Vlasov-Poisson solvers and database of I-V characteristics for electron-emitting objects of cylindrical and elliptic cross-sections immersed in Maxwellian plasmas

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The E.T.PACK Project

Title: Electrodynamic Tether Technology for Passive Consumable-less Deorbit Kit.

Acronym: E.T.PACK

Goal: develop a deorbit device based on Electrodynamic Tether (EDT) technology with **TRL 4**.

Call: H2020-FETOPEN-2018-2019-2020-01.

Duration: 45 months (1/3/2019 – 31/11/2022).

Budget: 3M€.

Partners: UC3M, IKTS, UNIPD, TUD, SENER, and ATD.

Potential Impact: Europe having access to a reversible and consumable-less propulsion technology.

Electrodynamic Tethers (EDTs):

Long in-orbit conductors carrying an electric current.

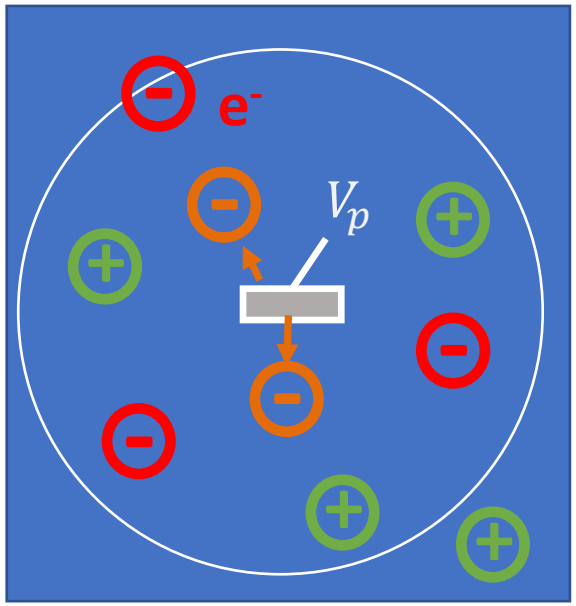
Applications

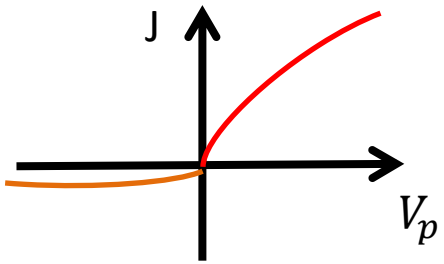
- Deorbiting of space-debris.
- Station-keeping.
- Science (Aurora, Planet exploration, …)

Advantages

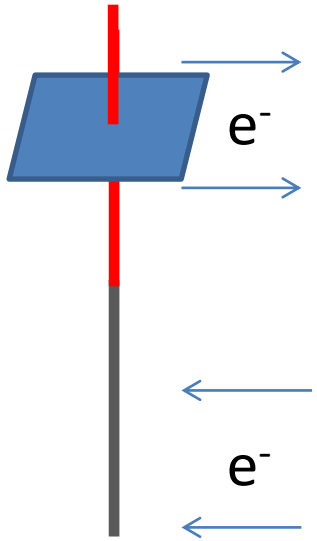
- Propellant-less
- Tethers are reversible devices: convert orbital energy into electric energy and viceversa.

Two EDTs are under developement in the E.T.PACK Project





Low-Work-Function Tether (LWT)



Williams , Sanmartin and Rand, IEEE Trans. Plasma Sci., 2012.

Sanchez-Arriaga and Chen, JPP, 2018.

The Vlasov-Poisson System

Hypotheses

- 2-dimensional object of contour Γ and bias V_p immersed in a stationary, collisionless plasma with no magnetic field.
- No trapped particles.
- Unperturbed plasma is Maxwellian (N_0, T_e, T_i, singly charged ions).
- Electrons are emitted from the object following a half-Maxwellian distribution function (N_{em0}, T_{em}) and a Richardson-Dushman law.

Parameters ($\alpha = e, i, em$)

- *Bias:* $\phi_p \equiv V_p / k_B T_e$
- *Temperatures:* $\delta_\alpha \equiv T_\alpha / T_e$
- *Charge:* $e_\alpha \equiv q_\alpha / e$.
- Emission level: $\beta \equiv N_{em0} / N_0 = f(\delta_{em})$,
- Geometry
  - Round tether: $r_p \equiv R / \lambda_{De}$.
  - Elliptic tether: e_p and $b_p = B / \lambda_{De}$

The Vlasov Poisson System

Vlasov-Poisson System

$$\boldsymbol{v} \cdot \nabla f_\alpha - \frac{e_\alpha}{2\delta_\alpha} \nabla\phi \cdot \nabla_v f_\alpha = 0,$$

$$\Delta\phi = -\rho \equiv n_e + \beta n_{em} - e_i n_i,$$

where

$$n_\alpha(\boldsymbol{r}) = \int_{-\infty}^{+\infty} f_\alpha(\boldsymbol{r}, \boldsymbol{v}) d\boldsymbol{v}.$$

Boundary conditions

$$\phi(\Gamma) = \phi_P$$

$$f_{em}(\Gamma, \boldsymbol{v} \cdot \boldsymbol{u}_n > 0) = f_{HM} \equiv \frac{2}{\pi} \exp\{-(\boldsymbol{v} \cdot \boldsymbol{v})\}$$

$$\phi(r \to \infty) \to 0$$

$$f_{e,i}(r \to \infty, \boldsymbol{v}) \to f_M \equiv \frac{1}{\pi} \exp\{-(\boldsymbol{v} \cdot \boldsymbol{v})\}$$

Parameters $(\alpha = e, i, em)$

- *Bias:* $\phi_p \equiv V_p / k_B T_e$
- *Temperatures:* $\delta_\alpha \equiv T_\alpha / T_e$
- *Charge:* $e_\alpha \equiv q_\alpha / e$.
- Emission level: $\beta \equiv N_{em0} / N_0 = f(\delta_{em})$,
- Geometry
  - Round tether: $r_p \equiv R / \lambda_{De}$.
  - Elliptic tether: e_p and $b_p = B / \lambda_{De}$

FONKS-C: Free of Noise Kinetic Solver for Cylindrical Geometry[1]

FONKS-C

[1]Shahsavani and Sánchez-Arriaga, Parametrization of current-voltage characteristics and operation domains of cylindrical emissive probes in collisionless Maxwellian plasmas at rest (submitted).

