Electron modeling in hybrid plasma thruster plume simulations

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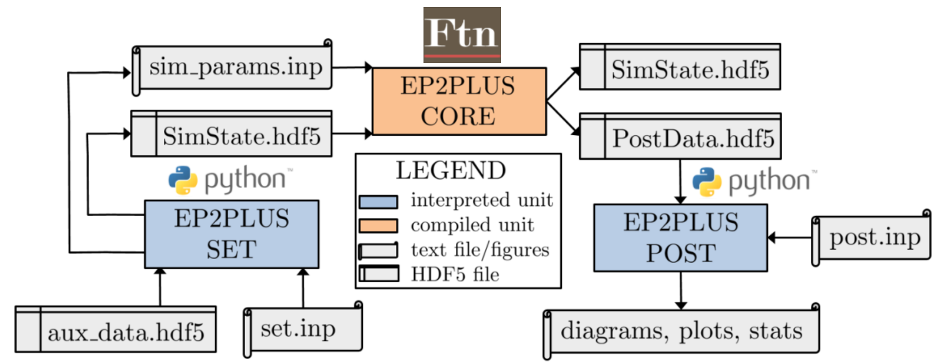
Contents

EP2PLUS: Extensible Parallel Plasma PLUme Simulator

- Hybrid PIC/fluid code
  - PIC ions/neutrals, fluid electrons
- 3D code → asymmetric physics simulation
  - Interaction of plasma thruster plume with the satellite and any downstream object
  - Plume from a cluster of thrusters
  - Asymmetric magnetic field effects
- Industry-level standards
  - HDF5 format I/O files
  - Parallelization with OpenMP
  - Test Driven Design (TDD)

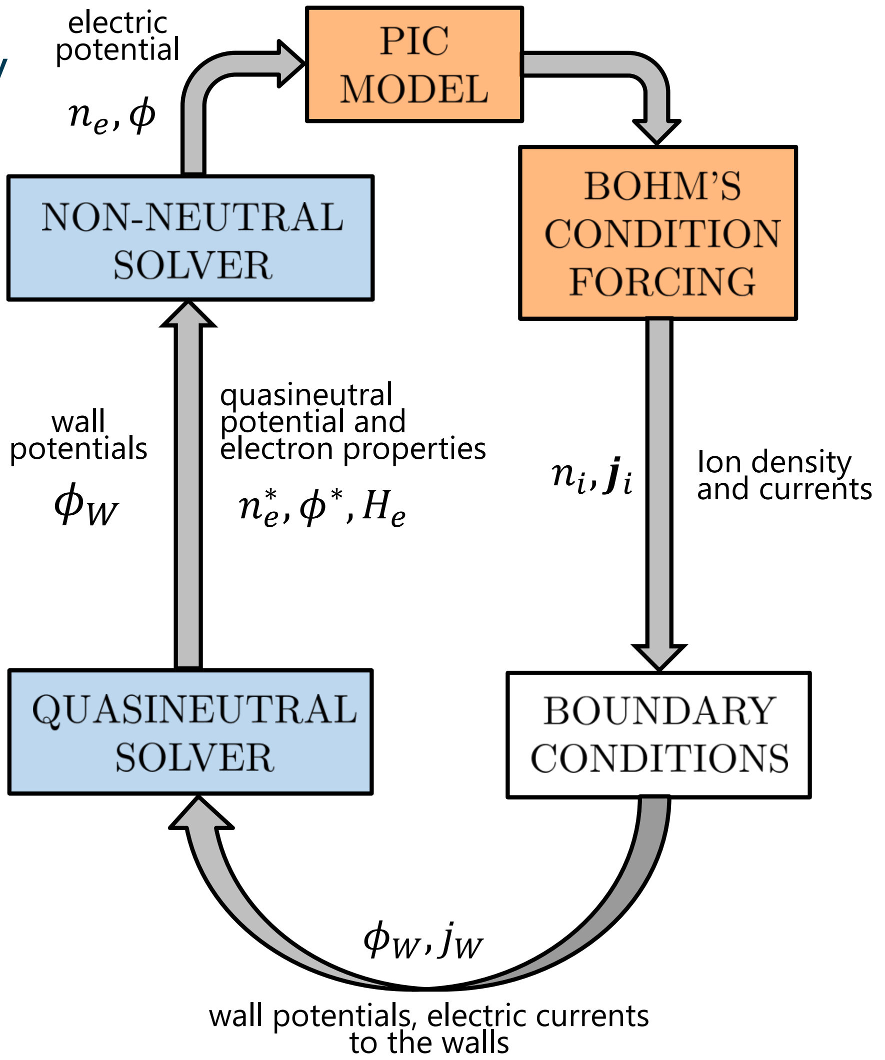


- Most distinguishing features:
  - Electron model enables the computation of both **electric currents** and **magnetic field effects** (ongoing)
  - **Non-neutral code** → interaction with the satellite, peripheral plume regions
  - **Deformed structured meshes** → interaction with cylindrical debris, simulation of plasma thruster discharges, etc…
  - **Easily adaptable to full-PIC**

EP2PLUS: the hybrid loop

1. PIC sub-model provides ion density and currents
   - ❑ Bohm's condition is enforced if necessary
2. Sheath and circuit model compute the electric current and potential at the walls
   - ❑ Dielectric and conductive walls
3. Quasi-neutral solver computes the new electric potential, assuming quasineutrality and boundary electric currents
   - ❑ Collisions and magnetization can be taken into account as a part of the Bernoulli function $-\boldsymbol{H_e/e}$
4. The electric potential is selectively recomputed with the non-linear Poisson solver in non-neutral regions
   - ❑ Non-neutrality criterion

THE HYBRID CODE LOOP



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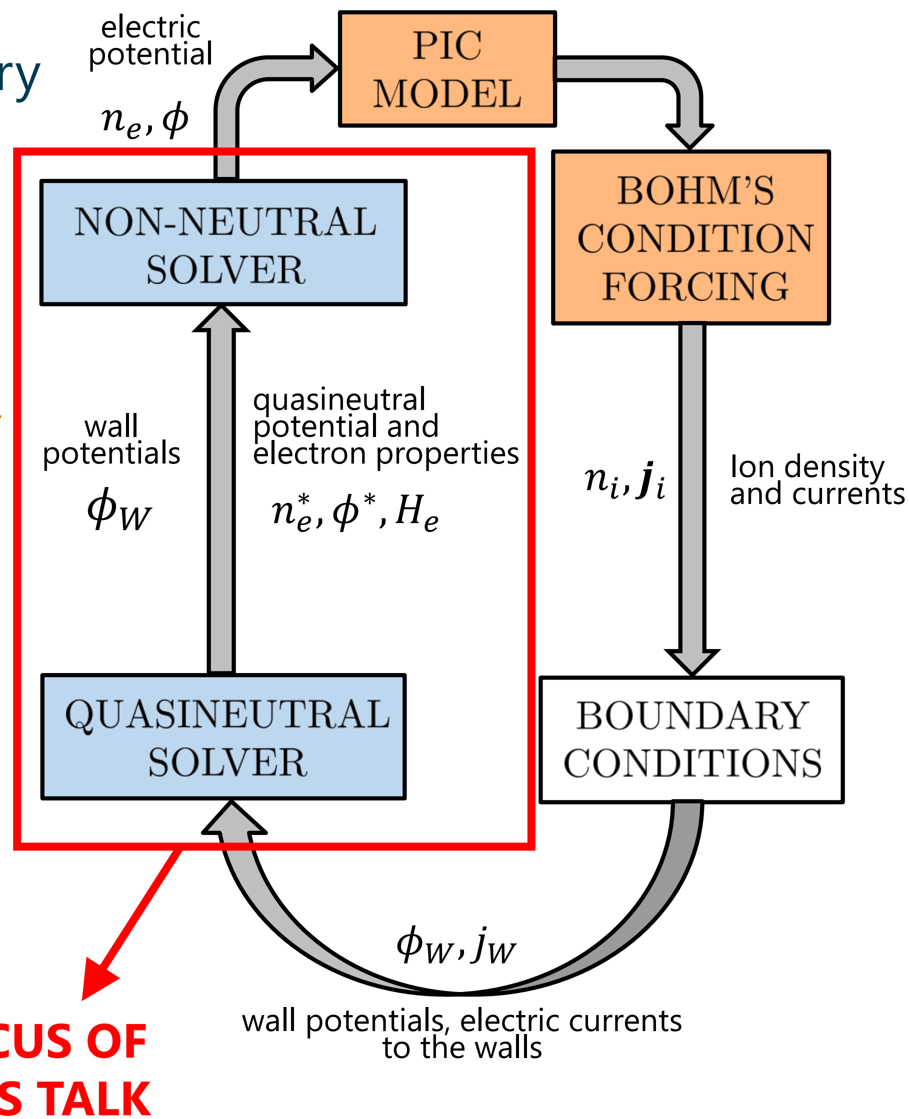
THE HYBRID CODE LOOP



Contents

The electron fluid model

- Assumptions
  - Stationary plume properties: $\frac{\partial}{\partial t} = 0$
  - Inertialess electrons: $m_e \ll m_i$
  - Barotropic electron fluid: $T_e = T_e(n_e) \rightarrow$ BAROTROPY GRADIENT $\boxed{\nabla h_e = \nabla p_e / n_e} \rightarrow h_e = h_e(n_e)$
  - Isotropic electrons: $\bar{\bar{P}}_e = p_e \bar{\bar{I}}$

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Electron momentum balance
+
Electric current continuity

