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# *Traversable Wormholes*

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# History of Wormhole Physics

In 1916, L. Flamm recognized that the Schwarzschild solution of Einstein's field equations represents a wormhole.

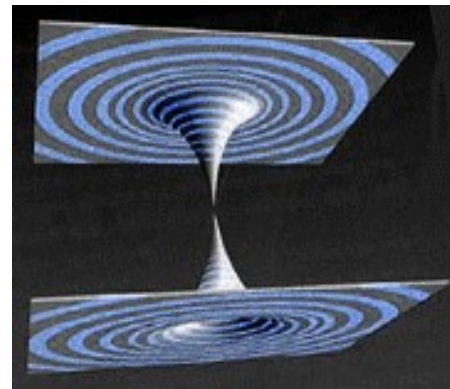
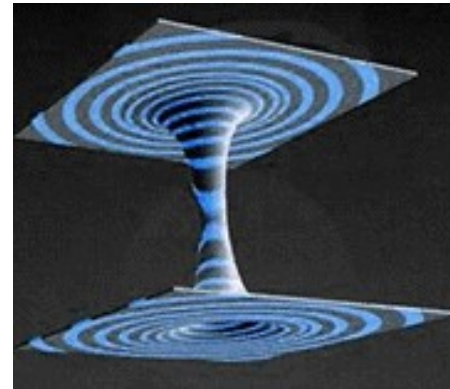
In 1935, A. Einstein and N. Rosen published a paper in Physical Review 48, 73 (1935) showing that implicit in the general relativity formalism is a curved-space structure that can join two distant regions of space-time through a tunnel-like curved spatial shortcut. The purpose of the paper of was *not* to promote faster-than-light or inter- universe travel, but to attempt to explain fundamental particles like electrons as space-tunnels threaded by electric lines of force.

Their particle model was subsequently shown to be invalid when it was realized that the smallest possible mass-energy of such a curved-space topology is a Planck mass, far larger than the mass-energy of an electron. Their spatial shortcut subsequently became known as an *Einstein-Rosen Bridge*, rechristened “*wormhole*” by John Wheeler.

# History of Wormhole Physics

In 1962 John Wheeler and a collaborator discovered that the Einstein-Rosen bridge space-time structure, which Wheeler re-christened as a "wormhole," was dynamically unstable in field-free space.

They showed that if such a wormhole somehow opened, it would close up again before even a single photon could be transmitted through it, thereby preserving Einsteinian causality.

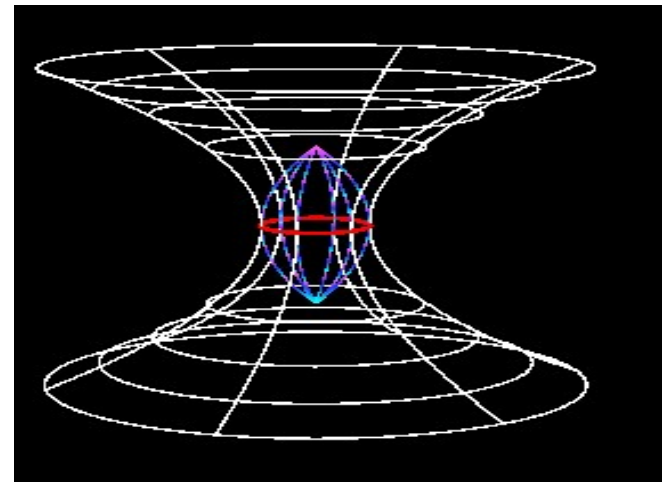
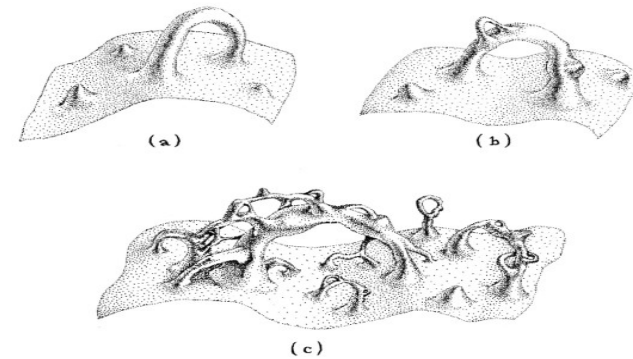


# History of Wormhole Physics

In 1988 Kip Thorne and his graduate student Mike Morris showed that a wormhole might be snatched from the quantum foam and stabilized by a region of space containing negative mass-energy.