FONKS-C: Architecture

Conserved quantities along Vlasov Characteristics

- Collisionless plasma -> Distribution function is conserved
- Stationary plasma -> Energy is conserved
- Azimuthal symmetry -> Angular momentum is conserved

(1)

$$\rho(\phi) = \int g(\phi) dE$$

Insert in Poisson Equation

$$F(\phi) \equiv \Delta\phi + \rho(\phi) = 0$$

Solve with a Newton-Raphson Method

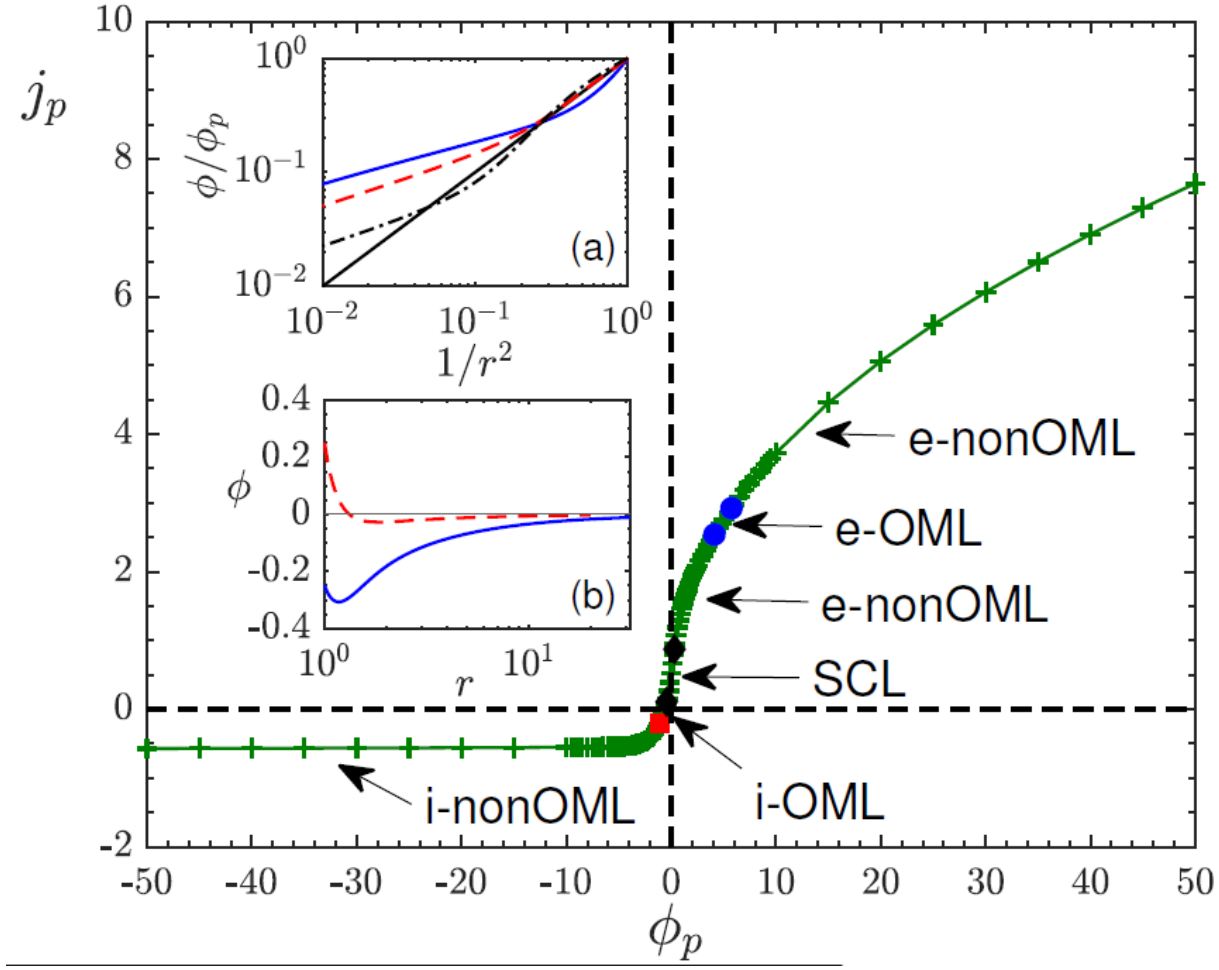
$$f_\alpha(r, \boldsymbol{v})$$

$$\phi(r), n_\alpha(r), \boldsymbol{j}_\alpha(r), T_\alpha(r),$$

Bottleneck of the algorithm: computation of the Jacobian matrix of $F(\phi)$

[1]**Exact** result. Find details in Chen and Sanchez-Arriaga, *Orbital Motion Theory and Operational Regimes for Cylindrical Emissive Probes*, Phys. of Plasmas, 2017.