Poisson's equation

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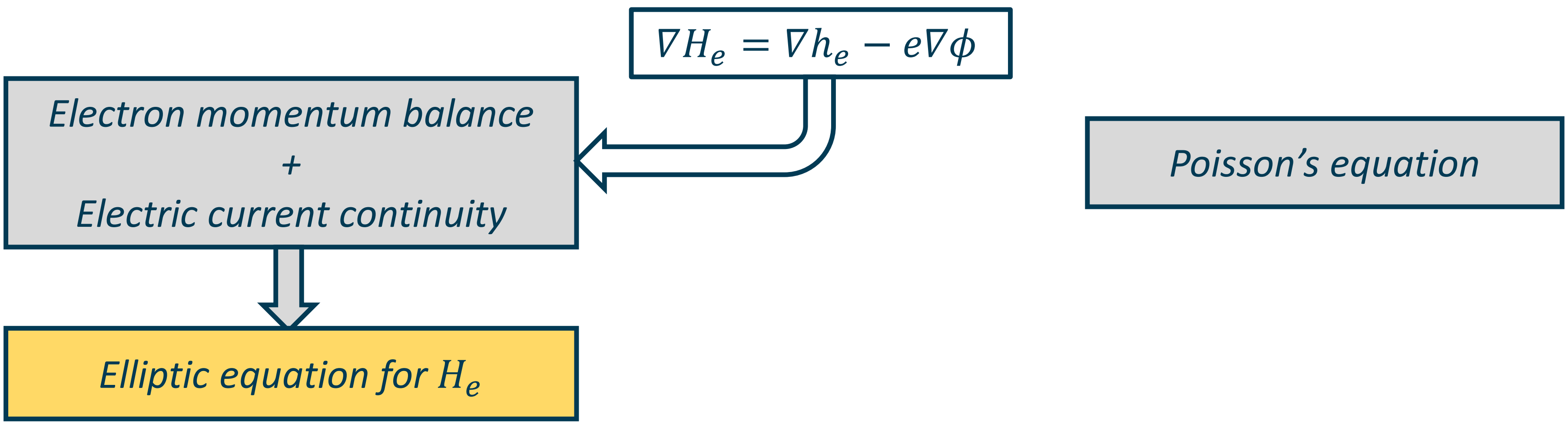

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- Stationary plume properties: $\frac{\partial}{\partial t} = 0$
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- Isotropic electrons: $\bar{\bar{P}}_e = p_e \bar{\bar{I}}$

$$\nabla H_e = \nabla h_e - e\nabla\phi$$

Electron momentum balance
+
Electric current continuity

Poisson's equation

Elliptic equation for H_e ← $j_W, n_i, \boldsymbol{j}_i$

quasineutral electron density

$$\boldsymbol{j} = \bar{\bar{K}} \cdot \frac{\nabla H_e}{e} + \boldsymbol{j}_{PIC} \quad ; \quad \phi^* = \frac{h_e(n_e^*) - H_e}{e}$$

ELECTRIC CURRENT AND QUASINEUTRAL POTENTIAL

The electron fluid model

Assumptions

- Stationary plume properties: $\frac{\partial}{\partial t} = 0$
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BAROTROPY GRADIENT: $\nabla h_e = \nabla p_e / n_e$

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Electron momentum balance
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Elliptic equation for H_e ← $j_W, n_i, \boldsymbol{j}_i$

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ELECTRIC CURRENT AND QUASINEUTRAL POTENTIAL

$n_e(\phi, H_e)$

ϕ_W, n_i

Poisson's equation

quasineutral electron density

$$\nabla^2\phi = -\frac{e}{\epsilon_0}[n_e^* - n_e(\phi, H_e)]$$

The electron fluid model

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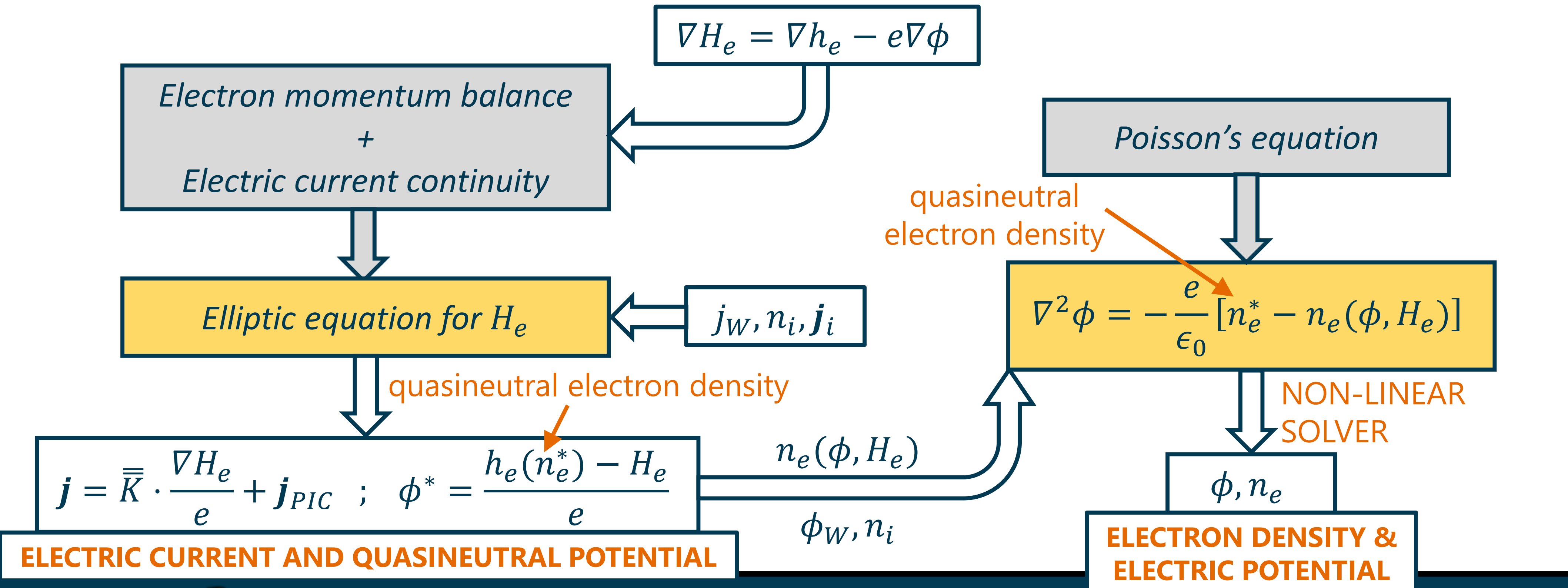
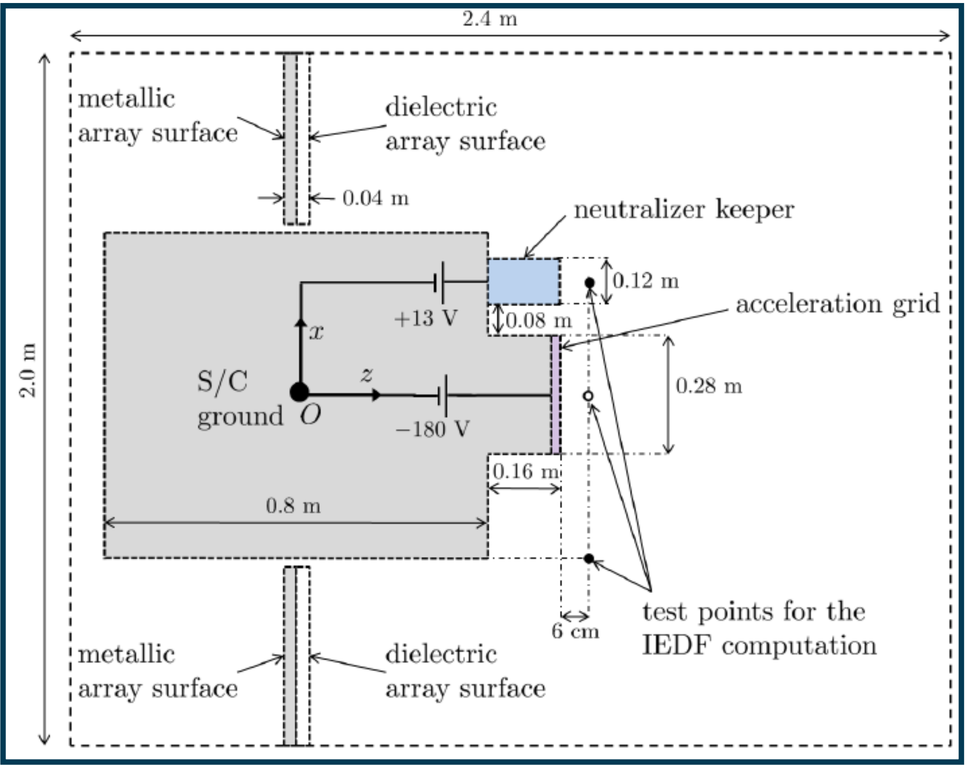


