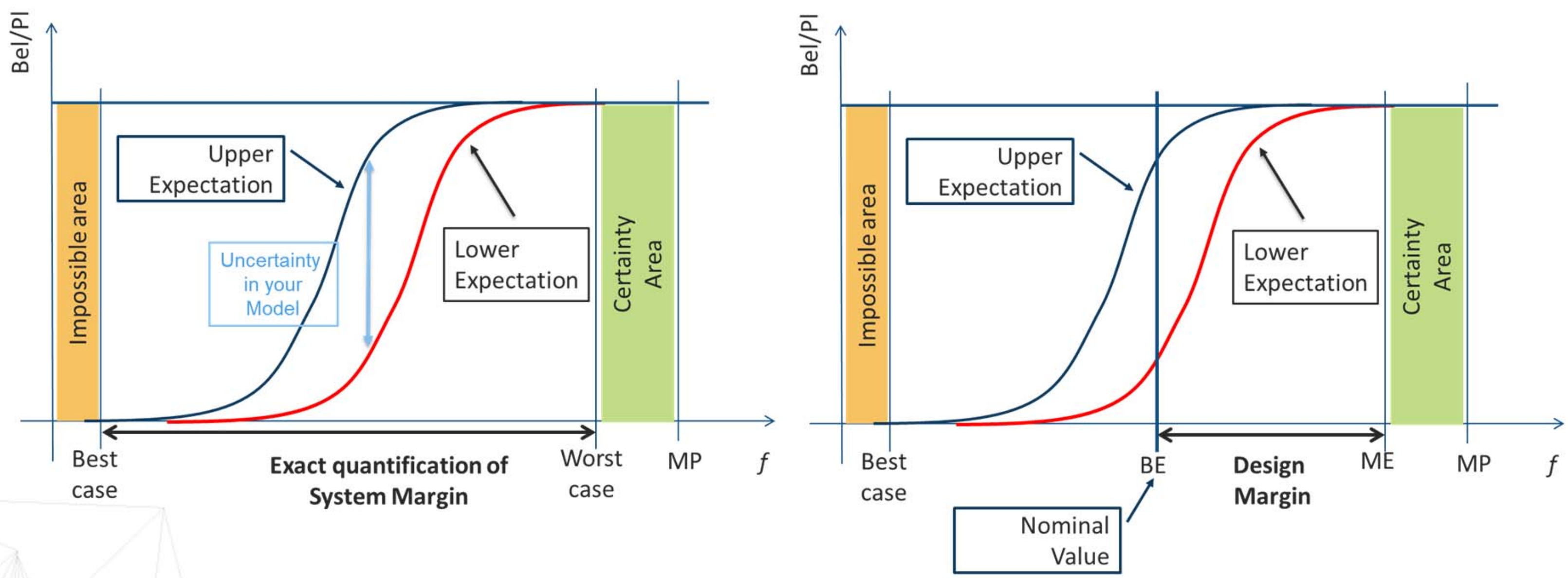
Origin and Results

- Origin:
  - Past activities of University of Strathclyde (including ITI proof-of-concept)
  - Worst case optimisation with Min-Max approach to find total margin of budget, margin of best estimate, and robust design optimisation problem
- Innovation content and benefits:
  - Developed more efficient strategies to reduce computational effort
  - Techniques included in software tool for multidisciplinary design optimisation
  - Still building blocks, but in tool for complex multidisciplinary optimisation, with design exploration and data analysis. Different engineering software can be plugged-in as black-boxes
  - Margin can be quantified, design and system characterisation can be done

Scenarios: Problems Addressed

- Min-Max Optimisation
  - Minimum possible solution for worst possible case given the uncertainty
- Min-Min Optimisation
  - Optimistic optimum
- Reconstruction of Belief and Plausibility curves
  - To characterise nominal value design or system



Algorithms Delivered

- Min-Max algorithm for single and multi-objective optimisation
  - Various heuristics (e.g. local search mechanism to prevent convergence rate issues but costly, or efficient global optimiser to reduce number of function evaluations in SO)
- Min-Min algorithm for single and multi-objective optimisation
  - To be compared with standard algorithms
- Reconstruction of Belief/Plausibility curves with a decomposition mechanism
  - Reduction of computational cost w.r.t. exact curve reconstruction
  - Conservative approximations (e.g. Belief reconstructed with decomposition approach approximates the exact Belief from below)
  - Constraint-handling strategy (not included in prototype tool yet)

Min-Max Algorithms

- Single and multi-objective cases
- Bi-level optimisation, with a SUPER algorithm where different solvers can be plugged-in for the OUTER and INNER level computations
- Yet, sequential approach (*cheaper* than nested one!):
  - OUTER: *minimise* over design variables
  - INNER: *maximise* over epistemic variables; in MO case: each objective is maximised separately
- Various heuristics specifically developed for the project:
  - Local search mechanism to prevent convergence rate issues, but costly
  - Efficient global optimiser to reduce number of evaluations (single obj. case)
- Constraint-handling strategy (not included in prototype tool yet)

Mathematical Benchmarks

- Benchmark on SO and MO Min-Max problems

Single objective

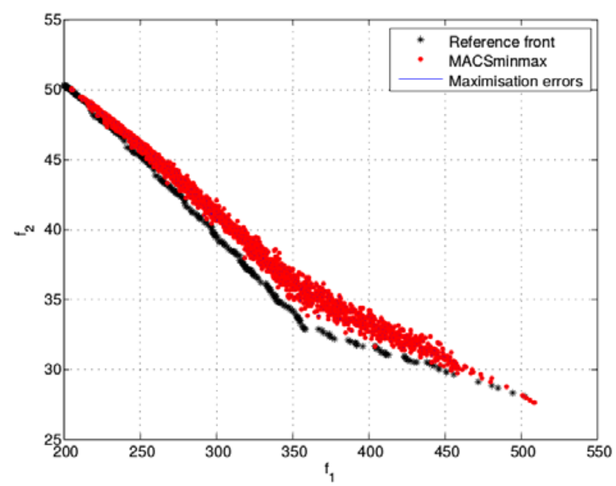
Problem	#feval (mean)	#feval (std)	Δf (mean)	Δf (std)
P1	5790	14	7.0e-7	2.7e-6
P2	5711	25	8.0e-4	3.1e-3
P3	16261	312	7.5e-7	6.9e-5
P4	7048	16	1.1e-3	1.7e-3
P5	4608	16	6.5e-5	5.7e-5
P6	53300	48	9.7e-7	1.7e-6
P7	96509	75	2.2e-3	2.2e-3
P8	574	54	1.0e-16	1.9e-16
P9	4259	42	6.4e-5	2.5e-4
P10	5509	211	7.7e-6	4.9e-5
P11	5501	179	1.4e-4	4.2e-4
P12	2391	92	5.3e-15	1.0e-15
P13	11665	980	1.3e-4	8.8e-4

Efficient single objective

Problem	#feval (mean)	#feval (std)	Δf (mean)	Δf (std)
P8	38	4	-5.7e-6	1.8e-5
P9	472	87	-1.6e-3	2.2e-3
P10	225	49	-3.1e-4	2.1e-3
P11	208	61	-5.2e-4	1.7e-3
P12	83	5	7.6e-6	4.3e-5
P13	142	18	1.5e-4	3.3e-4

Multi objective

Test-case	#feval (mean)	#feval (std)	M_{conv} (mean)	M_{conv} (std)	M_{spr} (mean)	M_{spr} (std)
TC1	120039	7893	5.2e-3	1.5e-3	1.5e-2	1.7e-3
TC2	1402813	288534	2.8e-2	4.2e-3	3.3e-2	2.3e-3
TC3	873041	58311	6.6e-3	7.1e-4	8.0e-3	8.2e-4
TC4	415383	42183	2.6e-2	1.2e-2	5.7e-2	1.6e-2
TC5	393985	35120	4.9e-2	3.1e-2	1.3e-1	6.1e-2
TC6	74434	14101	5.5e-3	2.8e-3	3.6e-2	1.4e-2
TC7	877227	95108	2.3e-2	3.7e-3	4.4e-2	8.1e-3



Min-Min Algorithm

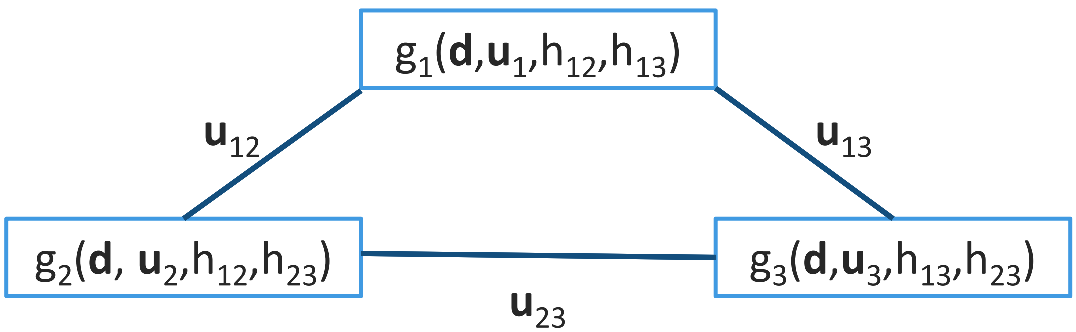
- Single and multi-objective optimisation
  - To be compared with standard algorithms

Belief/Plausibility Reconstruction with Decomposition

- Reduction of computational cost w.r.t. exact curve reconstruction
- Conservative approximations (e.g. Belief reconstructed with decomposition approximates exact Belief from below)
- Reconstruction of Belief/Plausibility curves from:
  - previously found worst/best case configurations
  - baseline design configuration
  - scratch

Evidence Network Model

- Complex system as network of interconnected subsystems
- Distinction between exclusive and exchange epistemic variables
- Assumptions:
  1. Exchange variable contributions through scalar functions h_{ij}
  2. All g_i are positive semi-definite functions
  3. All g_i are monotonically increasing functions of h_{ij} for every coupled system
- Example:



Total budget: $F=g_1(\mathbf{d},\mathbf{u}_1,h_{12},h_{13})+g_2(\mathbf{d},\mathbf{u}_2,h_{12},h_{23})+g_3(\mathbf{d},\mathbf{u}_3,h_{13},h_{23})$

Steps of Reconstruction through Decomposition

1. Solution of optimal worst/best case scenario
2. Maximisation over exchange variables (for fixed exclusive and design variables) and computation of partial Belief/Plausibility curves
3. Sampling of partial curves and maximization over exclusive variables
4. Reconstruction of whole approximate Belief/Plausibility curves

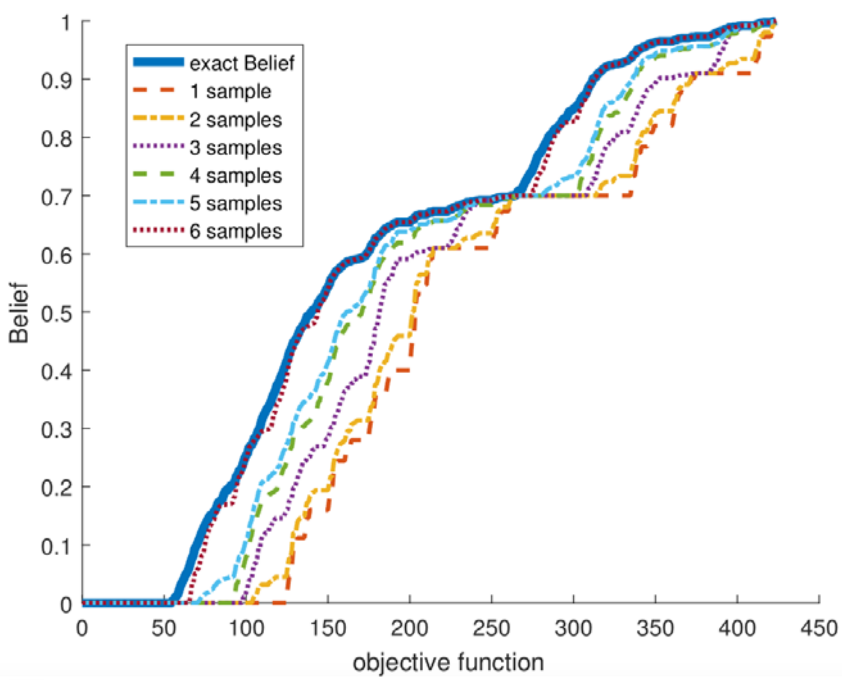
Number of optimisations required: $$N_{FE} = \prod_{k=1}^{m} N_k$$ *(Exact)*

$$N_{FE}^{Dec} = N_s \sum_{i=1}^{m_u} N_{FE,i}^{u} + \sum_{i=1}^{m_c} N_{FE,i}^{c}$$