They suggested that an “advanced civilization” capable of manipulating planet-scale quantities of mass-energy might use the Casimir effect to produce such a region of negative mass energy and, starting with vacuum fluctuations, might create stable wormholes.

M. Visser,  
*Lorentzian Wormholes: From Einstein to Hawking* (American Institute of Physics, New York, 1995).



# *Introduction*

- A wormhole can be represented by two asymptotically flat regions joined by a bridge
- One very simple and at the same time fundamental example of wormhole is represented by the Schwarzschild solution of the Einstein's field equations.
- One of the prerogatives of a wormhole is its ability to connect two distant points in space-time. In this amazing perspective, it is immediate to recognize the possibility of traveling crossing wormholes as a short-cut in space and time.
- A Schwarzschild wormhole does not possess this property.



Traversable wormholes

# The traversable wormhole metric

M. S. Morris and K. S. Thorne, Am. J. Phys. 56, 395 (1988).

$$ds^2 = -\exp(-2\phi(r))dt^2 + \frac{dr^2}{1 - \frac{b(r)}{r}} + r^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2$$

Condition

$b(r)$  is the shape function

$\phi(r)$  is the redshift function

$$r \in [r_0, +\infty)$$

$$b_{\pm}(r_0) = r_0$$

$$b_{\pm}(r) < r$$

Proper radial  
distance

$$l(r) = \pm \int_{r_0}^r \frac{dr'}{\sqrt{1 - b_{\pm}(r')/r'}}$$

$$\lim_{r \rightarrow \infty} b_{\pm}(r) = b_{\pm} \quad \text{Appropriate asymptotic}$$

$$\lim_{r \rightarrow \infty} \phi_{\pm}(r) = \phi_{\pm} \quad \text{limits}$$

# *Einstein Field Equations*

Orthonormal frame

$$b'(r) = 8\pi G \rho c^2 r^2$$

$$\phi'(r) = \frac{b + 8\pi G p_r r^3}{2r^2(1 - b(r)/r)} \quad \tau(r) = -p_r$$

$$p'_r(r) = \frac{2}{r}(p_t(r) - p_r(r)) - (\rho(r) + p_r(r))\phi'(r)$$

*Exotic Energy*

$$\rho(r) + p_r(r) < 0 \quad r \in [r_0, r_0 + \varepsilon] \quad \longleftrightarrow \quad b'(r) < b(r)/r \quad r \in [r_0, r_0 + \varepsilon]$$

Flare-Out Condition

Candidate



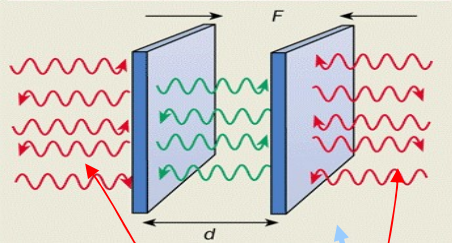
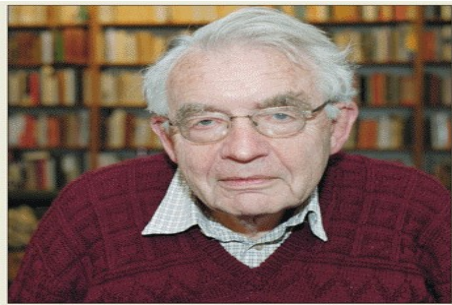
Casimir Energy

# Casimir Effect

Hendrik Casimir 1909-2000

H.B.G. Casimir and D. Polder,

Phys. Rev., 73, 360, 1948



(ZPE) responsible for the Casimir effect. This was predicted by Casimir [1] and confirmed experimentally in the Philips laboratories†. This is induced when the presence of electrical conductors distorts the zero-point energy of the quantum electrodynamics vacuum. Two parallel conducting surfaces, in a vacuum environment, attract one another by a very weak force that varies inversely as the fourth power of the distance between them. This kind of energy is a purely quantum effect; no real particles are involved, only virtual ones. The difference between the stress-energy computed in the presence and in the absence of the plates with the same boundary conditions gives

$$\Delta \langle T^{\mu\nu} \rangle = \langle T^{\mu\nu} \rangle_{\text{vac}}^p - \langle T^{\mu\nu} \rangle_{\text{vac}} = \frac{\pi^2}{720\alpha^4} \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -3 \end{pmatrix}. \quad (1)$$

