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University of Colorado **Boulder**

Plasma flow around a satellite with an ionosphere: Theoretical models of the moon-magnetosphere interaction

Frank Crary

University of Colorado

Laboratory for Atmospheric and Space Physics

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Overview

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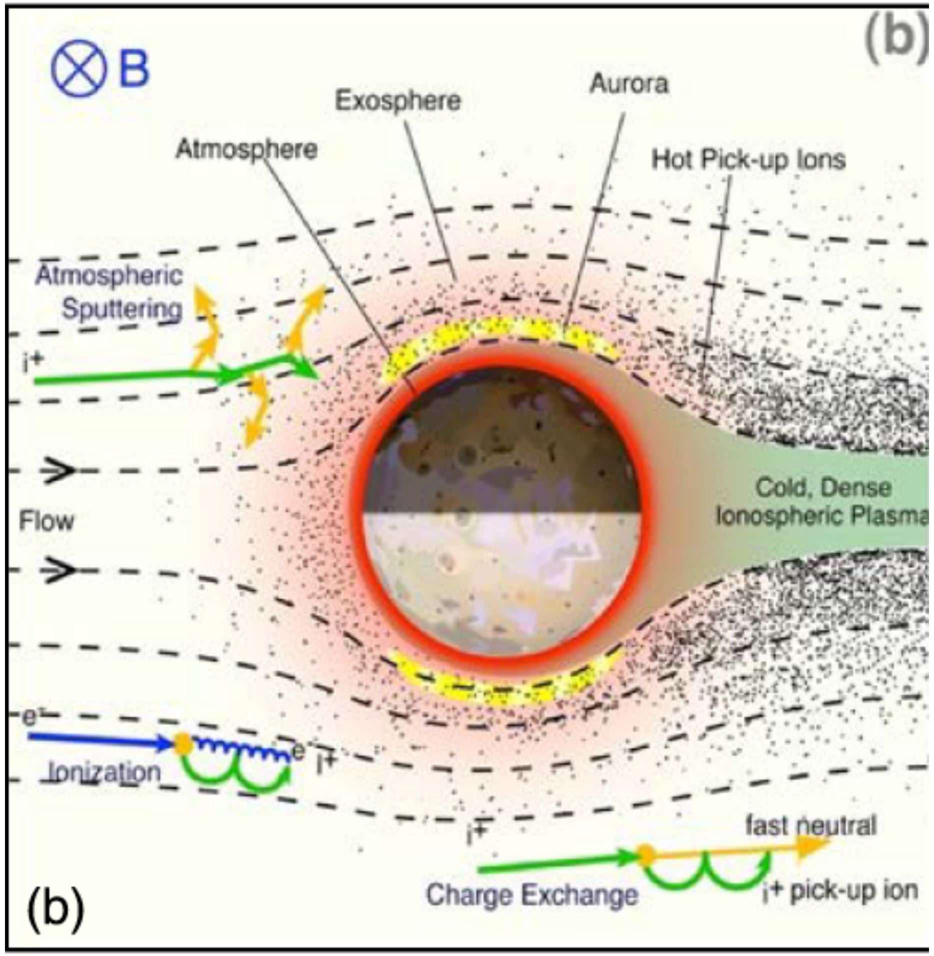
- Is there a good (but approximate) analytic description of the plasma flow past a conducting moon?
- Motivation and applications of a analytic flow field
  - Limits of the Neubauer/Saur solutions
  - Why not MHD (with a digression on boundary layers)
- Mathematical formalism
  - With a digression on more general solutions
- Solutions for a finite thickness ionosphere
- Examples of the resulting flow field

Motivation and Applications

- There are many aspects of the interaction which depend on the flow field
  - Atmospheric sputtering
  - Charge exchange/fast neutral production
  - Access of plasma & energetic particles to surface
  - See poster by Dols et. al later today
- Simple, analytic solution is convenient
  - Easy to vary parameters and study many cases
  - Approximate solutions may be fine
- Existing ("standard") solution has limits
  - Based on Neubauer, 1980 & Saur et al. 2004
  - Assumes constant conductivity of the body and zero conductivity outside the body
  - Overestimates plasma reaching the lower atmosphere and velocity of plasma along flanks



From Schneider and Bagenal, 2007

Why not use an MHD simulation?