FONKS-C: Results



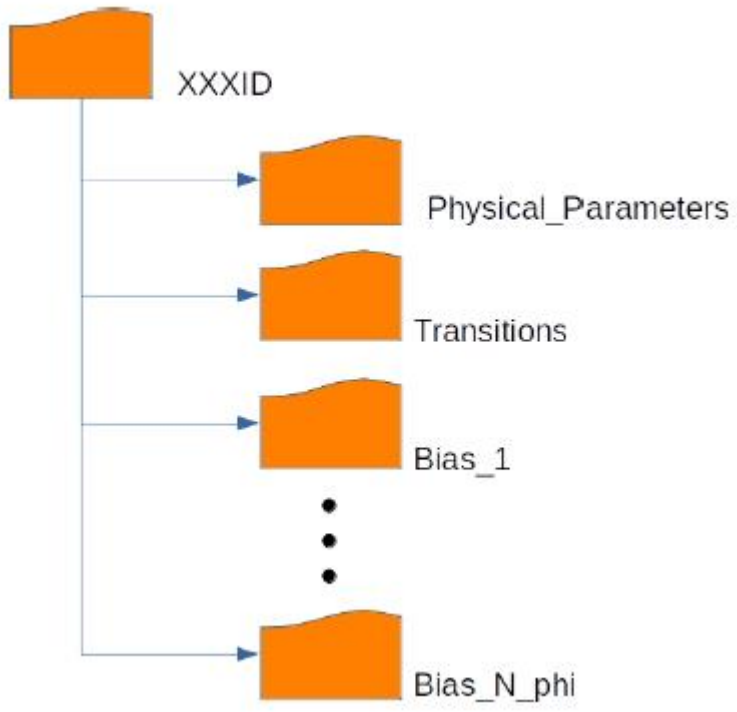
FONKS-C: The Database

Database

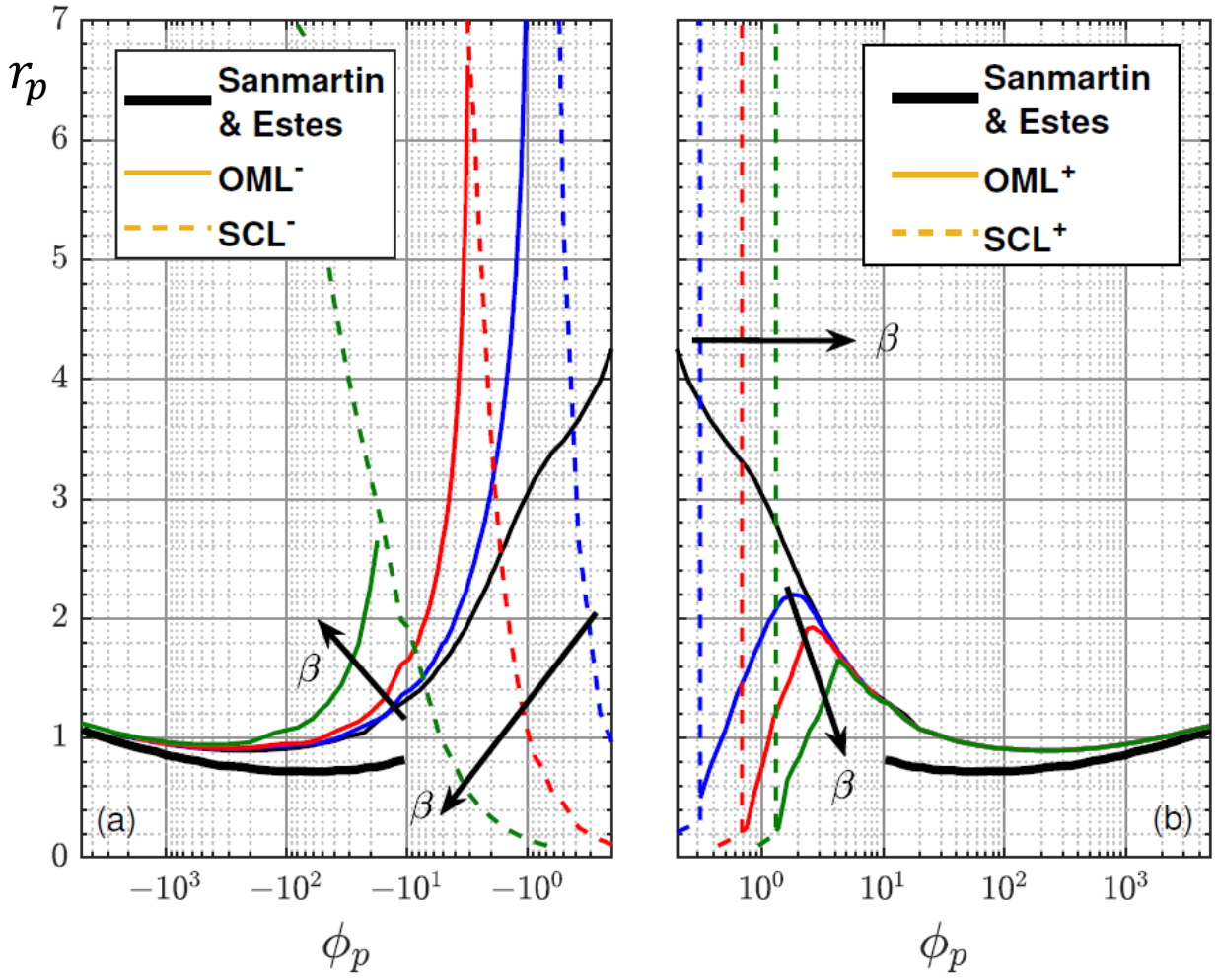
- More than 18,000 $J_p - \phi_p$ curves were computed.
- Find at **https://doi:org=10:21950=MBTLKG** the following information:
  - 140 $J_p - \phi_p$ curves in HDF5 format covering the following parameter domain

    β=[0.1, 0.3, 1, 3, 30, 100], δ_i=[0.1, 0.5, 1, 3]

    r_p=[0.1, 0.5, 1, 3, 10], $-1000 \leq \phi_p \leq 1000$.
  - Six main Matlab files to plot $J_p - \phi_p$, macroscopic quantities, distribution functions and domains of operation in parameter space.