It is evident that separately, each contribution coming from the summation over all possible resonance frequencies of the cavities is divergent and devoid of physical meaning but the *difference* between them in the two situations (with and without the plates) is well defined. Note that the energy density

$$\rho = E/V = \Delta \langle T^{00} \rangle = -\frac{\pi^2}{720\alpha^4} \quad (2)$$

Only wavelength less than d

Any wavelength is possible



# *Parallel Conducting Planes*

This example shows how the Q.E.D. vacuum reacts in presence of conducting surfaces. We consider two parallel perfectly conducting plates separated by a distance  $a$  at zero temperature.

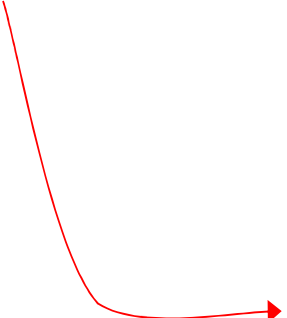
To proceed we imagine the electromagnetic field as a infinite set of harmonic oscillators with frequencies

$$\left\{ \begin{array}{ll} \omega_J = c\sqrt{\mathbf{k}^2} & J = (k_1, k_2, k_3) \quad \text{continuous} \\ \omega_J = \omega_{\mathbf{k}_\perp, n} = c\sqrt{k_1^2 + k_2^2 + \left(\frac{\pi n}{a}\right)^2} & J = \left(k_1, k_2, \frac{\pi n}{a}\right) = \left(\mathbf{k}_\perp^2, \frac{\pi n}{a}\right) \quad \text{boundaries} \end{array} \right.$$

# Parallel Conducting Planes

$$\begin{aligned}
 E_0^{\text{ren}}(a) &= \lim_{\delta \rightarrow 0} \frac{\hbar}{2} \int \frac{dk_1 dk_2}{(2\pi)^2} \left( \sum_{n=-\infty}^{\infty} \omega_{\mathbf{k}_{\perp},n} e^{-\delta \omega_{\mathbf{k}_{\perp},n}} - 2a \int \frac{dk_3}{2\pi} \omega_{\mathbf{k}} e^{-\delta \omega_{\mathbf{k}}} \right) S \\
 &= \frac{c\hbar\pi}{a} \lim_{\delta \rightarrow 0} \int \frac{dk_1 dk_2}{(2\pi)^2} \left( \sum_{n=0}^{\infty} \sqrt{\frac{k_{\perp}^2 a^2}{\pi^2} + n^2} e^{\delta \omega_{\mathbf{k}_{\perp},n}} \right. \\
 &\quad \left. - \int_0^{\infty} dt \sqrt{\frac{k_{\perp}^2 a^2}{\pi^2} + t^2} e^{-\delta \omega_{\mathbf{k}}} - \frac{k_{\perp} a}{2\pi} \right) S,
 \end{aligned}$$

where  $k_{\perp}^2 \equiv k_1^2 + k_2^2$ ,  $t \equiv ak_3/\pi$ .



$$= -\frac{c\hbar\pi^2}{720a^3} S$$

# *The role of the Casimir effect in different fields of physics*

☞ In QFT, the Casimir effect gives contributions to the total nucleon energy in QCD bag model of hadrons.

Spontaneous compactification of extra spatial dimensions in Kaluza-Klein field theories.

Gauge theories, SUSY, SUGRA, and string theory.

☞ In Condensed Matter Physics, the Casimir effect leads to attractive and repulsive forces between the closely spaced material boundaries which depend on the configuration geometry, on temperature, and on the electrical and mechanical properties of the boundary surface.

It is responsible for some properties of thin films and should be taken into account in investigations of surface tension and latent heat.

bulk and surface critical phenomena.