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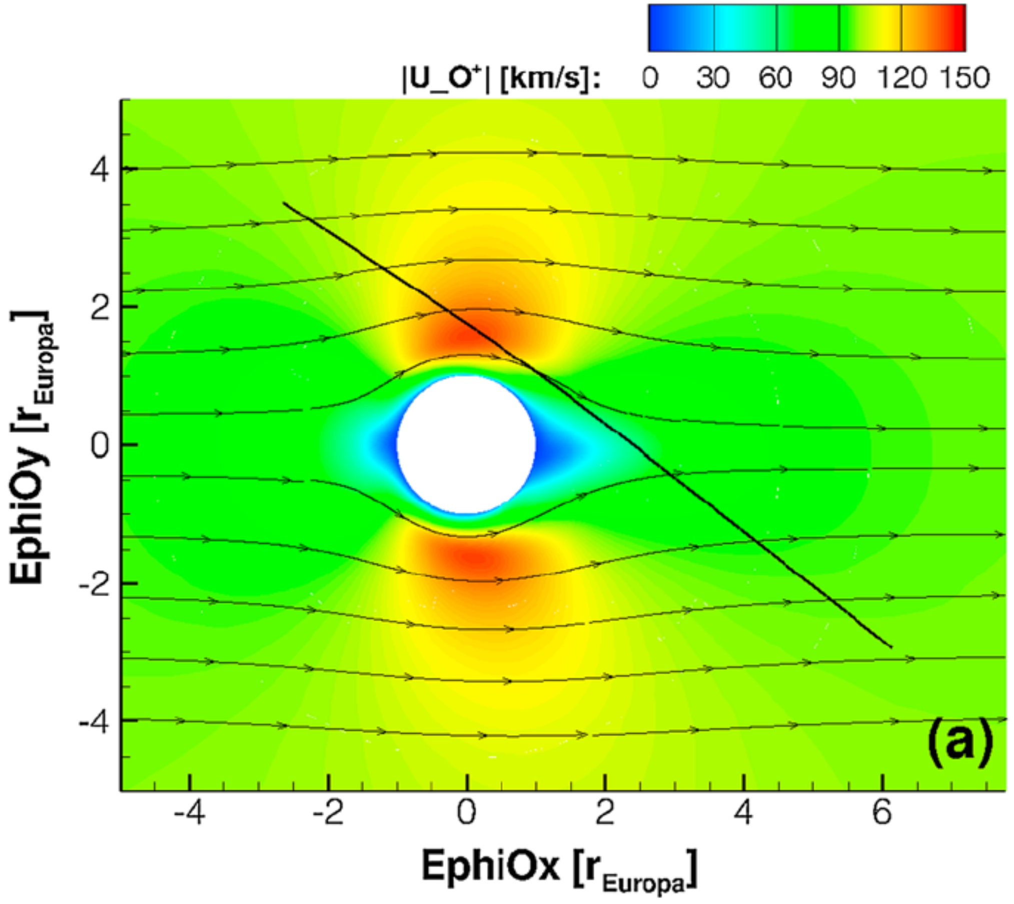
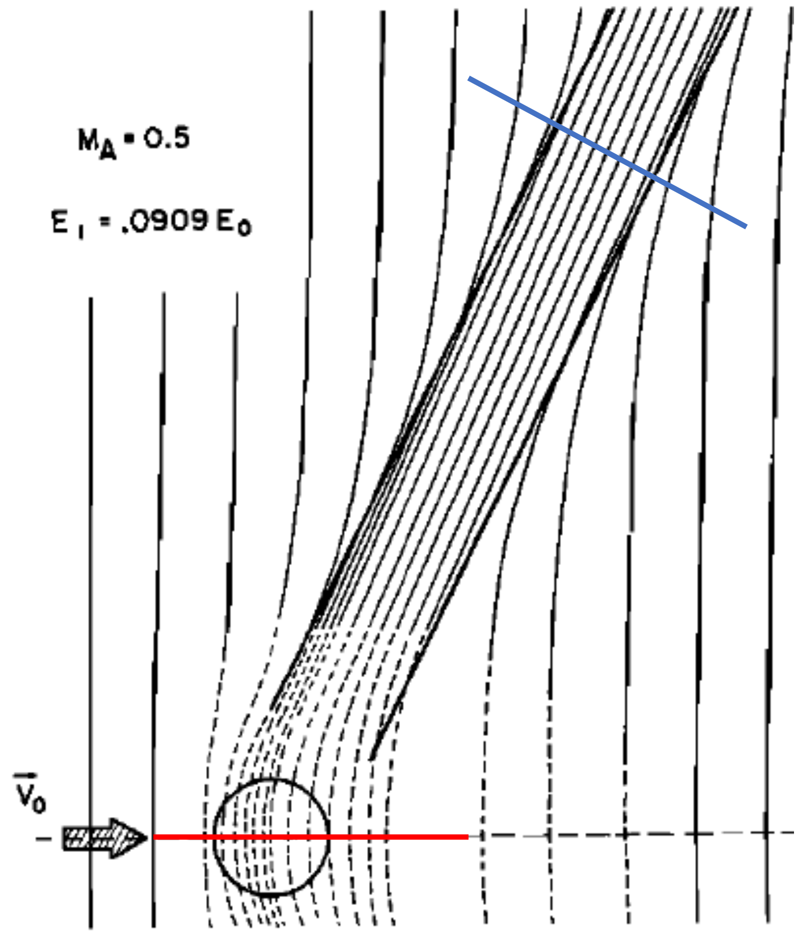
- Must be rerun for every case
  - No easy vary upstream conditions or ionospheric properties
- May use simplified ionospheric chemistry
- Full flow field may not be available for published results
  - Color figures can not be incorporated into future work
- The near surface flow is of interest
  - Most sensitive to boundary conditions
  - Flow in the ionosphere may be affected
  - If numerical diffusion could be taken literally, $\nu = \Delta x^2/\Delta t$, there would be a "boundary layer", $\delta/R \sim 5\sqrt{\frac{\Delta x}{R\,M_A}}$, which might comparable to H