Contents

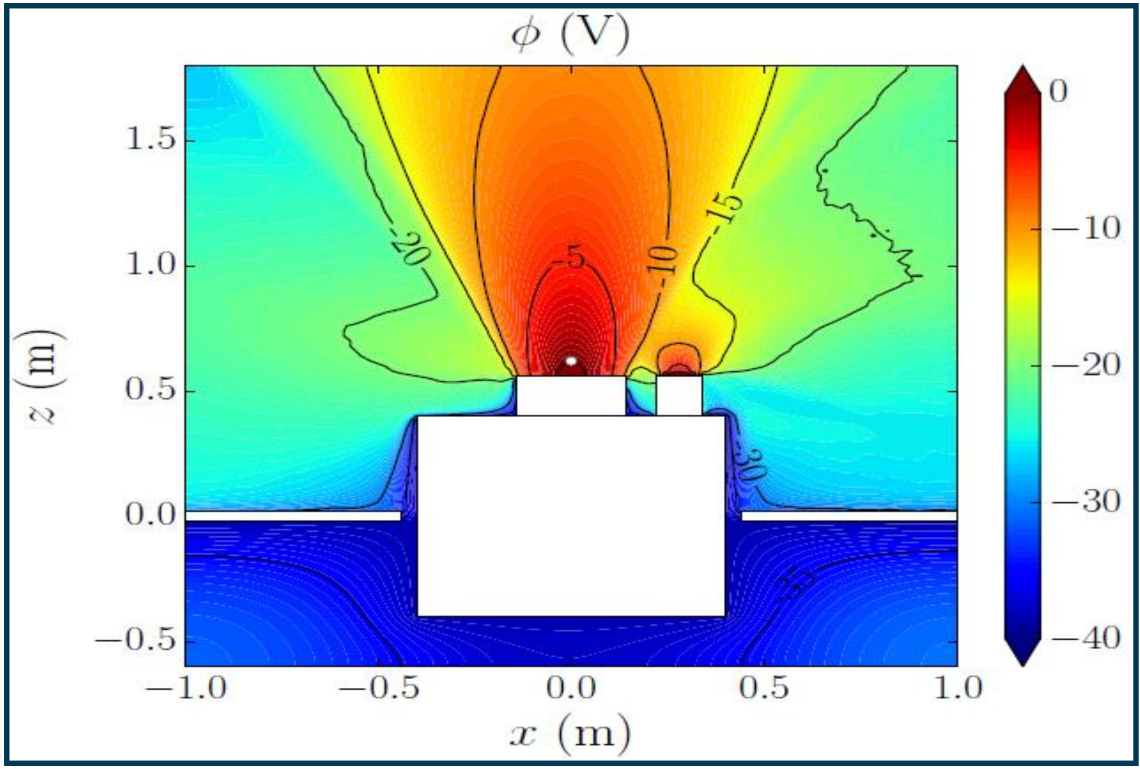
GIT plume neutralization (1)

- Study of the interaction of a plasma plume from a GIT with the satellite and an external neutralizer (**Cichocki PSST 2017**)
  - NSTAR thruster and a neutralizer
  - No magnetic field

EQUIVALENT CIRCUIT



ELECTRIC POTENTIAL

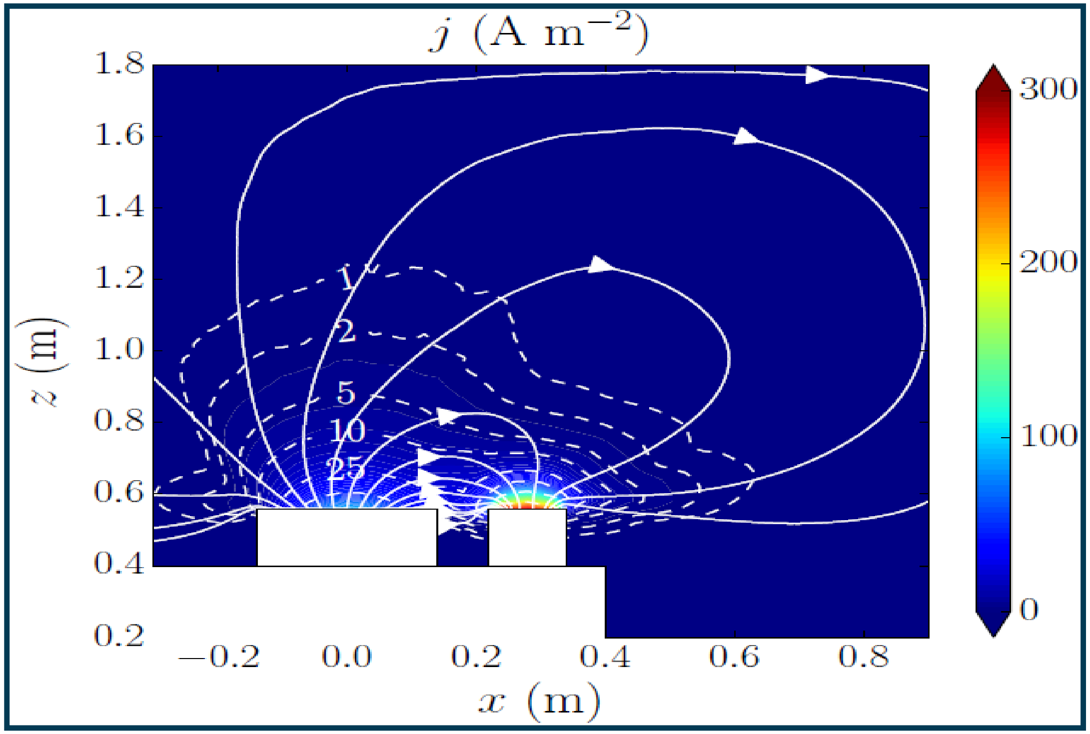


GIT plume neutralization (2)

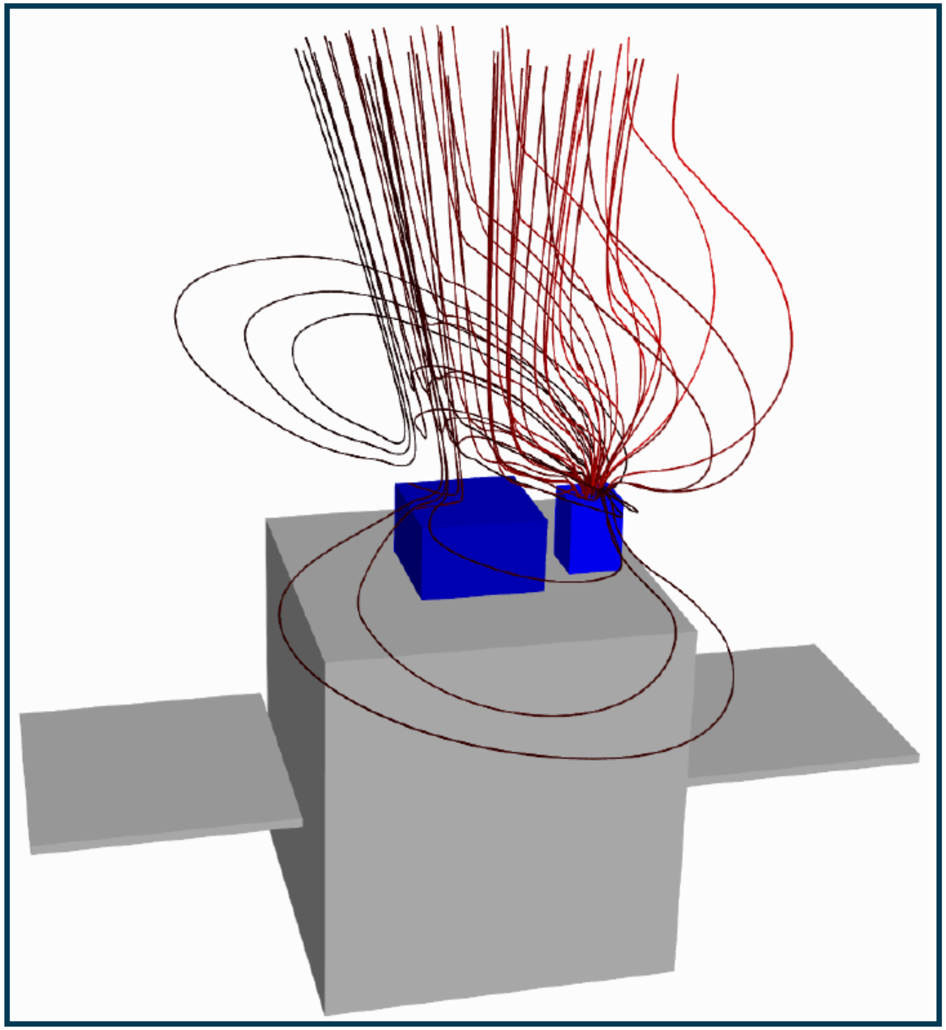
➢ GIT current neutralization was studied:

- Plume current negligible after 0.5 m
- 3D electron streamlines follow minimum resistance path

TOTAL ELECTRIC CURRENT



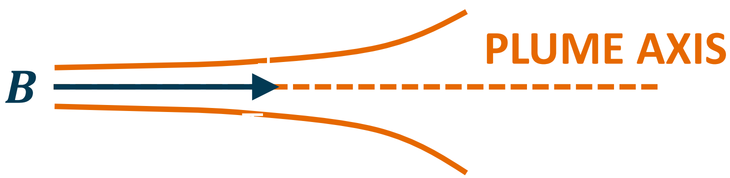
3D ELECTRON STREAMLINES



Contents

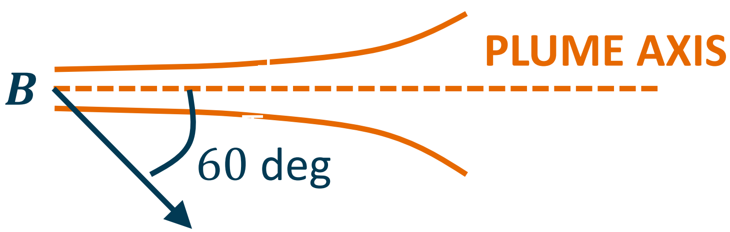
Plasma plume with uniform magnetic field (1)