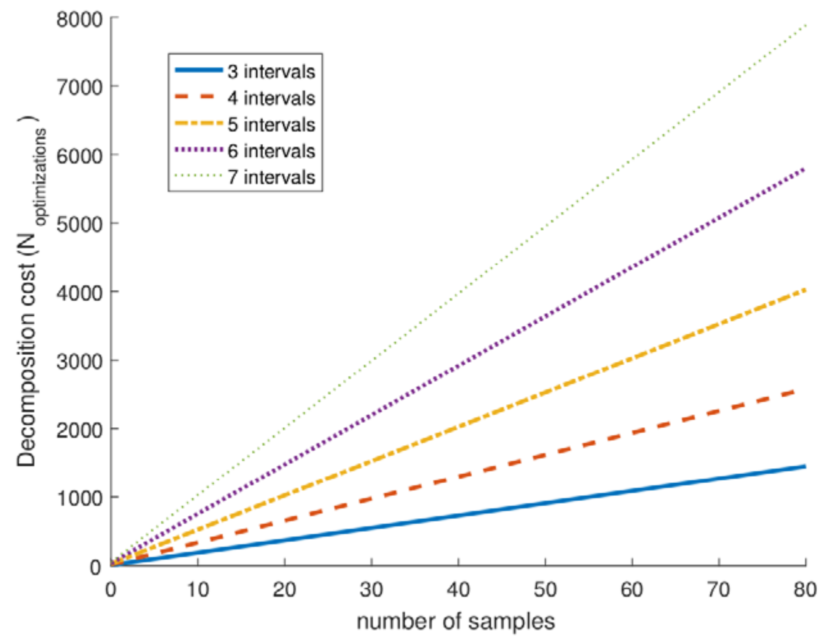
Mathematical Benchmarks

$$\begin{cases} F = g_1 + g_2 \\ g_1 = 10u_1^2 + |u_2|u_5^2 + \frac{u_6^4}{100} + d_1|d_2| \\ g_2 = |u_3| + u_4^2\frac{|u_5|}{10} + u_6^2 + |d_1| \end{cases}$$

Convergence with samples



Cost vs. samples & intervals



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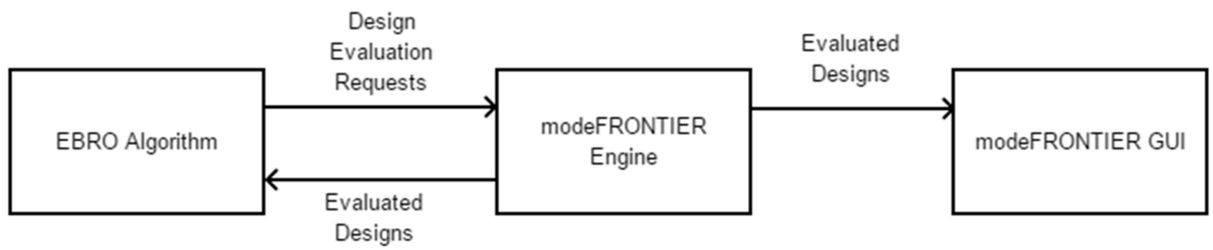
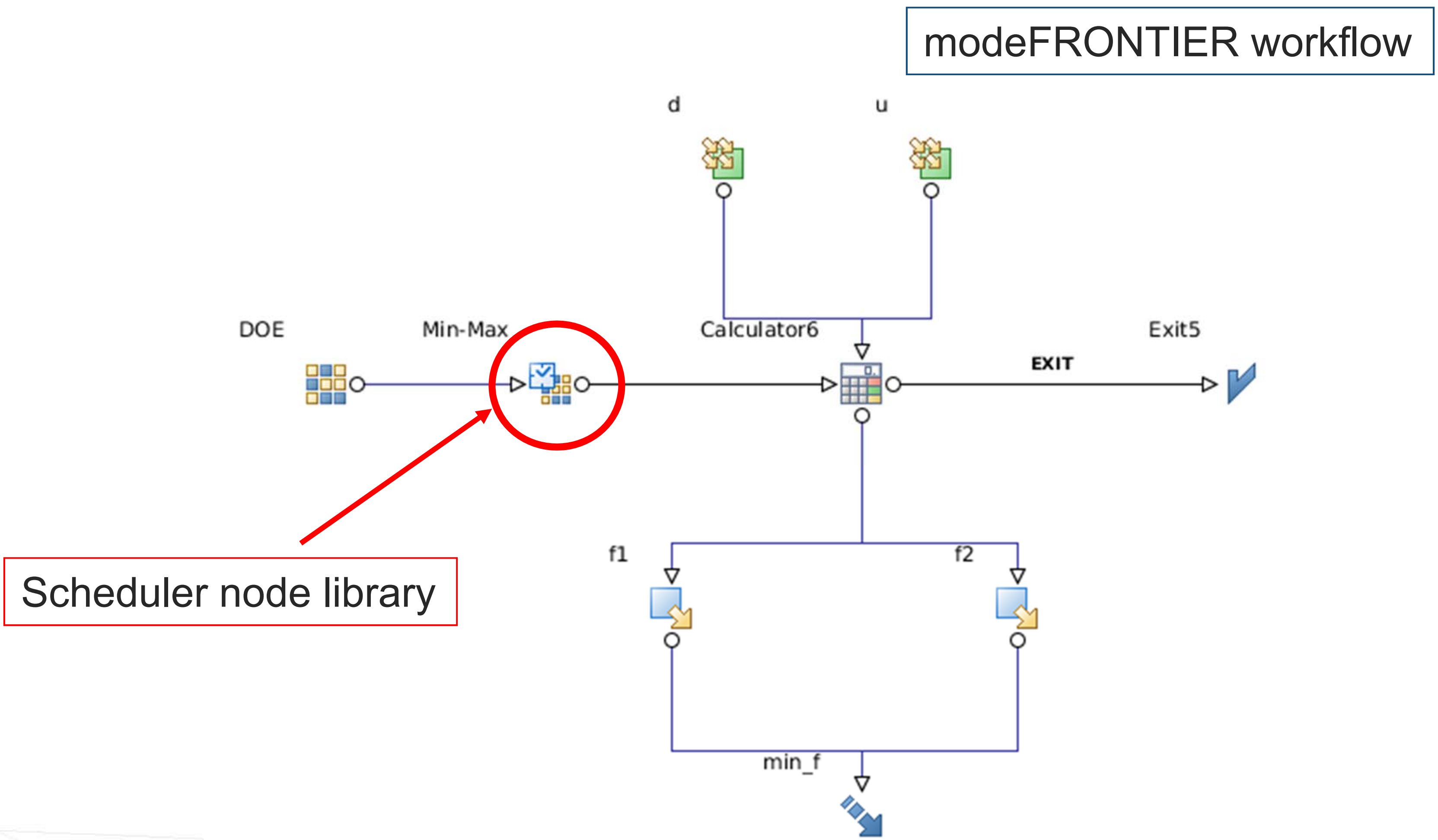
Evidence Based Robust Optimisation Plug-In

Prototype Plug-In Implementation

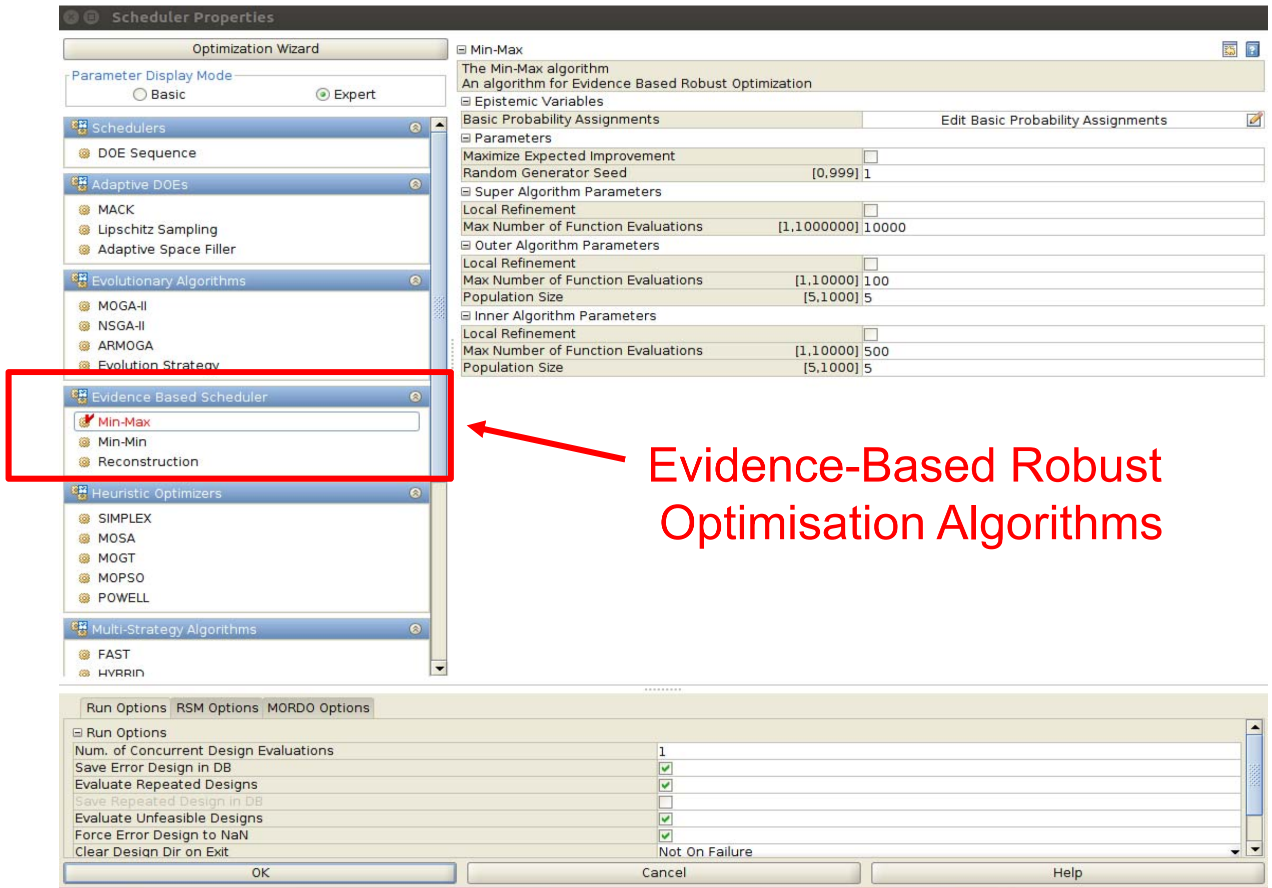


- Java implementation of setup for epistemic variable definition and characterisation + decomposition characterisation
- Development of dedicate MATLAB bridge for Evidence Based Robust Optimisation (EBRO)
- Development of GUI solutions

EBRO add-ons for modeFRONTIER 2017R5

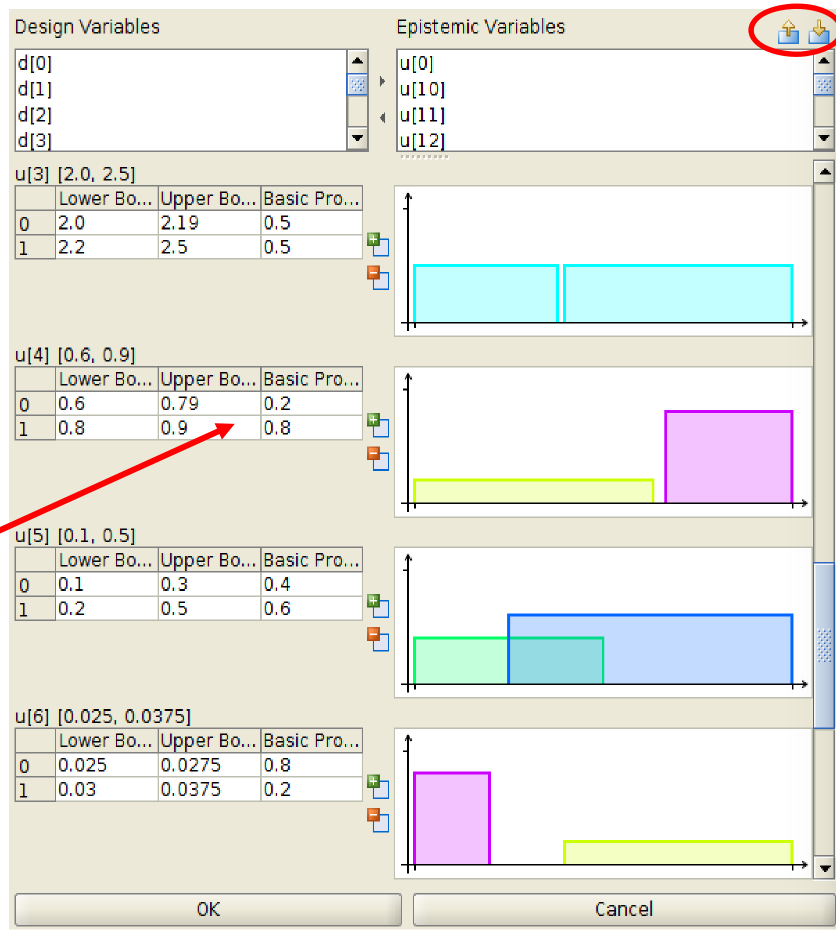


EBRO Algorithm Library



Evidence-Based Robust Optimisation Algorithms

Basic Probability Assignment



Buttons to export/import intervals and basic probabilities for variables in the Epistemic Variables environment