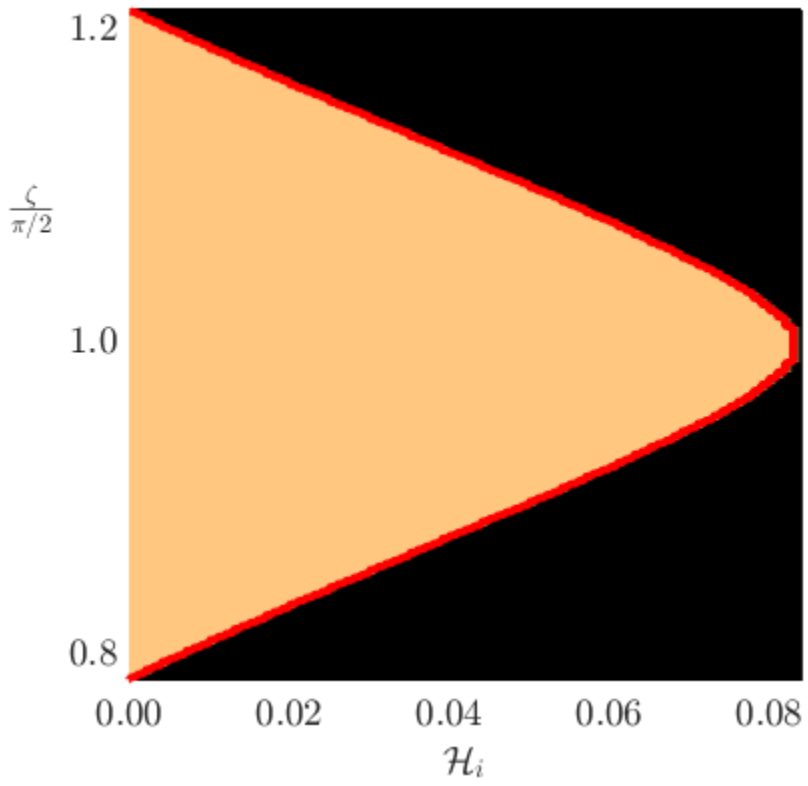
FONKS-C: Some selected results



OML: Orbital Motion Limited. **SCL**: Space-Charge-Limited

FONKS-C: Discussion about integrability

- The phase space of the characteristics has dimension 4: x,y, v_x, v_y
- There are two conserved quantities: energy and angular momentum.
- Dimension - # invariants = 4 – 2 = 2 ->
  - orbits are regular
  - empty/full boundaries of the distribution function are smooth curves.



$$\zeta \equiv \tan^{-1}\left(\frac{\boldsymbol{v}\cdot\boldsymbol{t}}{\boldsymbol{v}\cdot\boldsymbol{n}}\right)$$

$\mathcal{H}_i$ Hamiltonian

Black: full

Orange: Empty

Ion distribution function in the energy-angle space at (x,y) = (3.05,0) for $\phi_p \equiv -4,\ \delta_i \equiv 1,\ \beta \equiv 0, r_p = 1.$

FONKS-G: Free of Noise Kinetic Solver for Generic Geometries[1,2]

[1]Chiabó and Sánchez-Arriaga, Limitations of stationary Vlasov-Poisson solvers in probe theory, Journal of Computational Physics, 438, 110366, 2021.

[2]Chiabó, Shahsavani, and Sánchez-Arriaga, Kinetic Analysis of the Plasma sheath Around an Electron-Emitting Object With Elliptical Cross-section (to be submitted).

FONKS-G: Architecture

Conserved quantities along Vlasov Characteristics

- Collisionless plasma -> Distribution function is conserved
- Stationary plasma -> Energy is conserved
- Arbitrary geometry -> **Angular momentum is not conserved**

(1)

$$\rho(\phi) = \int g(\phi) dE$$

Insert in Poisson Equation

$$F(\phi) \equiv \Delta\phi + \rho(\phi) = 0$$

Solve with a Newton-Raphson Method

$$f_\alpha(r, \boldsymbol{v})$$

$$\phi(r), n_\alpha(r), \boldsymbol{j}_\alpha(r), T_\alpha(r),$$

Bottleneck of the algorithm: computation of the Jacobian matrix of $F(\phi)$

[1]**It requires the <u>numerical</u> integration of Vlasov's characteristic equations** -> **Backward Liouville Scheme**