- For a typical plasma thruster plume, various magnetic fields were considered (**Cichocki SP 2018**):
  - Unmagnetized case $B = 0$ for reference
  - Plume with axial magnetic field



SYMMETRIC COMPRESSION, NOT SHOWN HERE

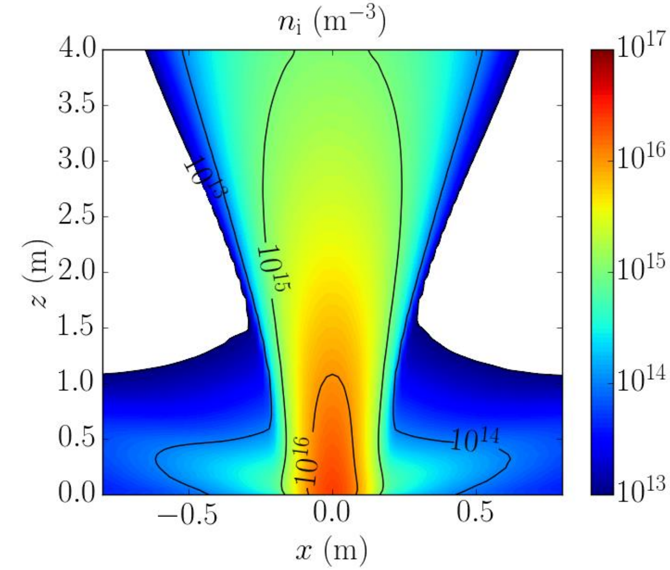
  - Plume with magnetic field forming an angle of 60 deg with the plume axis



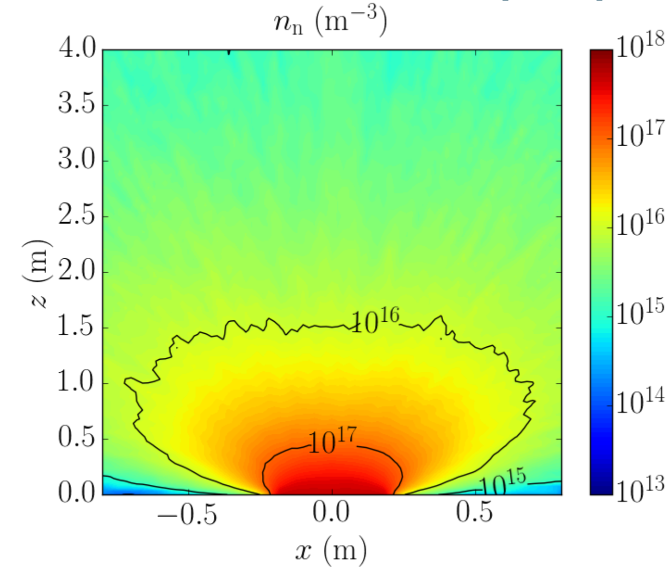
PREDICTED DISTORSION

- Magnetic field intensity: **0.1 Gauss**

ION DENSITY (B=0)



NEUTRAL DENSITY (B=0)

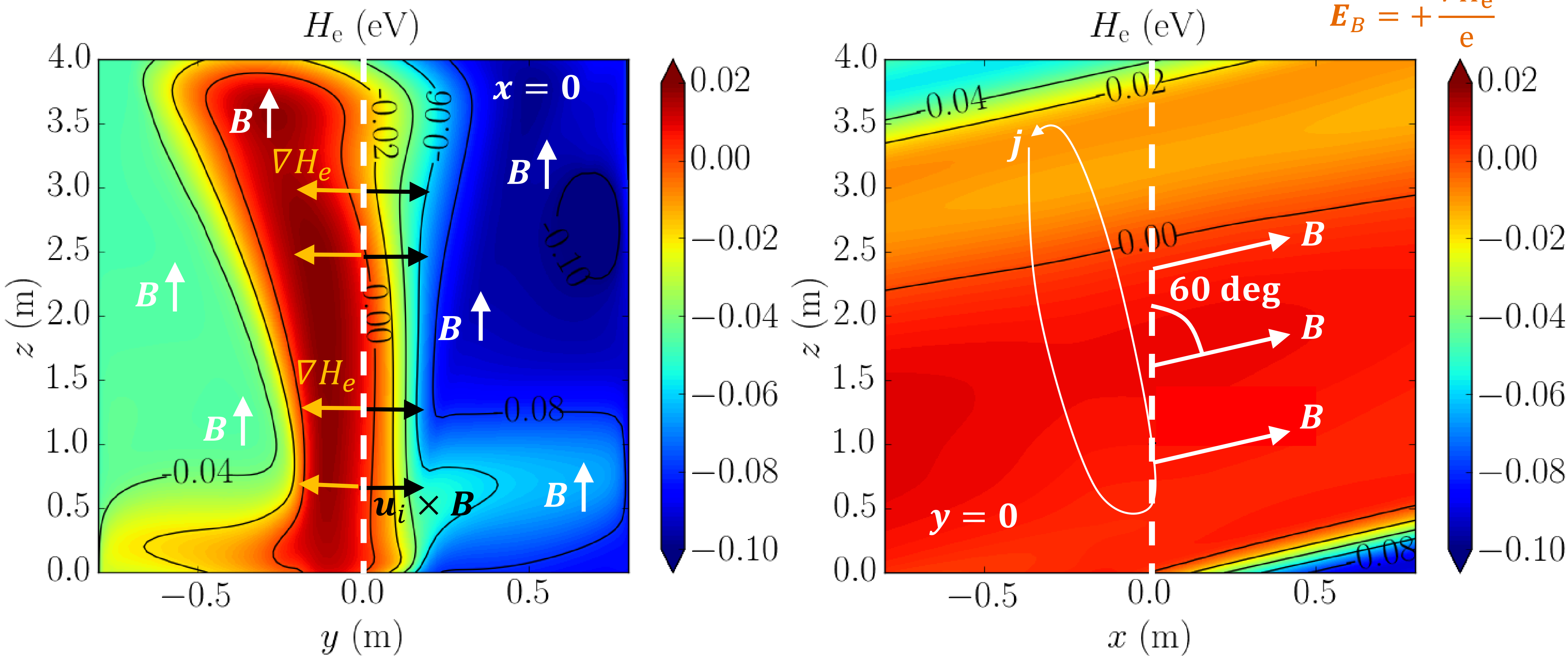


Plasma plume with uniform magnetic field (2)

OBLIQUE MAGNETIC FIELD (60 DEG ANGLE)

ELECTRIC FIELD CORRECTION DUE TO B:

$$\boldsymbol{E}_B = +\frac{\nabla H_e}{e}$$

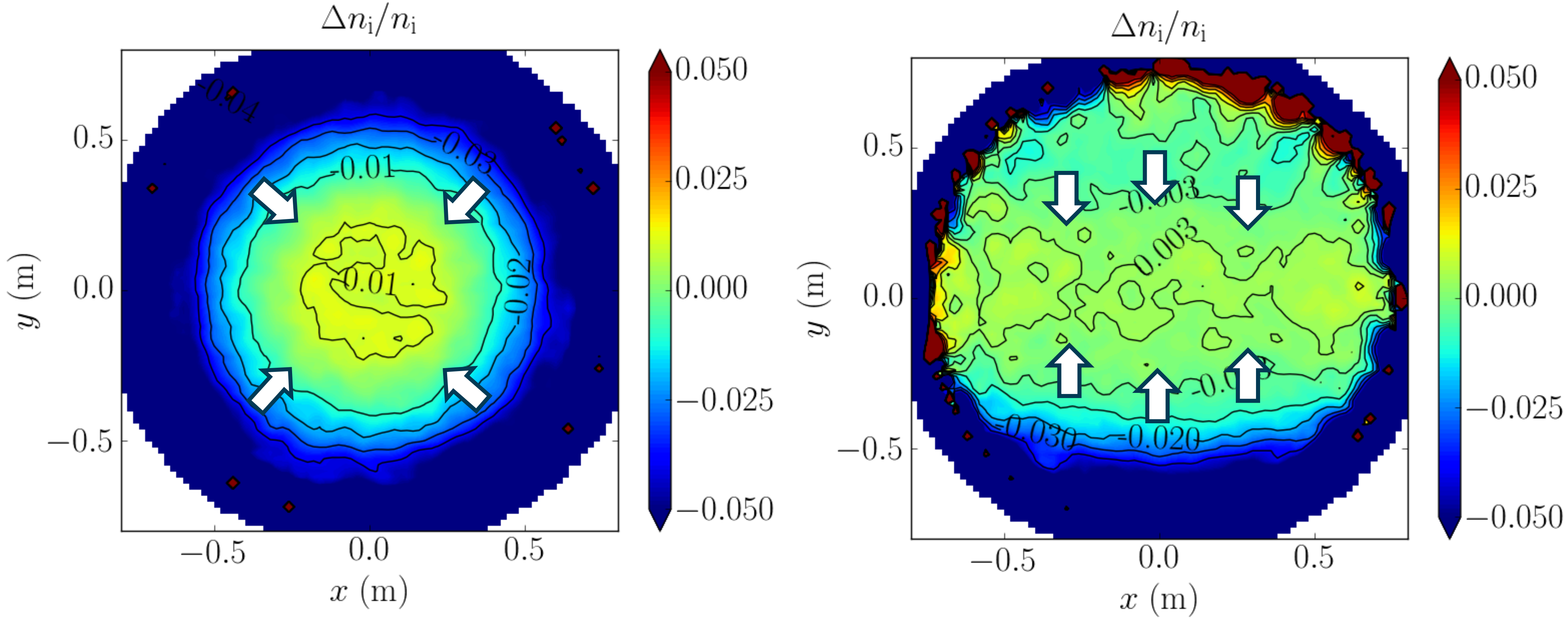


- $\frac{\partial H_e}{dy} \gg \frac{\partial H_e}{dx}$: compression is only in plane $y - z$ → distorsion of plume cross section
- Along plume centerline: $\boldsymbol{u}_i \times \boldsymbol{B} \approx -\nabla H_e / e$ → no net deflection of plasma plume