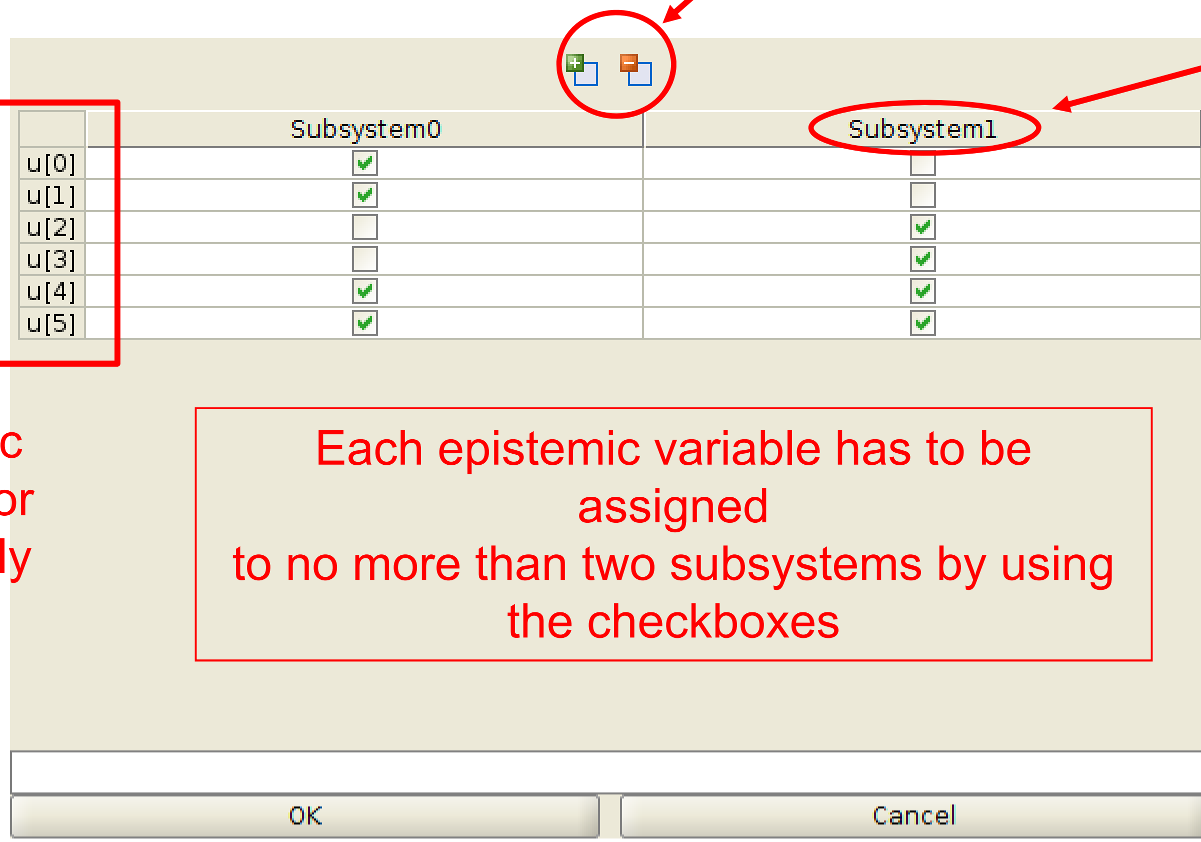
Heights and color of shaded regions depend on BPA values (no precise distributions)

For Reconstruction also probability values are required

Reconstruction Decomposition Editor

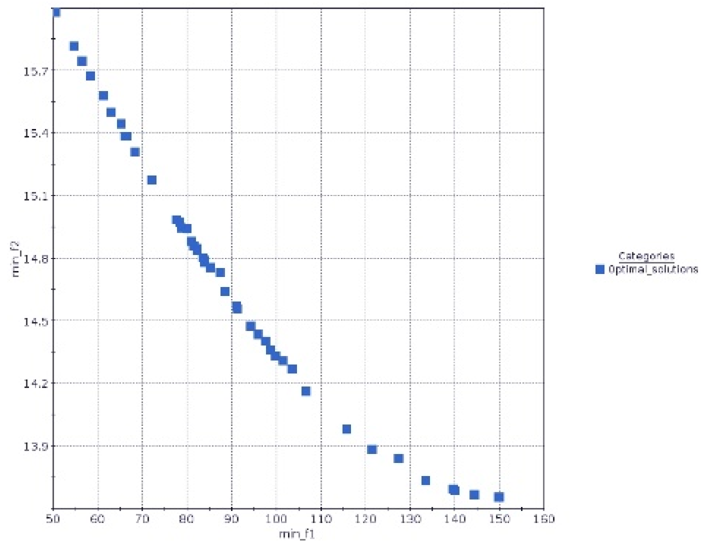
Add/remove subsystem

Subsystem names are editable



All variables set as epistemic in the BPA editor are automatically displayed

Results Tables and Charts (I)



MinMax Results

ID	M	CATEGORY	d_d		u_min_f1_u		u_min_f2_u		min_f1	min_f2
			d_d[0]	d_d[1]	u_min_f...	u_min_f...	u_min_f...	u_min_f...		
0	☐		1.0000E0	1.3481E0	-5.0000E0	-5.0000E0	1.7872E-10	9.5040E-2	5.8703E1	1.5668E1
1	☐		1.4023E0	1.8695E0	-5.0000E0	-5.0000E0	1.1135E-1	2.7090E-1	8.1793E1	1.4869E1
2	☐		2.2915E0	2.8016E0	-5.0000E0	-5.0000E0	4.4494E-1	6.8653E-1	1.2733E2	1.3843E1
3	☐		2.3550E0	1.9802E0	-5.0000E0	-5.0000E0	4.7343E-1	3.1385E-1	1.0838E2	1.4147E1
4	☐		2.6398E0	2.2306E0	-5.0000E0	-5.0000E0	6.0723E-1	4.1815E-1	1.2176E2	1.3904E1
5	☐		2.6773E0	2.8044E0	-5.0000E0	-5.0000E0	6.2544E-1	6.8790E-1	1.3704E2	1.3707E1
6	☐		3.0770E0	2.9966E0	-5.0000E0	-5.0000E0	8.2386E-1	7.8368E-1	1.5184E2	1.3659E1

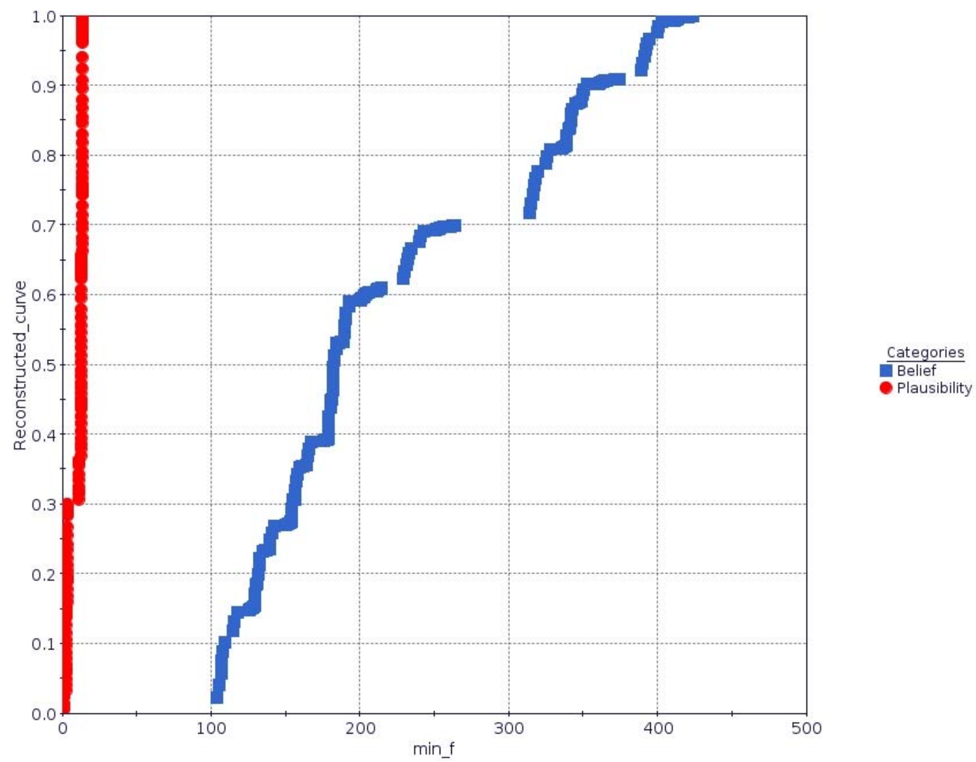
Results Tables and Charts (II)

Reconstructed Belief Table

ID	M	CATEGORY	min_f	Bel
0	☐		1.0381E2	2.3296E-2
1	☐		1.0481E2	4.0768E-2
2	☐		1.0631E2	5.8240E-2
3	☐		1.0681E2	7.5712E-2
4	☐		1.0731E2	8.8816E-2
5	☐		1.0931E2	1.0192E-1
6	☐		1.1431E2	1.1939E-1
7	☐		1.1531E2	1.3250E-1
8	☐		1.1731E2	1.4560E-1
9	☐		1.2525E2	1.4790E-1
10	☐		1.2625E2	1.4963E-1

Reconstructed Plausibility Table

ID	M	CATEGORY	min_f	Pl
0	☐		1.0110E0	6.8040E-3
1	☐		1.0110E0	8.1000E-3
2	☐		1.1110E0	1.4904E-2
3	☐		1.1110E0	2.3976E-2
4	☐		1.1110E0	2.5272E-2
5	☐		1.1110E0	2.7000E-2
6	☐		2.0110E0	3.3804E-2
7	☐		2.0110E0	4.2876E-2
8	☐		2.0110E0	4.4172E-2
9	☐		2.0110E0	4.5900E-2
10	☐		2.0110E0	5.2704E-2



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Validation

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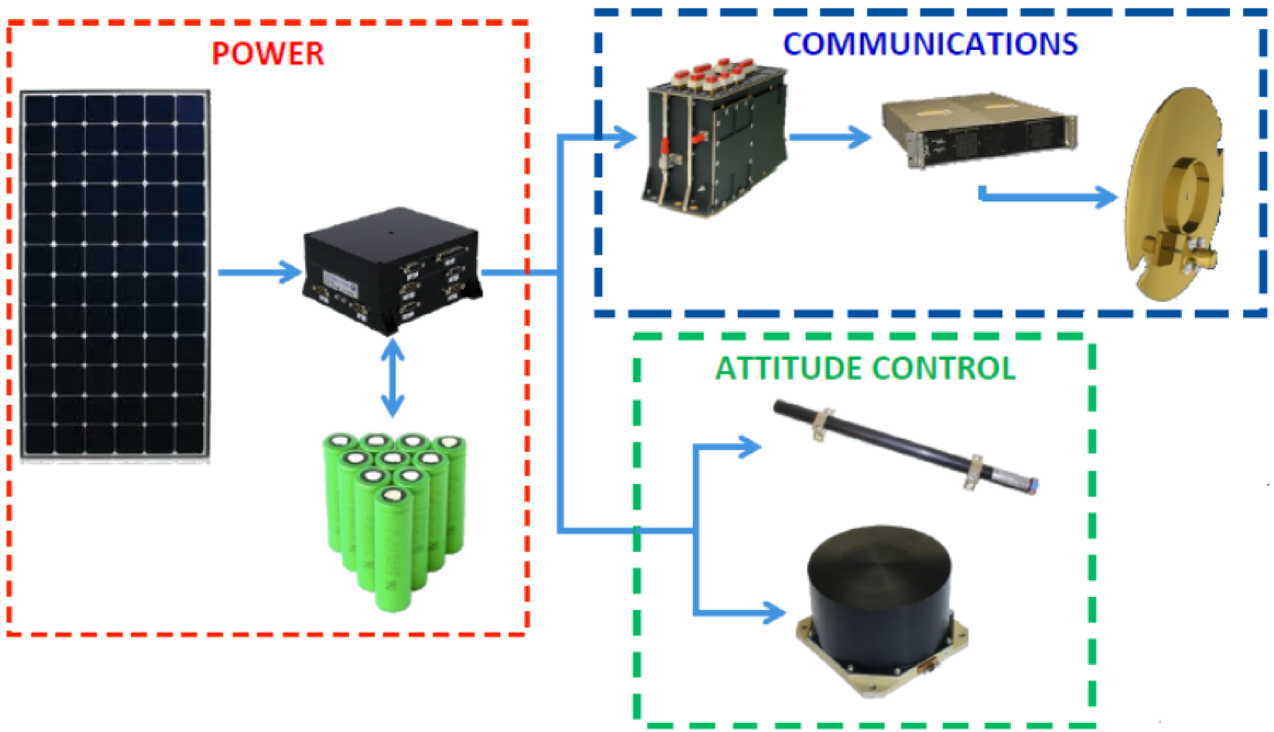
CubeSat example

CubeSat Sizing Problem

- Sizing of three subsystems
- Budget: total mass



- Alicino S. and Vasile M., Evidence-based Preliminary Design of Spacecraft. 6th International Conference on Systems & Concurrent Engineering for Space Applications. SECESA 2014
- Vasile M., Filippi G., Ortega Absil C. and Riccardi A., Fast Belief Estimation in Evidence Network Models. Presented at EUROGEN 2017

Variables

- 8 design variables, 12 epistemic variables (2 confidence intervals each), several fixed parameters

AOCS			TTC			EPS		
d[0]	ϕ_{slew}(deg)	[10, 60]	d[2]	f(GHz)	[7, 10]	d[5]	E_{cell} (Wh)	[135, 145]
d[1]	t_{slew} (s)	[30, 90]	d[3]	modulation	[0, 1]	d[6]	$V_{bus}(V)$	[3, 5]
			d[4]	amplifier	[0, 1]	d[7]	V_{drop} %	[1, 3]