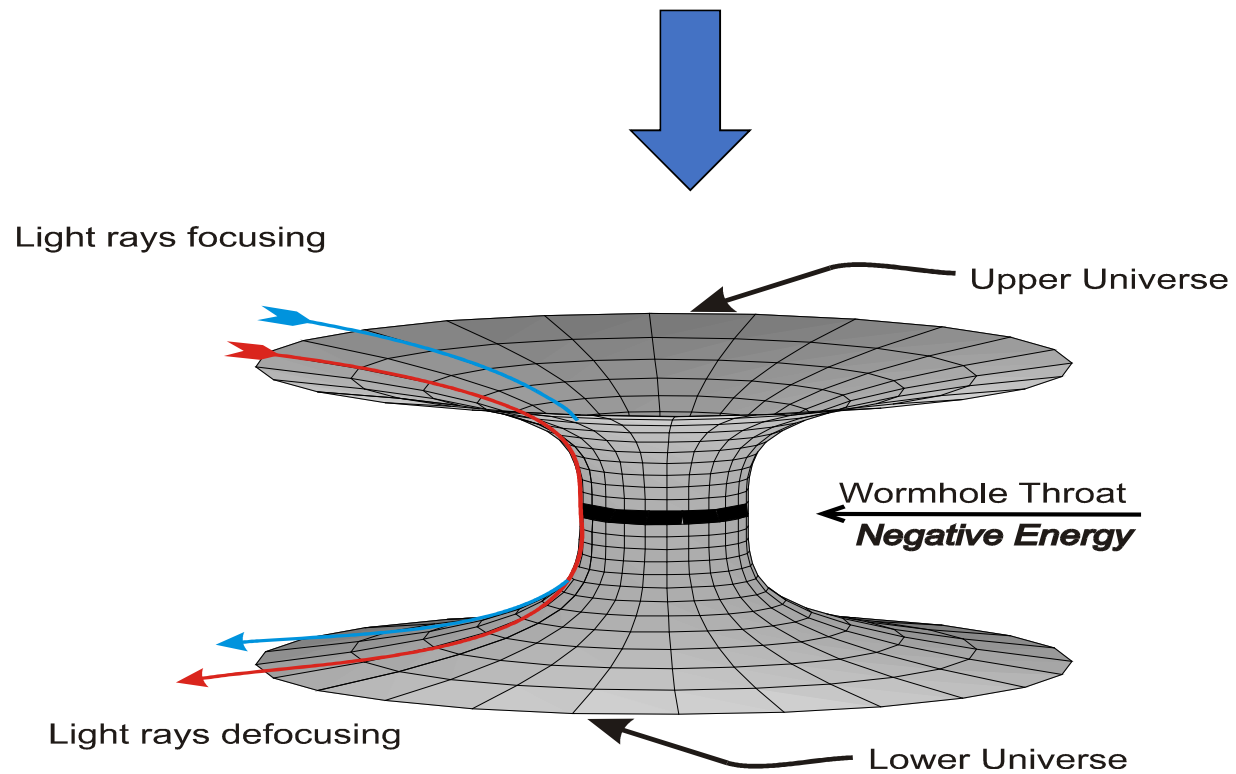
# *The role of the Casimir effect in different fields of physics*

☞ In Gravitation, Astrophysics and Cosmology, the Casimir effect arises in space-times with nontrivial topology. The vacuum polarization resulting from the Casimir effect can drive the inflation process. In the theory of structure formation of the Universe due to topological defects, the Casimir vacuum polarization near the cosmic strings may play an important role.

☞ In Atomic Physics, the long-range Casimir interaction leads to corrections to the energy levels of Rydberg states. A number of the Casimir-type effects arise in cavity Q.E.D. when the radiative processes and associated energy shifts are modified by the presence of the cavity walls.

☞ In Mathematical Physics, the Casimir effect has contributed to develop zeta functions and heat kernel expansion techniques for the regularization and renormalization of infinities.