Plasma plume with uniform magnetic field (3)

- Ion density at a final cross section (4 m downstream) was compared with the unmagnetized case. The plume cross section
  - compresses in the axial field case, and
  - distorts in the oblique field case

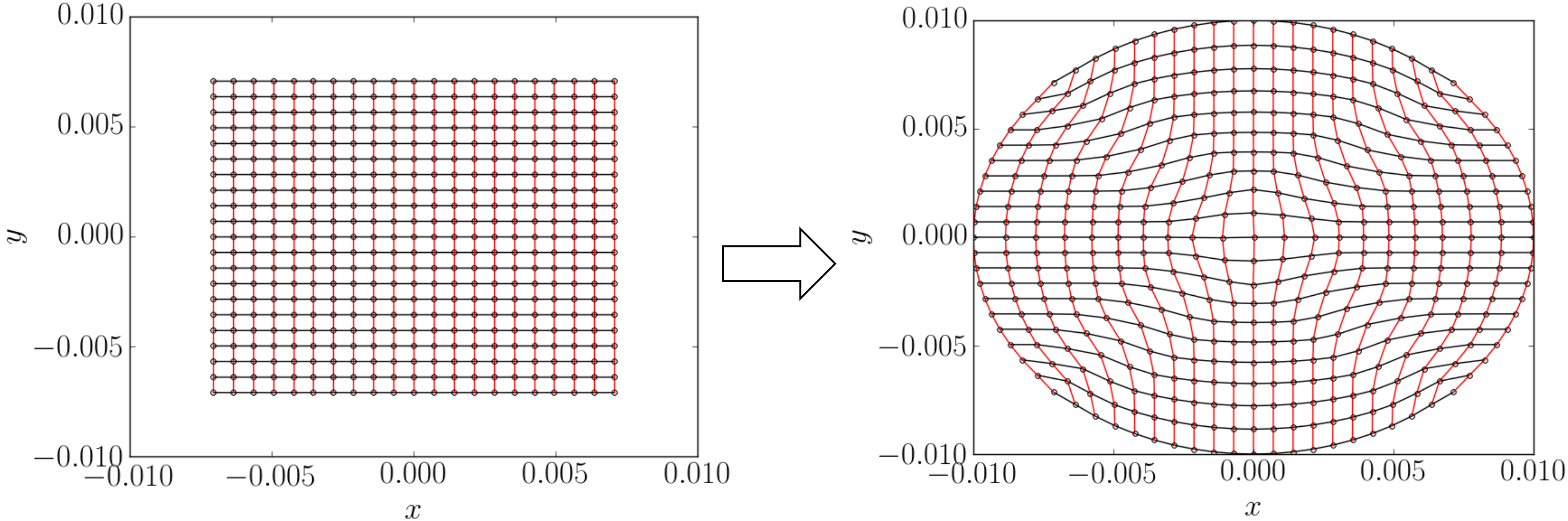
AXIAL FIELD | **OBLIQUE FIELD (60 DEG)**



Contents

Deformed meshes

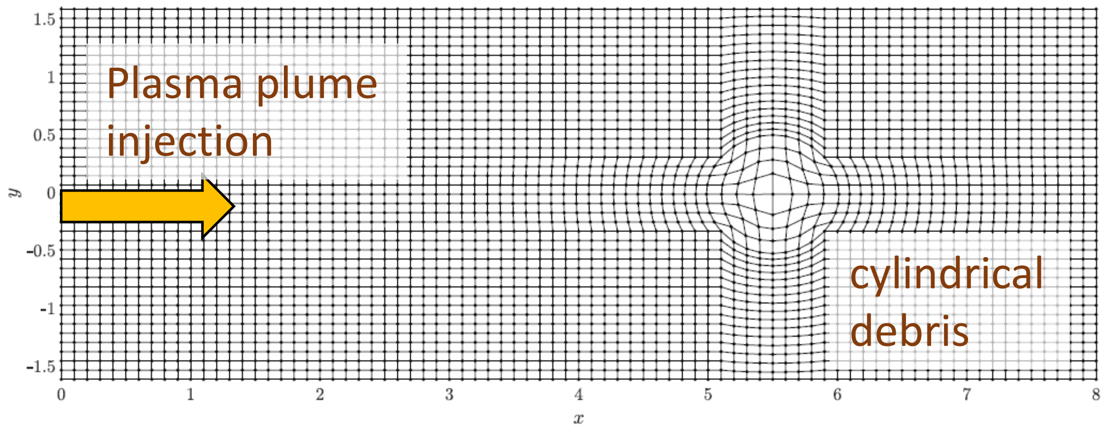
- A cylinder cross section is obtained from the deformation of a square (**J.M. Catalan's Master thesis**)
  - Coordinate parallel to the axis is unperturbed

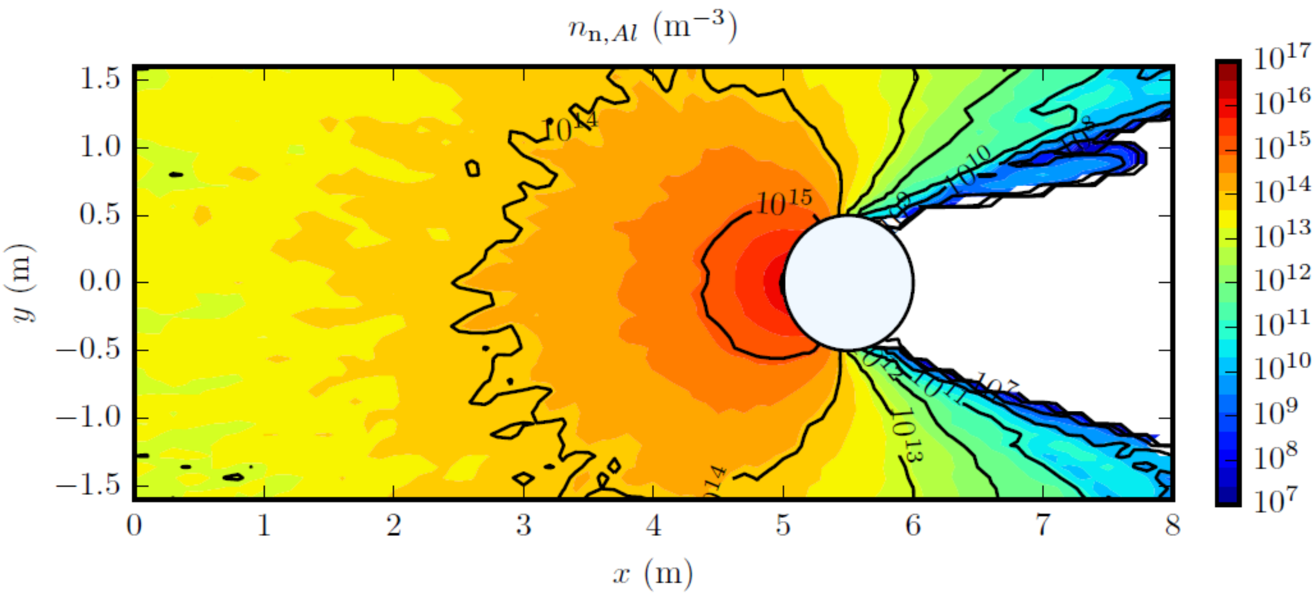


- New types of simulations are enabled
  - Plasma thruster discharge simulations (inside a cylinder)
  - Ion-beam-shepherd interaction with a cylindrical debris
  - Ion beam optics with circular holes

Deformed mesh applications

- The interaction of a plasma plume with a cylindrical debris can be studied with such a deformed mesh
  - Improvement of simulation capabilities for the IBS scenario (**Cichocki AA 2018)**