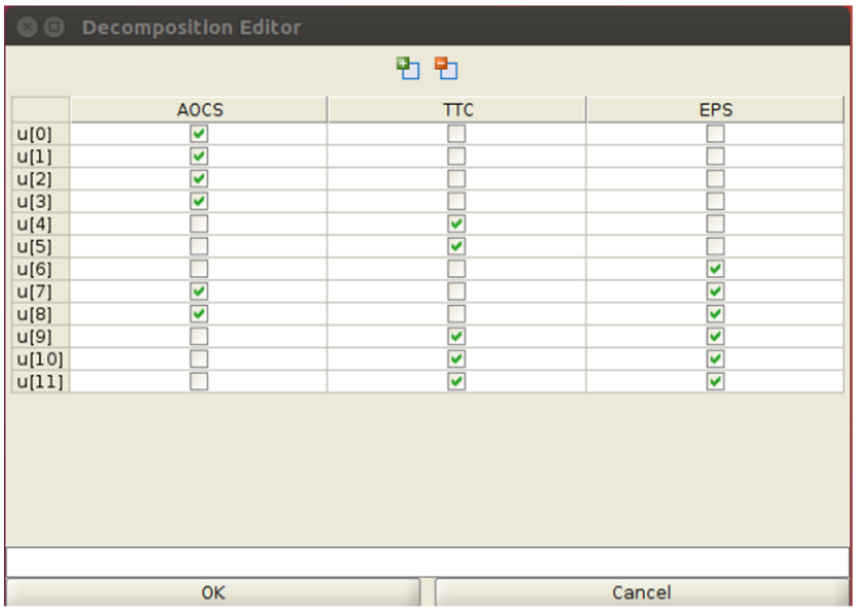
AOCS		
u[0], l(m)	[0.005, 0.01]	0.5
	[0.01, 0.02]	0.5
u[1], A(m^2)	[0.034, 0.0885]	0.5
	[0.0885, 0.15]	0.5
u[2], q	[0.5, 0.6]	0.5
	[0.6, 0.7]	0.5
u[3], C_D	[2, 2.2]	0.5
	[2.2, 2.5]	0.5
u[7], m(mA/m^2)	[0.5, 1]	0.5
	[1, 1.5]	0.5
u[8], δI(%)	[−10, 5]	0.4
	[5, 10]	0.6

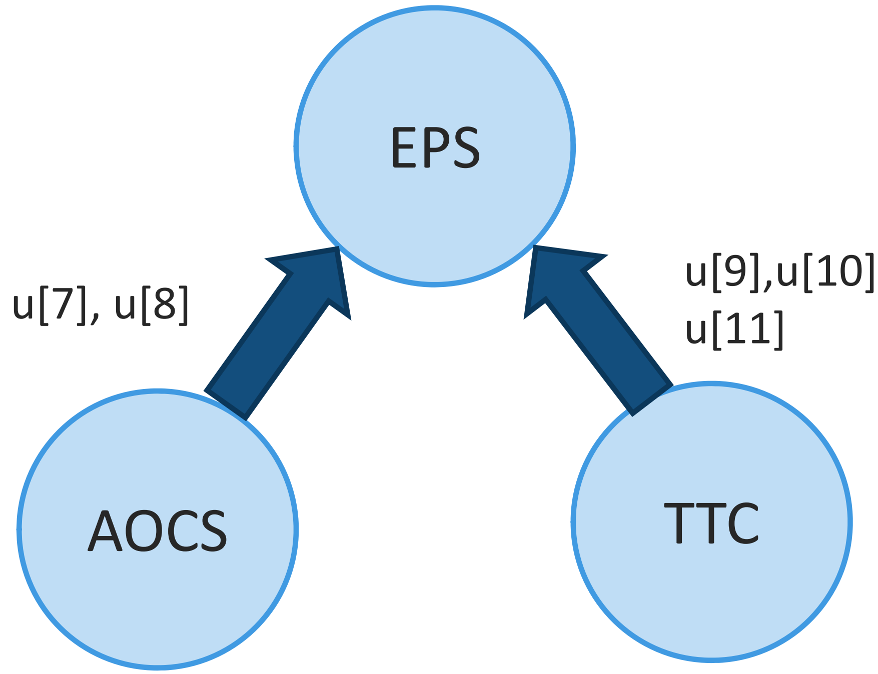
TTC		
u[4], η_{ant}	[0.6, 0.8]	0.2
	[0.8, 0.9]	0.8
u[5], M_{rfdn}(kg)	[0.1, 0.3]	0.4
	[0.2, 0.5]	0.6
u[9], G_t(dB)	[1, 3]	0.3
	[3, 5]	0.7
u[10], L_t(dB)	[0.1, 0.5]	0.3
	[0.5, 1]	0.7
u[11], L_{other}(dB)	[0.5, 1.5]	0.4
	[1.5, 2]	0.6

EPS		
u[6], η_a	[0.8, 0.85]	0.4
	[0.85, 0.9]	0.6

Decomposition

- *Exact* computation of Belief: 4096 optimisations
- Decomposition method: 12+Ns*22 optimisations





modeFRONTIER on CubeSat: Benchmark

Min-Max solution

Design Variables

	MinMax
d[0]	10.0
d[1]	90.0
d[2]	7.0
d[3]	0.3898551414874
d[4]	0.2581577955672
d[5]	145.0
d[6]	3.0
d[7]	1.0

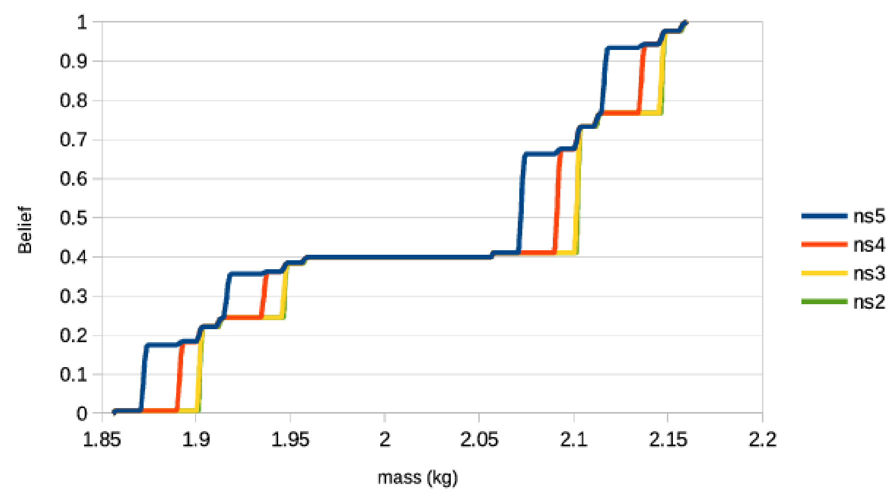
Epistemic Variables

	MinMax
u[0]	0.02
u[1]	0.15
u[2]	0.7
u[3]	2.5
u[4]	0.6
u[5]	0.5
u[6]	0.836241313729441
u[7]	1.5
u[8]	10.0
u[9]	1.0
u[10]	1.0
u[11]	2.0