FONKS-G: Numerical Shemes

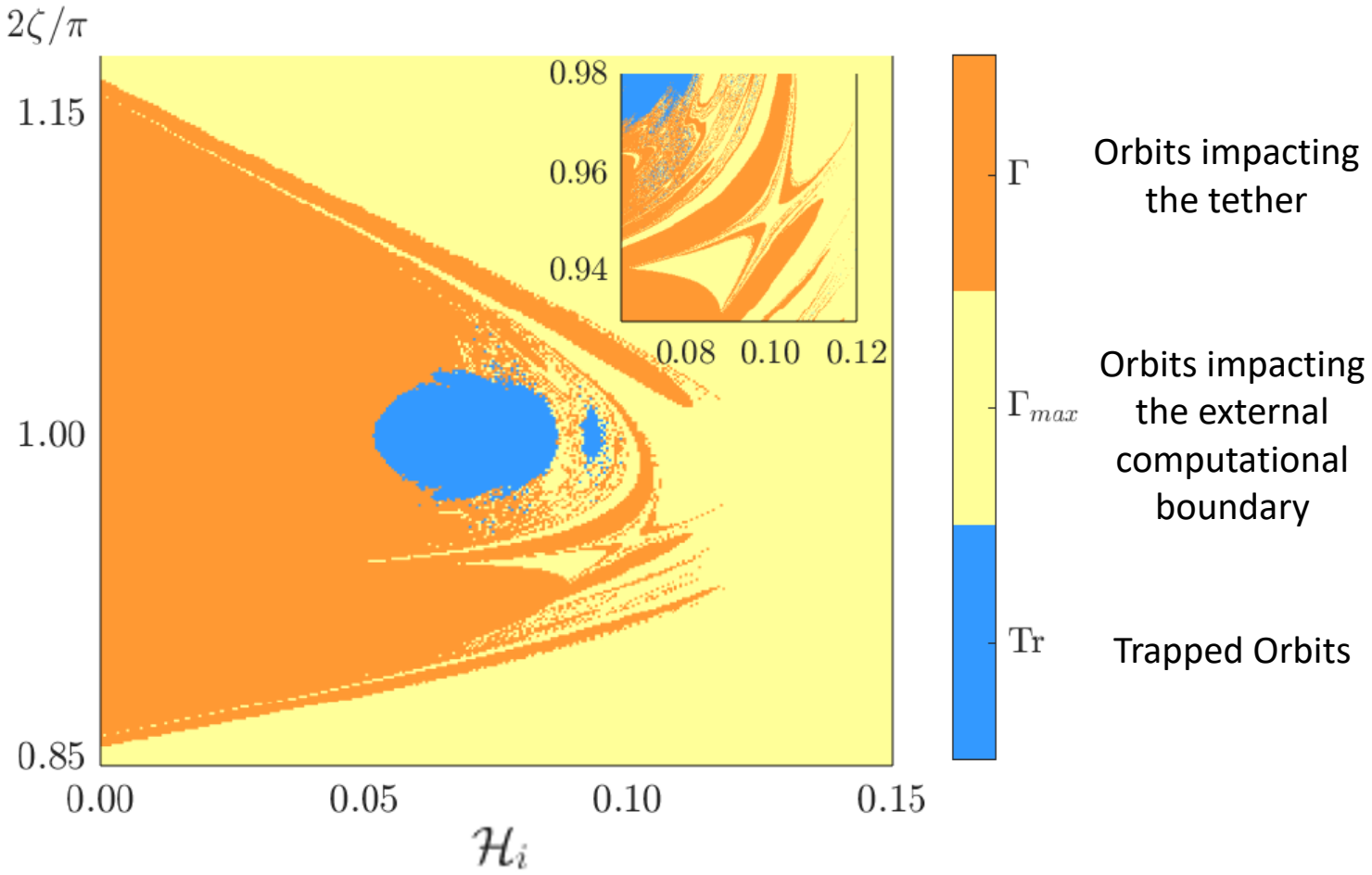
Name	Mesh	Poisson solver	Integrator	$\boldsymbol{E}$ reconstruction
FEM-RK	Unstructured	FEM [26]	Runge-Kutta [30]	Polynomial fitting + Natural neighbor interpolation [32]
FDM-RK	Structured	FDM	Runge-Kutta [30]	FDM + Bilinear interpolation [30]
FEM-LF	Unstructured	FEM [26]	Leapfrog [30]	Polynomial fitting + Natural neighbor interpolation [32]
FDM-LF	Structured	FDM	Leapfrog [30]	FDM + Bilinear interpolation [30]
FDM-CN	Structured	FDM	Crank-Nicolson [31]	B-spline [31]

Acronyms: FEM (Finite Element Method), RK (Runge-Kutta), FDM (Finite Differences Method), CN (Crank-Nicolson).

Crank-Nicolson: hybrid algorithm that propagates velocities in Cartesian space and position in logical curvilinear coordinates. It conserves the energy exactly. It is based on works by Chacon et al (2013, 2016) and Delzanno (2013).