From Rubin et al., 2015

Mathematical formalism



Neubauer, 1980

- Based on mapping field-aligned currents
- Far up the Alfvén wing

$$j_{||} = \Sigma_A \nabla_\perp^2 \Phi, \quad \Sigma_A = \frac{1}{\mu_0 V_A}$$

- In the vicinity of the moon

$$j_{||} = -\vec{\nabla}_\perp \cdot \vec{J}_\perp = \vec{\nabla}_\perp \cdot \Sigma_1 \left(\vec{\nabla}_\perp \Phi\right)$$

  - Integrated along field lines, not vertically
  - Conductance can be Pedersen or pickup
  - Can be generalized to include Σ_{Hall}
- Assumes north-south symmetry and no compression in the distant Alfvén wing

Mathematical formalism (II)

$$\vec{\nabla}_{\perp} \cdot \Sigma_1 \left(\vec{\nabla}_{\perp} \Phi \right) = \Sigma_A \nabla_{\perp}^2 \Phi$$

- Assumes 1:1 mapping from moon’s equator to Alfvén wing
  - Approximate unless $M_A = 0$ (no draping of field lines)
  - Compression near moon, mass loading also change mapping
  - But perhaps not to bad except in the wake
- Could (?) be generalized by including different mapping?
  - But how to define a self-consistent mapping (I don’t know)
- “Standard” solution assumes
  - m=1 ($\Phi \propto e^{i\varphi}$)
  - $\Sigma_1 = \begin{cases} constant & r < R \\ 0 & r > R \end{cases}$
  - Requires continuity at R=0

Solution for finite-thickness ionosphere

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$$\Phi = \Phi_0 f(r) e^{i\varphi}, \qquad \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f}{\partial r}\right) - \frac{1}{r^2} f + \frac{\frac{\partial \Sigma_1}{\partial r}}{\Sigma_A + \Sigma_1}\frac{\partial f}{\partial r} = 0$$