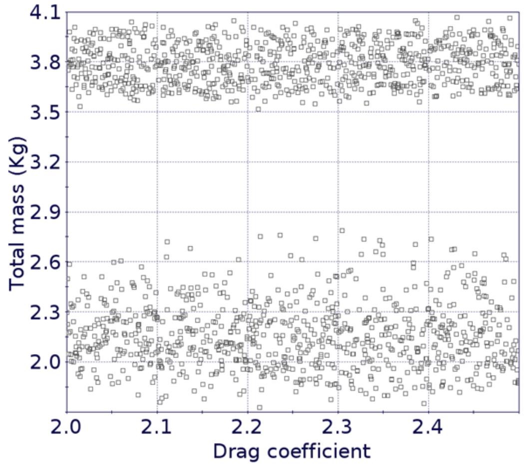
Objective

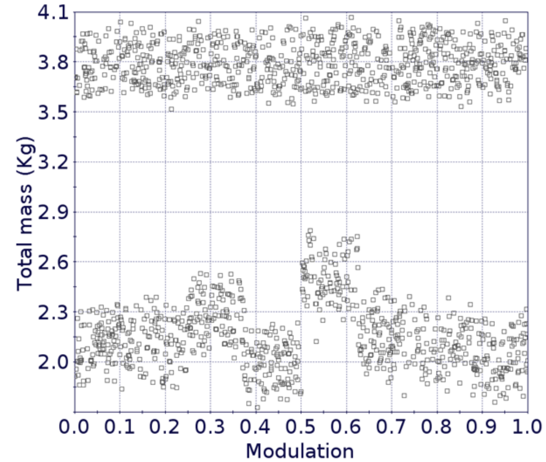
	MinMax
min_mass	2.159651961357106

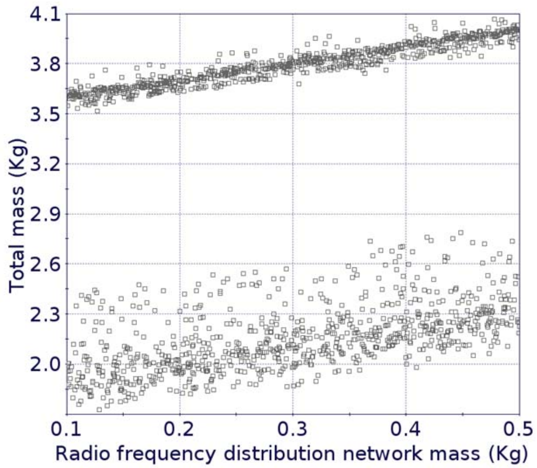
Belief convergence study

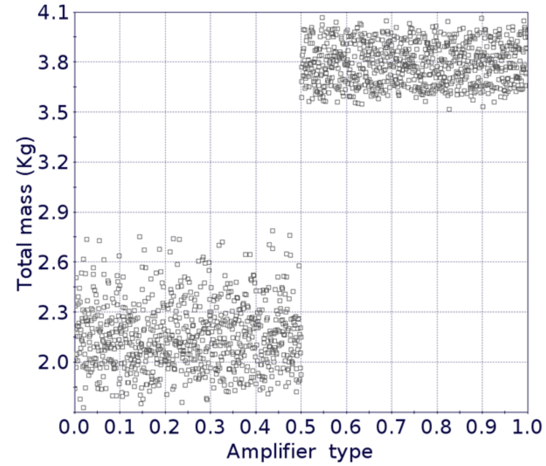


Variable Screening

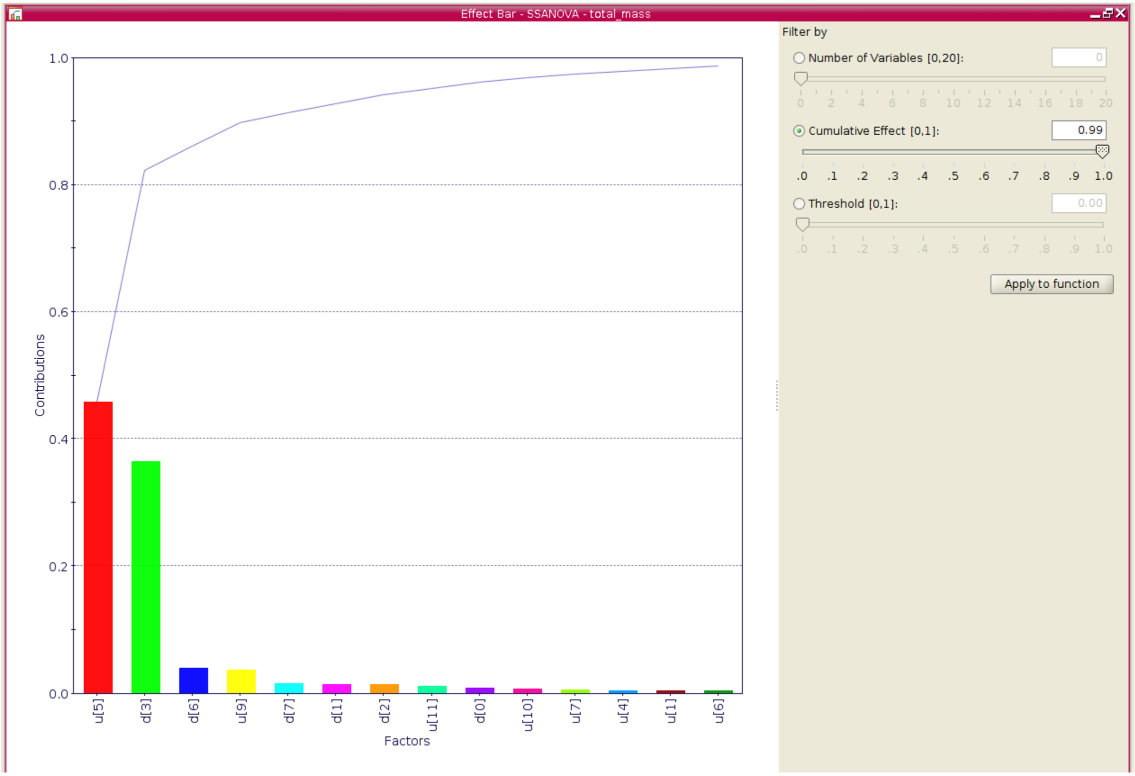








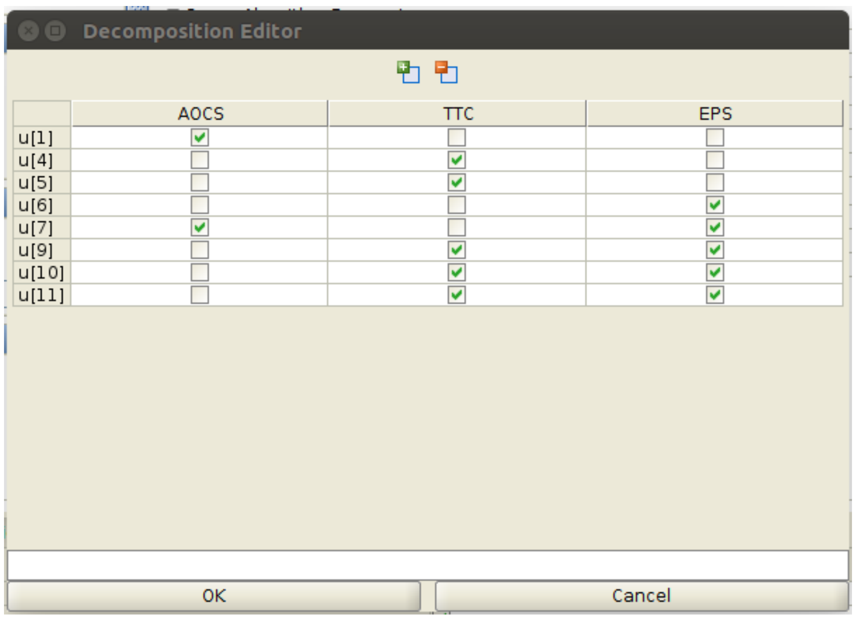
SS-ANOVA Sensitivity Analysis



Belief Reconstruction for Reduced System

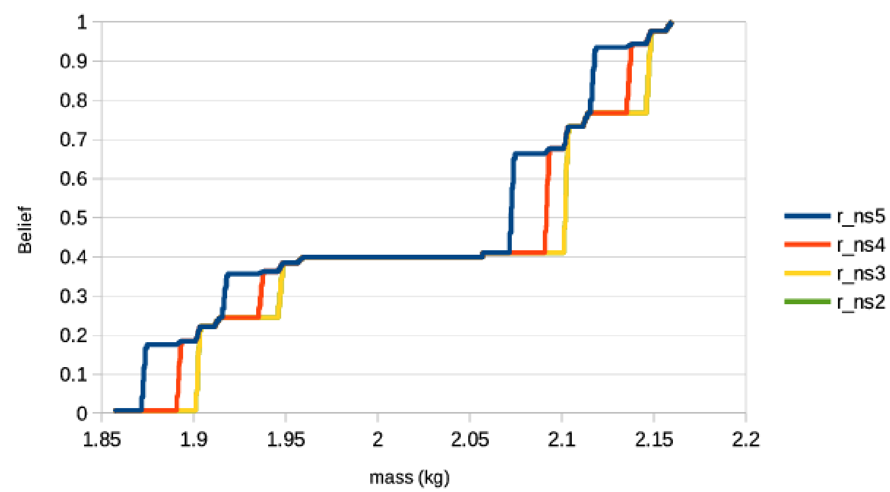
- Full system: 12+Ns*22 optimisations
- Reduced system: 10+Ns*8 optimisations

Decomposition after sensitivity



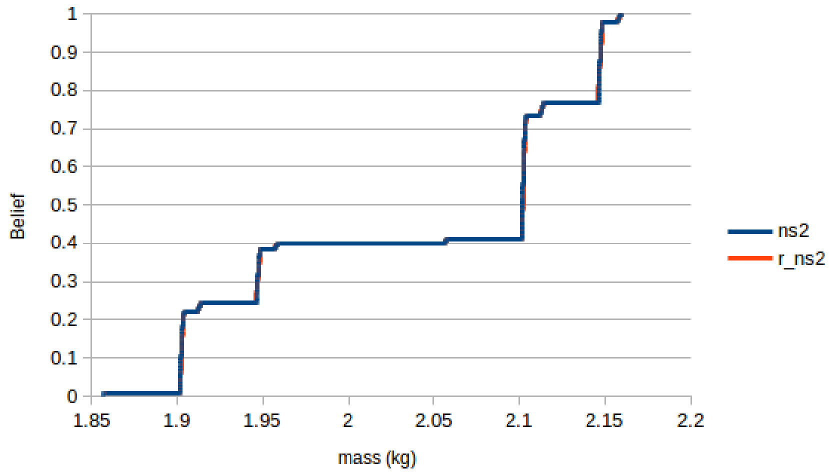
	AOCS	TTC	EPS
u[1]	✓		
u[4]		✓	
u[5]		✓	
u[6]			✓
u[7]	✓		✓
u[9]		✓	✓
u[10]		✓	✓
u[11]		✓	✓

Belief convergence study

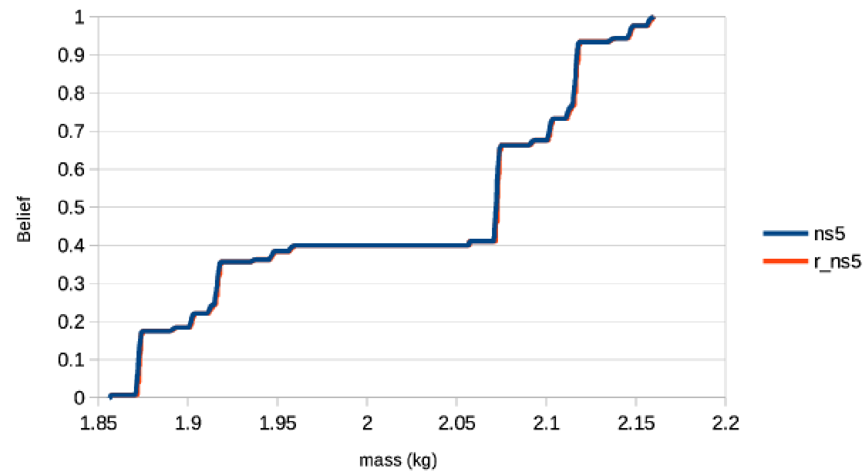


Comparison of Full and Reduced System Beliefs

2 samples



5 samples



EXPLORE DESIGN PERFECTION

SSTL Test Cases

esteco.com

Battery Sizing

- Goal: ensure spacecraft autonomy in transfer to geostationary orbit
- Objective: minimise battery mass
- Depth of Discharge fixed to maximum possible value
- Design variables:
  - type of battery (selected from a list)
  - bus voltage
  - insertion time (any day of 2019 at 7:00 A.M.)
- Uncertain variables: orbital parameters after LAE burn (uncertainty in transfer model) and battery efficiency (operational uncertainty)

Transfer Model

- Input: time insertion by the launcher and a parameter table

Semimajor axis [km]	Eccentricity [-]	Inclination [°]	RAAN [°]	Arg. of Perigee [°]	True Anomaly [°]	Time of Arrival [hrs.]	Burn duration [hrs.]
68500.3	0.902	22.81	86.63	180.10	0.00	0.0	0.0
73250.2	0.77866	9.12	86.79	180.06	180.08	24.7	0.6
86065.5	0.51300	1.09	85.96	180.81	180.84	79.9	0.8
49646.4	0.15392	0.36	86.85	180.97	4.25	114.7	0.6
42049.0	0.00100	0.05	270.0	0.00	359.95	145.2	0.3

- Output: a table with ON/OFF battery times

Battery ON [hrs.]	Battery OFF [hrs.]
24.1000	24.7000
48.2458	49.5174
79.1000	79.9000
114.1000	114.7000
144.9000	145.2000

Battery Model

Inputs:

- Design variables:
  - Battery (A,B,C,D)
  - BUS voltage
- Other parameters:
  - Cell capacity
  - Cell voltage
  - DoD allowed

Assumptions:

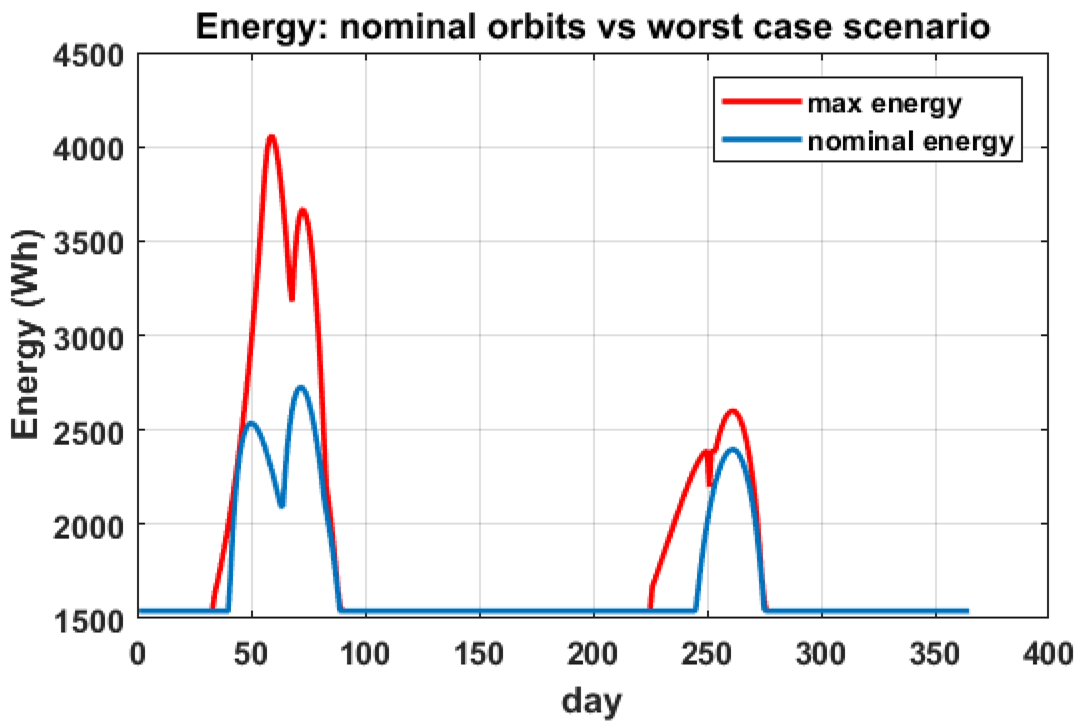
- Battery discharges while ON and charges while OFF, with different linear rates
- Stops charging at SoC=90%

List of Batteries

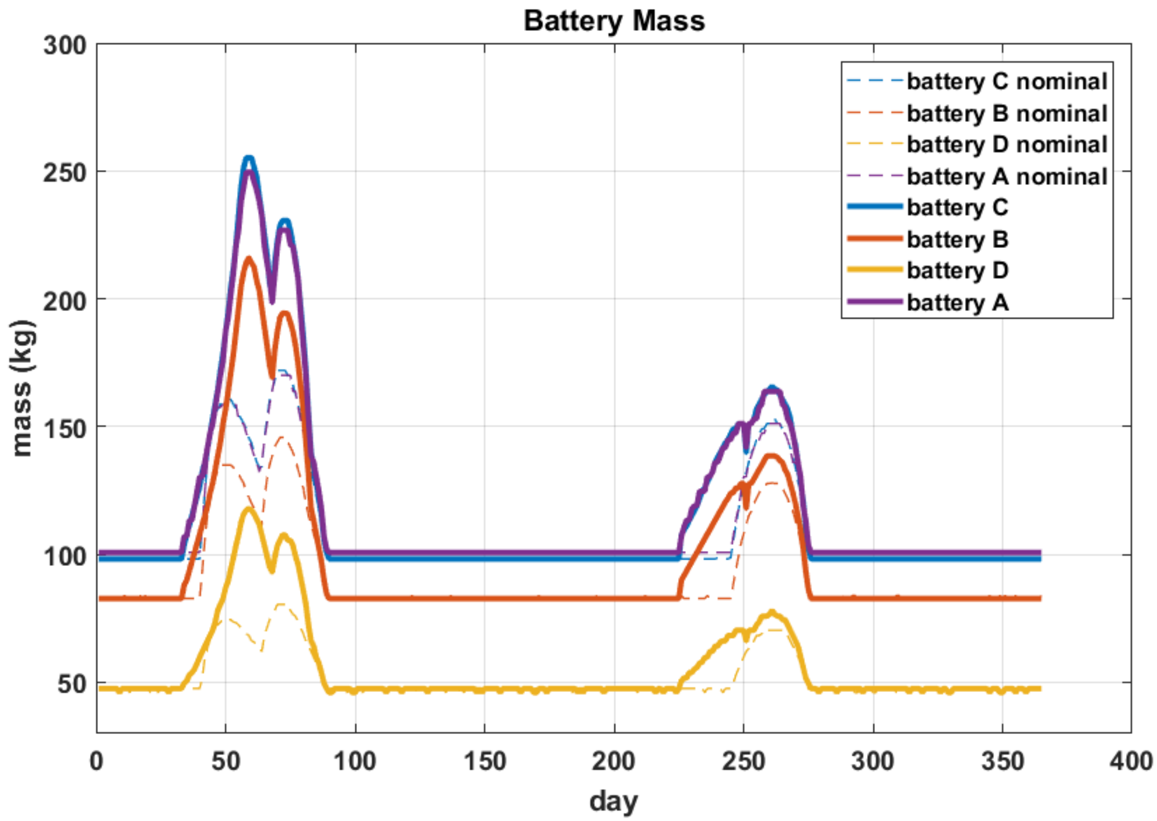
BATTERY	Battery A	Battery B	Battery C	Battery D
Cell nameplate capacity	16 Wh	5 Wh	5 Wh	8.5 Wh
Minimum battery voltage	36 V	25 V	25 V	25 V
Max DoD	80%	75%	75%	75%
Pack nameplate energy	2816 Wh	2640 Wh	3688 Wh	3756 Wh
Number of string failures	1	1	1	1
Available energy, BoL, after failure	10908 Wh	10513 Wh	14693 Wh	14929 Wh
Number of modules	4	4	4	4
Number of series cells	11	9	11	11
Number of parallel strings	16	56	64	40
Total mass	112 kg	100 kg	148 kg	100 kg