# Traversable Wormhole



# Traversable Wormholes

## Example

$$ds^2 = -dt^2 + dl^2 + (r_0^2 + l^2)(d\theta^2 + \sin^2 \theta d\varphi^2)$$

M. S. Morris and K. S. Thorne, Am. J. Phys. 56, 395 (1988).

H. G. Ellis, J. Math. Phys. 14, 104 (1973). K.A. Bronnikov, Acta Phys. Pol. B 4, 251 (1973).

The new coordinate  $l$  covers the range  $-\infty < l < +\infty$ . The constant time hypersurface  $\Sigma$  is an Einstein-Rosen bridge with wormhole topology  $S^2 \times \mathbb{R}^1$ . The Einstein-Rosen bridge defines a bifurcation surface dividing  $\Sigma$  in two parts denoted by  $\Sigma_+$  and  $\Sigma_-$ .

In Schwarzschild coordinates becomes

$$ds^2 = -dt^2 + \frac{dr^2}{1 - \frac{r_0^2}{r^2}} + r^2(d\theta^2 + \sin^2 \theta d\varphi^2)$$

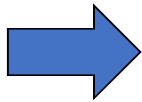
$$b(r_0) = r_0 \quad \text{Throat Condition}$$

$$\text{Minimum at the throat} \Rightarrow \frac{d^2 r}{dl^2} > 0 \Leftrightarrow \frac{b(r)}{r} > b'(r)$$

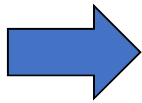
# Features of the EB Wormhole

$$M(r) = \frac{r_0^2}{2Gr} \rightarrow 0 \text{ when } r \rightarrow \infty \text{ Zero Mass Wormhole}$$

$$\text{But } M^P(r) = \pm \int_{r_0}^r \frac{4\pi\rho(r')r'^2}{\sqrt{1-b(r')/r'}} dr' \rightarrow \mp \frac{\pi r_0}{4G} \text{ when } r \rightarrow \infty$$

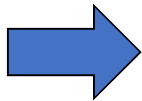


$$E_G(r) = M(r) - M^P(r) = \pm \frac{\pi r_0}{4G} \text{ Total Energy}$$

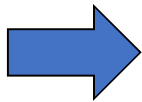


$$l(r) = \pm \sqrt{r^2 - r_0^2}$$

Proper length



$$p_r(r) = -\frac{b(r)}{8\pi Gr^3} = -\frac{r_0^2}{8\pi Gr^4} \Rightarrow -\frac{1}{8\pi Gr_0^2} \text{ On the throat}$$



$$p_t(r) = -\frac{b'(r)r - b(r)}{16\pi Gr^3} = \frac{r_0^2}{8\pi Gr^4} \Rightarrow \frac{1}{8\pi Gr_0^2} \text{ On the throat}$$

# *Traversable Wormholes*

## *Einstein Field Equations*

*Eq. of State*

$$p_r = \omega \rho$$

$$b'(r) = 8\pi G \rho(r) r^2$$

$$\phi'(r) = \frac{b + 8\pi G p_r r^3}{2r^2(1 - b(r)/r)} = \frac{b + \omega b' r}{2r^2(1 - b(r)/r)}$$

*N.E.C. Violation*  $\rho + p < 0$

*EoS*  $\rho(1 + \omega) < 0$

*Asymptotic flatness*

$$\lim_{r \rightarrow \infty} \frac{b(r)}{r} = 0 \Rightarrow \frac{\omega + 1}{\omega} > 0 \begin{cases} \omega > 0 \\ \omega < -1 \end{cases}$$

$$b(r) = r_0 \left( \frac{r_0}{r} \right)^{\frac{1}{\omega}}$$

$$\phi'(r) = 0$$

$$ds^2 = -A dt^2 + \frac{dr^2}{1 - \left( \frac{r_0}{r} \right)^{1 + \frac{1}{\omega}}} + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2)$$

$$b'(r)|_{r=r_0} = -\frac{1}{\omega} < 1$$



# *Take seriously the Casimir Energy*

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \quad \longrightarrow \quad G_{\mu\nu} = \frac{8\pi G}{c^4} \langle T_{\mu\nu} \rangle^{Ren}$$

See also

[M.S. Morris](#), [K.S. Thorne](#), [U. Yurtsever](#) ([Caltech](#)). 1988. 4 pp.

Published in Phys.Rev.Lett. 61 (1988) 1446-1449

AND

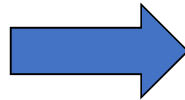
M. Visser, Lorentzian Wormholes: From Einstein to Hawking  
(American Institute of Physics, New York), 1995.