BACKSPUTTERING FLUX EVALUATION FOR A MORE REALISTIC GEOMETRY

Contents

Conclusions and future studies

- EP2PLUS electron fluid model permits computing
  - Electric currents
  - Electric potential correction due to both collisions and magnetization
- Benchmark simulations for the electron model:
  - Neutralization of the current in an unmagnetized GIT plume
  - Effects of a uniform background magnetic field on plume expansion
- Newly developed deformed meshes extend the applications of the code
  - Cylindrical discharge chambers and target debris objects (IBS)
- Future work
  - Study of the near plume of HETs
  - Use Akiles opensource kinetic electron model
  - Full-PIC simulations of the plume expansion

https://github.com/ep2lab/akiles

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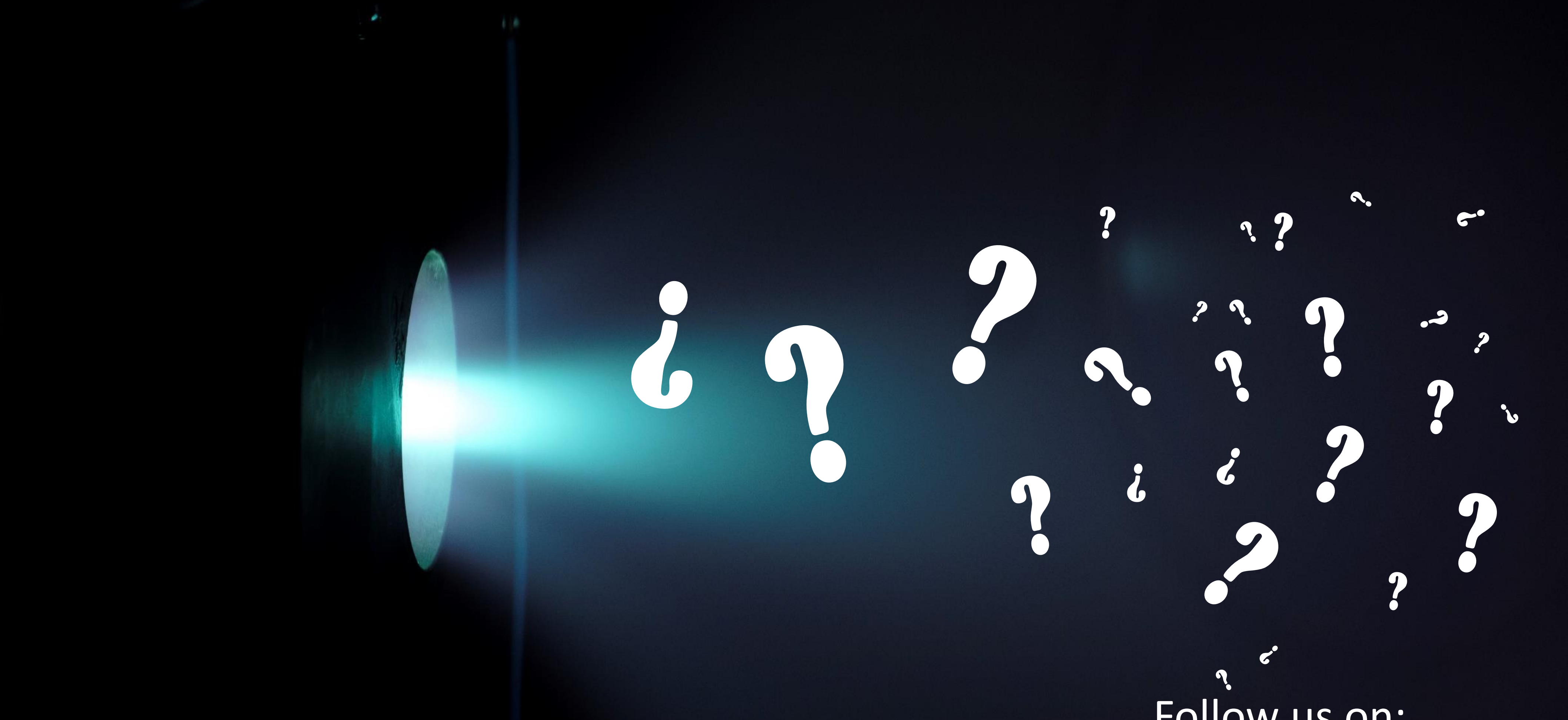
Thank you! Questions?



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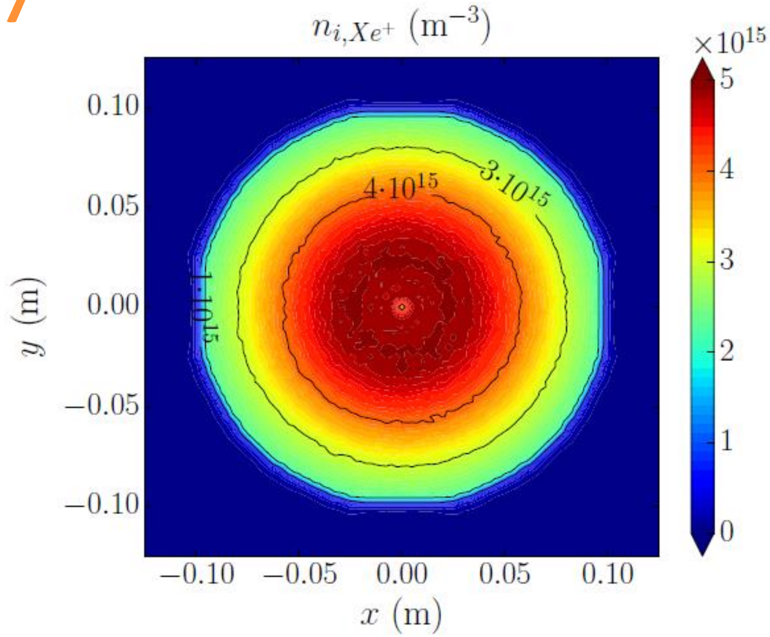
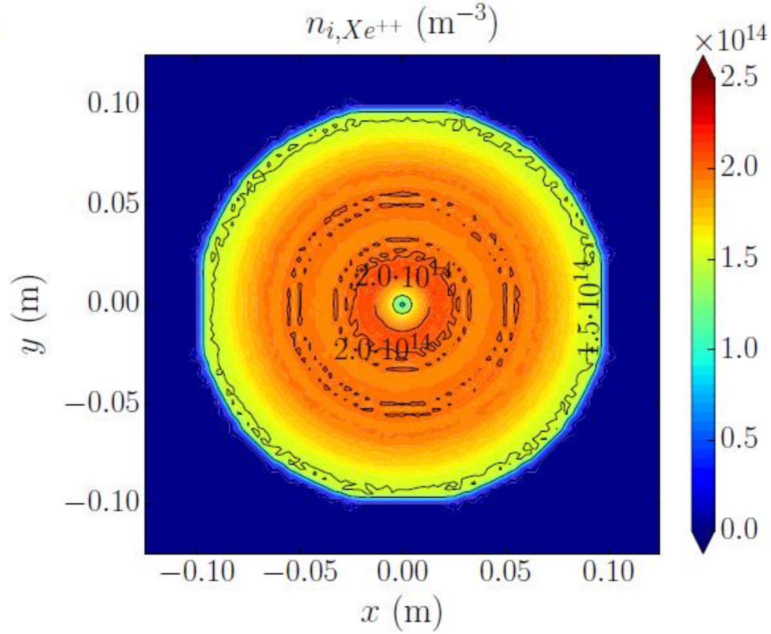
Simulation of a HEMPT thruster plume

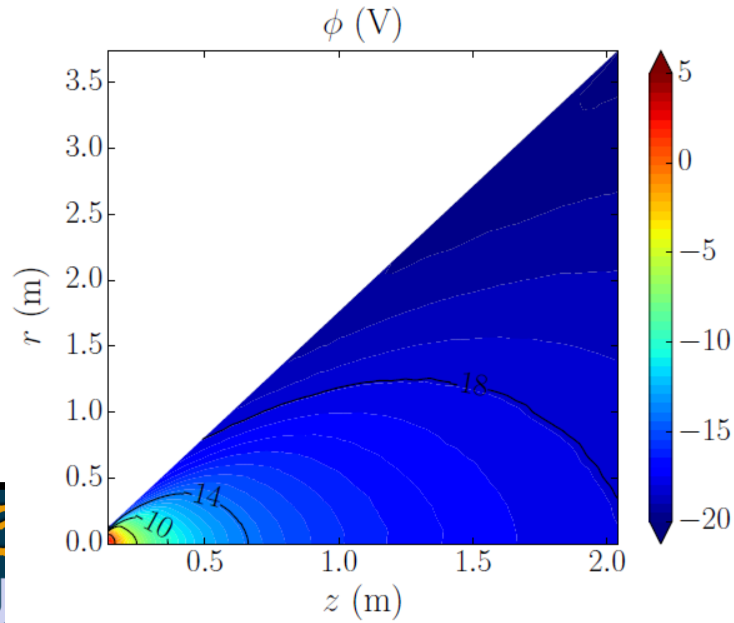
Kahnfeld
IEPC 2017

ION DENSITY PROFILES AT INITIAL CROSS SECTION (HOLLOW)

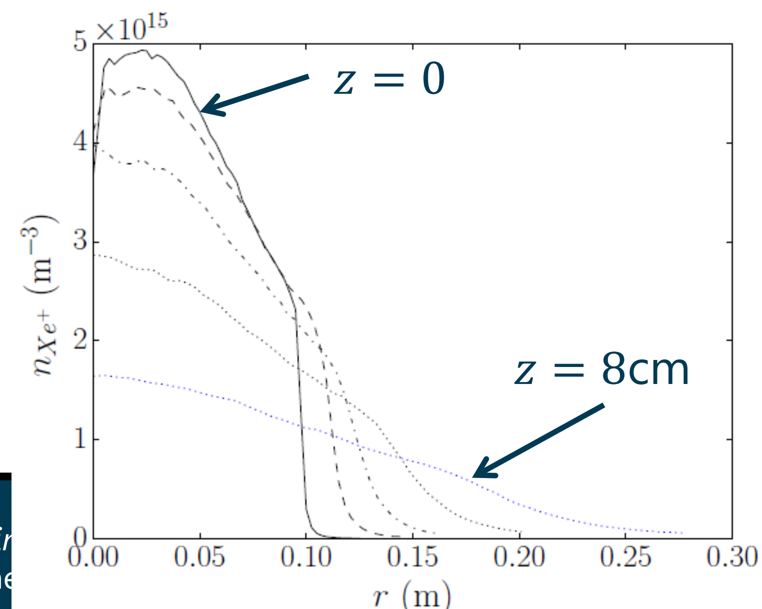




ELECTRIC POTENTIAL $z - r$ PROFILE



ION RADIAL PROFILES AT VARIOUS z



The electron fluid model (1)