If

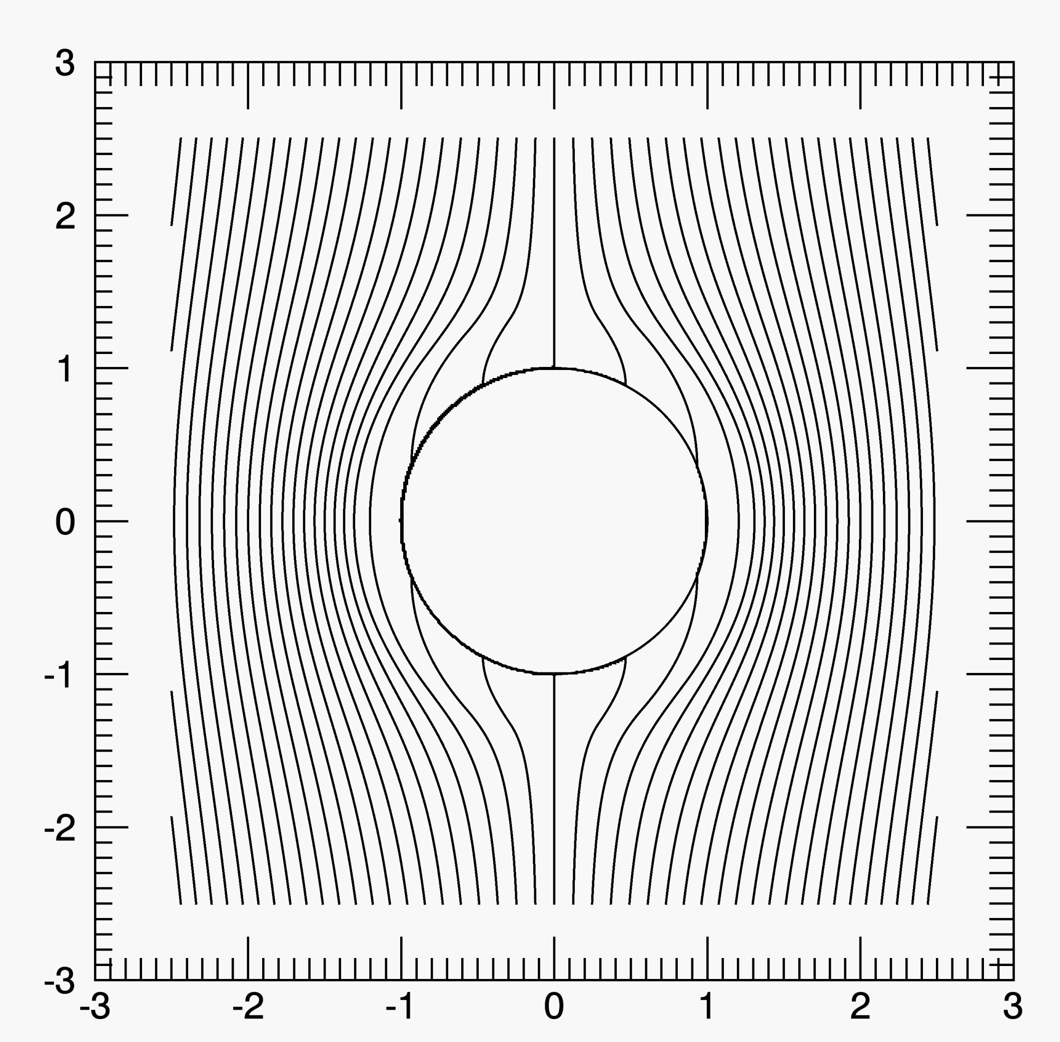
$$\Sigma_1 = \begin{cases} \Sigma_1 & r < R \\ (\Sigma_A + \Sigma_1) e^{-r-R/H} - \Sigma_A & R < r < R + H \ln\left(1 + \Sigma_1/\Sigma_A\right) \\ 0 & r > R + H \ln\left(1 + \Sigma_1/\Sigma_A\right) \end{cases}$$