# *Take seriously the Casimir Energy*

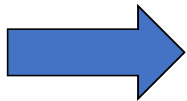
$$\rho(a) = -\frac{\hbar c \pi^2}{720 a^4} \quad p_r(a) = -3 \frac{\hbar c \pi^2}{720 a^4} \quad p_t(a) = \frac{\hbar c \pi^2}{720 a^4}$$

The Casimir Tensor  
is traceless and  
divergenceless

$$b'(r) = \frac{8\pi G}{c^4} \rho(a) r^2$$



$$b(r) - b(r_0) = \frac{8\pi G}{c^4} \int_{r_0}^r \left( -\frac{c \hbar \pi^2}{720 a^4} \right) r'^2 dr'$$



$$b(r) = r_0 - \frac{\pi^3 G \hbar}{270 c^3 a^4} (r^3 - r_0^3)$$

This is not a TW because there is no A. Flatness.

It is Asymptotically de Sitter

It can be transformed into a TW with the junction condition method matching the solution with the Schwarzschild metric at some point  $r=c$

# Take seriously the Casimir Energy

Zero Tidal Forces  $\phi(r) = 0$   $a \rightarrow r$

$$\rho(r) = -\frac{\hbar c \pi^2}{720 r^4} \quad p_r(r) = -3 \frac{\hbar c \pi^2}{720 r^4} \quad p_t(r) = \frac{\hbar c \pi^2}{720 r^4}$$

If we impose the EoS

$$p_r(r) = \omega \rho(r) \quad \omega = 3$$

$$\Rightarrow ds^2 = -dt^2 + \frac{dr^2}{1 - \left(\frac{r_0}{r}\right)^{\frac{4}{3}}} + r^2 d\Omega^2$$




No Connection with the original Casimir Stress-Energy tensor

$$T_{\mu\nu} = \frac{1}{\kappa r^2} \left(\frac{r_0}{r}\right)^{\frac{\omega+1}{\omega}} \left( -\frac{1}{\omega}, -1, \frac{\omega+1}{\omega}, \frac{\omega+1}{\omega} \right)$$


# Take seriously the Casimir Energy


Strategy  $\Rightarrow$  Impose an EoS

$$b'(r) = \frac{8\pi G}{c^4} \rho(r) r^2 \quad \rho(r) = -\frac{c\hbar\pi^2}{720r^4} \quad \text{Energy Density}$$




$$b(r) = b(r_0) + \frac{8\pi G}{c^4} \int_{r_0}^r \left( -\frac{c\hbar\pi^2}{720r'^4} \right) r'^2 dr' = r_0 - \frac{\pi^3 G\hbar}{90c^3} \left( \frac{1}{r_0} - \frac{1}{r} \right)$$



$$r_1^2 = \frac{\pi^3 G\hbar}{90 c^3} = \frac{\pi^3}{90} l_p^2$$


$$b(r) = r_0 - r_1^2 \left( \frac{1}{r_0} - \frac{1}{r} \right)$$



$$p_r(r) = \omega \rho(r) \quad \text{EoS}$$

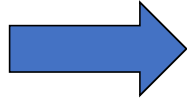
With the second EFE, we can compute  $\phi(r)$



Depending on the value of  $\omega$  we can have BH, TW or a singularity

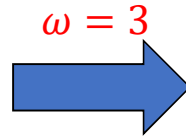
# *Take seriously the Casimir Energy*

When  $\omega = \frac{r_0^2}{r_1^2}$



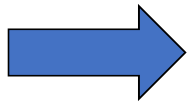
Traversable Wormhole

$$\phi(r) = \frac{1}{2}(\omega - 1) \ln \left( \frac{r(\omega + 1)}{(\omega r + r_0)} \right)$$