Effect of Transfer Model Uncertainty

- Required energy maximization over orbital parameters:



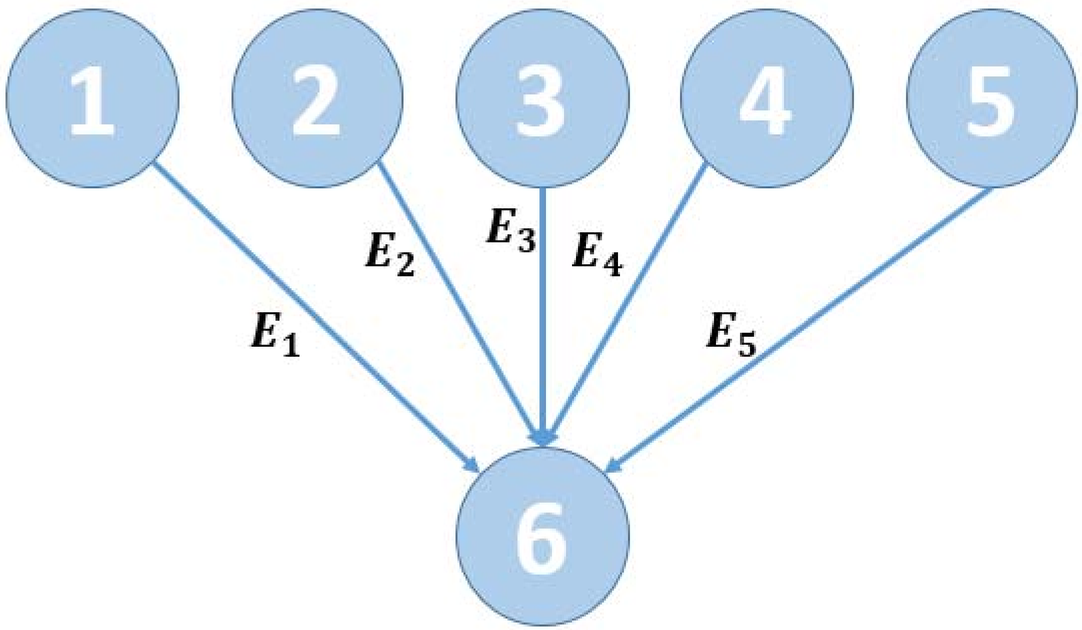
Worst-case Analysis of Battery Performances



- Day 59:
  - Design vector = [battery D, 36.9 V];
  - Objective function = 126.3 kg;
- Day 261:
  - Design vector = [battery D, 40.2 V];
  - Objective function = 82.8 kg;

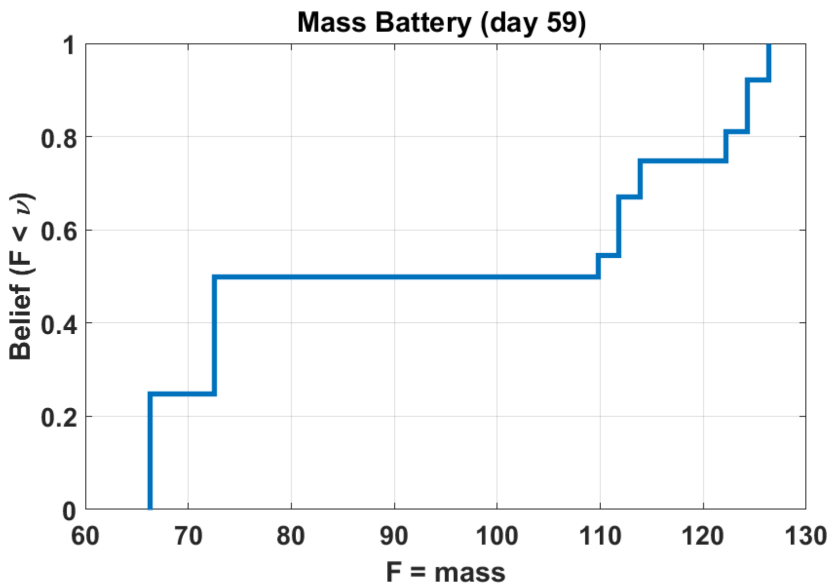
System Decomposition

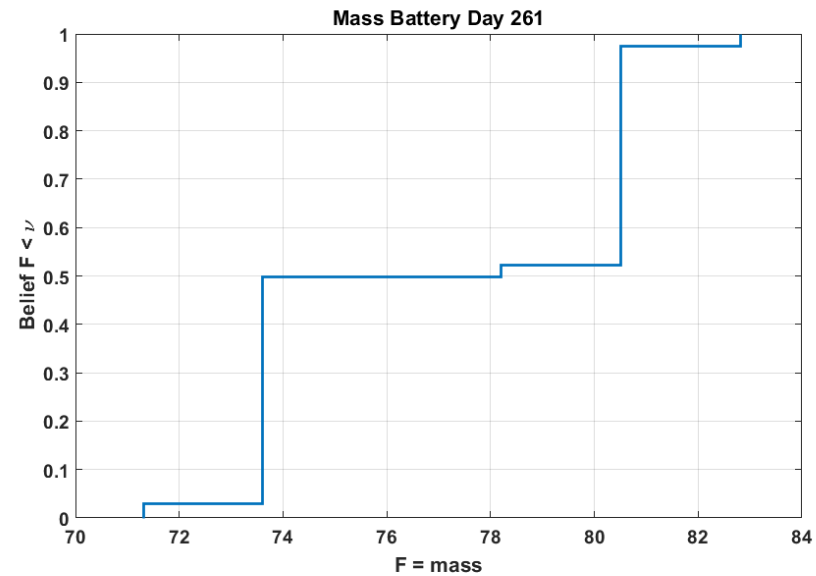
Full problem requires 2^{31} optimisations



Decomposed problem requires around 300 optimisations

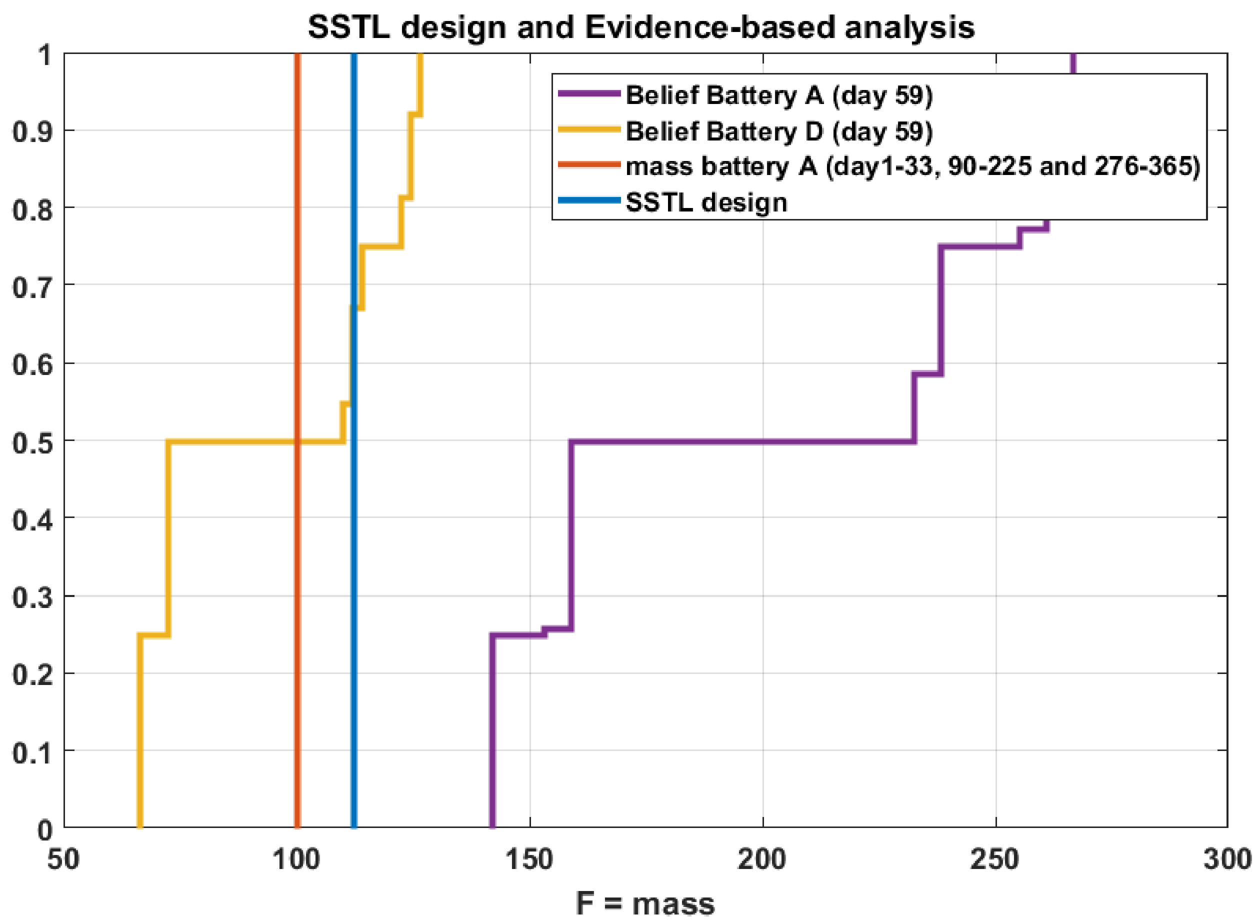
Belief Curve Reconstruction from Worst Case



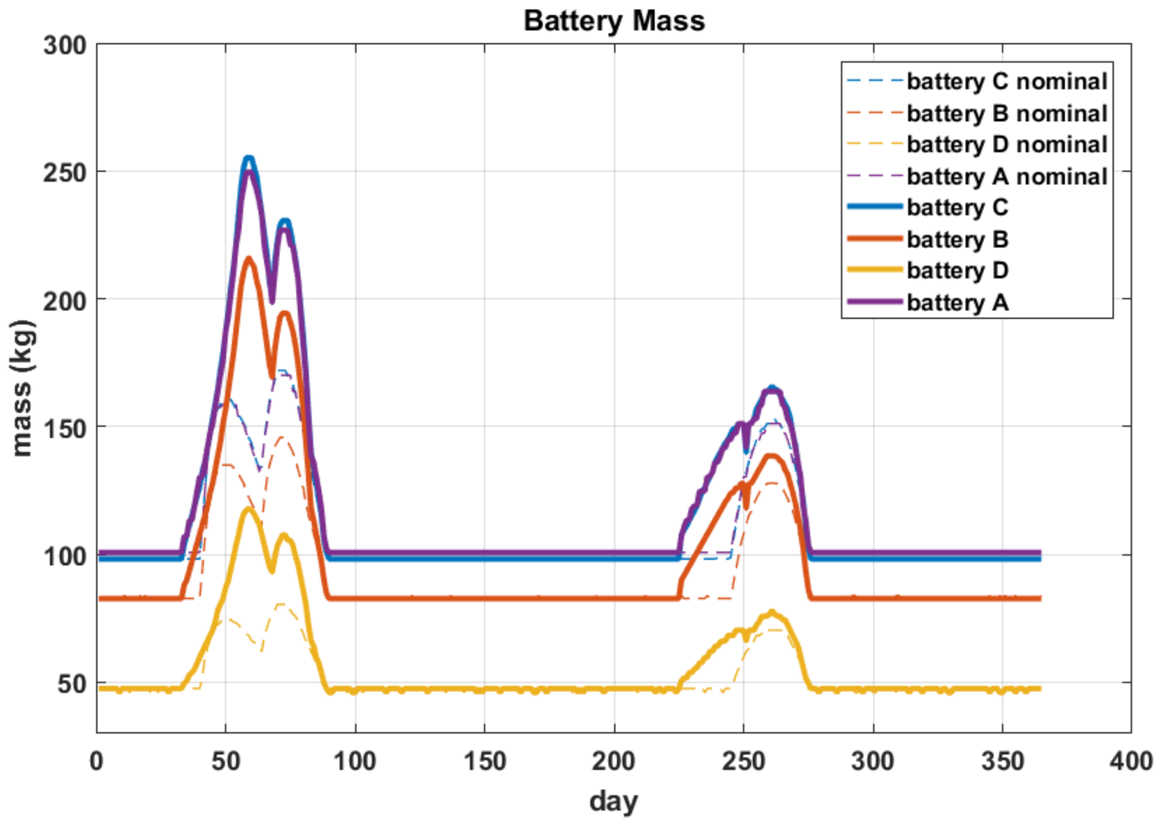


- Day 59:
  - Design vector = [battery D, 36.9 V];
  - Objective function = 126.3 kg;
- Day 261:
  - Design vector = [battery D, 40.2 V];
  - Objective function = 82.8 kg;