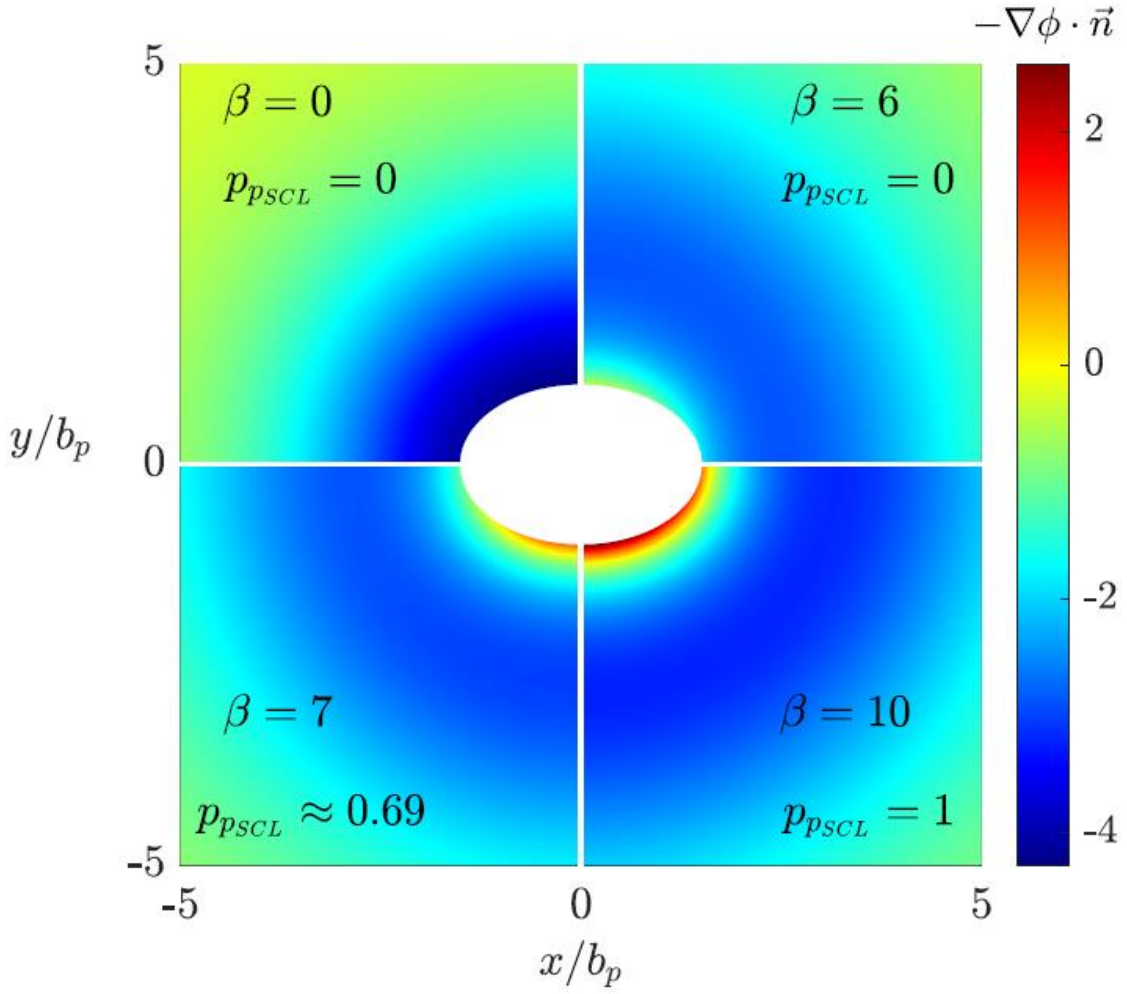
FONKS-G: Discussion about integrability

- The phase space of the characteristics has dimension 4: x,y, v_x, v_y
- There is only one conserved quantity: the energy.
- Dimension - # invariants = 4 – 1 = 3 ->
  - Chaotic orbits can appear
  - empty/full boundaries of the distribution function are fractals.

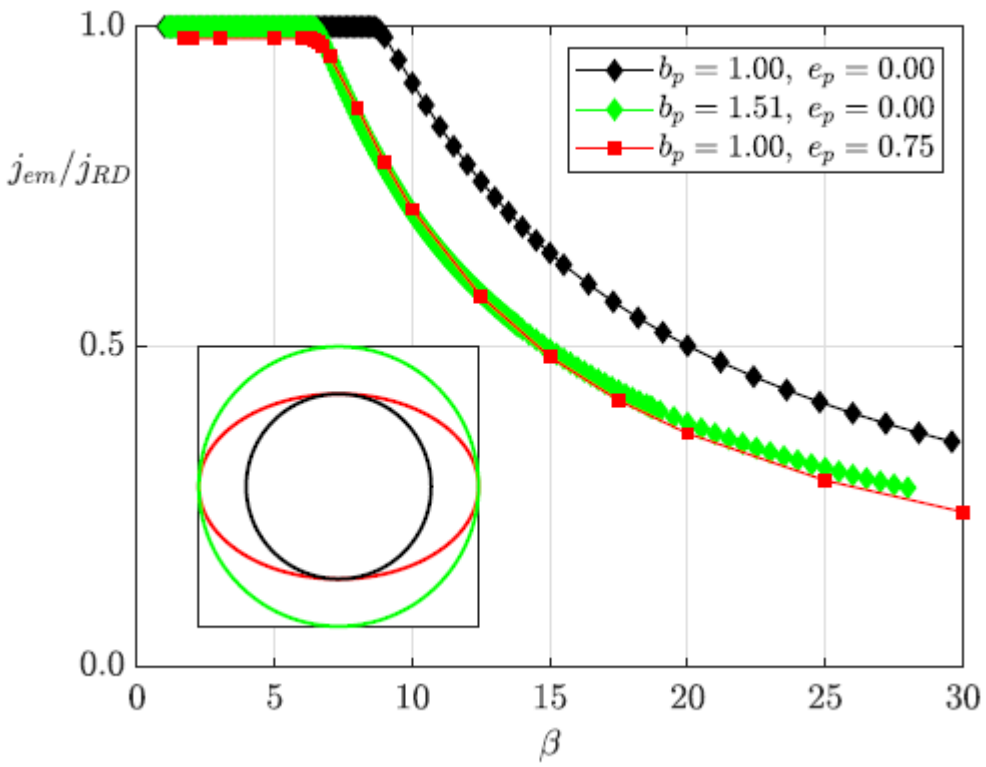


Ion distribution function in the energy-angle space at (x,y) = (3.37,0) for $\phi_p \equiv -4,\ \delta_i \equiv 1,\ \beta \equiv 0, e_p = 0.95$ and $b_p = 1$.

FONKS-G: Analysis of Space Charge Limited Conditions



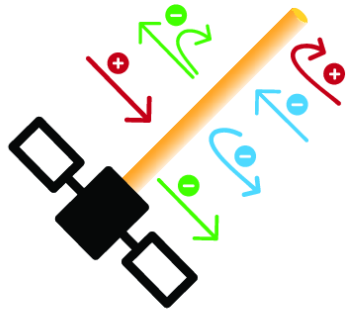
Normal electric field component for an elliptic probe with $\phi_p \equiv -4,\ \delta_i \equiv 1,\ e_p = 0.75$ and $b_p = 1$.