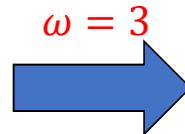


$$\phi(r) = \ln \left( \frac{4r}{(3r + r_0)} \right)$$

Planckian



$$b(r) = \left(1 - \frac{1}{\omega}\right) r_0 + \frac{r_0^2}{\omega r}$$

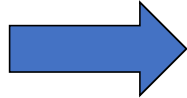


$$b(r) = \frac{2}{3} r_0 + \frac{r_0^2}{3r}$$

$$SET \quad T_{\mu\nu} = \left( \frac{r_0^2}{3kr^4} \right) \left[ \text{diag}(-1, -3, 1, 1) + \left( \frac{6r}{3r + r_0} \right) \text{diag}(0, 0, 1, 1) \right]$$

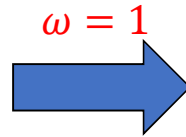
# *Take seriously the Casimir Energy*

When  $\omega = \frac{r_0^2}{r_1^2}$



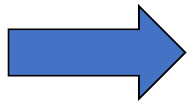
Traversable Wormhole

$$\phi(r) = \frac{1}{2}(\omega - 1) \ln \left( \frac{r(\omega + 1)}{(\omega r + r_0)} \right)$$

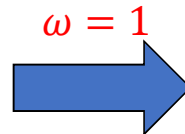


$$\phi(r) = 0$$

Sub-Planckian



$$b(r) = \left(1 - \frac{1}{\omega}\right) r_0 + \frac{r_0^2}{\omega r}$$



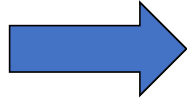
$$b(r) = \frac{r_0^2}{r}$$

EB Wormhole

$$SET \quad T_{\mu\nu} = \left( \frac{r_0^2}{\kappa r^4} \right) [\text{diag}(-1, -3, 1, 1) + 2 \text{diag}(0, 1, 0, 0)] = \left( \frac{r_0^2}{\kappa r^4} \right) [\text{diag}(-1, -1, 1, 1)]$$

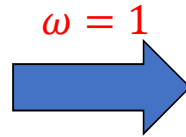
# Take seriously the Casimir Energy

When  $\omega = \frac{r_0^2}{r_1^2}$



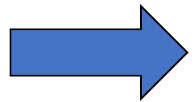
Traversable Wormhole

$$\phi(r) = \frac{1}{2}(\omega - 1) \ln \left( \frac{r(\omega + 1)}{(\omega r + r_0)} \right)$$

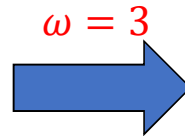


$$\phi(r) = 0$$

Sub-Planckian



$$b(r) = \left(1 - \frac{1}{\omega}\right) r_0 + \frac{r_0^2}{\omega r}$$



$$b(r) = \frac{r_0^2}{r}$$

EB Wormhole

$$SET \quad T_{\mu\nu} = \left( \frac{r_0^2}{2\kappa r^4} \right) [diag(-1, -3, 1, 1) + diag(-1, 1, 1, 1)] = \left( \frac{r_0^2}{\kappa r^4} \right) [diag(-1, -1, 1, 1)]$$

Canonical  
Decomposition

# *Effective Einstein Equations*

$$G_{\mu\nu} = \kappa T_{\mu\nu}$$

- $G_{\mu\nu}$  is the Einstein tensor,
- $\kappa = 8\pi G$ ,
- $T_{\mu\nu}$  is the stress-energy tensor.

Hochberg, Popov and Sushkov considered a self-consistent solution of the semiclassical Einstein equations corresponding to a Lorentzian wormhole coupled with a quantum scalar field

[Hochberg D, Popov A and Sushkov S V 1997 *Phys. Rev. Lett.* **78** 2050 (*Preprint* gr-qc/9701064)]

Khusnutdinov and Sushkov fixed their attention to the computation of the ground state of a massive scalar field in a wormhole background. They tried to see if a self-consistent solution restricted to the energy component appears in this configuration

[Khusnutdinov N R and Sushkov S V 2002 *Phys. Rev. D* **65** 084028 (*Preprint* hep-th/0202068)]



# *General setting for self sustained traversable wormholes*

R.Garattini, C.Q.Grav. 22 (2005) 1105 ArXiv:gr-qc/0501105

Instead of  $G_{\mu\nu} = \kappa T_{\mu\nu}$  consider  $G_{\mu\nu} = \kappa \langle T_{\mu\nu} \rangle^{ren}$

where  $\langle T_{\mu\nu} \rangle^{ren}$  renormalized expectation value  
of the stress-energy tensor operator  
of the quantized field