Comparison with SSTL Choice



Worst-case Analysis of Battery Performances



- Day 59:
  - Design vector = [battery D, 36.9 V];
  - Objective function = 126.3 kg;
- Day 261:
  - Design vector = [battery D, 40.2 V];
  - Objective function = 82.8 kg;

Model Validation

- Comparison of eclipse time yielded by the University of Strathclyde model and GMAT analysis
- Comparison for nominal and worst case orbits, on day 59 and 261

Model Validation: Day 59

Nominal orbit

- GMAT eclipse time: 1.18 h
- Strathclyde model eclipse time: 1.19 h

N orbit	SEMIAXIS (km)	ECCENTRICITY (-)	INCLINATION (°)	RAAN (°)	ARG. OF PERIGEE (°)	TRUE ANOMALY (°)
N = 2	73250	0.77866	9.12	86.79	180.06	180.08

Worst-case orbit

- GMAT eclipse time: 2.13 h
- Strathclyde model eclipse time: 2.11 h

N orbit	SEMIAXIS (km)	ECCENTRICITY (-)	INCLINATION (°)	RAAN (°)	ARG. OF PERIGEE (°)	TRUE ANOMALY (°)
N = 2	73252.58	0.7779	9.077	115.85	180.84	180.07

Model Validation: Day 261

Nominal orbit

- GMAT eclipse time: 1.263 h
- Strathclyde model eclipse time: 1.245 h

N orbit	SEMIAXIS (km)	ECCENTRICITY (-)	INCLINATION (°)	RAAN (°)	ARG. OF PERIGEE (°)	TRUE ANOMALY (°)
N = 4	49646.4	0.15392	0.36	86.85	180.97	4.25

Worst-case orbit

- GMAT eclipse time: 1.369 h
- Strathclyde model eclipse time: 1.351 h

N orbit	SEMIAXIS (km)	ECCENTRICITY (-)	INCLINATION (°)	RAAN (°)	ARG. OF PERIGEE (°)	TRUE ANOMALY (°)
N = 4	49639.16	0. 0.15413	0.37044	57.139	181.085	4.238

EXPLORE DESIGN PERFECTION

ESA Test Cases

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Allocation Problem

- Allocation of components (subsystems) in a spacecraft
- Subsystems represented by rectangular boxes
- Design variables: relative position of barycentres of boxes
- Epistemic variables:
  - size and mass of the components
  - tolerance in box positions
  - (thermal model parameters)
- Constraints:
  - non-intersection of boxes
  - (minimum and maximum operational temperature)
  - (bounds on admissible moments of inertia)
- Objective: minimise horizontal barycentre displacement from z-axis
- No decomposition is possible → simplifications to make Belief reconstruction practicable

Thermal Model (I)

- Model uncertainty in thermal model
  - Parameters of the equivalent model (conductivity and cross section areas)
  - Load case definition (heat source and external heat of subsystems)
- Thermal constraints violated if any component has a temperature below minimum or above maximum operational temperature
- Thermal load case: hot case/cold case
- Thermal network: finite difference/lumped parameter formulation
  - Nodes: temperature and heat source (internal dissipation/external heat load)
  - N nodes (subsystems) + 1 (radiator)
  - Some links: conduction only
  - Stationary model

Thermal Model (II)

- Transient heat balance equation for lumped parameter scheme:

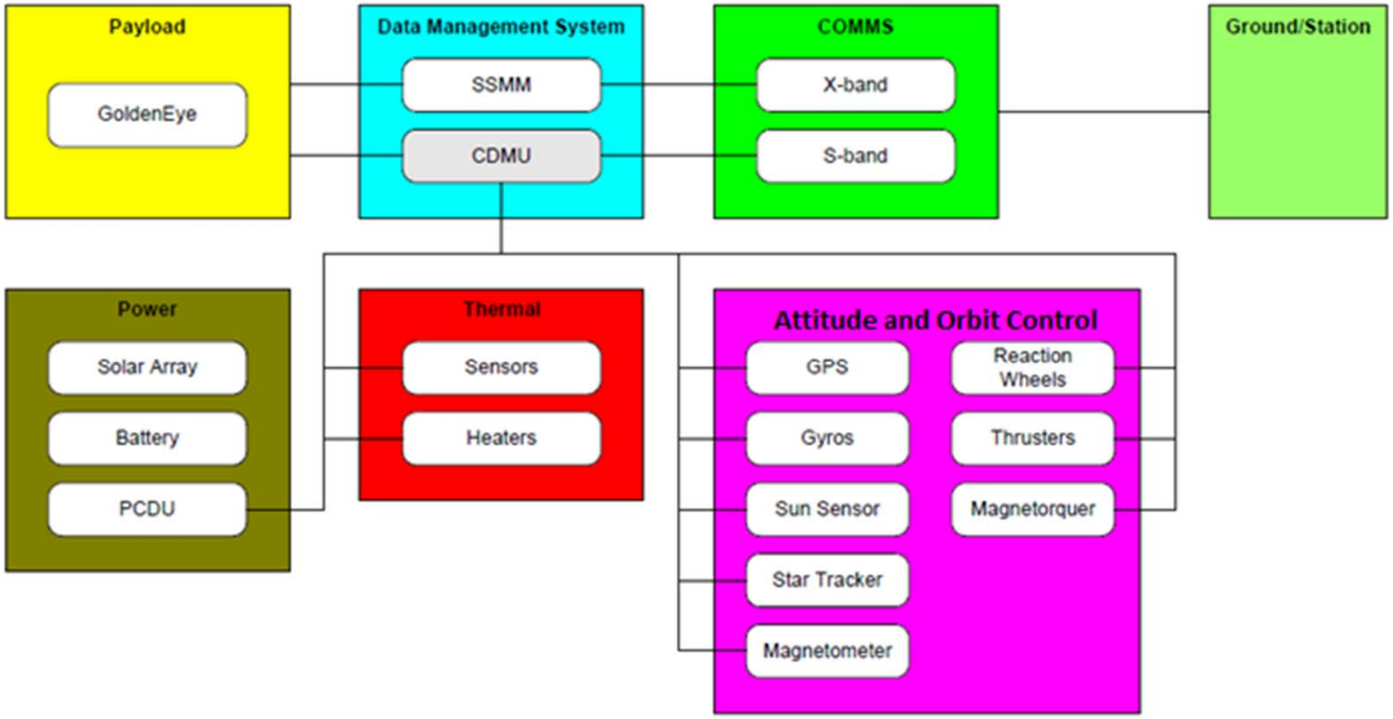
$$m_i c_i \frac{dT_i}{dt} = \sum_{i \neq j} K_{ij}(T_j - T_i) + \sum_{i \neq j} R_{ij}(T_j - T_i) + Q_i = 0$$

- Steady-state model with conduction only:

$$\sum_{i \neq j} K_{ij}(T_j - T_i) + Q_i = 0$$

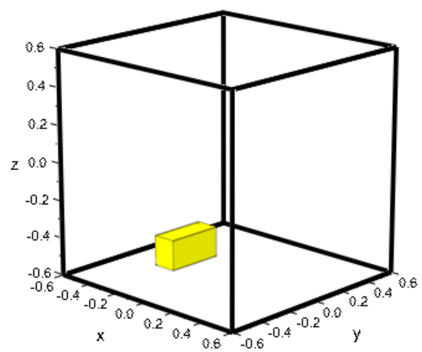
- Thermal model as a black-box
  - Input: component positions
  - Parameters: (non-deterministic) thermal model parameters
  - Output: constraint violation

Example: EagleEye Subsystem Structure

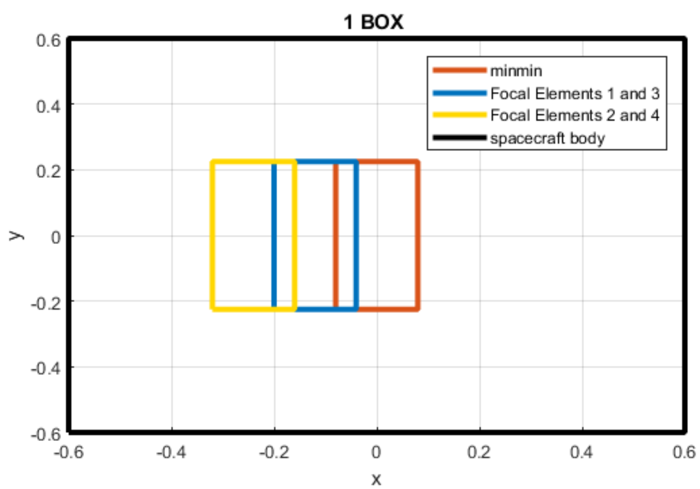


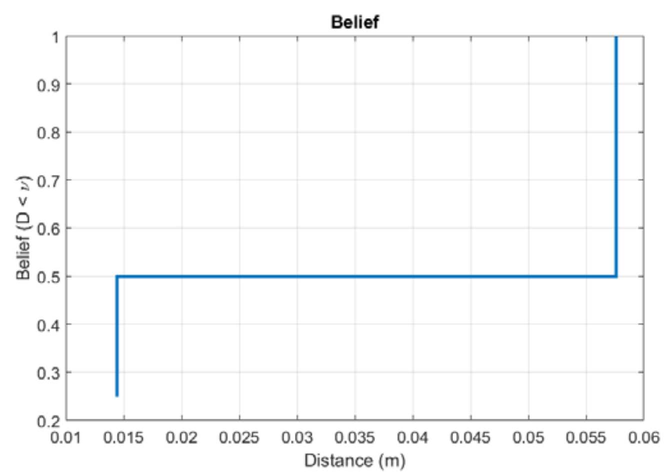
But objective function cannot be decomposed (no exchange variables)!

Case of a Single Box



- 2 design parameters
  (x and y distance from z axis)
- 5 epistemic parameters
  (mass, size of the box along x,y,z, and barycenter position tolerance)
- 32 focal elements





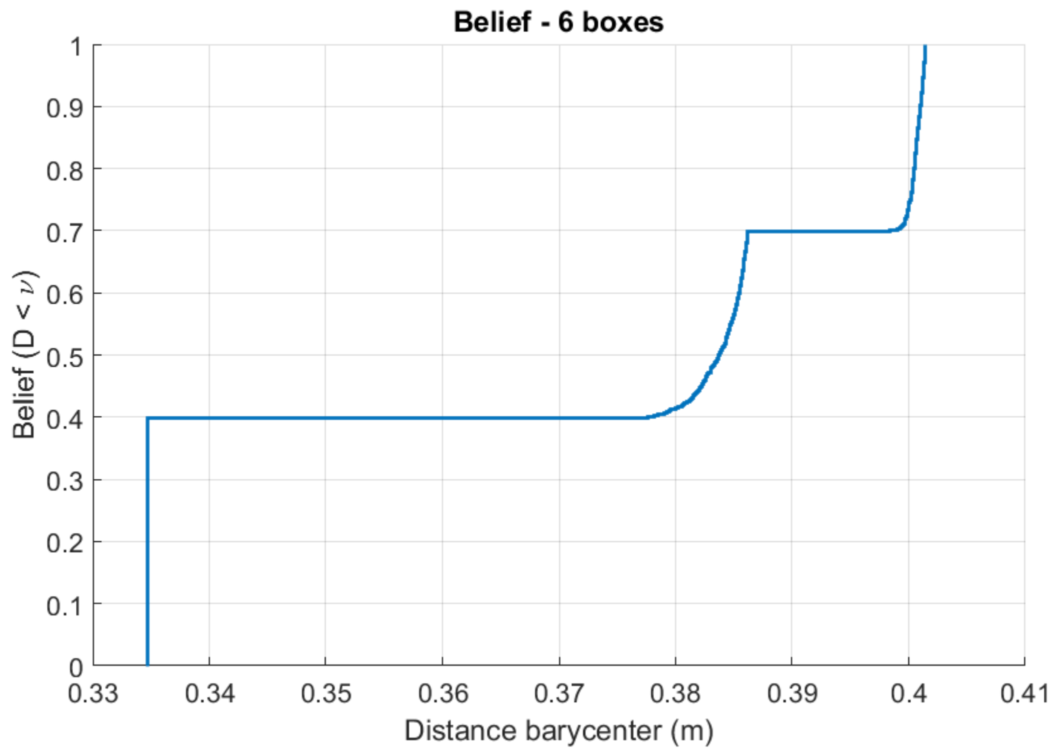
Case of 6 Boxes

Full problem:

- 12 design parameters
  (x and y distance from z axis)
- 30 epistemic parameters
  ((mass, x,y,z box sizes, tolerance)*6)
- 2^{30} focal elements

Simplified problem:

- 12 design parameters
  (x and y distance from z axis)
- 12 epistemic parameters
  ((x,y tolerances)*6)
- 2^{12}=4096 focal elements



Conclusions & Future Developments

Conclusions

- Evidence Theory to quantify epistemic uncertainty
- Plug-in for Evidence Based Robust Optimisation in modeFRONTIER
- Efficient methods for worst case optimisation & exact quantification of system and subsystem margins
- Decomposition method to reduce computational effort
- Algorithms and prototype validated on several test-cases

Future Developments

- Constraint handling in the prototype
- Use prototype for Robust Design Optimisation Problems (how to combine the building blocks)
- Improvement of Decision Making tool
- Resilient reliability problem
- Treatment of dynamic responses
- Validation on examples at Systems Engineering level
- Exploitation of modeFRONTIER integration capabilities
- Technical aspect: improvement of techniques to reduce number of function evaluations/computational effort