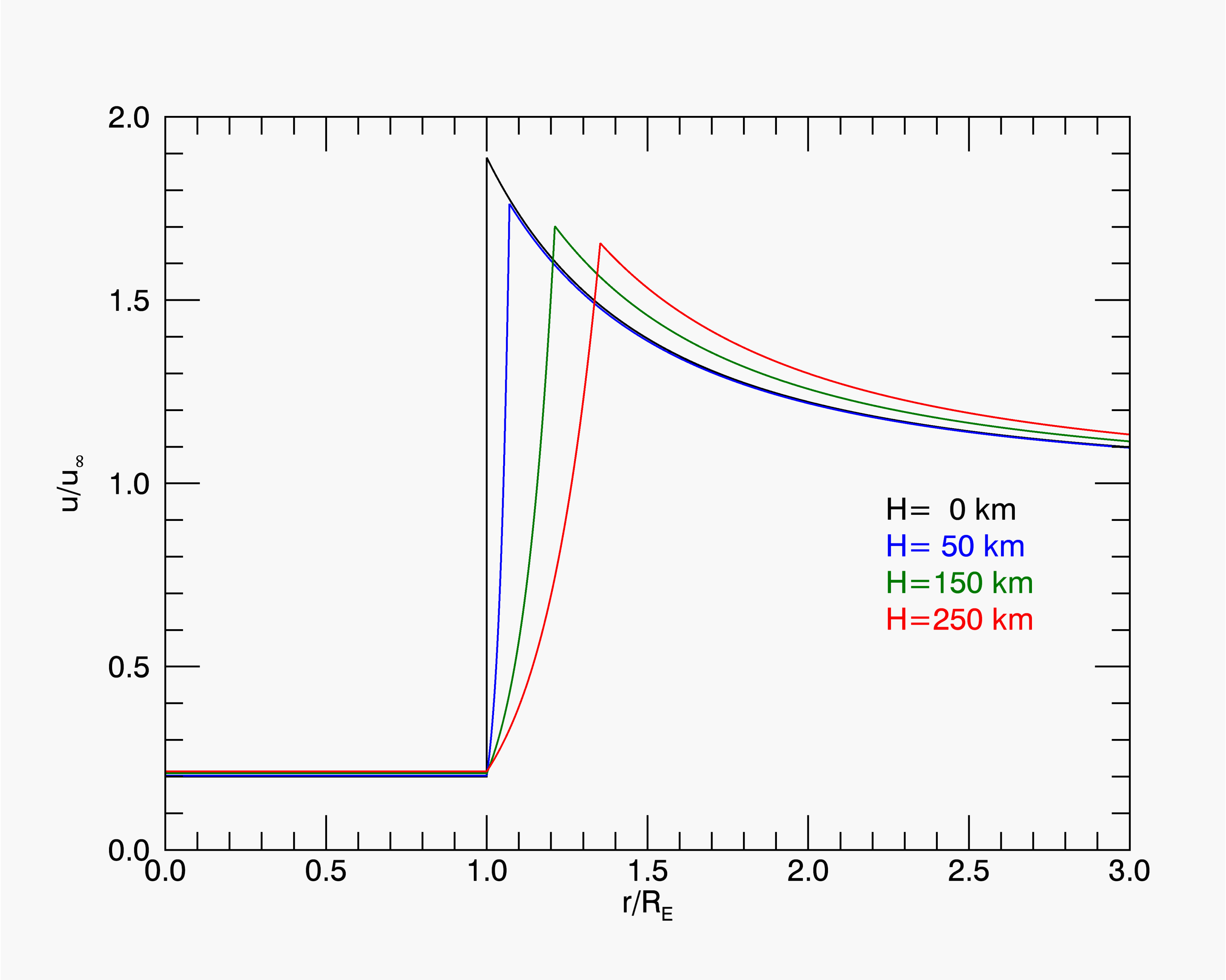
then

$$f = \begin{cases} C_{10}\dfrac{r}{R} & r < R \\ C_{11}\dfrac{R}{r} e^{r/H} - C_{21}\left(\dfrac{H}{R} - \dfrac{H^2}{R\,r}\right) & R < r < R + H \ln\left(1 + \Sigma_1/\Sigma_A\right) \\ \dfrac{r}{R} + C_{22}\dfrac{R}{r} & r > R + H \ln\left(1 + \Sigma_1/\Sigma_A\right) \end{cases}$$

Solution for finite-thickness ionosphere (II)

- Solve for C_{10}, C_{11}, C_{21}, C_{22}, based on continuity to get a closed-form solution for the flow field
- Note flow is excluded from lower ionosphere
- Similar solutions for a $1/r^n$ conductivity profile were found by Simon, 2015
  - Only applied to Enceladus and magnetic field data
  - "Standard" solution implied unobserved discontinuities