- Assumptions
  - Stationary plume properties: $\frac{\partial}{\partial t} = 0$
  - Inertialess electrons: $m_e \ll m_i$
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  - Isotropic electrons: $\bar{\bar{P}}_e = p_e \bar{\bar{I}}$

- Solved equations:

Electron momentum balance + electric current continuity:

$$\mathbf{0} = -\nabla p_e - e n_e(\boldsymbol{E} + \boldsymbol{u}_e \times \boldsymbol{B}) - \sum_{s=1}^{L} \nu_{es} m_e n_e(\boldsymbol{u}_e - \boldsymbol{u}_s) \quad \cap \quad \nabla \cdot \boldsymbol{j} = 0$$

Poisson's equation for the electric potential:

$\nabla^2 \phi = -\frac{e}{\epsilon_0}(n_e^* - n_e)$ where $n_e^* = \sum_{s=1}^{L} Z_s n_s$ is the **quasineutral** electron density

The electron fluid model (2)

- In terms of a Bernoulli's function H_e, momentum balance becomes:

Conductivity tensor

$$\nabla H_e = -e\nabla\phi + \nabla h_e \quad \rightarrow \quad \boldsymbol{j} = \bar{\bar{\boldsymbol{K}}} \cdot \frac{\nabla H_e}{e} + \boldsymbol{j}_{PIC}$$

function of heavy species properties and magnetic field

- Applying current continuity, we get an elliptic equation for H_e:

$$\nabla \cdot \boldsymbol{j} = 0 \rightarrow \bar{\bar{\boldsymbol{K}}} : \nabla\nabla H_e + \nabla H_e \cdot \left(\nabla \cdot \bar{\bar{\boldsymbol{K}}}\right) = -e\nabla \cdot \boldsymbol{j}_{PIC}$$

- Once H_e is solved for, quasineutral electric potential is then obtained as:

$$\phi^* = \frac{h_e(n_e^*) - H_e}{e}$$

$\boldsymbol{H_e \equiv 0}$ for collisionless unmagnetized electrons!!

- Electric potential and electron densities are recomputed in non-neutral regions with a non-linear Poisson's solver:
  - Quasineutral nodes are considered as Dirichlet points
  - $\nabla^2\phi = -\frac{e}{\epsilon_0}[n_e^* - n_e(\phi, H_e)]$ where $n_e(\phi, H_e) = h_e^{-1}(e\phi + H_e)$

from PIC

Detailed electron fluid model (1)

- Considered equations:
  - **Current continuity**

Stationary conditions

$$\nabla \cdot \boldsymbol{j} = -\frac{\partial \rho_c}{\partial t} = 0$$

  - **Electron momentum balance equation**

collision frequency with the s[th] heavy particle population

$$\mathbf{0} = -\nabla p_e - e n_e(\boldsymbol{E} + \boldsymbol{u}_e \times \boldsymbol{B}) - \sum_{s=1}^{L} \nu_{es} m_e n_e (\boldsymbol{u}_e - \boldsymbol{u}_s)$$

- Introducing
  - The gyrofrequency $\omega_{ce} = eB/m_e$
  - The Hall parameter $\chi = \omega_{ce}/\nu_e$
  - The scalar electron conductivity $\sigma_e = \frac{e^2 n_e}{m_e \nu_e}$
  - The driving current density $\boldsymbol{j}_d = \boldsymbol{j}_i - \frac{e n_e}{\nu_e} \sum_{s=1}^{L} \nu_{es} \boldsymbol{u}_s$

Total electron collision frequency $\nu_e = \sum_{L=1}^{S} \nu_{es}$

- The electron momentum balance can be written as:

unit vector along the magnetic field

$$\boldsymbol{j} + \chi(\boldsymbol{j} \times \mathbf{1}_b) = \sigma_e \left(\frac{\nabla p_e}{e n_e} - \nabla \phi \right) + (\boldsymbol{j}_d + \chi \boldsymbol{j}_i \times \mathbf{1}_b)$$

Detailed electron fluid model (2)

- Solving for $\boldsymbol{j}$ and introducing the Bernoulli function H_e:

$$H_e = h_e - e\phi$$

- We finally obtain:

$$\boldsymbol{j} = \boxed{\bar{\bar{\boldsymbol{K}}}} \cdot \frac{\nabla H_e}{e} + \boxed{\frac{\bar{\bar{\boldsymbol{K}}}}{\sigma_e} \cdot (\boldsymbol{j}_d + \chi \boldsymbol{j}_i \times \mathbf{1}_b)}$$

Conductivity tensor — we call this $\boldsymbol{j}_{PIC}$

CONDUCTIVITY TENSOR

$$\bar{\bar{\boldsymbol{K}}} = \sigma_e \begin{bmatrix} 1 & \chi b_z & -\chi b_y \\ -\chi b_z & 1 & \chi b_x \\ \chi b_y & -\chi b_x & 1 \end{bmatrix}^{-1}$$

- With the use of the current continuity $\nabla \cdot \boldsymbol{j} = 0$, we finally obtain:

ELLIPTIC PDE FOR H_e

$$\nabla \cdot \boldsymbol{j} = 0 \rightarrow \bar{\bar{\boldsymbol{K}}} : \nabla \nabla H_e + \nabla H_e \cdot \left(\nabla \cdot \bar{\bar{\boldsymbol{K}}}\right) = -e \nabla \cdot \boldsymbol{j}_{PIC}$$

- Once H_e is known, we obtain the electric potential as:

$$\phi = \frac{h_e - \boxed{H_e}}{e}$$

Correction with respect to the collisionless, unmagnetized solution

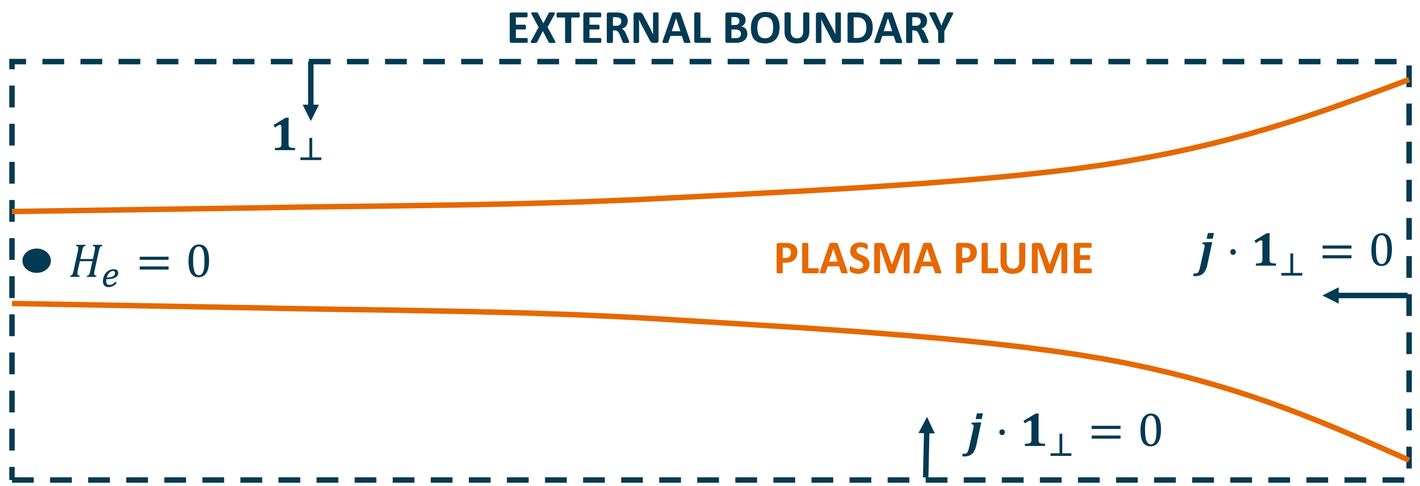
- Parameters affecting the solution:
  - Hall parameter χ
  - Scalar conductivity σ_e

Detailed electron fluid model (3)

- $\nabla \cdot \boldsymbol{j} = 0$ urges (through Gauss theorem) that:
  - Total net current through simulation boundaries be zero