Thank you for your attention

EXPLORE DESIGN PERFECTION

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Min-Max Algorithms

- SUPER algorithm: IDEAminmax
- Single-objective:
- OUTER is MACS algorithm for single objective case and MPAIDEA for multiobjective case
- INNER is MPAIDEA algorithm (for both single and multiobjective cases)

Single-Objective Bi-level Optimisation Problems

- Min-Min (points of minimum Plausibility):

$$\nu_{\min} = \min_{\mathbf{d}\in D,\mathbf{u}\in\bar{U}} f(\mathbf{d},\mathbf{u})$$

- Min-Max (minimum budget with maximum belief):

$$\nu_{\max} = \min_{\mathbf{d}\in D} \max_{\mathbf{u}\in\bar{U}} f(\mathbf{d},\mathbf{u})$$

Multiobjective Bi-level Optimisation Problems

- Min-Min (points of minimum Plausibility):

$$\min_{\mathbf{d}\in D}\left[\min_{\mathbf{u}\in\bar{U}} f_1(\mathbf{d},\mathbf{u}), \min_{\mathbf{u}\in\bar{U}} f_2(\mathbf{d},\mathbf{u}), \ldots, \min_{\mathbf{u}\in\bar{U}} f_m(\mathbf{d},\mathbf{u})\right]^T$$

- Min-Max (minimum budget with maximum belief):

$$\min_{\mathbf{d}\in D}\left[\max_{\mathbf{u}\in\bar{U}} f_1(\mathbf{d},\mathbf{u}), \max_{\mathbf{u}\in\bar{U}} f_2(\mathbf{d},\mathbf{u}), \ldots, \max_{\mathbf{u}\in\bar{U}} f_m(\mathbf{d},\mathbf{u})\right]^T$$

- Minimisation over the whole space: front lies inbetween

$$\min_{\mathbf{d}\in D}\left[\max_{\mathbf{u}\in\bar{U}} f_1(\mathbf{d},\mathbf{u}), \max_{\mathbf{u}\in\bar{U}} f_2(\mathbf{d},\mathbf{u}), \ldots, \max_{\mathbf{u}\in\bar{U}} f_m(\mathbf{d},\mathbf{u})\right]^T$$

Multiobjective Metrics

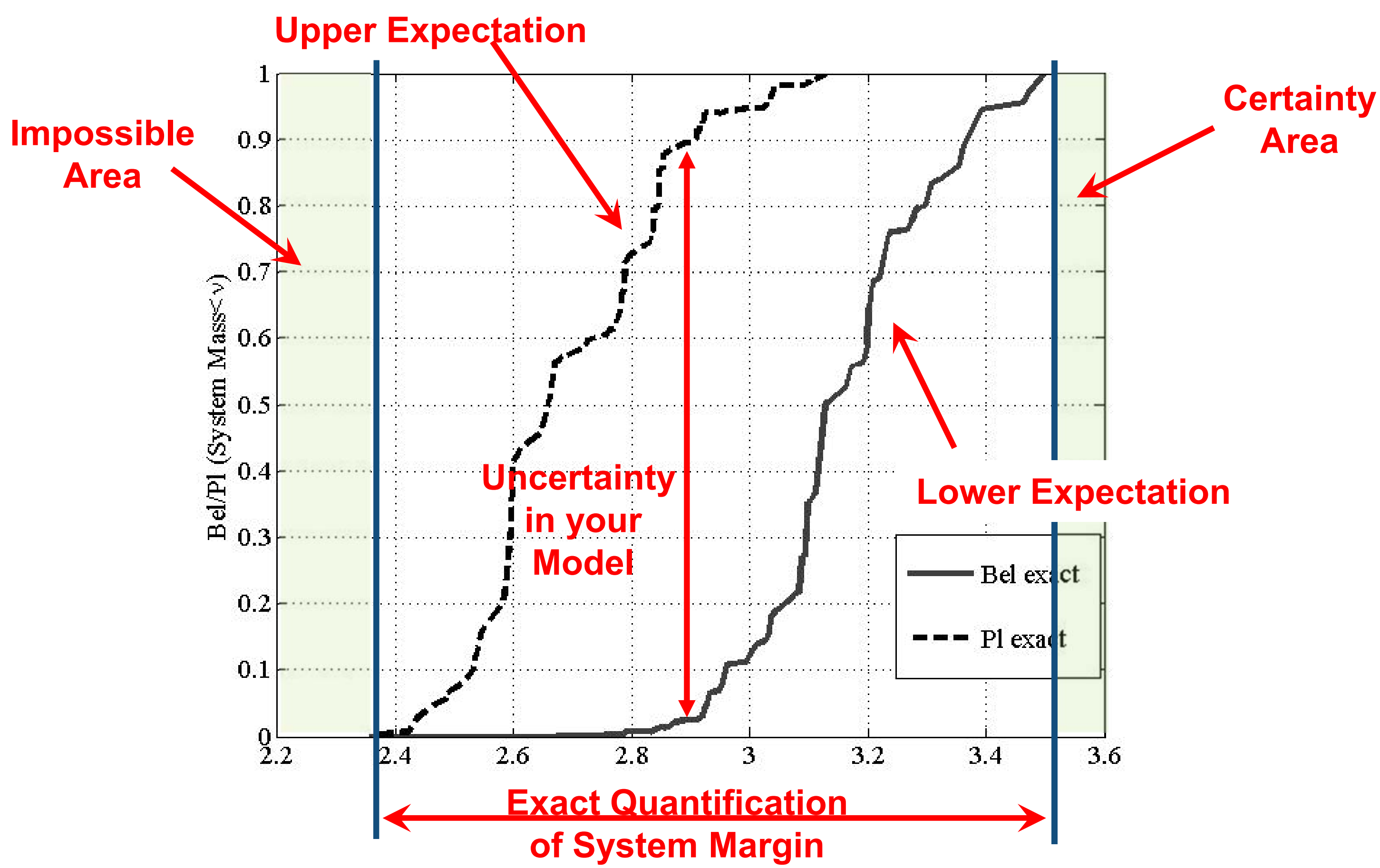
$$M_{conv} = \frac{1}{N_p} \sum_{i=1}^{N_p} \min_{j \in M_p} \left\| g_j - f_i \right\|$$

Front convergence

$$M_{spr} = \frac{1}{M_p} \sum_{i=1}^{M_p} \min_{j \in N_p} \left\| f_i - g_j \right\|$$

Front spreading

Solution: Evidence-Based UQ



Relaxation Mechanism

Algorithm 1 Relaxation procedure (single-objective)

Initialise archive $A_u = \{\mathbf{u}_1\}$ and $i = 1$

while the termination condition is not met **do**

$\quad \mathbf{d}_i \leftarrow \arg\min_{\mathbf{d} \in D} \left\{ \max_{\mathbf{u} \in A_u} f(\mathbf{d}, \mathbf{u}) \right\}$

$\quad \mathbf{u}_{i+1} \leftarrow \arg\max_{\mathbf{u} \in U} f(\mathbf{d}_i, \mathbf{u})$

$\quad A_u \leftarrow A_u \cup \mathbf{u}_{i+1}$

$\quad i \leftarrow i + 1$

end while

return $\{\mathbf{d}_i, \mathbf{u}_{i+1}, f(\mathbf{d}_i, \mathbf{u}_{i+1})\}$

Min-Max

Algorithm 2 MACSminmax

$n_{feval} = 0$
Initialise archive $A_d = \{d_1, d_2, \dots, d_{n_{d,0}}\}$
for all $\mathbf{d}_j \in A_d$ **do**
  for all $l \in \{1, \dots, n_f\}$ **do**
    $\mathbf{u}^l_{max,j} \leftarrow \arg\max_{\mathbf{u} \in U} f^l(\mathbf{d}_j, \mathbf{u})$ **[MPAIDEA]**
    $A^l_u \leftarrow A^l_u \cup \{\mathbf{u}^l_{max,j}\}$
  end for
end for
$A_f \leftarrow \{[\max_{\mathbf{u}^l \in A^l_u} f^l(\mathbf{d}_j, \mathbf{u}^l), l \in \{1, \dots, n_f\}], \mathbf{d}_j \in A_d\}$
while $n_{feval} < n_{feval,max}$ **do**
  $\mathbf{d}^A_{min} \leftarrow \arg\min_{\mathbf{d} \in D} [\max_{\mathbf{u}^l \in A^l_u} f^l(\mathbf{d}, \mathbf{u}^l), l \in \{1, \dots, n_f\}]^T$ **[MACS]**
  Fit surrogate $\mathbf{S}(\mathbf{d})$ on data points $\{A_d, A_f\}$
  $\mathbf{d}^S_{min} \leftarrow \arg\min_{\mathbf{d} \in D} \mathbf{S}(\mathbf{d})$ **[MACS]**
  $A_d \leftarrow A_d \cup \mathbf{d}^A_{min} \cup \mathbf{d}^S_{min}$
  for all $\mathbf{d}_j \in \mathbf{d}^A_{min} \cup \mathbf{d}^S_{min}$ **do**
    for all $l \in \{1, \dots, n_f\}$ **do**
      $\mathbf{u}^l_{max,j} \leftarrow \arg\max_{\mathbf{u} \in U} f^l(\mathbf{d}_j, \mathbf{u})$ **[MPAIDEA]**
      if $\mathbf{d}_j \notin \mathbf{d}^A_{min}$ **or** $f^l(\mathbf{d}_j, \mathbf{u}^l_{max,j}) > \max_{\mathbf{u}^l \in A^l_u} f^l(\mathbf{d}_j, \mathbf{u}^l)$ **then**
        $A^l_u \leftarrow A^l_u \cup \{\mathbf{u}^l_{max,j}\}$
      end if
    end for
  end for
  $A_f \leftarrow \{[\max_{\mathbf{u}^l \in A^l_u} f^l(\mathbf{d}_j, \mathbf{u}^l), l \in \{1, \dots, n_f\}], \mathbf{d}_j \in A_d\}$
end while
if *local search refinement enabled* **then**
  $A^{ref.}_d \leftarrow \{\emptyset\}$, $A^{ND}_d \leftarrow$ {arguments $\mathbf{d}_j \in A_d$ of non-dominated entries $\mathbf{f}_j \in A_f$}
  while $A^{ND}_d \not\subset A^{ref.}_d$ **do**
    for all $\mathbf{d}_j \in A^{ND}_d \setminus A^{ref.}_d$ **do**
      for all $l \in \{1, \dots, n_f\}$ **do**
        Run multi-start local search with $\mathbf{X}_0 \subseteq A^l_u$ **[SQP]**
        Use result to refine $\mathbf{u}^l_{max,j}$ associated to $\mathbf{d}_j$ and update A_f
      end for
      $A^{ref.}_d \leftarrow A^{ref.}_d \cup \{\mathbf{d}_j\}$
    end for
    $A^{ND}_d \leftarrow$ {arguments $\mathbf{d}_j \in A_d$ of non-dominated entries $\mathbf{f}_j \in A_f$}
  end while
end if
return Non-dominated $\mathbf{f}^*_i \in A_f$, their arguments $\mathbf{d}^*_i \in A_d$ and associated $\mathbf{u}^{l,*}_{max,i}$, $l \in \{1, \dots, n_f\}$

Constraint Handling

Algorithm 1 Constrained minmax

Initialise $\bar{\mathbf{d}}$ at random and run $\mathbf{u}_a = \text{argmax}\, F(\bar{\mathbf{d}}, \mathbf{u})$ s.t. $C(\mathbf{d}_{min}, \mathbf{u}) \leq 0$
$A_u = A_u \cup \{\mathbf{u}_a\}$; $A_c = \emptyset$; $A_d = \emptyset$
while $N_{fval} < N_{fval}^{max}$ **do**
  Outer loop:
  $\mathbf{d}_{min} = \text{argmin}_{d \in D}\{\max_{\mathbf{u} \in A_u \cup A_c} F(\mathbf{d}, \mathbf{u})\}$ s.t. $\max_{u \in A_u \cup A_c} C(\mathbf{d}, \mathbf{u}) \leq 0$
  $A_d = A_d \cup \{\mathbf{d}_{min}\}$
  Inner loop:
  $\mathbf{u}_{a,F} = \text{argmax}_{\mathbf{u} \in \mathbf{U}} F(\mathbf{d}_{min}, \mathbf{u})$ s.t. $C(\mathbf{d}_{min}, \mathbf{u}) \leq 0$
  $\mathbf{u}_{a,C} = \text{argmax}_{u \in U} C(\mathbf{d}_{min}, \mathbf{u})$
  $A_u = A_u \cup \{\mathbf{u}_{a,F}\}$
  if $N_{fval} < N_{fval}^{relaxation} \vee$ $\exists \mathbf{d} \in A_d$ t.c. $\max_{u \in U} C(\mathbf{d}, \mathbf{u}) \leq 0$ **then**
    if $\max_{u \in U} C(\mathbf{d}_{min}, \mathbf{u}) > 0$ **then**
      $A_c = A_c \cup \{\mathbf{u}_{a,C}\}$
    end if
  else
    update ϵ
    $A_c = \{A_c \setminus \mathbf{u}_{a,C} \mid C(\mathbf{d}_{min}, \mathbf{u}) \leq \epsilon\}$
    if $\max_{u \in U} C(\mathbf{d}_{min}, \mathbf{u}) > \epsilon$ **then**
      $A_c = A_c \cup \{\mathbf{u}_{a,C}\}$
    end if
  end if
end while