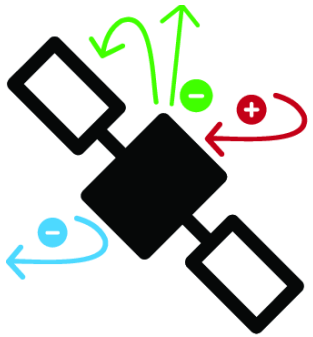
Emitted electron current-to-RD current versus β for $\phi_p \equiv -4,\ \delta_i \equiv 1,\ \delta_{em} \equiv 0.32$

Conclusions

- Two Vlasov-Poisson solvers to study the plasma structure around 2D electron emitting objects have been developed.
- Their most important features are
  - Free of statistical noise.
  - Invariants, i.e. the energy or the energy and the angular momentum, are conserved exactly.
  - Suitable to study in very detail kinetic features such as the filamentation of the distribution function.



FONKS-C



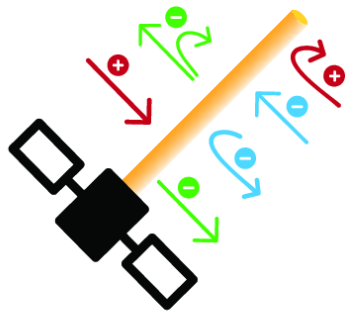
FONKS-G

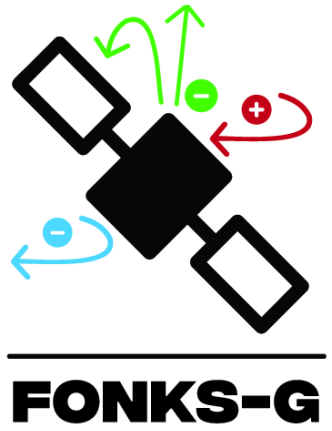
Results

- FONKS-C
  - I-V Database and visualization software are available at

    https://doi:org=10:21950=MBTLKG

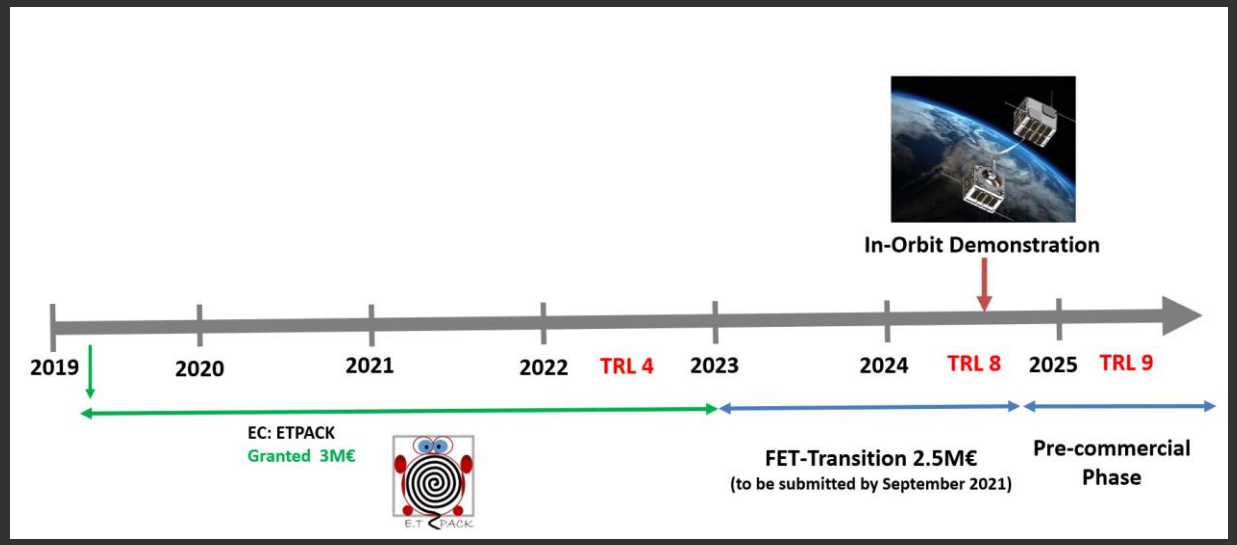
  - Analytical fitting laws were found for transition boundaries.
  - Experimental methods were revisited.
- **FONKS-G**:
  - Physical and numerical limitations of stationary Vlasov-Poisson solvers were studied.
  - The plasma sheaths around elliptic objects were investigated as well as intetesting kinetic features like the filamentation of the distribution functions.

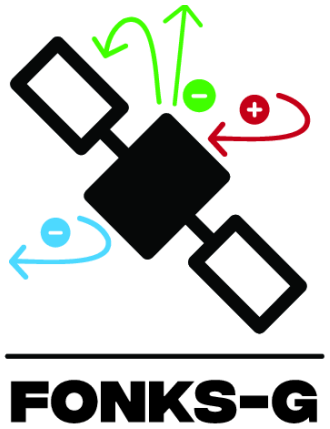




Applications/Collaborations

- Use the database of FONKS-C to create a software to extract plasma parameters from experimental I-V curves.
- Use **FONKS-C** and **FONKS-G** to benchmark other spacecraft charging software.
- Incorporate **FONKS-C, FONKS-G**, or the database of I-V curves to SPIS ?
- E.T.PACK's roadmap is an excellent opportunity for SPINE community





Thank you for your attention !

More information at www.etpack.eu

Contact: gonzalo.sanchez@uc3m.es

FET OPEN PROJECT FUNDED BY THE EUROPEAN UNION'S

HORIZON 2020 RESEARCH AND INNOVATION PROGRAMME

FONKS-C: Some selected results

Other results

- Analytical fitting laws for the operational boundaries in parameterspace
- Analytical fitting laws for the current drops outside the OML regime and inside the SCL regime.
- Discussion of experimental methods:
    - Floating potential
    - Separation point
    - Inflection point.