S.S.T.W.



$$E^{Classical} = -E^{TT}$$

If the matter field source is absent  $\langle T_{\mu\nu} \rangle^{ren} = -\frac{1}{\kappa} \langle \Delta G_{\mu\nu}(\tilde{g}_{\alpha\beta}, h_{\alpha\beta}) \rangle^{ren}$

$$G_{\mu\nu}(g_{\alpha\beta}) = G_{\mu\nu}(\tilde{g}_{\alpha\beta}) + \Delta G_{\mu\nu}(\tilde{g}_{\alpha\beta}, h_{\alpha\beta}) \quad g_{\alpha\beta} = \tilde{g}_{\alpha\beta} + h_{\alpha\beta}$$

$\Delta G_{\mu\nu}(\tilde{g}_{\alpha\beta}, h_{\alpha\beta})$  is a perturbation series in terms of  $g_{\alpha\beta}$

The Einstein tensor  $G_{\mu\nu}$  can be divided into a part which is unperturbed related to the background geometry and a part related to quantum fluctuations like the metric  $g_{\alpha\beta}$

# Finding the wormhole radius with phantom energy

[R.G. Class.Quant.Grav.24:1189-1210,2007 gr-qc/0701019]

Eq. of State

$$p_r = \omega \rho \quad \omega < 0$$

$$b(r) = r_0 \left( \frac{r_0}{r} \right)^{\frac{1}{\omega}}$$

$$\phi'(r) = \frac{b(r) + \omega b'(r)r}{2r^2(1 - b(r)/r)}$$

Solution

Asymptotic flatness

$$\lim_{r \rightarrow \infty} \frac{b(r)}{r} = 0 \Rightarrow \frac{\omega + 1}{\omega} > 0 \quad \begin{cases} \omega > 0 \\ \omega < -1 \end{cases}$$

For  $\omega = 1$  one simply gets

$$ds^2 = -dt^2 + dl^2 + (r_0^2 + l^2)(d\theta^2 + \sin^2 \theta d\varphi^2)$$

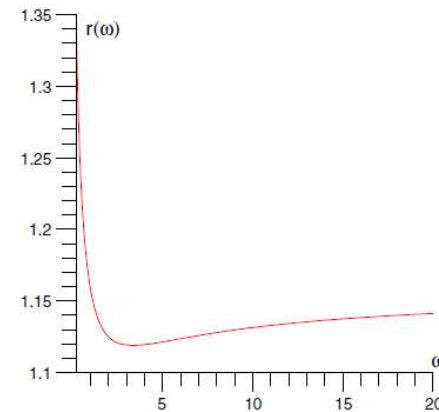


Figure 1. Plot of the wormhole throat  $\bar{r}_l$  as a function of  $\omega$  in the positive range.

# Conclusions and Perspectives

- Casimir energy is the only source of exotic matter that can be generated in laboratory.
- Traversable wormholes can be sustained by Casimir Energy.
- The Wormhole is traversable in principle but not in practice.
- At this stage the TW is completely useless, we need an amplification mechanism.
- Self Sustained Casimir TW?!?
- TW relevant for GW as BH mimickers.

Thank You for Your Attention

## Other Traversable Wormholes Papers

- ◆ R. G. and F. S. N. Lobo *Self sustained phantom wormholes in semi-classical gravity* CQG 24 (2007) 2401 gr-qc/0701020
- ◆ R. G. and F. S. N. Lobo *Self-sustained traversable wormholes in noncommutative geometry* PLB 671 (2009) 146 arXiv:0811.0919 [gr-qc]
- ◆ R. G. and F. S. N. Lobo *Self-sustained wormholes in modified dispersion relations* PRD 85 (2012) 024043 arXiv:1111.5729 [gr-qc]
- ◆ R.G. and F.S.N. Lobo, *Gravity`s Rainbow induces topology change* Eur.Phys.J. C74 (2014) 2884 arXiv:1303.5566 [gr-qc]
- ◆ R. G. and F. S. N. Lobo *Self-Sustained Traversable Wormholes* Fundam.Theor.Phys. 189 (2017) 111-135

# *Outlook*