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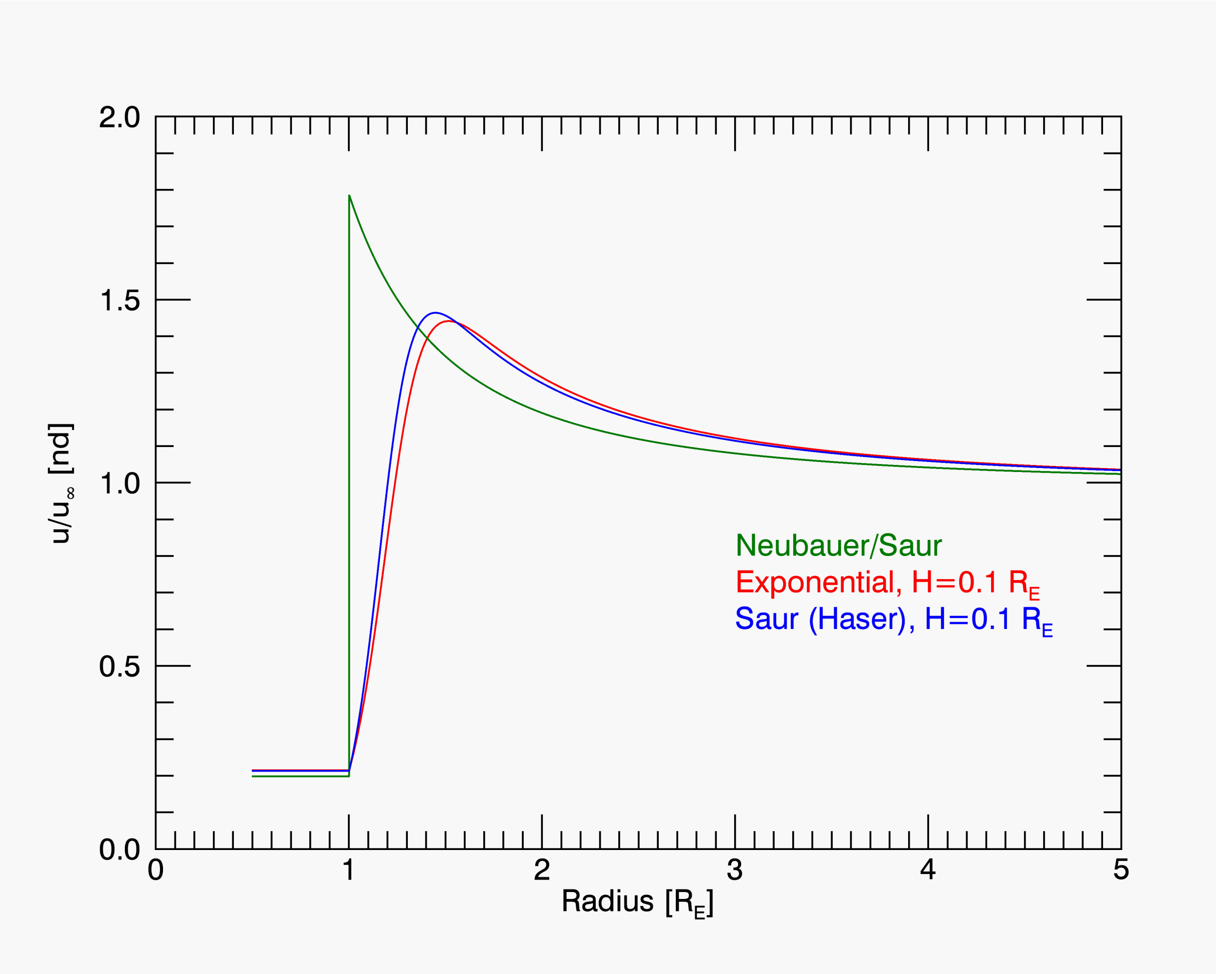
Flow speed along the flanks

Solutions for other profiles require some numerics

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f}{\partial r}\right) - \frac{1}{r^2}f + \frac{\frac{\partial \Sigma_1}{\partial r}}{\Sigma_A + \Sigma_1}\frac{\partial f}{\partial r} = 0$$

- Is a fairly simple equation to solve numerically for any Σ_1
- Much easier than a full MHD (or other) simulation
- Examples shown for a full exponential profile and a Haser (1956) profile, $\frac{1}{r^2}e^{-r/H}$



Conclusions

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- An analytic solution for the flow field is convenient
  - Easily incorporated into other models of the ionosphere, e.g. atmospheric sputtering, charge exchange, fast neutral production and particle access to the surface
- This presentation described as solution for an exponentially decreasing conductance outside the body
- Plasma is excluded from the lower ionosphere
  - Reducing production of fast neutrals and atmospheric sputtering
- Flow along flanks is slower than predicted by standard model
- This flow field can be incorporated into other calculations of the interaction (e.g. Dols et al., this meeting.)