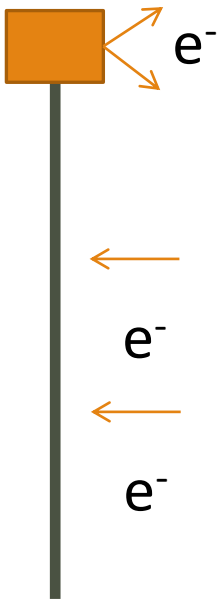
Find details at

Shahsavani and Sánchez-Arriaga, *Parametrization of current-voltage characteristics and operation domains of cylindrical emissive probes in collisionless Maxwellian plasmas at rest* (submitted).

Countries with recent missions and/or preparing missions on tethers

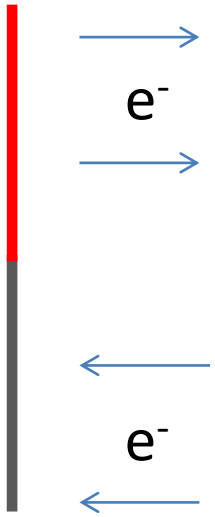
Two EDTs are under developement in the E.T.PACK Project

Bare Tether equipped with active electron emitter



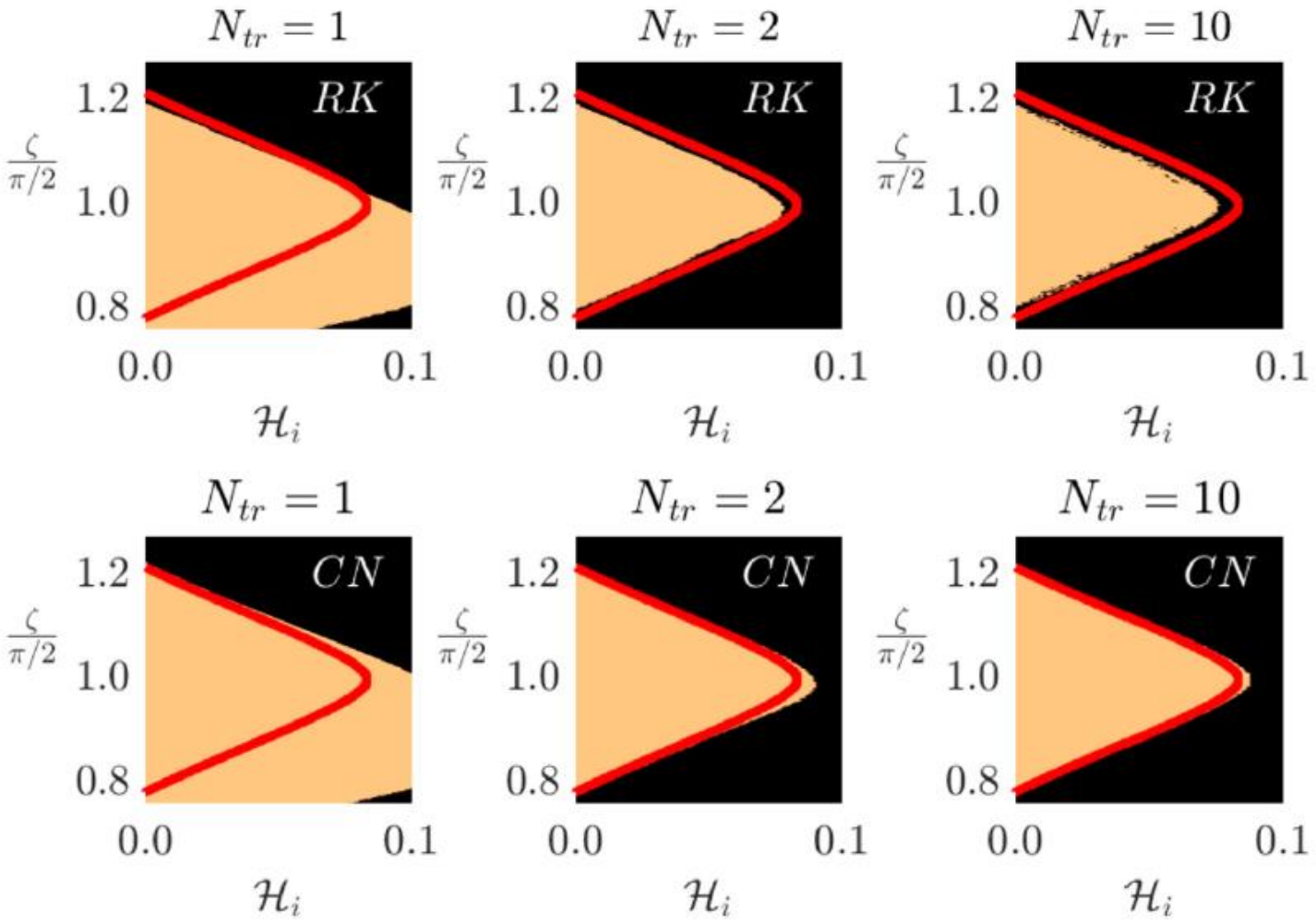
Sanmartin, Martínez-Sánchez and Ahedo, JPP, 1993

Low-Work-Function Tether (LWT)



Williams , Sanmartin and Rand, IEEE Trans. Plasma Sci., 2012.
Sanchez-Arriaga and Chen, JPP, 2018.

Comparison of FONKS-C and FONKS-G: Effect of the integrator and the number of loops (N_{tr}) to be considered as trapped



Black: full

Orange: Empty

Red: Exact boundary

Ion distribution function in the energy-angle space at (x,y) = (3.05,0) for $\phi_p \equiv -4,\ \delta_i \equiv 1,\ \beta \equiv 0, r_p = 1.$