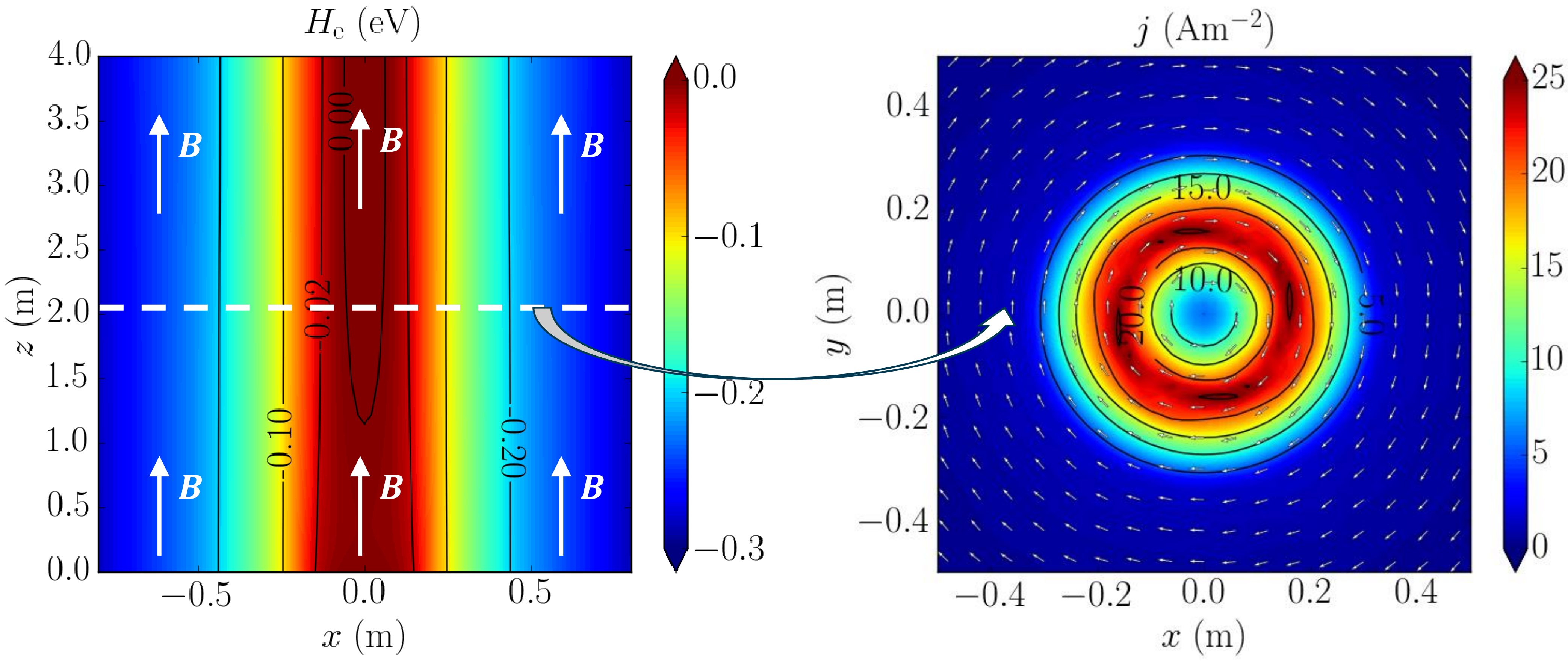
- We consider a stronger closure:
  - Zero normal current at all boundary nodes: $\boldsymbol{j} \cdot \mathbf{1}_{\perp} = 0$

- So, H_e is solved with the following boudary conditions:
  - $H_e = 0$ at a reference point where $\phi = 0$, $n_e = n_{e0}$, $T_e = T_{e0}$
  - $\left(\bar{\bar{\boldsymbol{K}}} \cdot \nabla H_e\right) \cdot \mathbf{1}_{\perp} = -e\boldsymbol{j}_{PIC} \cdot \mathbf{1}_{\perp}$ at the external boundaries



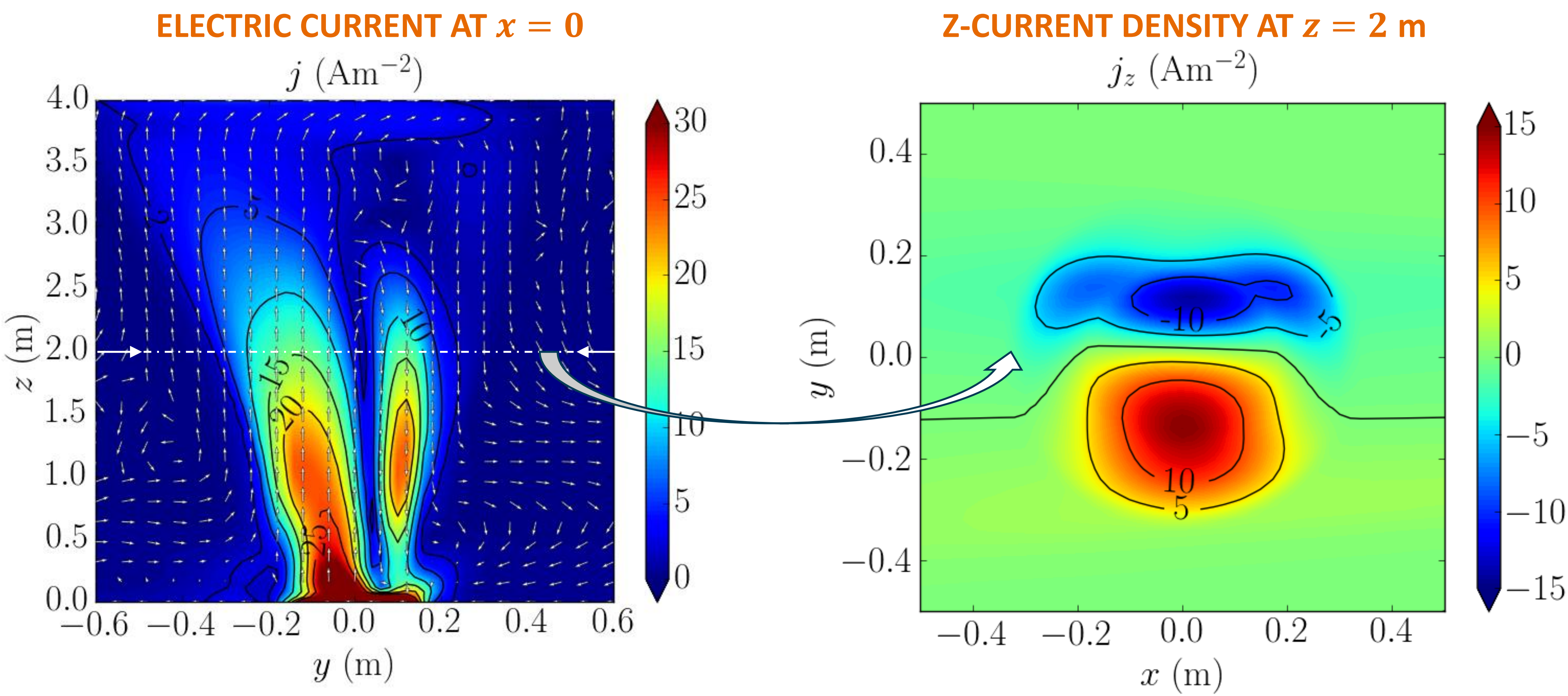
Axial field case of the plasma plume expansion

AXIAL MAGNETIC FIELD



- The potential correction is axisymmetric → diamagnetic current loop
- Lower radial potential fall → **plume compression** (radial confinement)

Currents in the oblique magnetic field case (60 deg)



- Boundary conditions affect the currents nearby
  - Need to define a sufficiently large simulation box

Non-neutrality criteria in EP2PLUS

➢ Non-neutrality criterion for plasma nodes:

RELATIVE CHARGE DENSITY ESTIMATION

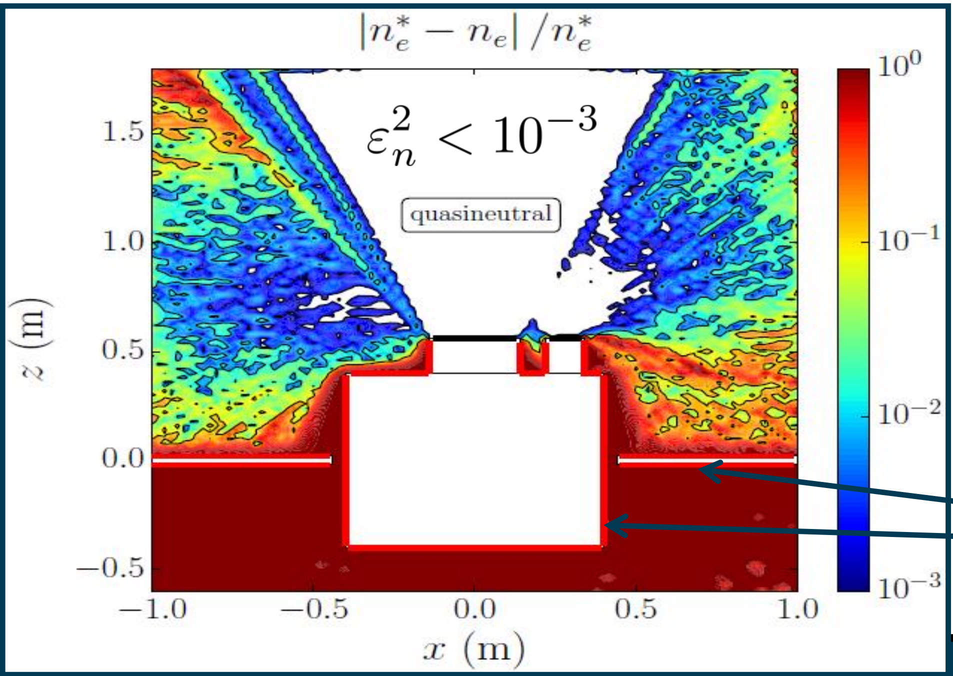
$$\varepsilon_n = \left| \frac{n_e^* - n_e}{n_e^*} \right|^{1/2} = \left| \frac{\epsilon_0 \nabla^2 \phi^*}{e n_e^*} \right|^{1/2} < \varepsilon_{max}$$

➢ Non-neutrality criterion for material cell-faces:

DEBYE LENGTH TO CELL SIZE RATIO

$$\varepsilon_f = \frac{1}{\Delta l} \sqrt{\frac{\epsilon_0 T_e^*}{e^2 n_e^*}} < \varepsilon_{max}$$

RELATIVE CHARGE DENSITY



NON NEUTRAL BOUNDARIES